Energy 74 (2014) 12–26

Contents lists available at ScienceDirect

Energy

journal homepage: www.elsevier.com/locate/energy

A detailed MILP optimization model for combined cooling, heat and power system operation planning

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ARTICLE INFO

Article history:
Received 28 October 2013
Received in revised form
1 February 2014
Accepted 11 February 2014
Available online 13 April 2014

Keywords:
Combined cooling heat and power (CCHP)
Planning optimization
MILP
Off-design
Heat storage

ABSTRACT

A detailed optimization model is presented for planning the short-term operation of combined cooling, heat and power (CCHP) energy systems. The purpose is, given the design of a cogeneration system, to determine an operating schedule that minimizes the total operating and maintenance costs minus the revenue due to the electricity sold to the grid, while taking into account time-varying loads, tariffs and ambient conditions. The model considers the simultaneous use of different prime movers (generating electricity and heat), boilers, compression heat pumps and chillers, and absorption chillers to satisfy given electricity, heat and cooling demands. Heat and cooling load can be stored in storage tanks. Units can have one or two operative variables, highly nonlinear performance curves describing their off-design behavior, and limitations or penalizations affecting their start-up/shut-down operations. To exploit the effectiveness of state-of-the-art Mixed Integer Linear Program (MILP) solvers, the resulting Mixed Integer Nonlinear Programming (MINLP) model is converted into a MILP by appropriate piecewise linear approximation of the nonlinear performance curves. The model, written in the AMPL modeling language, has been tested on several plant test cases. The computational results are discussed in terms of the quality of the solutions, the linearization accuracy and the computational time.

1. Introduction

Combined cooling, heat and power (CCHP) generation is an effective way to reduce at the same time primary energy consumption and carbon dioxide emissions [1,2]. Such systems make rational use of primary energy generating simultaneously heat, electric/mechanical power and refrigeration effect for a set of users. As in a cascade process, first primary energy (fuel) is converted into electric/mechanical power by a thermodynamic cycle, then the heat discharged by the thermodynamic cycle is used to satisfy the user's heat demand and/or to generate refrigeration power with absorption chillers. Simple thermodynamic considerations can prove that such cascade energy flow ensures a saving of primary energy with respect to not-integrated solutions. Nowadays cogeneration systems are also favored by the incentive policies adopted by several European [3] and North American countries [4].

Several types of prime movers are suitable for co-generative applications, ranging from micro-turbines to gas-steam turbine combined cycles [5]. They can be classified on the basis of their number of independent operative variables (here called "degrees of freedom"). One-degree of freedom units feature a single operative variable (i.e., decision variable related to their operation mode) which determines the amount of heat recovered at a given temperature as well as the electric power output and the energy consumption. Examples of "one-degree of freedom" units are internal combustion engines and simple cycle gas turbines whose electric power output and co-generated thermal power depend on the fuel mass flow rate (which is considered here as their operative variable). In "two-degree of freedom" units, the operation mode (i.e., the electric power output and the thermal power output) depends on two independent operative variables. Examples are gas turbines with post-firing, and extraction-condensing steam turbines with a single turbine bleed [6]. Systems with more than two degrees of freedom are typically used in chemical plants and refineries, where a combined heat and power (CHP) steam cycle features a steam network with multiple steam and heat extractions. However, it is important to note that such systems are the minority of the existing

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http://dx.doi.org/10.1016/j.energy.2014.02.042
0360-5442/

cogeneration units. Indeed, most cogeneration units can be classified as either one or two-degree of freedom prime movers.

It is worth noting that heat can be recovered from the above-mentioned prime movers at different temperature levels, depending on both customer requirement and plant design. High temperature heat is typically recovered as saturated steam above 120 °C, while low temperature heat is recovered as hot water in the temperature range of 40–90 °C. Typically, high temperature heat is needed by industrial processes while low temperature heat is used by domestic users for residential heating (e.g., district heating networks). In addition to cogeneration units, systems such as heat pumps, refrigeration units and conventional boilers are typically included, the latter as back-up units.

The operation of CCHP systems poses a number of challenges due to the fact that (1) in many instances electric power, heat and refrigeration power demands do not follow the same trend, (2) different units can be used to satisfy the user demands. As a consequence of (1), the prime mover load cannot be regulated to exactly match all the three power demands, and a compromise solution must be found. For example, if the location is not remote, electricity can be purchased from the electric grid, while heat surplus can be either wasted (rejected to the environment) or stored in a heat storage tank to make it available during peak hours. Another solution to improve the CCHP system flexibility could be the downgrade of high temperature heat into low temperature heat. Due to the large number of decision variables and the necessity of determining trade-off solutions, the operation planning of CCHP plants requires the development of specific optimization tools.

1.1. Relevant literature

The multi-period optimization of production planning strategies for cogeneration energy conversion systems has been tackled by several research groups in the last decades. Optimization tools have been proposed for both short-term scheduling problems [7–13] as well as lifetime analyses aimed at addressing variability in operating conditions on a design level [14–20]. In the following we focus mainly on approaches tackling short-term scheduling problems.

A general version of the short-term operation planning problem can be summarized as follows: given a set of production units comprising prime movers generating only electricity or electricity and heat (CHP), boilers, heat pumps, compression and absorption refrigeration units and optional heat storage systems, determine for each time period t which units must be switched on and the value of their operative variables in order to minimize a relevant objective function (such as the total operating cost) while satisfying the demands of thermal power, cooling load and electricity, and while dealing with the operational features and constraints of the units (e.g., performance curves at off-design conditions, minimum and maximum allowed loads, average start-up time, maximum number of start-ups per day, etc.). The rigorous formulation of the above-mentioned optimization problem turns out to be a Mixed Integer NonLinear Program (MINLP), where binary variables are needed to switch on/off the units, continuous variables are related to the unit loads and user demands, and the performance curves describing the part load performance of the units introduce nonlinear equations. The large number of integer variables, which originate when considering multiple interconnected units and a reasonable time horizon (e.g., 1–7 days) with a hour-basis discretization, can make the MINLP extremely challenging, even for state-of-the-art optimization software. Thus, a number of approaches (e.g. [7,17]) have been proposed to convert the original MINLP into an approximated Mixed Integer Linear Program (MILP). Such problem linearization is considerably advantageous because of the convergence guarantees on the solution and the extremely fast and effective commercially available MILP solvers (e.g., CPLEX [21], Gurobi [22], Xpress [23]).

From the energy system point of view, the approaches for tackling short-term scheduling problem can be classified into two groups:

- Data-driven black-box approaches: the operating conditions of the involved energy systems (i.e., gas turbines, steam cycle power plants, boilers, refrigeration cycles, internal combustion engines, etc.) are defined by their performance curves which can be obtained by interpolating experimental data with linear, piecewise linear or polynomial functions. Examples are the methods proposed by Ladhelma and Hakonen [8], Makkonen and Lahdelma [9]; Christidis et al. [12]; Zhou et al. [17]. This is the preferred approach for one-degree of freedom units.
- First-principle thermodynamic approaches: the involved energy systems are decomposed into simple components (e.g., turbine, compressor, pumps, heat exchangers, etc.) with known performance curves, and mass/energy balance equations are imposed and included into the optimization problem to determine the plant operating conditions. Such approach is typically used to deal with complex CHP steam cycles as well as combined cycles featuring multiple independent operative variables, like those used in chemical plants and refineries. Examples are the works by Seeger et al. [7], Dvořák and Havel [11] and Mitra et al. [13].

It is worth noting that, in both approaches, logic constraints aimed at modeling the commitment of each individual unit, maximum load ramp rate and start-up/shut-down costs can be included.

Lahdelma and Hakonen [8] model the hourly operation of CHP plants as a Linear Program (LP) by representing the feasible region of operation with a convex combination of extreme points in the space of heat output, power output and cost (convex-hull representation). Binary variables to account for unit switch on/off and minimum load are not included. Later, Makkonen and Lahdelma [9] modify the above-mentioned linear model to handle plants with nonconvex feasible region of operation. Christidis et al. [12] use the convex-hull representation [8] and include binary variable to model start-up/shut-down operation and minimum/maximum load of units. The resulting MILP model accounts also for minimum downtime intervals of units. Zhou et al. [17] propose a MILP model which approximates the performance curves of one-degree of freedom units (an internal combustion engine, a boiler and an absorption chiller) with piecewise linear functions. The model is used to optimize the design of the cogeneration system while taking into account its lifetime operation and investment cost. The total annual cost is then compared with that corresponding to a solution found by a simpler MILP model using linear performance curves.

Dvořák and Havel [11] and Mitra et al. [13] propose fairly detailed linear thermodynamic models for steam cycle based CHP power plants with arbitrary number of degrees of freedom. The steam cycle is decomposed into its main components (i.e., turbine, steam generator, heat exchangers, condenser, deareator, headers), linear equations are derived to describe the behavior of each component, and mass/energy balance equations together with interconnection are imposed. More in detail, to preserve linearity, Dvořák and Havel [11] approximate the extraction-condensing turbine and steam generator efficiency with a piecewise linear function of the fuel mass flow rate, and the bilinear terms $m \cdot h$ (where m denotes the stream mass flow rates and h the enthalpies) with piecewise linear functions of m. Mitra et al. [13] develop a detailed mode model which can track the state of each plant component, determine the best operating mode (e.g., “on”, “off”, “warm start-up”, “cold start-up”, “unfired” or “supplementary fired” for the heat recovery steam generator (HRSG)), and account for transitional behavior (pre-defined sequence of mode, pre-defined trajectories during start-up, ramping constraints, etc.). Similarly to the other approaches, approximated relations and the piecewise linearization of component equations are used to avoid nonlinearities and obtain a MILP.

1.2. Contribution

In this work we present a data-driven MILP model for tackling the short-term scheduling problem which aims at minimizing the operating cost of a CCHP system comprising multiple cogeneration prime movers, boilers, heat pumps and refrigeration cycles. The proposed model can take into account all the following issues: (i) performance curves of units, derived from experimental data or provided by the manufacturer, may be highly nonlinear and non-concave functions of one or two operative variables, (ii) performance curves of units may depend on ambient temperature, (iii) units may have one or two degrees of freedom (i.e., one or two independent operative variables), and (iv) units can generate electric power and/or thermal power and/or refrigeration power starting from fuel, electricity or heat (i.e., units may be prime movers engines, units producing just electricity, boilers, compression heat pumps, compression and absorption chillers), (v) the start-up phase of some units requires a non-negligible time and also implies a significant energy penalization due to the warm-up phase of the machines, (vi) the number of start-up operations per day of each unit must be limited as they have a detrimental effect on the unit life.

