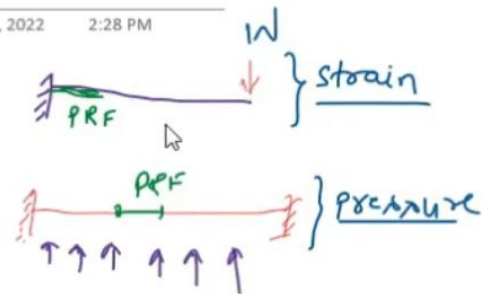


Tuesday, February 15, 2022 2:28 PM



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SMI-L-09

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strain

pressure

physical quantities

$$\frac{\Delta R/R}{\epsilon} = \frac{\partial \rho}{\rho \epsilon} + (1+2\nu)$$

geometrical property
material property

D

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$\downarrow W$

strain

$\uparrow \uparrow \uparrow \uparrow \uparrow$

pressure

physical quantities

$$\frac{\Delta R/R}{\epsilon} = \frac{\partial P}{P \epsilon} + (1+2\nu)$$

geometrical property
material property

Materials

① Metals

② Semi-conductors
(Silicon, Ge) → 10%

} functionally equivalent

0.1% stretching

D

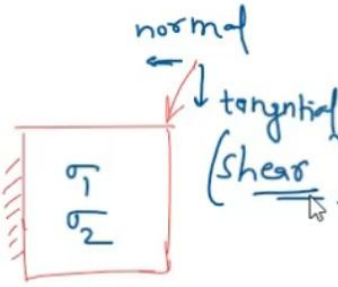
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② Semi-conductors (Silicon, Ge) → 10% functionally equivalent stretching

$$\frac{\Delta R}{R} = \frac{G}{E} \sigma = \pi \sigma$$

$$\frac{\Delta \rho}{\rho} = \pi_1 \sigma_1 + \pi_2 \sigma_2$$



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SMI-L-09

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thickness (microns)

physical quantities

$$\frac{\Delta R/R}{\epsilon} = \frac{\partial P}{P \epsilon} + (1+2\nu)$$

Material property
geometrical property

Materials

① Metals

② Semi-conductors (Silicon, Ge) → 10%

functionally equivalent

0.1% stretching

normal



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② Semi-conductors (Silicon, Ge) → 10% functionally equivalent stretching

$$\frac{\Delta R}{R} = \frac{G}{E} \sigma = \pi \sigma$$

$$\frac{\Delta \rho}{\rho} = \pi_1 \sigma_1 + \pi_2 \sigma_2$$

$$= \pi_L \sigma_L + \pi_T \sigma_T$$

longitudinal radial

normal
tangential (shear)

2-D loading behaviour

cylindrical

σ, θ, z



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② Semi-conductors (Silicon, Ge) → 10% } functionally equivalent sketching

$$\frac{\Delta R}{R} = \frac{G}{E} \sigma = \pi \sigma \left. \begin{array}{l} \text{linear} \end{array} \right\}$$

$$\frac{\Delta \rho}{\rho} = \pi_1 \sigma_1 + \pi_2 \sigma_2 \left. \begin{array}{l} \text{linear} \end{array} \right\}$$

$$= \pi_L \sigma_L + \pi_T \sigma_T$$

longitudinal radial

normal tangential (shear)

2-D loading behaviour

cylindrical

σ, θ, z



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② Semi-conductors (Silicon, Ge) → 10% } functionally equivalent stretching

$$\frac{\Delta R}{R} = \frac{G}{E} \sigma = \pi \sigma \left\{ \begin{array}{l} \text{linear} \end{array} \right.$$

$$\frac{\Delta P}{P} = \pi_1 \sigma_1 + \pi_2 \sigma_2 \left\{ \begin{array}{l} \text{linear} \end{array} \right.$$

$$= \bar{\pi}_L \sigma_L + \bar{\pi}_T \sigma_T$$

2-D loading behaviour

normal
tangential (shear)

longitudinal radial

cylindrical

nonlinearity

- piezo-resistive
- crystal
- structural

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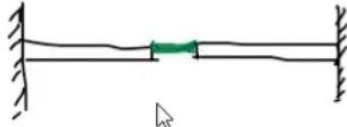
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Shapes Ink to Shape

