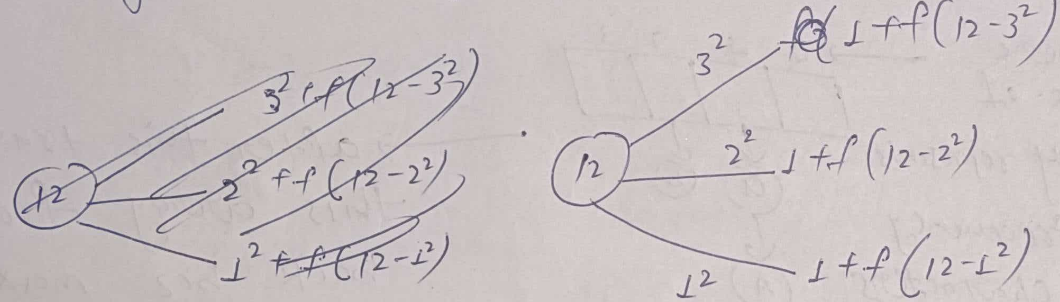
279. Perfect Squares

ex

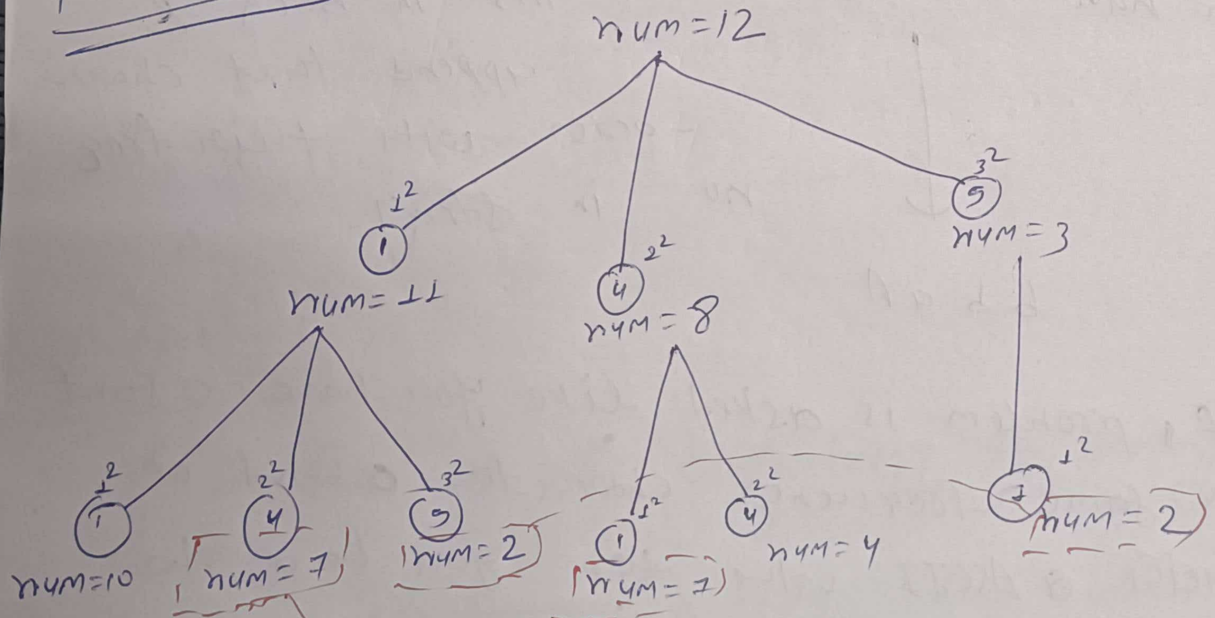
$n = 12$

output = 3 $\longrightarrow$ $12 = 4+4+4$ (perfect square)

$\Rightarrow$ Try for all possible perfect square, at every step & try to make number == 0