Unlike in the approaches based on convex-hull representations, we consider piecewise linear approximation of the nonlinear performance curves. For each unit, the user can select, based on the available performance data, a piecewise linear approximation with an appropriate number of intervals. Since ambient temperature affects unit performance, the range as well as the shape of the performance curves can vary with temperature during the planning horizon. Start-up and subsequent warm-up operations are penalized according to criteria derived from the industrial practice. The objective function can be replaced by any linear function related to energy consumption or environmental index (e.g., CO_2 emissions).

Compared to previous works, the proposed MILP model for short-term scheduling optimization includes not only all the features of the above-mentioned data-driven approaches (i.e., heat storage systems as in Christidis et al. [12] and Zhou et al. [17], binary variables to model the on/off status of units and operational constraints as in Christidis et al. [12] and Zhou et al. [17], performance curves handled via either a convex-hull approach as in Lahdelma and Hakonen [8] or a linear approximation of one-degree of freedom units as in Zhou et al. [17], ambient temperature effect on the unit performance as in Christidis et al. [12]), but also the following new items:

- piecewise linearization of nonlinear performance curves of two-degree of freedom units,
- a strategy to limit (in terms of maximum allowed number per day) and effectively penalize the start-up operations of units without considerably increasing the computational complexity of the optimization problem, and
- three thermal power networks: High Temperature (HT), Low Temperature (LT) heat and refrigeration power.

2. Problem statement

The short-term scheduling optimization problem we want to tackle can be stated as follows. Given:

- one and/or two-degree of freedom generation units with fixed size, performance curves and start-up/warm-up times;
- low temperature storage tank with fixed capacity and constant loss rate;
- time-dependent demands of low and high temperature thermal power;
- time-dependent price of electricity;
- time-dependent ambient temperatures;

determine for each time period t of a time horizon T:

- the set of units to be switched on
- the value of the operative variables of each unit
- the storage tank level

which minimize the operating cost while satisfying the demands of low and high temperature heat.

3. Model formulation

In this section, we first describe the arrangement of the heat and power distribution network connecting the generation units with the storage system and the final users, and then detail the mathematical model providing all the related equations.

3.1. Heat and power distribution network

A schematic representation of the heat and power distribution network is represented in Fig. 1.

The general system under consideration is composed of four networks for the distribution of electric energy (el), refrigeration (q_{cold}), high and low temperature thermal energy (q_{high} and q_{low}). The HT heat network is included to model a steam network for an industrial heat user, while the LT heat network is thought to model a district heating network. Optionally, storage tanks can be connected to the LT heat network as well as to the refrigeration power network. The electric power generated by the engines can be used to fulfill the customers' requirements and, at the same time, drive the compression heat pumps and compression chillers, and provide the electricity needs of the absorption chillers. Electric power can be sold/purchased to/from the electric grid according to time-dependent prices. It is worth noting that purchasing and selling prices may differ. The HT and LT heat networks are interconnected in order to have the possibility to downgrade high temperature heat down to the low temperature heat network. Finally, each unit of LT heat and cooling load can, if needed, be wasted through a dedicated heat exchanger.

Six main classes of generation units are defined.

- One-degree of freedom prime movers cogenerating electricity and heat. This class comprises gas turbines, internal combustion engines, back-pressure steam cycles, fuel cells.
- Two-degree of freedom prime movers cogenerating electricity and heat (depending on two independent variables). This class includes for instance gas turbines with supplementary firing in the heat recovery section, steam cycles with extraction-condensing turbine, combined cycles with supplementary firing in the HRSG and back-pressure bottoming cycle.
- Boilers (i.e., one-degree of freedom systems generating only heat from fuel).

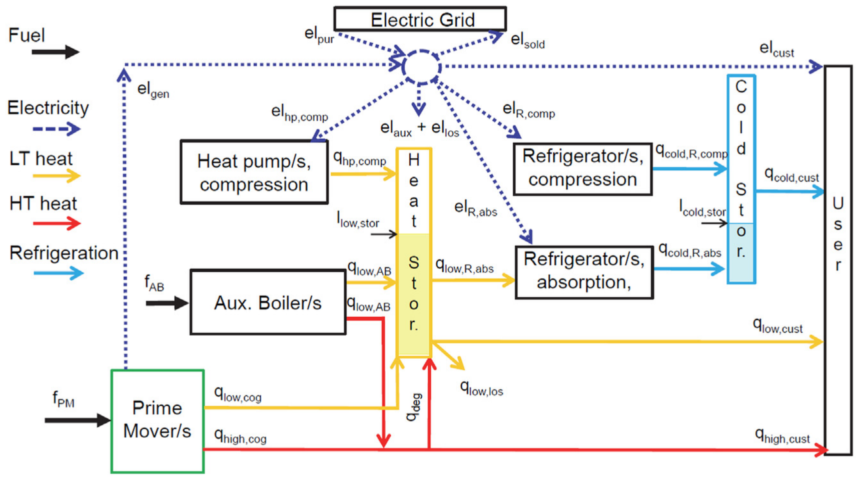

Fig. 1. Schematic representation of the heat and power distribution network connecting the generation units with the storage tanks, the electric grid and the users. Red and yellow arrows represent, respectively, the high and low temperature thermal power flows, light blue arrows represent the cooling load flows, blue dotted arrows represent the electric power and the black ones the fuel thermal power consumed by each unit. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

- Compression heat pumps (i.e., one-degree of freedom systems generating only heat from electricity).
- Compression chillers (i.e., one-degree of freedom systems generating only refrigeration power from electricity).
- Absorption chillers (i.e., one-degree of freedom systems generating only cooling load from heat and electric power).

The above-mentioned six classes of units allow to envisage any type of energy conversion systems, except for complex power plants with more than two degrees of freedom (i.e., more than two independent variables), such as combined cycles with supplementary firing in the HRSG and extraction-condensing bottoming cycle (whose operating conditions are defined by three independent decision variables), and steam cycles designed to provide steam at different temperature/pressure levels to chemical processes.

3.2. Mathematical model description

To start with the description of the MILP model, we first introduce the parameters, that are constant and known a priori, and the variables that are subject to optimization. Then we describe step by step the mathematical model by presenting the objective function and all the constraints.

First of all, we introduce the set T of hours of the day $T=\{1,2,3,4,\ldots,24\}$. The subscript t indicates a variable or parameter corresponding to the t-th period of the day.

As far as the units are concerned, we classify them on the basis of three criteria: the useful effect they generate, the type of energy they utilize, and the number of degrees of freedom. Thus, the following sets have been defined to distinguish units on the basis of the type of useful effect.

- P: set of units which produce electric energy.
- HT: set of units which produce high temperature heat.
- LT: set of units which produce low temperature heat.
- $Cold$: set of units which produce cooling load.

In addition, the following sets and subsets allow to classify units on the basis of type of energy they utilize and the number of degrees of freedom:

- $I=I_1\cup I_2$: set of units which consume fuel (prime movers and auxiliary boilers):
  - I_1: Subset of units with 1 degree of freedom,
  - I_2: Subset of units with 2 degrees of freedom,
- El: set of units consuming electricity,
- Th: set of units consuming heat.

Using this classification, the behavior of each unit can be described in terms of the set(s) to which it belongs. For example, CHP internal combustion engines belong to the sets $P\cap LT\cap HT\cap I_1$, boilers (generating both HT and LT heat) to sets $LT\cap HT\cap I_1$, CHP gas turbines with supplementary firing to $P\cap LT\cap HT\cap I_2$, heat pumps to $El\cap LT$, compression chillers to $El\cap Cold$ and the absorption units (which consume heat and some electric power to generate refrigeration power) to $El\cap Th\cap Cold$.

It is important to note that this framework, based on sets, is well suited for the implementation of the model in the AMPL modeling language.

3.2.1. Parameters

The following parameters are affected by fluctuations of the market and demand or ambient temperature. Thus, they are considered time-varying, as they have a different value for each time period t:

- $\widehat{T}_{0,t}$: ambient temperature in time period t, with $t\in T$,
- $\widehat{el}_t$: required electric energy in time period t, with $t\in T$,
- $\widehat{q}_{high,t}$: required high temperature heat in time period t with $t\in T$,
- $\widehat{q}_{low,t}$: required low temperature heat in time period t, with $t\in T$,
- $\widehat{q}_{cold,t}$: required cooling load in time period t, with $t\in T$,
- $\widehat{c}_{fuel,i,t}$: fuel cost for the unit i in time period t, with $i\in I$, $t\in T$,
- $\widehat{c}_{el,purch,t}$: purchase cost of electric energy in time period t, with $t\in T$,
- $\widehat{c}_{el,sold,t}$: selling price of electric energy in time period t, with $t\in T$,
- $\widehat{f}^{min}_{i,t}$, $\widehat{f}^{max}_{i,t}$: minimum and maximum limits of consumed fuel for the unit i, with $i\in I$, $t\in T$,
- $\widehat{el}^{min}_{i,t}$, $\widehat{el}^{max}_{i,t}$: minimum and maximum limits of consumed electricity for the unit i, with $i\in El$, $t\in T$,
- $\widehat{q}^{min}_{low,i,t}$, $\widehat{q}^{max}_{low,i,t}$: minimum and maximum limits of consumed low temperature heat for the unit i, with $i\in Th$, $t\in T$.

Parameters that are constant, or may change over time periods, are listed below:

- $\hat{c}_{\text{O\&M,cons},i}$: operation and maintenance cost proportional to the amount of fuel, heat or electricity utilized by unit i, with $i \in I \cup El \cup Th$ (note that for the absorption units consuming both heat and electricity, just the heat consumption is taken into consideration for this purpose);
- $\hat{c}_{\text{O\&M,time},i}$: operation and maintenance cost of unit i proportional to the number of hours the unit is working, with $i \in I \cup El \cup Th$;
- $\hat{c}_{\text{O\&M,on/off},i}$: operation and maintenance cost of unit i proportional to its number of start-up operations, with $i \in I \cup El \cup Th$;
- $\hat{N}_{\text{S},i}$: maximum number of daily start-ups for unit i, with $i \in I \cup El \cup Th$;
- $\hat{c}_{\text{fuel,start},i}$: start-up cost of unit i proportional to the amount of consumed fuel, with $i \in I$;
- $\hat{c}_{\text{el,start},i}$: start-up cost of unit i proportional to the amount of consumed electricity, with $i \in El$;
- $\hat{c}_{\text{low,start},i}$: start-up cost of absorption unit i proportional to the amount of low temperature heat utilized by the unit, with $i \in Th$;
- $\hat{l}_{\text{low,stor,max}}$: maximum low temperature heat storage level depending on the storage tank size;
- $\hat{l}_{\text{cold,stor,max}}$: maximum cooling load storage level depending on the storage tank size;
- $\hat{u}_{\text{low,los}}$: low temperature heat storage loss, assumed as a fixed percentage of the hourly heat storage level and depending on the storage tank characteristics like size and thermal insulations;
- $\hat{u}_{\text{cold,los}}$: cooling load storage loss, assumed as a fixed percentage of the hourly cooling load storage level and depending on the storage tank characteristics like size and thermal insulation;
- $\widehat{\text{el}}_{\text{abs},i}$: specific electric consumption (i.e., electricity consumed per unit of cooling load) by the absorption unit i, with $i \in Th \cup El \cap Cold$.