→ structured] [[0,0,2

Modelling & Development of Piezo-resistive Pressure sensor



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undo redo link unlink shapes ink to shape

piezo-resistive
circuit
structural } applications

ir

cylindrical

σ, θ, z

Modelling & Development of Piezo-resistive Pressure sensor

① Voltage out \rightarrow sensitivity of the sensor

② Amount of strains

membrane plate fixed end



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↶ ↷ I + Shapes Ink to Shape

piezo-resistive
circuit } application
structural

cylindrical
 (σ, θ, z)

Modelling & Development of Piezo-resistive Pressure sensor


① Voltage out → sensitivity of the sensor

② Amount of strains

thickness of plate (h)
deflection of plate (δ)

membrane plate fixed end deflection

(σ, z)



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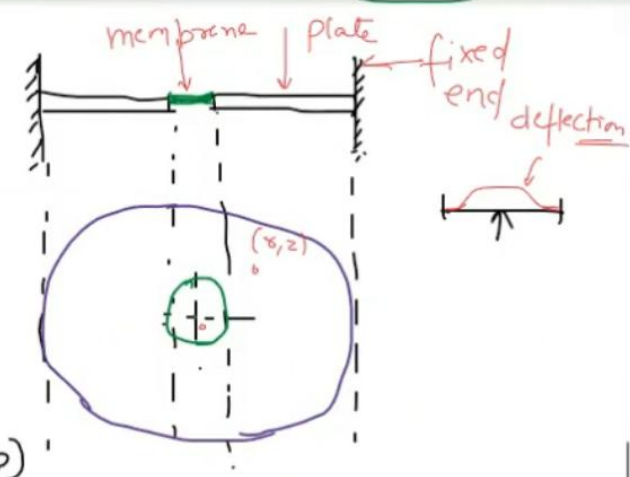
① Voltage out \rightarrow sensitivity of the sensor

② Amount of strains

thickness of plate (h)
 deflection of plate (δ)
 uniformly distributed pressure (p)

* Assumption
 $\delta \ll h/5$

a differential eqⁿ to describe the deflection / elastic behaviour

$$\nabla^4 \delta = \frac{p}{k} \quad \left\{ k = \frac{Eh^3}{12(1-\nu^2)} \right.$$


The diagram shows a horizontal plate fixed at both ends. A green membrane is attached to the top surface. A red arrow indicates the direction of deflection. A coordinate system (x, z) is shown in the center of the plate. A small inset shows a cross-section of the plate with a red arrow indicating deflection.

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① Voltage out → sensitivity of the sensor

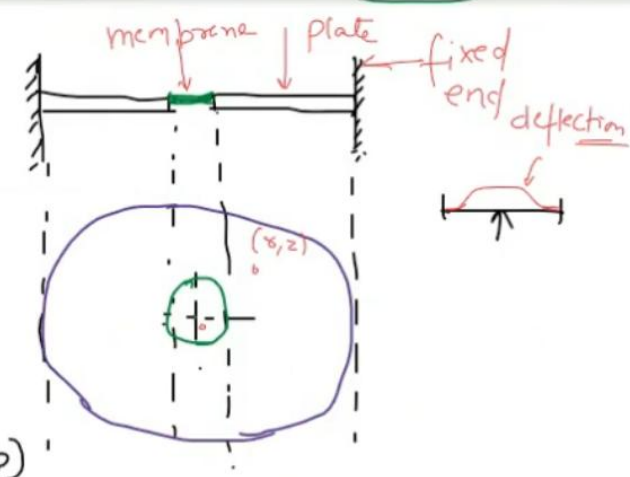
② Amount of strains

thickness of plate (h)
 deflection of plate (δ)
 uniformly distributed pressure (p)

* Assumption
 $\delta \ll h/5$

a differential eqⁿ to describe the deflection / elastic behaviour

$\frac{D^4 \delta}{dt^4} = \frac{p}{k}$ $\left\{ k = \frac{Eh^3}{12(1-\nu^2)} \right.$
 Stiffness



The diagram shows a horizontal plate of thickness h fixed at both ends. A membrane is shown on top of the plate. A coordinate system (x, z) is centered on the plate. A deflection curve is shown on the right, labeled 'fixed end deflection'.

