

Que.: Unique Paths

(1) Why apply recursion?

⇒ Bcz we have to try all possible ways to get answer so ~~only~~ we use recursion.

(2) How to write Recurrence relation?

⇒ (i) Express in term of index here is a 2D matrix so express in $\begin{pmatrix} i & j \end{pmatrix}$
row col

(ii) Explore/Do all stuffs on that grid.

(iii) If Question is count ways → sum up all the stuff

max → max (all the stuff)
min → min (all the stuff)

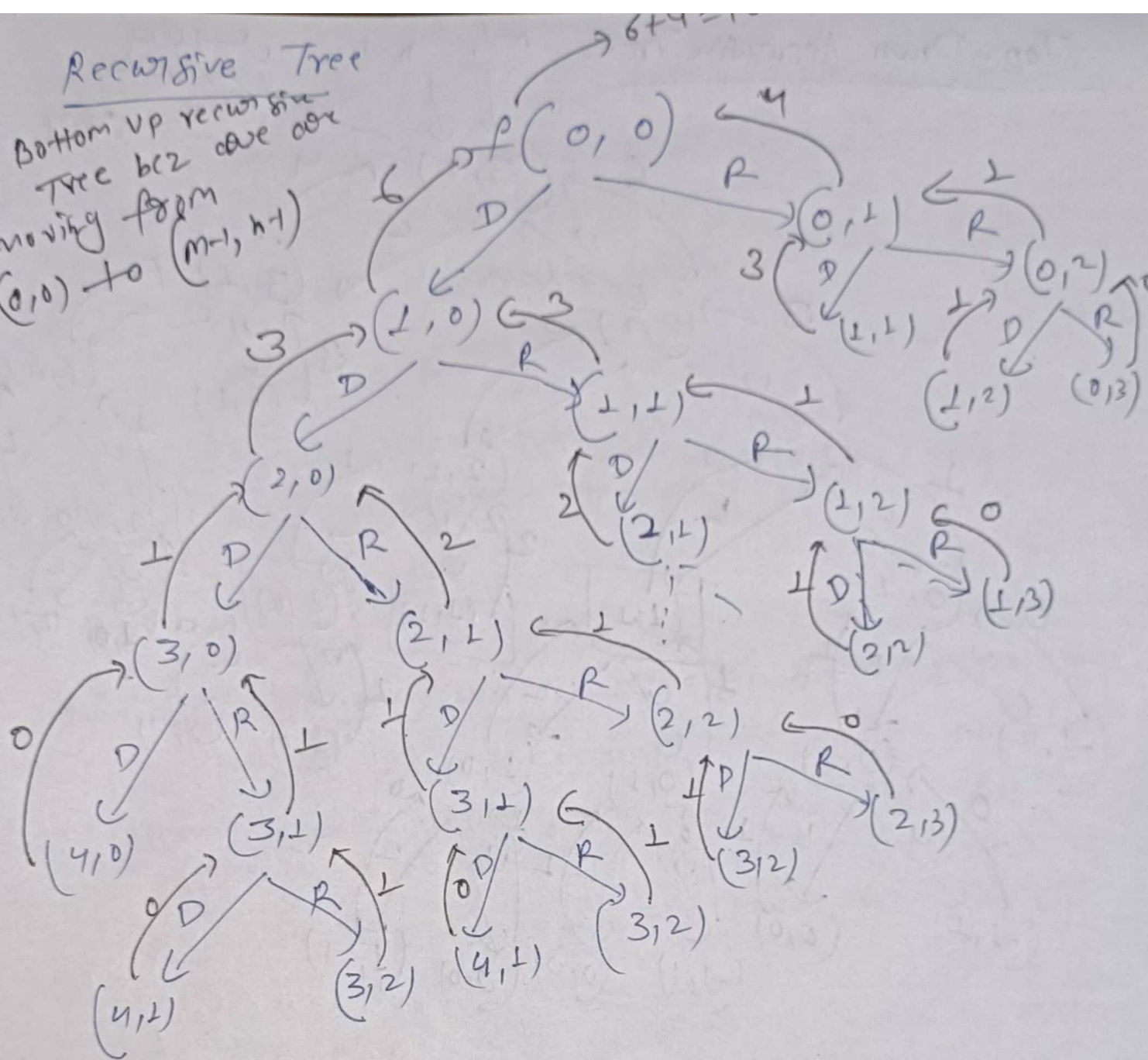
Take an example

$m=4, n=3$

2D Grid / 2D matrix

	0	1	2
0	Start (0,0)	(0,1)	(0,2)
1	(1,0)	(1,1)	(1,2)
2	(2,0)	(2,1)	(2,2)
3	(3,0)	(3,1)	Destination (3,2)