It is important to point out that the three above-mentioned start-up costs have been estimated for each unit on the basis of the available data relative to the start-up and load ramp operations.

3.2.2. Continuous (real) variables

- $f_{i,t} \in \mathbb{R}$: mass flow rate of fuel for unit i in time period $i \in I$, $t \in T$, with $i \in I$, $t \in T$ and $f_{i,t}^{\min} \le f_{i,t} \le f_{i,t}^{\max}$;
- $y_{i,t} \in \mathbb{R}$: the second operative variable, in addition to the fuel consumption, for the units with two degrees of freedom, with $i \in I_2$, $t \in T$ and $0 \le y_{i,t} \le y_{i,t}^{\max}$;

The upper bound $y_{i,t}^{\max}$ may not be constant and depend on the first operative variable. As an example, in the case of a gas turbine with post-firing, the dependency of the maximum amount of post-firing fuel depends from the GT fuel and it may be easily approximated as a linear function of $f_{i,t}$.

- $\text{el}_{i,t} \in \mathbb{R}$: electric energy consumed by unit i in time period t, with $i \in El \cup Th$, $t \in T$ and $\widehat{\text{el}}_{i,t}^{\min} \le \text{el}_{i,t} \le \widehat{\text{el}}_{i,t}^{\max}$ (this variable involves compression heat pumps and chillers, as well as the small amount of electricity consumed by the absorption units);
- $q_{\text{low},i,t} \in \mathbb{R}$: low temperature heat consumed by the absorption unit i in time period t, with $i \in Th$, $t \in T$ and $\hat{q}_{\text{low},i,t}^{\min} \le q_{\text{low},i,t} \le \hat{q}_{\text{low},i,t}^{\max}$;
- $\text{el}_{i,t} \in \mathbb{R}$: electric energy generated by prime mover i in time period t, with $i \in I$, $t \in T$ and $\text{el}_{i,t} \ge 0$;
- $q_{\text{high},i,t} \in \mathbb{R}$: high temperature thermal energy generated (auxiliary boiler) or co-generated (prime mover) by the unit i at time t, with $i \in I$, $t \in T$ and $q_{\text{high},i,t} \ge 0$;
- $q_{\text{low},i,t} \in \mathbb{R}$: low temperature thermal energy generated (auxiliary boiler) or co-generated (prime mover) by the unit i in time period t, with $i \in I \cup \text{El} \cup \text{Th}$, $t \in T$ and $q_{\text{low,i,t}} \ge 0$;
- $q_{\text{cold},i,t} \in \mathbb{R}$: cooling load generated by the unit i in time period t, with $i \in \text{El} \cup \text{Th}$, $t \in T$ and $q_{\text{cold,i,t}} \ge 0$;
- $l_{\text{low},t} \in \mathbb{R}$: level of low temperature heat accumulated in the storage tank at the beginning of time period t, with $t \in T$;
- $l_{\text{cold},t} \in \mathbb{R}$: level of cooling load accumulated in the storage tank at the beginning of time period t, with $t \in T$;
- $q_{\text{deg},t} \in \mathbb{R}$: thermal power degraded from high temperature down to low temperature at time t, with $t \in T$ and $q_{\text{deg,t}} \ge 0$.

3.2.3. Binary variables

- $z_{i,t} \in \{0,1\}$: binary variable utilized to turn on/off each unit, with $i \in I \cup El \cup Th$ and $t \in T$;
- $s_t \in \{0,1\}$: binary variable utilized to keep into account whether the electricity is sold or purchased from the grid, with $t \in T$;
- $\Delta_{i,t} \in \{0,1\}$: binary variable utilized to keep into account whether the unit has been switched on at the beginning of time period t, with $i \in I \cup El \cup Th$ and $t \in T$. Note that, in the model, variables $\Delta_{i,t}$ do not need to be explicitly declared to be binary: their integrality is implied by Eqs. (23)–(25).

3.2.4. Objective function

The objective function is the daily operating cost:

$$\sum_{i \in I}^{24} C_{f,\text{tot},i} + \sum_{t=1}^{24} C_{\text{O\&M,tot},t} + \sum_{t=1}^{24} C_{\text{on/off,tot},t} - \sum_{t=1}^{24} \text{El}_{\text{tot},t} \tag{1}$$

where $C_{\text{f,tot},t}$ denotes the cost of the total fuel amount consumed by all the units during the t time period, $C_{\text{O\&M,tot},t}$ denotes the sum of the operation and maintenance costs of all the units during time *t, and* $C_{\text{on/off,tot},t}$ denotes the total cost of the extra fuel or electric power or heat required by all the units during the t time period in order to start the unit up to production of its useful effect. $\text{El}_{\text{tot},t}$ denotes the revenue due to the electricity export to the electric grid during the t time period – note that $\text{El}_{\text{tot},t}$ can also be negative, in case we need to import *from* the electric grid. The objective function can be integrated with other linear terms in order to keep into account more factors influencing the operational strategy (e.g., credits/taxes related to the emissions of CO_2 and pollutants).

The cost of the fuel consumed in each period t is given by the sum of the amount of fuel each device is utilizing multiplied by its specific cost. Note that the fuel may be different depending on the unit.

$$C_{f,\text{tot},t} = \sum_{i \in I} \hat{c}_{\text{fuel},i} \cdot f_{i,t} \tag{2}$$

The operation and maintenance costs depend on three factors. The first factor is the consumption rate of the units (i.e., the load), and it was assumed to be a linear function of the energy input (fuel, electric power or heat, depending on the unit type). The second factor is the number of operative hours. The third factor takes into account the O&M penalties related to the start-up operations. As a result, the O&M costs which will be taken into consideration are formulated as below:

$$C_{\mathrm{O\&M,tot},t} = \sum_{i\in I}\hat{c}_{\mathrm{O\&M,cons},i}f_{i,t} + \sum_{i\in El}\hat{c}_{\mathrm{O\&M,cons},i}\mathrm{el}_{i,t} + \sum_{i\in Th}\hat{c}_{\mathrm{O\&M,cons},i}q_{\mathrm{low},i,t} + \sum_{i\in I\cup El\cup Th}\hat{c}_{\mathrm{O\&M,time},i}z_{i,t} + \sum_{i\in I\cup El\cup Th}\hat{c}_{\mathrm{O\&M,on/off},i}\Delta_{i,t}, \tag{3}$$

and the extra cost associated to the start-up procedure, is calculated as

$$C_{\mathrm{on/off},t} = \sum_{i\in I}\hat{c}_{\mathrm{fuel,start},i}\Delta_{i,t} + \sum_{i\in \mathrm{El}}\hat{c}_{\mathrm{el,start},i}\Delta_{i,t} + \sum_{i\in \mathrm{Th}}\hat{c}_{\mathrm{heat,start},i}\Delta_{i,t}. \tag{4}$$

In Eq. (1) $\mathrm{El}_{\mathrm{tot},t}$, is calculated multiplying the amount of electric energy hourly purchased or sold to the grid $\mathrm{el}_{\mathrm{grid},t}$, by the electricity cost, see Eqs. (7) and (8). The price for selling and purchasing electrical energy is not the same: expressing such difference requires additional constraints that will be presented in detail in the following section.

3.2.5. Constraints

3.2.5.1. Electric energy. The electric energy which is either sold to the grid or bought from the grid at time *t* is given by the sum of all the electric energy produced by the prime movers at time *t* (belonging to the set *I*) minus the electric energy absorbed by the electrically driven units (belonging to the set *El*), and the electric energy required by the customers ($\widehat{\mathrm{el}}_t$).

$$\mathrm{el}_{\mathrm{grid},t} = \sum_{i\in P}\mathrm{el}_{i,t} - \sum_{i\in \mathrm{El}}\mathrm{el}_{i,t} - \widehat{\mathrm{el}}_t \tag{5}$$

The price of energy is different depending on whether you sell or purchase it. This can be modeled in terms of bilinear constraints, which can be linearized with the standard “big-M” terms, as follows. The binary variable s_t has value 1 if $\mathrm{el}_{\mathrm{grid},t}$ is positive, meaning that electricity is sold to the grid, and 0 if $\mathrm{el}_{\mathrm{grid},t}$ is negative, meaning that electricity is purchased from the grid. The price of the energy bought from the grid is $\widehat{c}_{\mathrm{el,purch},t}$, while that sold to the grid has price $\widehat{c}_{\mathrm{el,sold},t}$. This is guaranteed by the following linear constraints:

$$(s_t - 1)\cdot M_{4,t} \le \mathrm{el}_{\mathrm{grid},t} \le s_t\cdot M_{3,t} \tag{6}$$

$$\mathrm{El}_{\mathrm{tot},t} \le \mathrm{el}_{\mathrm{grid},t}\cdot\widehat{c}_{\mathrm{el,sold},t} + M_{1,t}\cdot(1-s_t) \tag{7}$$

$$\mathrm{El}_{\mathrm{tot},t} \le \mathrm{el}_{\mathrm{grid},t}\cdot\widehat{c}_{\mathrm{el,purch},t} + s_t\cdot M_{2,t}, \tag{8}$$

where the parameters $M_{1,\mathrm{t}}$, $M_{2,\mathrm{t}}$, $M_{3,\mathrm{t}}$ and $M_{4,\mathrm{t}}$ are large enough to activate and deactivate the respective constraint. In particular, we take:

$$M_{1,t} = M_{4,i,t}\cdot\widehat{c}_{\mathrm{el,purch},t}, t\in T, \tag{9}$$

$$M_{2,t} = M_{3,i,t}\cdot\widehat{c}_{\mathrm{el,sold},t}, t\in T, \tag{10}$$

$$M_{3,t} = \left(\sum_{i\in P}\hat{\mathrm{el}}_{max,i,t}\right)\cdot 1.5, t\in T, \tag{11}$$

$$M_{4,\mathrm{t}} = \left(\hat{\mathrm{el}}_t + \sum_{i\in El}\hat{\mathrm{el}}_{max,i,t}\right)\cdot 1.5, t\in T. \tag{12}$$

3.2.5.2. Heat and cooling. The constraint on the high temperature thermal power imposes that the customer high temperature heat requirements need to be fulfilled by the high temperature heat produced, both considering the co-generated and not co-generated (e.g. auxiliary boiler, not co-generative post-firing). If an amount of high temperature heat larger than required is produced, this can be downgraded to low temperature heat $q_{\mathrm{deg},t}$, as

$$\sum_{i\in I} q_{\mathrm{high},i,t} - q_{\mathrm{deg},t} \ge \hat{q}_{\mathrm{high},t}, \forall t\in T \tag{13}$$