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① Voltage out \rightarrow sensitivity of the sensor

② Amount of strains

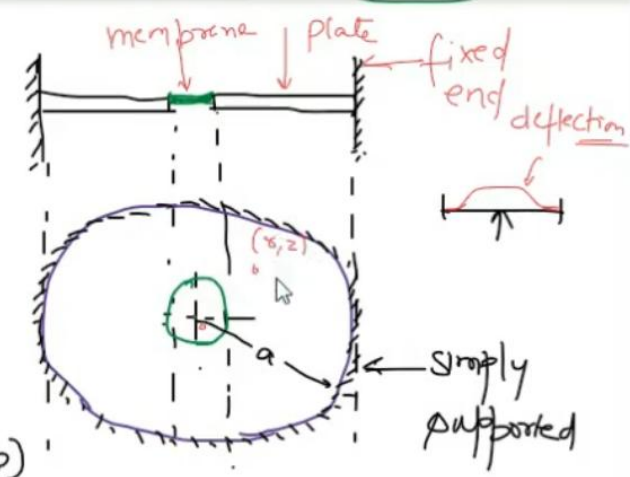
thickness of plate (h)
deflection of plate (δ)
uniformly distributed pressure (p)

* Assumption
 $\delta \ll h/5$

a differential eqⁿ to describe the deflection / elastic behaviour

$$\nabla^4 \delta = \frac{p}{k} \quad \left\{ k = \frac{Eh^3}{12(1-\nu^2)} \right.$$

Stiffness

$$\delta = \frac{p(a^2 - r^2)}{15 + \nu} (a^2 - r^2)$$


The diagram illustrates a circular plate with a central hole of radius a . The plate is fixed at the outer edge, and the deflection is labeled as δ . A smaller diagram shows a cross-section of the plate with a fixed end and a deflection curve. The plate is labeled as 'membrane' and 'plate'. The deflection is labeled as 'fixed end deflection' and 'simply supported'.

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$$\nabla^4 \delta = \frac{p}{k}$$

$$\left\{ k = \frac{E h^3}{12(1-\nu^2)} \right.$$

$$\delta = \frac{p(a^2 - r^2)}{64k} \left[\left(\frac{5+v}{1+v} \right) (a^2 - r^2) \right]$$



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Shapes Ink to Shape

Modelling & Development of Piezo-resistive Pressure sensor

- ① Voltage out \rightarrow sensitivity of the sensor
- ② Amount of strains

Thickness of plate (h)
 deflection of plate (δ)
 uniformly distributed pressure (p)

~~Assumption~~
 $\delta \ll h/5$
 a differential eqⁿ to describe the deflection / elastic behaviour

$\nabla^4 \delta = \frac{p}{k}$ $k = \frac{Eh^3}{12(1-\nu^2)}$



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41:16

1:12:26



18°C
Haze



ENG IN 11:46 PM 3/2/2022

strain \rightarrow radial
 \rightarrow tangential

Radial strain

$$\epsilon_r(r, z) = -\frac{kz}{Eh^{3/2}} \left(\frac{d^2 s}{dr^2} + \frac{v}{r} \frac{ds}{dr} \right)$$

$$\epsilon_r(r, z) = \frac{3}{8} \left[\frac{Pa^2(h_p - h_m)(3 + \nu)}{F} \right]$$


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strain \rightarrow radial
 strain \rightarrow tangential

Radial strain

$$\epsilon_r(r, z) = -\frac{kz}{Eh^{3/2}} \left(\frac{d^2s}{dr^2} + \frac{v}{r} \frac{ds}{dr} \right)$$

$$\epsilon_r(r, z) = \left[\frac{3}{8} \left[\frac{Pa^2(h_p - h_m)(3 + \nu)}{E(h_p + h_m)^3} \right] \right] \left[1 - \frac{r^2}{a^2} \right]$$

$$\epsilon_t = \left[\frac{3}{8} \left[\frac{Pa^2(h_p - h_m)(3 + \nu)}{E(h_p + h_m)^3} \right] \right] \left[1 - \left(\frac{\nu + 1}{3 + \nu} \right) \left(\frac{rL}{a^2} \right) \right]$$

$\epsilon_0 \rightarrow$ max. strain acting at the centre of the circular plate



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strain \rightarrow radial
 \rightarrow tangential

$$\epsilon_r(r, z) = -\frac{kz}{Eh^{3/2}} \left(\frac{d^2 s}{dr^2} + \frac{v}{r} \frac{ds}{dr} \right)$$