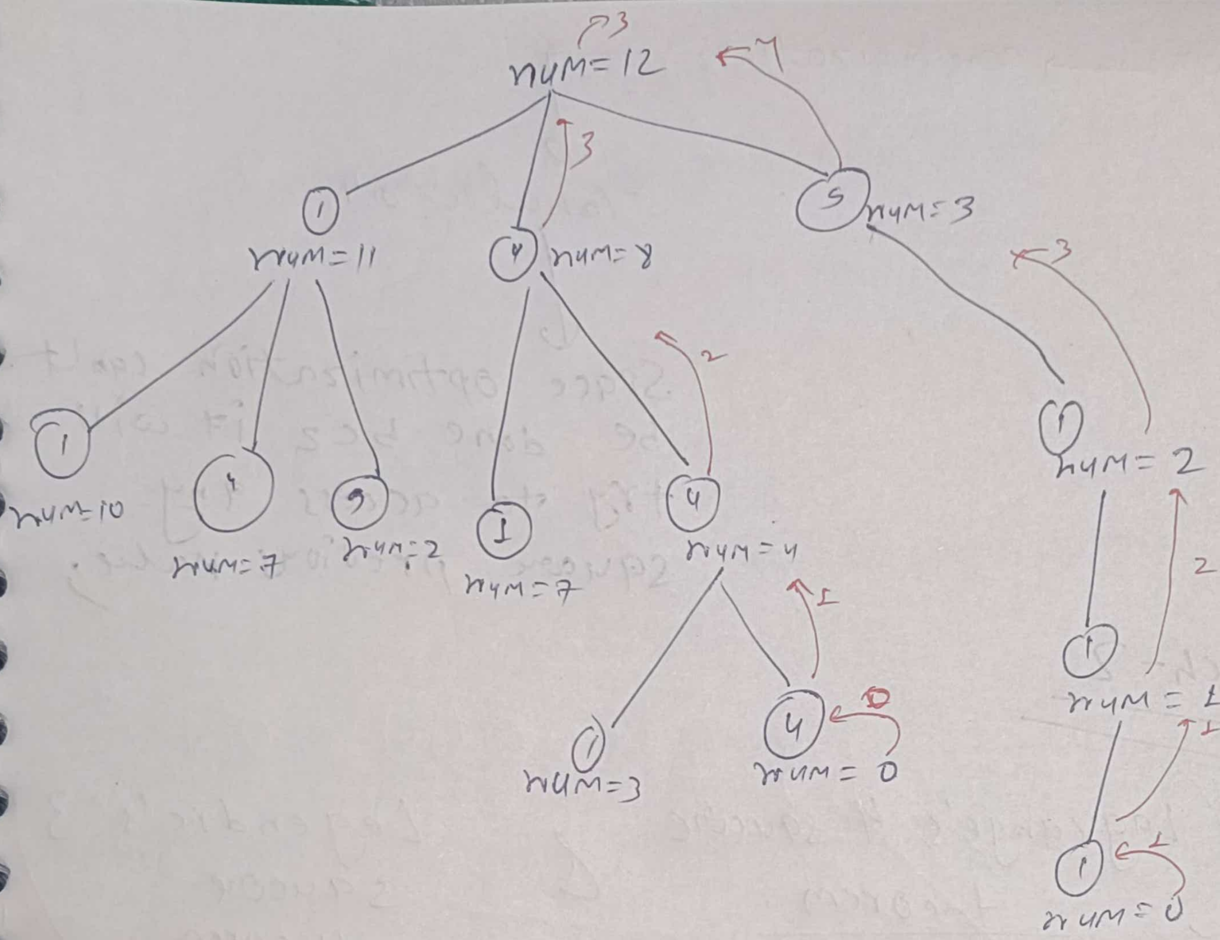


Recursion Tree



repeating subproblem

$\hookrightarrow$ DP



Why not greedy →

$12 - 9 = 3$ → for this ans=3

$1 + 3 = \cancel{4}$ $4 \neq 3$

Recursion → $\boxed{T.C. = (\sqrt{n})^n \quad S.C. = n}$

memoization → $\boxed{T.C. = O(n\sqrt{n}) \quad S.C. = O(n)}$

Recursion → memoization → Tabulation → Space optimization can't be done bcz it will try to access any square previous value;

Approch-2

math (Lagrange's 4 square theorem & Legendre's 3 square theorem)

Lagrange's 4 square theorem → Bachet's Conjection → Every Natural Number can be reperesented as the sum of '4' integers square.

$$P = a_0^2 + a_1^2 + a_2^2 + a_3^2$$

$\{a_0, a_1, a_2, a_3\}$ are integers

eg:-

$3 = 1^2 + 1^2 + 1^2 + 0^2 =$

$31^2 = 5^2 + 2^2 + 1^2 + 1^2$

⇒ Lagrange's 4 square theorem, sets an upper bound that max Answer can be 4

~~but ever~~ so answer can be 1, 2, 3, 4.

Case-1 if $\boxed{ans == 1}$

number is the perfect square in itself

case-2 $\boxed{ans = 2}$

find 2 pairs of integers (i, j) such that

$i*i + j*j = n$

itrate on i & find j such that $n - i*i = j*j$

j shoub be perfect square.

Case-3 $\boxed{ans = 3}$

↓

Adrien - Marie Legendre completed 4 square theorem

↓

with his 3-square theorem

Any positive integer 'n' can be expressed as the sum of 3 squares, it also include 0^2

iff $\boxed{n \neq 4^k (8m+7)}$

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if $n = 4^k (8m+7)$

then answer = 4

$$\boxed{\begin{aligned} T.C. &= O(\sqrt{n}) \\ S.C. &= O(1) \end{aligned}}$$

Bfs

$$\boxed{\begin{aligned} T.C. &= O(\sqrt{n})^n \simeq O(\sqrt{n})^4 \\ &= O(n^2) \end{aligned}}$$