The constraint on the low temperature thermal power ensures that customer requirements of low temperature heat are always fulfilled by both co-generated and not co-generated heat (e.g. auxiliary boiler, not co-generative post-firing), together with the extra high temperature heat available at low temperature after a downgrade. Low temperature heat can also be consumed by absorption units. In addition, the thermal storage can be used either to fulfill the customer requirements, in case heat has been previously accumulated, or it can be filled with heat, when the low temperature heat production exceeds the customer requirements for that specific hour, *t*. Such logic is formulated with the following constraint:

$$\sum_{i\in I\cup El} q_{\mathrm{low},i,t} - \sum_{i\in Th} q_{\mathrm{low},i,t} + q_{\mathrm{deg},t} + (l_{\mathrm{low},t} - l_{\mathrm{low},t+1}) - l_{\mathrm{low},t}\hat{u}_{\mathrm{low,los}} \ge \hat{q}_{\mathrm{low},t} \ \forall t\in T \tag{14}$$

The level of accumulated heat has to be non-negative and not larger than the maximum storage value allowed:

$$l_{\mathrm{low},t} \ge 0, \forall t\in T \tag{15}$$

$$l_{\mathrm{low},t} \le \hat{l}_{\mathrm{low,max}}, \forall t\in T \tag{16}$$

Similarly to the HT and LT heat, also the cooling load balance has to be always fulfilled. The generated cooling load, both by compression and absorption units, needs to fulfill the customer requirements for each hour *t*. Thermal storage can be used with the same logic of the low temperature heat balance:

$$\sum_{i\in Cold} q_{\mathrm{cold},i,t} + (l_{\mathrm{cold},t} - l_{\mathrm{cold},t+1}) - l_{\mathrm{cold},t}\hat{u}_{\mathrm{cold,los}} \ge \hat{q}_{\mathrm{cold},t} \quad \forall t\in T. \tag{17}$$

Both for low temperature heat and the cooling load, the level of thermal storage at the end of the day must be equal to that at the beginning of the day.

3.2.5.3. Technical limits. Each unit has a minimum and maximum value of electric energy, high temperature heat, low temperature heat and cooling load it can produce in each hour. These technical limits are imposed by the following constraints, which also impose the values of electric and thermal power being zero in case the device is shut down, thanks to binary variables $z_{i,t}$:

$$z_{i,t}\hat{\mathrm{el}}_{\mathrm{min},i,t} \le \mathrm{el}_{i,t} \le z_{i,t}\hat{\mathrm{el}}_{\mathrm{max},i,t} \ \text{ with } i\in P \ \forall t\in T \tag{18}$$

$$z_{i,t}\hat{q}_{\text{high,min},i,t} \le q_{\text{high},i,t} \le z_{i,t}\hat{q}_{\text{high,max},i,t} \text{ with } i \in HT\ \forall t \in T \tag{19}$$

$$z_{i,t}\hat{q}_{\text{low,min},i,t} \le q_{\text{low},i,t} \le z_{i,t}\hat{q}_{\text{low,max},i,t} \text{ with } i \in LT\ \forall t \in T \tag{20}$$

$$z_{i,t}\hat{q}_{\text{cold, min},i,t} \le q_{\text{cold},i,t} \le z_{i,t}\hat{q}_{\text{cold, max},i,t} \text{ with } i \in Cold \text{ and } \forall\ t \in T \tag{21}$$

3.2.5.4. Start-ups. The "start-up" constraints are used in order to set a maximum number of start-up procedures that can be tolerated by each process unit on a daily basis, in order to avoid damages. These values depend on the characteristics of each single unit.

$$\sum_{T} \Delta_{i,t} \le \hat{N}_{s,i}, \text{ with } i \in I \cup El \cup Th \tag{22}$$

To ensure that the variable $\Delta_{i,t}$ has value 1 at time t if, and only if, unit t was off at time $t-1$ ($z_{i,t-1} = 0$) and is on at time t, we introduce the following constraints for each $i \in I \cup \mathrm{El} \cup \mathrm{Th}$ and $t \in T$:

$$\Delta_{i,t} \ge z_{i,t} - z_{i,t-1}, \tag{23}$$

$$\Delta_{i,t} \le 1 - z_{i,t-1}, \tag{24}$$

$$\Delta_{i,t} \le z_{i,t}. \tag{25}$$

Note that we impose that at the end of the day, the state of the system has to be exactly the same as the beginning of the day. Therefore, for $t = 1$, the previous time period $(t-1)$ is to be considered $t = 24$.

3.2.6. Linearization of performance curves

For each unit of the system, the input variables (consumed fuel and electricity) are related to the output variables (generated heat, electricity, cooling) by means of the performance curves $g_{i,t}$. In particular, the following performance curves are considered for each type of unit:

- Heat pumps, $i \in El \cap LT$
  - $q_{\text{low},i,t} = g_{i,t}(\mathrm{el}_{i,t})$
- Low temperature auxiliary boilers, $i \in I_1 \cap LT$
  - $q_{\text{low},i,t} = g_{i,t}(f_{i,t})$
- High temperature auxiliary boilers, $i \in I_1 \cap HT$
  - $q_{\text{high},i,t} = g_{i,t}(f_{i,t})$
- Prime movers with one-degree of freedom (e.g. internal combustion engines and gas turbines without post-firing), $i \in I_1 \cap HT \cap LT \cap P$
  - $q_{\text{low},i,t} = g'_{i,t}(f_{i,t})$
  - $q_{\text{high},i,t} = g''_{i,t}(f_{i,t})$
  - $\mathrm{el}_{i,t} = g'''_{i,t}(f_{i,t})$
- Prime movers with two degrees of freedom (e.g. gas turbines with post-firing), $i \in I_2 \cap HT \cap LT \cap P$
  - $q_{\text{low},i,t} = g'_{i,t}(f_{i,t}, y_{i,t})$
  - $q_{\text{high},i,t} = g''_{i,t}(f_{i,t}, y_{i,t})$
  - $\mathrm{el}_{i,t} = g'''_{i,t}(f_{i,t}, y_{i,t})$
- Compression chillers, $i \in El \cap Cold$
  - $q_{\text{cold},i,t} = g_{i,t}(\mathrm{el}_{i,t})$
- Absorption chillers, $i \in Th \cap El \cap Cold$
  - $q_{\text{cold},i,t} = g_{i,t}(q_{\text{low},i,t})$

The nonlinear performance curves are replaced by a piecewise linear lower approximation with additional auxiliary variables and constraints that, for the sake of simplicity of the model, we do not describe in details here. While the piecewise linear approximation of one-degree of freedom performance curves is relatively straightforward, particular attention must be paid to two-degree of freedom units. Indeed, several piecewise linear approximation approaches are available for 2-D functions, such as the "triangle" and the "rectangle" methods recently analyzed in D'Ambrosio et al. [24], which considerably differ in terms of accuracy of the approximation, mathematical properties (e.g., continuity), and computational cost of the resulting MILP. In particular, compared to the "rectangle" method, the "triangle" method provides a better approximation of the nonlinear function but with the disadvantage of a larger number of variables and constraints. In this first version of the model we have implemented the triangle method in order to maximize the approximation accuracy. Further evaluations will include a performance comparison with the rectangle and other piecewise linear approximation methods.

The model is written in the AMPL algebraic modeling language [25] and the resulting MILP is solved with IBM ILOG CPLEX Optimizer [21].

4. Computational tests

In this section we evaluate the proposed MILP model on four test cases characterized by different number and type of units. For the sake of simplicity, we consider the same user's demand profile and operative cost assumptions for all tests. The demand profile is represented in Fig. 2. It is the average demand profile of an industrial facility located in Northern Italy during winter, determined on the basis of data provided by the plant owner for the years 2008–2011. Although the prediction of the load demand profile is a quite challenging problem and specific approaches have been proposed [26], in this work we consider average loads over each period, since the main focus is on the MILP model.

The main operative cost assumptions are reported in Table 1, while the performance curves of the units are reported in Table 2. The inputs and output of the performance curves are normalized with respect to the nominal value measured at ISO ambient conditions. This allows us to account for units with different size by just changing their scale factors (i.e., their nominal power at ISO conditions). The unit sizes used in the test cases are reported in Table 3.

The performance curves are obtained from data provided by the manufacturers, simulated with dedicated software [27], or taken from published literature. In particular, the performance data for the gas turbine unit [28] corresponds to a nonlinear as well as nonsmooth function (see Fig. 3). This is due to the change of control

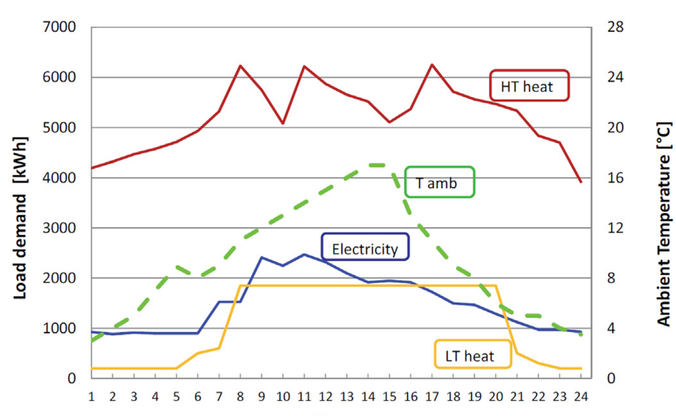

Fig. 2. User's demand of low temperature (LT) heat, high temperature (HT) heat, and electricity during the day. The ambient temperature profile is also reported.

Table 1
Fuel, O&M, heat storage losses and electricity cost assumptions.

Cost item	Basis	Value		
Fuel cost	Thermal energy, LHV basis [€/kWh]	0.06		
Load-dependent O&M cost	Energy input [€/kWh]	0.01		
Time-dependent O&M cost	Operating hours [€/h]	0.1		
On/off operation O&M cost	Number of start-ups [€/on]	0		
Storage heat loss penalty	Stored level [%]	0.25		
		Time periods		
		9th–19th hour	8th; 20th – 23rd hour	24th–07th hour
Electricity purchase prices	[€/kWh]	0.1577	0.1157	0.0877
Electricity sale prices	[€/kWh]	0.1261	0.0925	0.0701

strategy of the gas turbine (i.e., from the load control with the variable inlet guide vanes to the control with the firing temperature and fuel injection). In this case, we consider as performance curve either a third-degree polynomial fit of the available data, or a two-piece quadratic fit that accounts for the single non-differentiability point of the original curve (see Section 4.5).