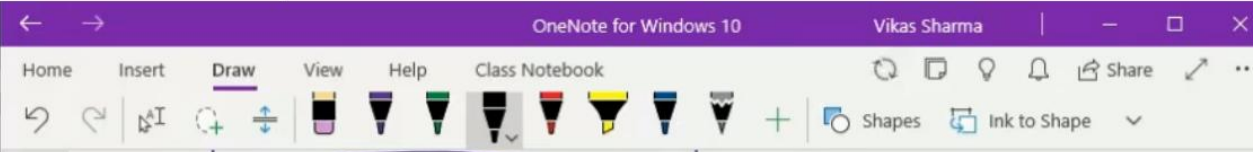
$$\epsilon_r(x, z) = \frac{3}{8} \left[\frac{\rho a^2 (h_p - h_m) (3 + \nu)}{E (h_p + h_m)^3} \right] \left[1 - \frac{r^2}{a^2} \right]$$

$$\epsilon_t = \frac{3}{8} \left[\frac{q a^2 (h_p - h_m) (3 + \nu)}{E (h_p + h_m)^3} \right] \left[1 - \left(\frac{\nu + 1}{3 + \nu} \right) \left(\frac{r_L}{a^2} \right) \right]$$

$\epsilon_0 \rightarrow$ max. strain acting at the centre of the circular plate



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$\epsilon_c \rightarrow$ max. strain acting at the centre of the circular plate

$$P/A = \sigma = E\epsilon$$

$$P/\epsilon = E \times A = \frac{\pi}{4} E r^2$$

$$P/\epsilon = r^2$$

Respective induced strains are fⁿ of radius (circular)

$$\bar{\epsilon} = \frac{\int_{r_1}^{r_2} \epsilon_r(r) dr}{\int_{r_1}^{r_2} dr}$$



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$$P/A = \sigma = E\epsilon$$

$$P/\epsilon = E \times A = \frac{\pi}{4} E r^2$$

$$P/\epsilon = r^2$$

Pressure induced strain are fⁿ of radius (circular)

$$\bar{\epsilon}_r = \frac{\int_{r_1}^{r_2} \epsilon_r(r) dr}{\int_{r_1}^{r_2} dr}$$

Mean radial strain = ϵ_0



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Pen Eraser Highlighter Pencil Ink Shapes Ink to Shape

Potential induced charge are fⁿ of radius (coulomb)

$$\bar{E} = \frac{\int_{r_1}^{r_0} \epsilon_r(r) dr}{\int_{r_1}^{r_0} dr}$$

Mean radial charge

$$= \epsilon_0 \left[1 - \frac{1}{3a^2} \left(\frac{r_0^3 - r_1^3}{r_0 - r_1} \right) \right]$$


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Pressure induced strains are fⁿ of radius (circular)

$$\bar{\epsilon}_r = \frac{\int_{r_i}^{r_o} \epsilon_r(r) dr}{\int_{r_i}^{r_o} dr}$$

Mean radial strain

$$= \epsilon_0 \left[1 - \frac{1}{3\alpha^2} \left(\frac{r_o^3 - r_i^3}{r_o - r_i} \right) \right]$$

Mean tangential strain

$$\bar{\epsilon}_t = \epsilon_0 \left[1 - \frac{1}{3\alpha^2} \left(\frac{3\nu+1}{3+\nu} \right) \left(\frac{\rho_o^3 - \rho_i^3}{\rho_o - \rho_i} \right) \right]$$



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$$\bar{\epsilon}_t = \epsilon_0 \left[1 - \frac{1}{3\alpha^2} \left(\frac{3\nu+1}{3+\nu} \right) \left(\frac{\rho_0^3 - \rho_i^3}{\rho_0 - \rho_i} \right) \right]$$

$$\text{Gauge factor} = \frac{\Delta R/R}{\epsilon}$$

$$\frac{\Delta R}{R} = \text{Gauge factor} \times \epsilon$$

$$\frac{\Delta R}{R}_r = G(\bar{\epsilon}_r + \epsilon_{\text{temp}}) \quad \frac{\Delta R}{R}_t = G(\bar{\epsilon}_t + \epsilon_{\text{temp}})$$



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55:03



1:12:26



18°C
Haze



ENG IN 11:53 PM 3/2/2022

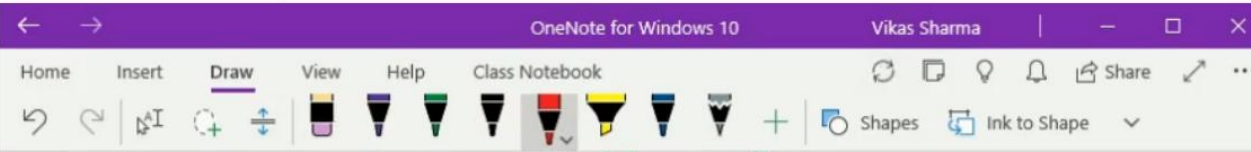
$$V_{out} = \frac{V_s \left(\frac{\Delta R_1}{R} - \frac{\Delta R_2}{R} \right)}{2 + \frac{\Delta R_1}{R} + \frac{\Delta R_2}{R}}$$

