23. In the given progressive wave,

$y = 5\sin(100\pi t - 0.4\pi x)$

where, y and x are in m, t is in seconds. What is the

(i) amplitude, (ii) wavelength,

(iii) frequency, (iv) wave velocity,

(v) particle velocity amplitude? **[NCERT Exemplar]**

Sol. Comparing with the standard form of equation

$$y = a\sin[\omega t - kx]$$

(i) Amplitude, $a = 5$ m **[1]**

(ii) $\omega = \dfrac{2\pi}{T} = 100\pi$

$$k = \frac{2\pi}{\lambda} = 0.4\pi \Rightarrow \lambda = \frac{2}{0.4} = \frac{20}{4} = 5 \text{ m}$$ **[1]**

(iii) $\omega = 2\pi\nu = 100\pi \Rightarrow \nu = 50$ Hz **[1]**

(iv) Wave velocity, $v = \nu\lambda = 50 \times 5 = 250$ m/s **[1]**

(v) Particle velocity $= \dfrac{dy}{dt} = a\omega\cos(\omega t - kx)$

$$\left(\frac{dy}{dt}\right)_{max} = a\omega = 5 \times 100\pi = 500\pi \text{ m/s}$$ **[1]**

11. A string of mass 2.5 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, the disturbance will reach the other end in ... **[NCERT Exemplar]**

Sol. Here, $\mu = \frac{2.5}{20}$ kg/m, $T = 200$ N [1]

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200}{2.5/20}} = \sqrt{\frac{200 \times 20}{2.5}}$$

$$= \sqrt{\frac{4 \times 10^4}{25}} = \frac{2 \times 10^2}{5} = \frac{20 \times 10}{5} = 40 \text{ m/s}$$

So, $t = \frac{l}{v} = \frac{20}{40} = 0.50$ s [1]

EXAMPLE |4| Open and Closed Flute

A pipe 30.0 cm long is opened at both ends. Which harmonic mode of the pipe resonates with a 1.1 kHz source? Will resonance with the same source be observed, if one end of the pipe is closed? Take the speed of sound in air as 330 ms^{-1}. [NCERT]

Sol. Here, $L = 30.0 \text{ cm} = 0.3 \text{ m}$

Let nth harmonic of open pipe resonate with 1·1 kHz source,

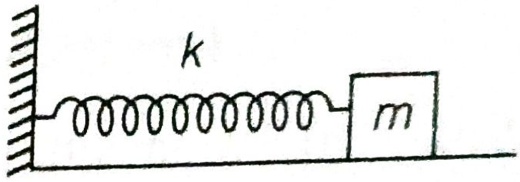
i.e. $$\nu_n = 1.1 \text{ kHz} = 1100 \text{ Hz}$$

As, $$\nu_n = \frac{n v}{2L}$$

$$\therefore \quad n = \frac{2 L \nu_n}{v} = \frac{2 \times 0.30 \times 1100}{330} = 2$$

i.e. **2nd harmonic** resonates with open pipe.

16. A spring of force constant 1200 Nm^{-1} is mounted on a horizontal table. A mass of 3.0 kg is attached to the free end of the spring, pulled sideways to a distance of 2.0 cm and then released.



(i) What is the frequency of oscillation of the mass?

(ii) What is the maximum acceleration of the mass?

(iii) What is the maximum speed of the mass?

[NCERT]

Sol. Here, $k = 1200\,\text{Nm}^{-1}$, $m = 3.0$ kg

and $A = 2.0\text{ cm} = 2.0 \times 10^{-2}$ m

(i) Frequency of oscillation of the mass,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2 \times 3.14}\sqrt{\frac{1200}{3.0}} = \frac{1}{2 \times 3.14} \times 20$$

$$= 3.18\,s^{-1} \approx 3.2\,s^{-1}$$ **[1]**

(ii) Angular frequency,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3.0}} = 20\,s^{-1}.$$

$\therefore$ Maximum acceleration of the mass

$$= \omega^2 A = (20)^2 \times 2.0 \times 10^{-2} = 8.0\,\text{ms}^{-2}.$$ **[1]**

(iii) Maximum speed of the mass

$$= \omega \cdot A = 20 \times 2.0 \times 10^{-2} = 0.40\,\text{ms}^{-1}.$$ **[1]**

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EXAMPLE |9| A Periodic Motion

A particle executes SHM with a time period of 2 s and amplitude 5 cm. Find

(i) displacement (ii) velocity and

(iii) acceleration after 1/3 s starting from the mean position.