The four test instances are described in detail in the following subsections. For each test case, the MILP optimization model is solved with different levels of accuracy of the piecewise linear (PWL) approximation of the nonlinear performance curves. The purpose is to evaluate the impact on the solution, the model complexity (number of variables and constraints), and the computational time. In Section 4.5 the non-smooth performance curve of the gas turbine is taken into account in order to show that the proposed MILP model is also able to deal with this feature.

4.1. Test Case A

The system includes three one-degree of freedom units: a compression heat pump, an auxiliary boiler generating low temperature heat and a boiler generating high temperature heat (see Fig. 4).

The performance curves and the nominal values of the units are reported, respectively, in Tables 2 and 3. It is worth emphasizing that the performance curve of the heat pump is highly nonlinear with respect to load and ambient temperature, as shown in Figs. 5 and 6.

Table 4 summarizes the main MILP model features and the results obtained by piecewise linear approximation of the unit performance curves with 5, 10 and 20 intervals. Clearly, the larger the number of intervals, the more accurate the approximation of the nonlinear performance curves, and the closer we expect the solution of our MILP model to be with respect to that of the original MINLP problem. This is obviously at the price of an increased number of (binary and continuous) variables and constraints. Due to the small number of units in this test case, the MILP solver is able to return the optimal solution in less than a second even when 20 intervals are considered.

The optimized short-term schedule obtained with 20 intervals is represented in Fig. 7. All the electricity is purchased because there

Table 2
Normalized performance curves of the units considered for all the Test Cases.

Unit type	Operative variables	Performance curves
GT	f_{GT}	*Third-degree polynomial (smooth curve)* Range: from 45% (minimum load) to 100% of the electric power output $q_{high,GT} = 1.5025 - 0.0227 \cdot T_0 - 4.2896 \cdot f_{GT} + 0.0001 \cdot T_0^2 + 0.0532 \cdot f_{GT} \cdot T_0 + 6.3670 \cdot f_{GT}^2 + 0.0000 \cdot T_0^3 - 0.0002 \cdot T_0^2 \cdot f_{GT}$ $- 0.0327 \cdot T_0 \cdot f_{GT}^2 - 2.5298 \cdot f_{GT}^3$ $q_{low,GT} = 4.4146 - 0.0469 \cdot T_0 - 12.3183 \cdot f_{GT} + 0.0000 \cdot T_0^2 + 0.0931 \cdot f_{GT} \cdot T_0 + 13.0429 \cdot f_{GT}^2 + 0.0000 \cdot T_0^3 + 0.0000 \cdot T_0^2 \cdot f_{GT}$ $- 0.0462 \cdot T_0 \cdot f_{GT}^2 - 4.1268 \cdot f_{GT}^3$ $el_{,GT} = 2.5748 - 0.0358 \cdot T_0 - 8.6217 \cdot f_{GT} + 0.0001 \cdot T_0^2 + 0.0833 \cdot f_{GT} \cdot T_0 + 10.8790 \cdot f_{GT}^2 + 0.0000 \cdot T_0^3 - 0.0002 \cdot T_0^2 \cdot f_{GT}$ $- 0.0465 \cdot T_0 \cdot f_{GT}^2 - 3.8297 \cdot f_{GT}^3$ *Two-piece quadratic fit (non-smooth curve)* Range 1: from 45% (minimum load) to 65% of the electric power output $q_{high,GT} = 2.6813 - 0.0449 \cdot T_0 - 10.3036 \cdot f_{GT} + 0.0003 \cdot T_0^2 + 0.1253 \cdot f_{GT} \cdot T_0 + 16.2263 \cdot f_{GT}^2 - 0.0000 \cdot T_0^3 - 0.0005 \cdot T_0^2 \cdot f_{GT}$ $- 0.0896 \cdot T_0 \cdot f_{GT}^2 - 7.7592 \cdot f_{GT}^3$ $q_{low,GT} = 0.1534 + 0.0057 \cdot T_0 + 2.6460 \cdot f_{GT} - 0.0001 \cdot T_0^2 - 0.0282 \cdot f_{GT} \cdot T_0 - 3.7139 \cdot f_{GT}^2 - 0.0000 \cdot T_0^3 + 0.0001 \cdot T_0^2 \cdot f_{GT}$ $+ 0.0201 \cdot T_0 \cdot f_{GT}^2 + 1.7472 \cdot f_{GT}^3$ $el_{,GT} = 3.7174 - 0.0638 \cdot T_0 - 15.6651 \cdot f_{GT} + 0.0004 \cdot T_0^2 + 0.1814 \cdot f_{GT} \cdot T_0 + 23.8038 \cdot f_{GT}^2 - 0.0000 \cdot T_0^3 - 0.0007 \cdot T_0^2 \cdot f_{GT}$ $- 0.1283 \cdot T_0 \cdot f_{GT}^2 - 11.2134 \cdot f_{GT}^3$ Range 2: from 65% to 100% of the electric power output $q_{high,GT} = - 21.3667 + 0.3028 \cdot T_0 + 66.0182 \cdot f_{GT} - 0.0011 \cdot T_0^2 - 0.6193 \cdot f_{GT} \cdot T_0 - 65.4818 \cdot f_{GT}^2 + 0.0000 \cdot T_0^3 + 0.0011 \cdot T_0^2 \cdot f_{GT}$ $+ 0.3137 \cdot T_0 \cdot f_{GT}^2 + 21.8797 \cdot f_{GT}^3$ $q_{low,GT} = - 24.2273 + 0.3418 \cdot T_0 + 73.9794 \cdot f_{GT} - 0.0013 \cdot T_0^2 - 0.6961 \cdot f_{GT} \cdot T_0 - 73.3136 \cdot f_{GT}^2 + 0.0000 \cdot T_0^3 + 0.0013 \cdot T_0^2 \cdot f_{GT}$ $+ 0.3532 \cdot T_0 \cdot f_{GT}^2 + 24.5818 \cdot f_{GT}^3$ $el_{,GT} = - 26.5692 + 0.3685 \cdot T_0 + 79.8152 \cdot f_{GT} - 0.0013 \cdot T_0^2 - 0.7428 \cdot f_{GT} \cdot T_0 - 78.2867 \cdot f_{GT}^2 + 0.0000 \cdot T_0^3 + 0.0013 \cdot T_0^2 \cdot f_{GT}$ $+ 0.3742 \cdot T_0 \cdot f_{GT}^2 - 26.0477 \cdot f_{GT}^3$
ICE	f_{ICE}	Range: from 50%(minimum load) to 100% of the electric power output $q_{high,ICE} = 0.1390 + 0.0000 \cdot T_0 + 0.7192 \cdot \mathrm{f}_{ICE} + 0.0000 \cdot T_0^2 + 0.0000 \cdot \mathrm{f}_{ICE} \cdot T_0 + 0.1418 \cdot \mathrm{f}_{ICE}^2$ $q_{low,ICE} = 0.1390 + 0.0000 \cdot T_0 + 0.7192 \cdot \mathrm{f}_{ICE} + 0.0000 \cdot T_0^2 + 0.0000 \cdot \mathrm{f}_{ICE} \cdot T_0 + 0.1418 \cdot \mathrm{f}_{ICE}^2$ $el_{,ICE} = - 0.1259 + 0.0029 \cdot T_0 + 1.1371 \cdot \mathrm{f}_{ICE} - 0.0001 \cdot T_0^2 - 0.0011 \cdot \mathrm{f}_{ICE} \cdot T_0 v 0.0194 \cdot \mathrm{f}_{ICE}^2$
HP	$el_{HP,comp}$	Range: from 12.5%(minimum load) to 100% of the thermal power $q_{low,HP,comp} = - 0.0219 + 0.0056 \cdot T_0 + 0.3632 \cdot el_{HP,comp} + 0.0003 \cdot T_0^2 + 0.0436 \cdot el_{HP,comp} \cdot T_0 + 1.5057 \cdot el_{HP,comp}^2 - 0.0000 \cdot T_0^3$ $- 0.0001 \cdot T_0^2 \cdot el_{HP,comp} - 0.0088 \cdot T_0 \cdot el_{HP,\ comp}^2 - 1.2770 \cdot el_{HP,\ comp}^3$
AB,HT	$f_{AB,HT}$	Range: from 0% to 100% of the thermal power $q_{high,AB} = - 0.0245 + 1.1378 \cdot f_{AB,HT} - 0.1150 \cdot f_{AB,HT}^2$
AB,LT	$f_{AB,LT}$	Range: from 0% to 100% of the thermal power $q_{low,AB} = - 0.0245 + 1.1378 \cdot f_{AB,LT} - 0.1150 \cdot f_{AB,LT}^2$

Table 3
Nominal values utilized to define the performance curves of the units considered in all the Test Cases.

Test Case	Unit type	Nominal value (power at ISO conditions)
A	HP	$el_{HP,comp,nom} = 560.00$
		$q_{low,HP,comp,nom} = 2632.00$
	AB,HT	$f_{AB,HT,nom} = 8522.73$
		$q_{high,AB,nom} = 7500.00$
	AB,LT	$f_{AB,LT,nom} = 2666.67$
		$q_{low,AB,nom} = 2400.00$
B	GT	$f_{GT,nom} = 15560.27$
		$q_{high,GT,nom} = 5475.88$
		$q_{low,GT,nom} = 2172.97$
		$el_{,GT,nom} = 4668.08$
		$f_{GT,PF,Not,cog,nom} = 5000.00$
		$q_{high,GT,PF,Not,cog,nom} = 4700.00$
	AB,HT	$f_{AB,HT,nom} = 6818.18$
		$q_{high,AB,nom} = 6000.00$
	AB,LT	$f_{AB,LT,nom} = 2666.67$
		$q_{low,AB,nom} = 2400.00$
C	GT	$f_{GT,nom} = 6765.33$
		$q_{high,GT,nom} = 2380.82$
		$q_{low,GT,nom} = 944.77$
		$el_{,GT,nom} = 2029.60$
	ICE	$f_{ICE,nom} = 5197.97$
		$q_{high,ICE,nom} = 1483.20$
		$q_{low,ICE,nom} = 988.80$
		$el_{,ICE,nom} = 2048.00$
	HP	$el_{HP,comp,nom} = 280.00$
		$q_{low,HP,comp,nom} = 1316.00$
	AB,HT	$f_{AB,HT,nom} = 6818.18$
		$q_{high,AB,nom} = 6000.00$
	AB,LT	$f_{AB,LT,nom} = 2666.67$
		$q_{low,AB,nom} = 2400.00$
D	GT	$f_{GT,nom} = 6765.33$
		$q_{high,GT,nom} = 2380.82$
		$q_{low,GT,nom} = 944.77$
		$el_{,GT,nom} = 2029.60$
		$f_{GT,PF,Not,cog,nom} = 2500.00$
		$q_{high,GT,PF,Not,cog,nom} = 2350.00$
	ICE	$f_{ICE,nom} = 5197.97$
		$q_{high,ICE,nom} = 1483.20$
		$q_{low,ICE,nom} = 988.80$
		$el_{,ICE,nom} = 2048.00$
	HP	$el_{HP,comp,nom} = 280.00$
		$q_{low,HP,comp,nom} = 1316.00$
	AB,HT	$f_{AB,HT,nom} = 6818.18$
		$q_{high,AB,nom} = 6000.00$
	AB,LT	$f_{AB,LT,nom} = 2666.67$
		$q_{low,AB,nom} = 2400.00$