$$\left(\frac{\Delta R}{R} \right) \begin{cases} = 1 \\ < 1 \\ > 1 \end{cases}$$

$$\frac{\Delta R}{R} \ll 1$$



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$$2 + \frac{\Delta R}{R} + \frac{\Delta R}{R}$$

$$\left(\frac{\Delta R}{R} \right) \begin{cases} = 1 \\ < 1 \\ > 1 \end{cases} \quad \frac{\Delta R}{R} \ll 1$$

$$V_{out} = \frac{V_o}{2} \cdot G (\bar{\epsilon}_r - \bar{\epsilon}_t)$$


$$V_{out} = \frac{G}{16} \frac{(h_p - h_m)(3 + \nu)}{E(h_p + h_m)^3} \times \left[\frac{3\nu + 1}{3 + \nu} \left(\frac{\rho_o^3 - \rho_i^3}{\rho_o - \rho_i} \right) - \left(\frac{r_o^3 - r_i^3}{r_o - r_i} \right) \right] P$$



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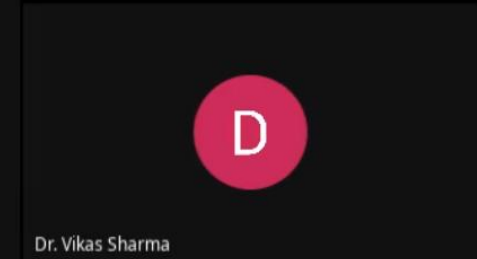

 $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$
 $\frac{V}{R} \rightarrow R < 1$

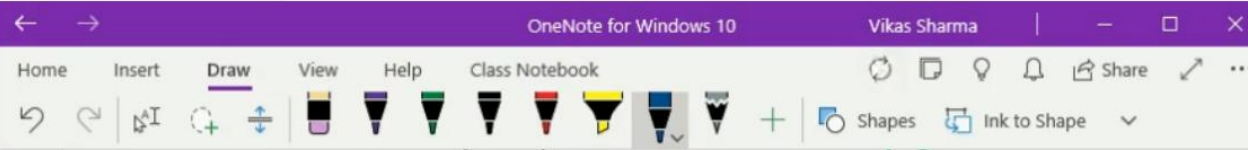
$$V_{out} = \frac{V_0}{2} \cdot G (\bar{\epsilon}_r - \bar{\epsilon}_t)$$

$$V_{out} = \frac{G(h_p - h_m)(3 + \nu)}{16 E (h_p + h_m)^3} \times \left[\frac{3\nu + 1}{3 + \nu} \left(\frac{\rho_0^3 - \rho_1^3}{\rho_0 - \rho_1} \right) - \left(\frac{r_0^3 - r_1^3}{r_0 - r_1} \right) \right] P$$

pressure

Sensitivity of a P.R. based pressure sensor (P)

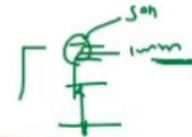
$$S = \frac{1}{V_s} \frac{d}{dP} V_{out}$$




Sensitivity of a P.R. based pressure sensor (P)

$$S = \frac{1}{V_s} \frac{d}{dP} V_{out}$$

$$S = \frac{G}{16} \frac{(h_p + h_m)(3 + \nu)}{E(h_p + h_m)^3} \times \left[\left(\frac{3\nu + 1}{3 + \nu} \right) \left(\frac{\rho_0^3 - \rho_i^3}{\rho_0 - \rho_i} \right) - \frac{\rho_0^3 - \rho_i^3}{r_0 - r_i} \right]$$



Material \rightarrow flexible

\rightarrow Elastic modulus (low) \downarrow



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$$\frac{\Delta R}{R} = \text{Gauge factor} \times \epsilon$$

doping
↓
n, p
} Silicon
→ G_E
Semiconductors

$$\underline{V_{out}} = \frac{V_S \left(\frac{\Delta R_1}{R} - \frac{\Delta R_2}{R} \right)}{2 + \frac{\Delta R_1}{R} + \frac{\Delta R_2}{R}}$$

$$\left(\frac{\Delta R}{P} \right) \begin{cases} = 1 \\ < 1 \end{cases} \quad \frac{\Delta R}{P} \rightarrow \ll 1$$