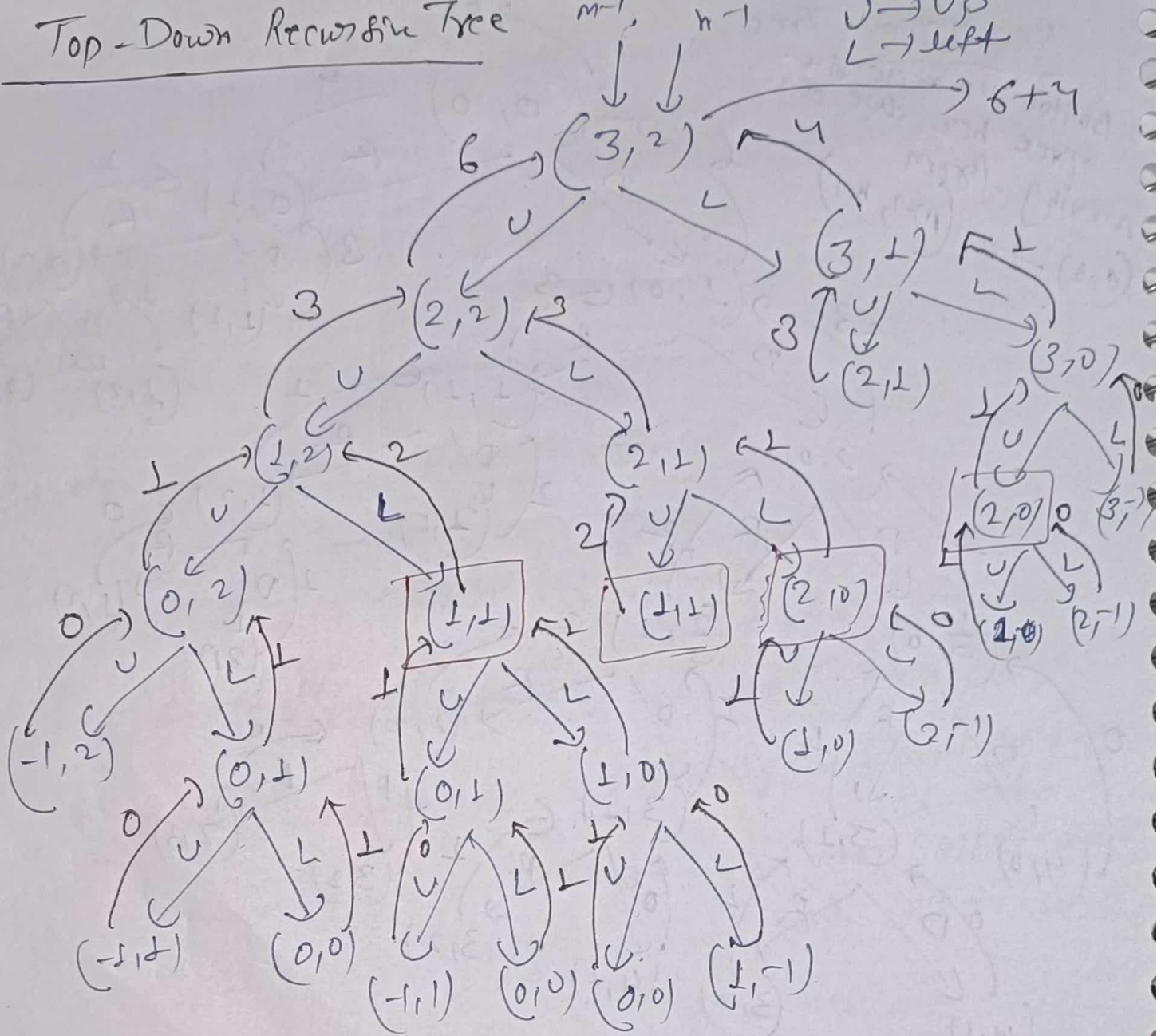
Recursive Tree
 Bottom up recursive
 Tree bcz we are
 moving from
 $(0,0)$ to $(m-1, n-1)$



$(3,1) \rightarrow$ return 1 that mean if we are at $(3,1)$ grid & we want to reach $(3,2)$ grid then there is only 1 unique path exist that's way $(3,1)$ returning 1

$f(m,n) \rightarrow$ return P that mean from (m,n) grid to reach $((m-1), (n-1))$ there are only P unique path exist.

Top-Down Recursion Tree



$(0, 2) \rightarrow$ returning 1 that mean if we start from $(0, 0)$ & want to reach at $(0, 2)$ so there is only 1 unique path

Recursion tree में जो हम देख सकते हैं कि हम $(0, 0)$ से value return करने शुरू करते हैं।

Top-Down Approach - it means first we are going down till the base case & after that we are computing result & returning values & go up wards.

In Recursive Tree we can see that there are multiple overlapping subproblems so we can apply Dynamic Programming.

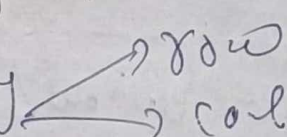
① First Decide the Data Structure.

So in order to apply DP first we have to use DP data structure i.e. what we should use like 1D array, 2D array, map or something else.

⇒ यहाँ हम 1D array use नहीं कर सकते bcz 1D array तब use करते हैं जब only one variable change होता हो क्योंकि उस 1 variable की सभी values के corresponding ans को हम 1D array में store कर सकते हैं।

But here are two variables m & n are changing simultaneously so we have to use 2D array so we can store the ans corresponding to (m, n) in this 2D array.

② Decide the size of the data structure.

→ 2D array 

row \rightarrow 0 to m & col = 0 to n

$DP[m][n]$

⇒ Can I optimize the space.

vector 1 → size N

Base value case

vector 2 size N

upRow

leftCol

	1	0	0
1	(0,0)	(0,1)	(0,2)
0	(1,0)	(1,1)	(1,2)
0	(2,0)	(2,1)	(2,2)
0	(3,0)	(3,1)	(3,2)

In tabulation method

$$dp[i][j] = dp[i-1][j] + dp[i][j-1]$$

In the most cases

$dp[i][j]$ depends on same column previous row $dp[i-1][j]$

" same row previous col $dp[i][j-1]$

So Rather than creating a $M \times N$ DP we create two vector of size M & N

Suppose you want to find (i, j) with unique path

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int down = upRow[j]; // dp[i-1][j]
int Right = leftCol[i]; // dp[i][j-1]
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upRow[j] = leftCol[i] = down + Right
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