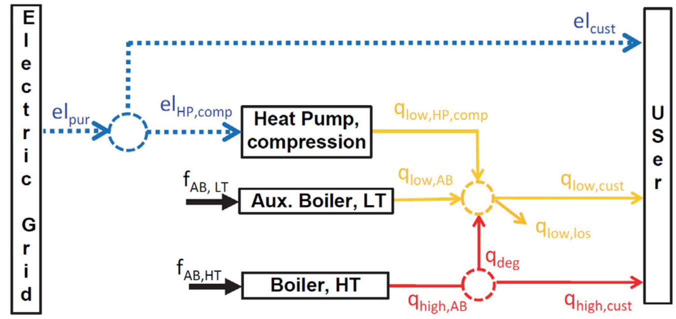

Fig. 4. Schematic representation of the system of units considered in Test Case A.

are no co-generative units and all the high temperature heat is produced by the high temperature boiler. All the low temperature heat is generated by the compression heat pump because of the high value of its COP and because the ambient temperature is not so cold, especially during the hours when the electricity is more expensive. During the first 5 h and the last 3 h, a small fraction of the low temperature heat produced by the heat pump is dissipated. This is due to the fact that the heat pump cannot work below a minimum load (defined as the 12.5% of the electric maximum power input, see Table 2). Therefore, the minimum amount of heat provided by the heat pump increases with the ambient temperature because of its influence on the maximum power and the coefficient of performance (Fig. 6).

From the solution quality point of view, Table 4 shows that increasing the number of intervals from 5 to 20 leads to a slightly smaller objective function value, which corresponds to a slight reduction in the total cost. Since the performance curves of the involved units are concave, a more accurate PWL lower approximation leads to a larger efficiency. Nevertheless, this improvement is negligible (less than 0.02%) when the number of intervals increases from 10 to 20. Despite the small difference in the objective function values, the two short-term planning solutions show some noticeable differences, as discussed in Section 4.5.

4.2. Test Case B

Test Case B includes a gas turbine with post-firing cogenerating electricity and both HT and LT heat, and two auxiliary boilers generating respectively HT and LT heat (see Fig. 8). The gas turbine

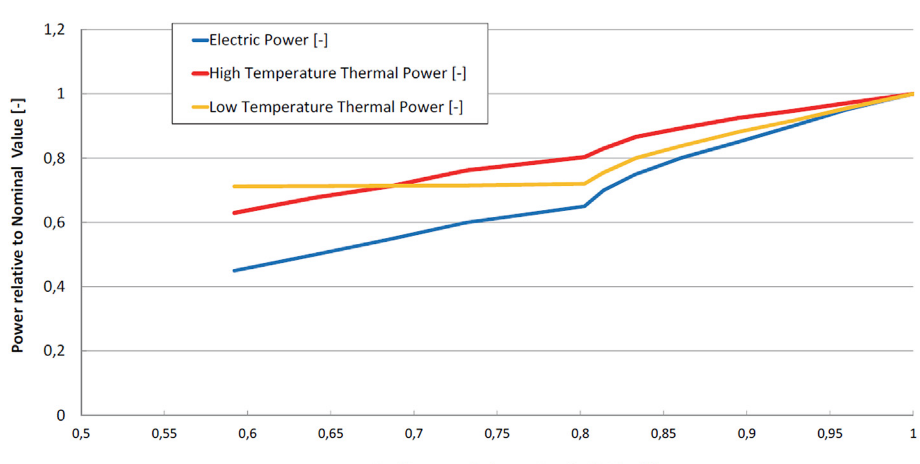

Fig. 3. Plot of the performance curves of the gas turbine, namely the electric power, the high temperature thermal power and the low temperature thermal power output relative to their nominal value expressed as a function of the fuel power relative to its nominal value at 15 °C.

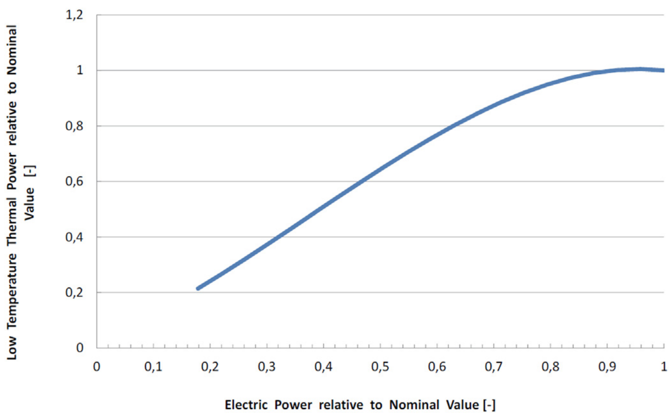

Fig. 5. Normalized performance curve of the heat pump at ambient temperature of 10 °C. The *x*-axis represents the ratio between the actual electric power input and its nominal value, and the *y*-axis represents the ratio between the actual thermal power and its nominal value.

unit is an example of two-degree of freedom unit (this clearly increases the number of independent variables), while the boilers are one-degree of freedom units. The performance curves and the nominal values of the units are reported, respectively, in Tables 2 and 3. More precisely, in this test case we consider smooth GT performance curves (i.e., third-degree polynomial).

The main MILP model features and the optimization results obtained with 5, 10 and 20 intervals are reported in Table 5. Compared to Test Case A, the MILP model involves a considerably larger number of variables and constraints. Even if the same number of units and approximation intervals are considered, additional variables and constraints (quadratic in the number of intervals) are needed to define two-dimensional piecewise linear approximations for the GT with post-firing (see Section 4.2). The computational times required with 10 and 20 intervals turn out to be, respectively, two and three orders of magnitude higher than those for Test Case A.

The short-term planning solution obtained by solving the proposed MILP model with 20 intervals is graphically represented in Fig. 9. Focusing on electricity balance, it can be noted that GT is switched on only from period 8–20, while it is off when the prices of electricity are low (see Table 1). In periods 8–20, the electricity generated is higher than the demand; hence part of it is sold to the

Table 4
Main MILP model features for Test Case A and optimization results for piecewise linear approximations with 5, 10 and 20 intervals.

	5 intervals	10 intervals	20 intervals
Number of binary variables	408	768	1488
Total number of variables	1392	2112	3552
Number of constraints	1371	1731	2451
Computational time (s)	0.108	0.120	0.328
Relative MILP gap[a] (%)	4E−5	9E−3	2.7E−3
Objective function value (€)	15346.26	15337.51	15334.12

[a] The relative MILP gap is defined as gap% = 100(cost$_{sol}$ − cost$_{LP\ bound}$/cost$_{sol}$), where cost$_{sol}$ is the objective function value of the best feasible solution found and cost$_{LP\ bound}$ is the value of the optimal solution of the linear relaxation. Since cost$_{LP\ bound}$ is a lower bound on the value of the optimal solution, this provides a guarantee that the solution found is within gap% percent from the optimal value.

grid. This is because the GT load is driven by the HT demand; simultaneously, a small quantity of LT heat has to be dissipated. The HT heat demand and the LT heat demand are covered by the GT and HT auxiliary boiler. The LT boiler is always off because the optimal strategy consists in downgrading the HT heat (the LT auxiliary boiler would work at partial load, with a lower efficiency). Finally, in periods 8 and 20, the GT operates at minimum load without post-firing, and the required HT heat is generated by the boiler. Indeed, if supplementary firing were used, it would not suffice to fulfill the whole HT request, implying the necessity to switch on the HT auxiliary boiler at a very low load with a substantial efficiency penalty.

As far as the accuracy is concerned, it is important to note that the difference in objective function value (cost) obtained with 10 and 20 interval approximations is negligible (below 0.001%), while the short-term planning solutions show noticeable differences. This issue is further discussed in Subsection 4.5.

4.3. Test Case C

This third test case is meant to show that the approach can be applied to systems with several units and heat storage. Test Case C, whose flow diagram is reported in Fig. 10, includes:

- An internal combustion engine (ICE) and a gas turbine, both producing electric power (el_{ICE} and el_{GT}), high temperature ($q_{high,ICE}$ and $q_{high,GT}$) and low temperature heat ($q_{low,ICE}$ and $q_{low,GT}$). Both the ICE and the GT have one-degree of freedom.

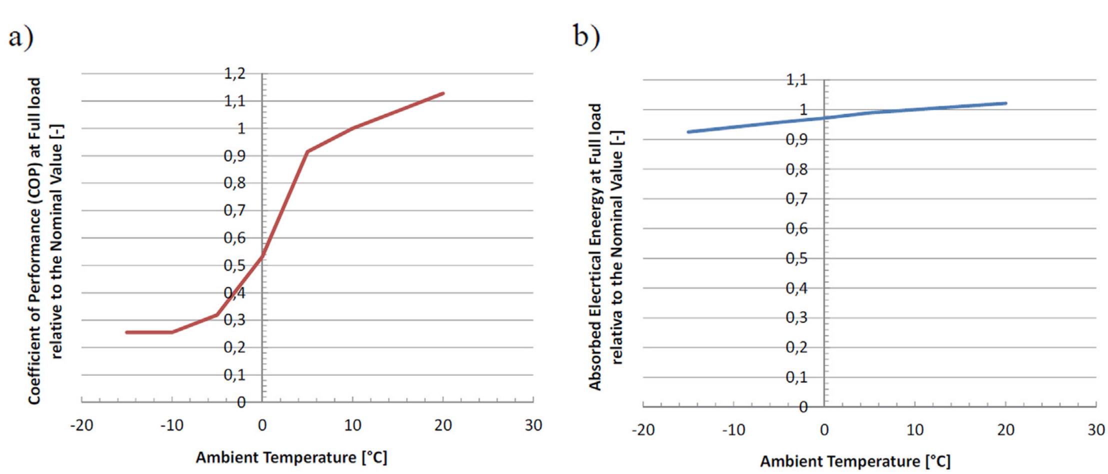

Fig. 6. Effect of the ambient temperature on the heat pump coefficient of performance (a) and electric power consumption (b) at full load. The plots refer to the ratio between the full load value at a certain ambient temperature and the nominal value at 10 °C.

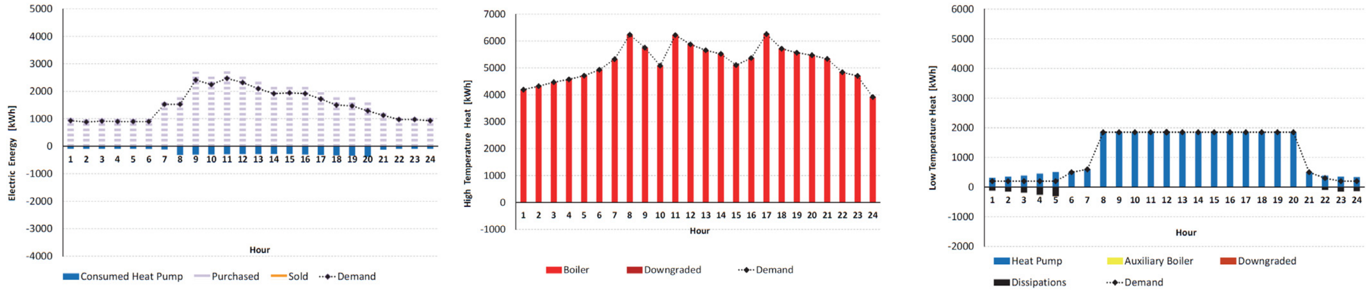

Fig. 7. Electricity (left), HT heat (middle) and LT heat (right) optimal schedules for Test Case A, obtained by 20 intervals piecewise linear approximation of the units' performance curves.

- A compression heat pump (HP), producing low temperature heat ($q_{low,HP,comp}$) and consuming electric power ($el_{HP,\ comp}$).
- Two auxiliary boilers, one producing high temperature heat ($q_{high,AB}$) and the other producing low temperature heat ($q_{low,AB}$).
- A LT heat storage system with capacity equal to 6000 kWh and heat loss rate proportional to the storage level.

The performance curves and the nominal values of the units are reported, respectively, in Tables 2 and 3. Also in this test case we consider smooth GT performance curves (i.e., third-degree polynomial).

In order to show the effectiveness of our approach to penalize the number of start-up/shut-down operations of the units, we compare the solutions obtained with and without penalties. The main MILP model features and the optimization results found with the piecewise linear approximation of the performance curves involving 5, 10 and 20 intervals are reported in Table 6, while the optimized short-term operation plans computed with 20 intervals are plotted in Figs. 11 and 12.

In spite of the large number of binary variables and constraints (up to 12,073 binary variables and 5845 constraints), the computational time required to solve the MILP model is only of the order of a few minutes, even when 20 intervals are considered. This confirms the excellent efficiency of the state-of-the-art MIP solvers, such as CPLEX. In both cases, with and without on/off penalties, the difference in objective function value between the solutions for MILP models with 10 and 20 interval approximation is below 0.015%. Also the difference in the short-term planning solutions is negligible.

As far as the optimized operation planning solutions are concerned, it is worth analyzing the differences between the cases with and without start-up penalty. In the case without start-up penalty, the GT is switched on twice (periods 9, 11 and 12, see Fig. 11) for a total operating time of only 3 h. This discontinuous operation of the GT may be detrimental for its lifetime and it may significantly increase its maintenance costs. Instead, in the case with start-up penalties (Fig. 12), the GT is never switched on because the demand can be covered by increasing the load of the other units and a different management of the heat storage. The heat pump is switched on twice in both cases, but this is due to the fact that its start-up penalty is very small compared to that of the gas turbine (the penalty was set equal to 10% of the nominal input, i.e., 28 kW to be added to the consumed electricity for the heat pump versus 676.53 kW to be added to the consumed fuel for the gas turbine).

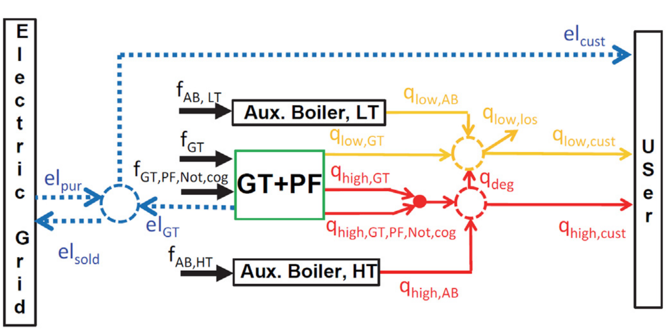

Fig. 8. Schematic representation of the system of units considered in Test Case B.

4.4. Test Case D

The fourth test case is composed of the same units as Test Case C and, in addition, has the possibility of using the post-firing burner of the gas turbine (i.e., the GT has two degrees of freedom). The main MILP model features and the optimization results obtained by approximating the performance curves with 5, 10 and 20 intervals are reported in Table 7.

Even in this test case, with larger number of binary variables and constraints (up to 21,744 binary variables and 19,085 constraints), the computational time required to determine the solution is of the order of 5 min even when considering 20 intervals.

The short-term planning solution found with 20 intervals is graphically represented in Fig. 13. Compared to the solutions of Test Case C with start-up penalties (where PF was not allowed and the GT was not used), in this instance the GT with PF runs for 11 h of the day replacing the HT boiler and exporting power to the electric grid. This operation is advantageous during the hours with high electricity price, from time period 9 to time period 19 (see Table 1). Unlike in Test Case C, here the gas turbine is advantaged by the post-firing contribution whose energy efficiency is larger than that of the boiler. In addition, the GT with post-firing is capable to fulfill the whole high temperature heat demand without the need of switching on the HT auxiliary boiler (which would run at partial load, thus with low efficiency).

Table 5
Main MILP model features for Test Case B and optimization results for piecewise linear approximations with 5, 10 and 20 intervals.

	5 intervals	10 intervals	20 intervals
Number of binary variables	1656	5616	20,736
Total number of variables	4800	12,120	37,560
Number of constraints	3747	7107	17,427
Computational time (s)	4.397	37.99	158.76
Relative MILP gap (%)	8E−3	6E−3	9E−3
Objective function value (€)	15375.68	15368.79	15368.93

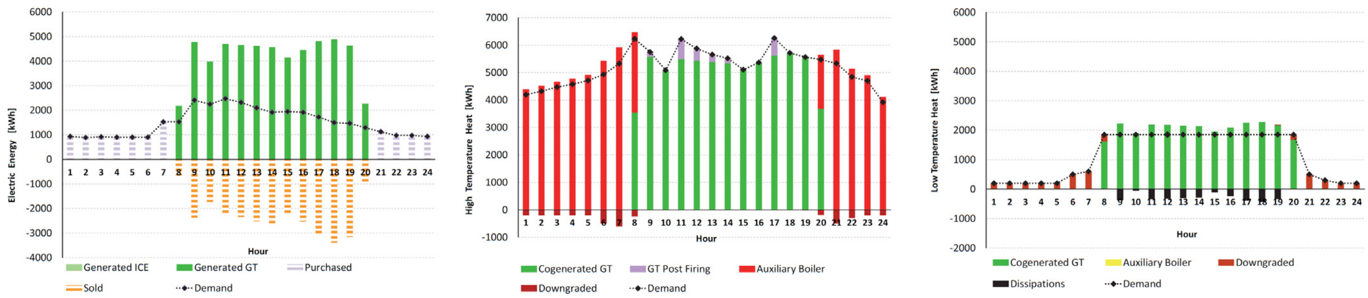

Fig. 9. Electricity (left), HT heat (middle) and LT heat (right) optimal schedules for Test Case B, obtained by 20 intervals piecewise linear approximation of the units' performance curves.

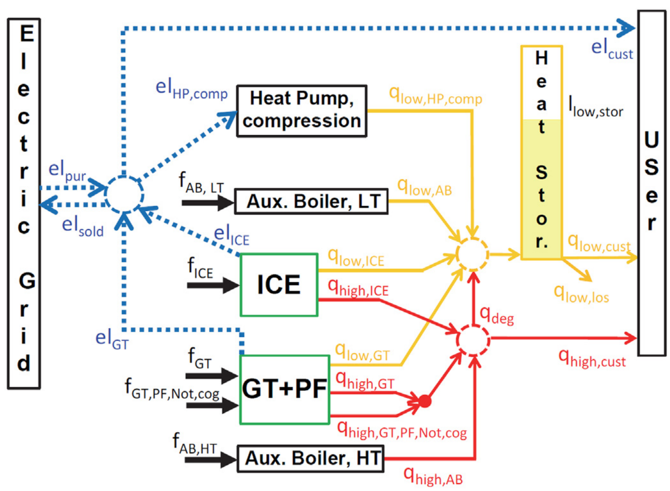

Fig. 10. Schematic representation of the system of units considered in Test Case C and D.

4.5. *Remarks on the linearization accuracy and non-smooth test cases*

In this subsection we first analyze the effects of the linearization accuracy on the short-term planning solutions returned by the MILP solver. We focus on Test Case D, for which the PWL approximation has a substantial influence on the optimal solution. Despite the rather small difference between the objective function values obtained with 5 and 20 PWL intervals (see Table 7), the two planning solutions differ on an hourly basis, as shown in Fig. 14.

Fig. 14 plots the heat pump load and the level of the low temperature heat storage as a function of time. Note that in the solution found with 5 intervals the heat pump is used for 5 h, while in the solution found with 20 intervals it is used for 4 h and at a higher load. Also the optimized heat storage starting level is slightly different. It is worth highlighting that the solution corresponding to 10 intervals has the same on–off variables and units' loads as the one obtained with 20 intervals, but different loads of the heat pump with differences up to 6.5%.

This may be due to the fact that the optimal loads of the heat pump fall strictly within an interval of the domain partitioning of the PWL approximation, while those of the GT and ICE (Internal Combustion Engine) are close to either the maximum load or a PWL interpolation point for both the 10 and 20 intervals approximations. Indeed, consider two PWL approximations of a unit performance curve with different number of interval points, and assume that the set of interpolation points of the finer approximation contains the interpolation points of the coarser one. If the unit optimal load is equal to its maximum or minimum load, the number of intervals in the approximation has no impact. If the unit optimal load is strictly contained within the load range, then the difference between the

Table 6
Main MILP model features for Test Case C and optimization results for piecewise linear approximations with 5, 10 and 20 intervals.

	5 intervals ON/OFF No penalty	5 intervals ON/OFF penalty	10 intervals ON/OFF No penalty	10 intervals ON/OFF penalty	20 intervals ON/OFF No penalty	20 intervals ON/OFF penalty
Number of binary variables	721	721	1321	1321	2521	2521
Total number of variables	2400	2400	3600	3600	6000	6000
Number of constraints	2477	2477	3077	3077	4277	4277
Computational time (s)	25.30	29.65	139.65	123.56	614.56	295.88
Relative MILP gap (%)	9E−3	9E−3	9E−3	9E−3	9E−3	9E−3
Objective function value (€)	14154.04	14210.26	14147.09	14202.79	14145.39	14201.30

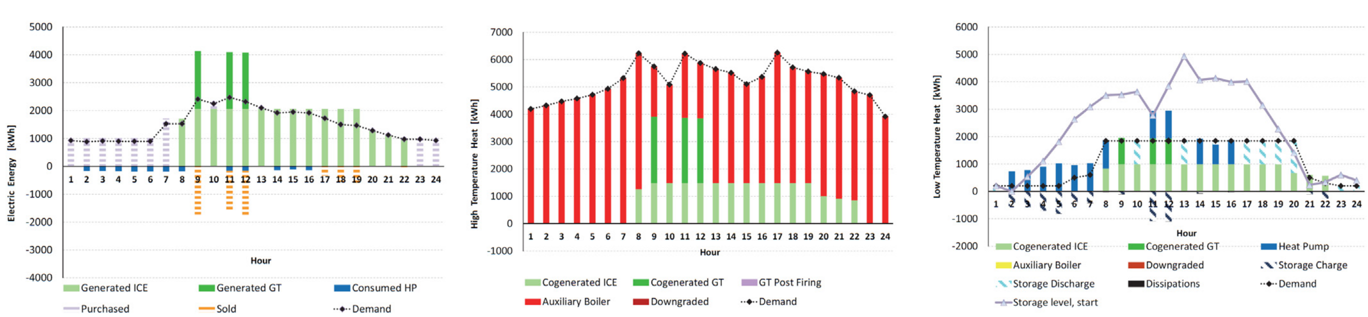

Fig. 11. Electricity (left), HT heat (middle) and LT heat (right) optimal schedules for Test Case C without start-up penalties, obtained by 20 intervals piecewise linear approximation of the units' performance curves.

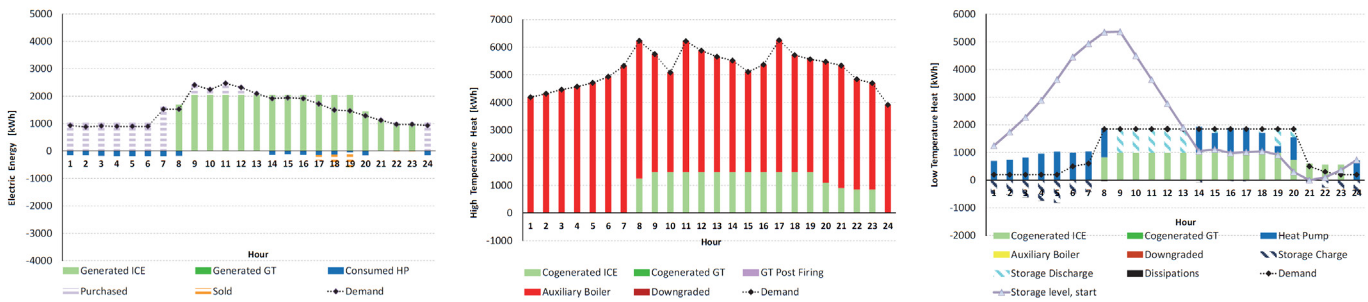

Fig. 12. Electricity (left), HT heat (middle) and LT heat (right) optimal schedules for Test Case C with start-up penalties, obtained by 20 intervals piecewise linear approximation of the units' performance curves.

values of the PWL approximation and the actual performance values clearly increases with the distance from the closest interpolation points.

A similar effect of the PWL approximation on the solutions can be observed in the other test cases. As previously mentioned, such differences do not have an appreciable impact on the objective function because they occur only in a few hours of the day, and the heat pump power is relatively small compared to the other units.

To show the capability of our model to handle non-smooth performance curves, we have also solved Test Cases B, C and D with a two-piece quadratic GT performance curve (see Table 2 and introduction of Section 5) so as to account for the non-differentiability point (located at about 80% of the fuel power). The computational results corresponding to the PWL approximation with 20 intervals are reported in Table 8. Concerning the computational time, a considerable increase only occurs for Test Case B (the time required to achieve the same MILP gap is about 10 times larger). Nevertheless, the computational time is still reasonably short.

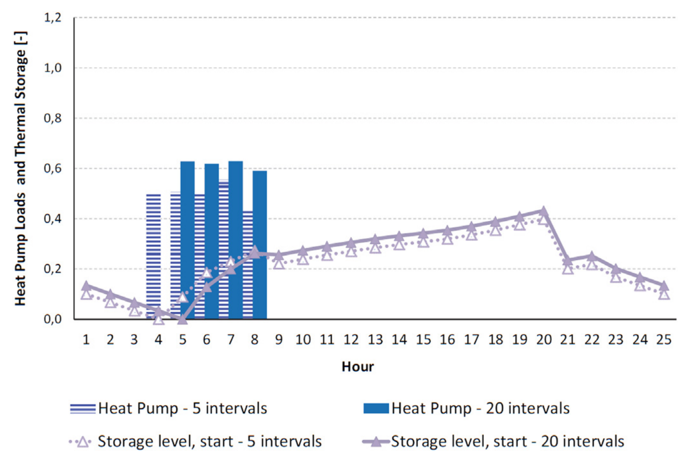

Fig. 14. Normalized values of the HP load and thermal storage level optimal schedules for Test Case D obtained both by 5 and 20 intervals piecewise linear approximation of the units' performance curves.

5. Conclusions

To cope with the increasing degree of complexity of energy systems and to capture with a higher level of accuracy the performance of the units, we have developed a detailed MILP optimization model for CCHP systems operation planning.

Table 7
Main MILP model features for Test Case D and optimization results for piecewise linear approximations with 5, 10 and 20 intervals.

	5 intervals	10 intervals	20 intervals
Number of binary variables	1944	6144	21,744
Total number of variables	5712	13,512	39,912
Number of constraints	4685	8285	19,085
Computational time (s)	22.69	68.28	293.86
Relative MILP gap (%)	9E−3	9E−3	9E−3
Objective function value (€)	14059.19	14055.43	14054.55

The MILP model, which allows to deal with problems involving the interaction of a large number of units, accounts for the possibility to buy and sell electric energy from and to the grid at different prices, to consider two temperature levels of heat generated, to degrade heat from high to low temperature, and to store low temperature heat. The model can handle units with nonlinear performance curves and two degrees of freedom. The piecewise linear approximation of the performance curves allows us to deal also with non-smooth points and discontinuities. Start-up penalties can be included in order to limit the number of on/off operations of selected units.

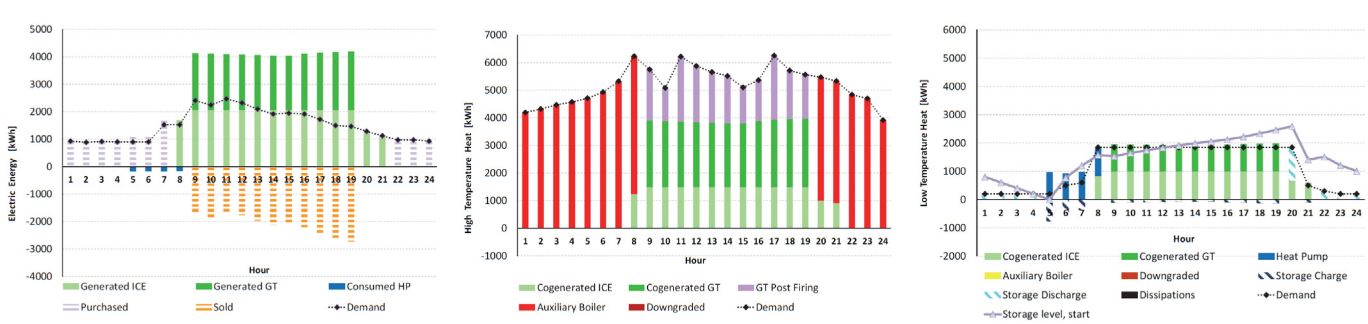

Fig. 13. Electricity (left), HT heat (middle) and LT heat (right) optimal schedules for Test Case D, obtained by 20 intervals piecewise linear approximation of the units' performance curves.

Table 8
Main MILP model features and optimization results for test cases B, C and D with non-smooth GT performance curves obtained by 20 interval piecewise linear approximation.

	B	C ON/OFF no penalty	C ON/OFF penalty	D
Number of binary variables	20,736	2521	2521	21,744
Total number of variables	37,560	6000	6000	39,912
Number of constraints	17,427	4277	4277	19,085
Computational time (s)	1735.54	454.38	439.88	332.99
Relative MILP gap (%)	9E−3	9E−3	9E−3	9E−3
Objective function value (€)	15349.30	14141.97	14201.23	14043.75

Four meaningful test cases have been considered in order to evaluate the quality of the solutions and the computational requirements. The results suggest that, if the piecewise linear approximations involve 5–20 intervals, close-to-optimal solutions can be found in short computational times (at most 5 min) even for challenging networks of units. Moreover, comparing solutions obtained with different number of intervals in the piecewise linear approximation indicates that 10 intervals are sufficient to provide accurate estimates of the optimal objective function values. The short-term planning solutions can, however, be further refined by increasing the number of intervals in the PWL approximation of the performance curves of the nonlinear units.

Nomenclature

Abbreviations and notations

C, c	cost
cold	units producing cooling load
CCHP	combine cooling heat and power
CHP	combined heat and power
COP	coefficient of performance
El, el	electricity
F, f	fuel [kWh]
g	units performance curve
GT	gas turbine
HP	heat pump
HT	high temperature
I	units consuming fuel
I_1	units consuming fuel with one-degree of freedom
I_2	units consuming fuel with two degrees of freedom
ICE	internal combustion engine
l	level of energy storage [kWh]
LHV	lower heating value
LP	linear program
LT	low temperature
M	constant parameters utilized for the “big-M” linearization
MILP	mixed integer linear program
MINLP	mixed integer nonlinear program
N_s	maximum number of start-ups
p	consumed electric energy
P	units producing electric energy
PF	post-firing
PWL	piecewise linear
Q, q	heat [kWh]
R	refrigeration
s	electricity sold/purchased
T	hours of the day
Th	consuming heat
y	second operative variable
z	units on/off status
Δ	start-ups number

Subscripts/superscripts

AB	auxiliary boiler
abs	absorption
aux	auxiliaries
chil	chiller
cog	co-generated
cold	cooling load
comp	compression
cons	constant
cust	customer
diss	dissipated
deg	degraded heat
fuel	fuel
grid	electric grid
i	i-th unit
hp	heat pump
heat	heat
high	high temperature
los	losses
low	low temperature
min	minimum
max	maximum
nom	nominal
on/off	start-up operation
O&M	operation and maintenance
pur	purchased
purch	purchased electric energy
sold	sold
start	start-up operation
stor	storage of heat
t	t-th hour of the day
time	number of hours
0	ambient conditions
$\hat{}$	parameters

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