Sol. Here, $T = 2\text{ s}$, $A = 5\text{ cm}$, $t = \frac{1}{3}\text{ s}$

(i) For the particle starting from mean position, (i.e. $\phi = 0$) displacement,

$$x = A\sin\omega t = A\sin\frac{2\pi}{T}t$$

$$= 5\sin\frac{2\pi}{2}\times\frac{1}{3} = 5\sin\frac{\pi}{3}$$

$$= 5\times\frac{\sqrt{3}}{2} = 4.33\text{ cm}$$

(ii) Velocity, $v = \frac{dx}{dt} = \frac{d(A\sin\omega t)}{dt} = A\omega\cos\omega t$

$$= \frac{2\pi A}{T}\cos\frac{2\pi}{T}t = \frac{2\pi\times 5}{2}\cos\frac{\pi}{3}$$

$$= 5\times 3.14\times 0.5 \qquad \left[\because \cos\frac{\pi}{3} = 0.5\right]$$

$$= 7.85\text{ cm s}^{-1}$$

(iii) Acceleration, $a = \frac{dv}{dt} = \frac{d(A\omega\cos\omega t)}{dt} = -A\omega^2\sin\omega t$

$$= -\frac{4\pi^2 A}{T^2}\sin\frac{2\pi}{T}t$$

$$= -\frac{4\times 9.87\times 5}{4}\sin\frac{\pi}{3}$$

$$= -9.87\times 5\times\frac{\sqrt{3}}{2} = -42.73\text{ cm s}^{-2}$$

$\therefore \quad |a| = 42.73\text{ cm s}^{-2}$

3. An electric heater supplies heat to a system at a rate of 100 W. If the system performs work at a rate of 75 J/s. At what rate, is the internal energy increasing? **[NCERT]**

Sol. Heat energy supplied per second by the heater

$$\Delta Q = 100 \text{ W} = 100 \text{ J/s}$$

Work done by the system $(\Delta W) = +75$ J/s

Rate of change in internal energy $(\Delta U) = ?$

According to first law of thermodynamics,

$$\Delta U = \Delta Q - \Delta W$$

$$= 100 - 75 = 25 \text{ J/s}$$

$$= 25 \text{ W}$$

a.

14. What amount of heat must be supplied to 2.0×10^{-2} kg of nitrogen (at room temperature) to raise its temperature by 45°C at constant pressure? (Molecular mass of $N_2 = 28$, $R = 8.3 \text{ J mol}^{-1}\text{K}^{-1}$) **[NCERT]**

Sol. Here, mass of gas, $m = 2 \times 10^{-2}$ kg = 20 g

Rise in temperature, $\Delta T = 45°\text{C}$

Heat required, $\Delta Q = ?$

Molecular mass, $M = 28$

Number of moles, $n = \dfrac{m}{M} = \dfrac{20}{28} = 0.714$

As nitrogen is a diatomic gas, molar specific heat at constant pressure is

$$C_p = \frac{7}{2}R = \frac{7}{2} \times 8.3 \text{ J mol}^{-1}\text{K}^{-1}$$

[1]

~~As~~ $\Delta Q = nC_p\,\Delta T$

$$\Delta Q = 0.714 \times \frac{7}{2} \times 8.3 \times 45 \text{ J} = 933.4 \text{ J}$$

[1]

kg of water to steam at 100°C.

EXAMPLE |4| Partial Melting 21

The ice of 0.15 kg mass at 0°C is mixed with 0.30 kg of water at 50°C in a container. The resulting temperature of the container (after mixing) is 6.7°C. Determine the heat of fusion of ice if specific heat (s_w) for the water is 4186 $Jkg^{-1}K^{-1}$.

[NCERT]

Sol. According to the calorimetry principle, we get

Heat lost by water = $ms_w(\theta_f - \theta_i)_w$

$$= (0.30\text{ kg})(4186\text{ Jkg}^{-1}\text{K}^{-1})(50.0 - 6.7)°\text{C} = 54376.14\text{ J}$$

Heat required to melt ice = mass of ice × L_f

$$= m_I L_f = (0.15\text{ kg})\, L_f$$

Heat required to raise temperature of ice water to final temperature (from the calorimetry)

$$= m_I s_w (\theta_f - \theta_i)_I$$

$$= (0.15\text{ kg})(4186\text{ Jkg}^{-1}\text{K}^{-1})(6.7°\text{C} - 0°\text{C}) = 4206.93\text{ J}$$

Principle of calorimetry i.e. heat lost = heat gained

$$\Rightarrow \quad 54376.14\text{ J} = (0.15\text{ kg})L_f + 4206.93\text{ J}$$

$$\Rightarrow \quad L_f = 3.34 \times 10^5\text{ Jkg}^{-1}$$

This is required value of heat of fusion of ice.

EXAMPLE |7| Cool the Steel Wire

Consider the steel wire having diameter 3 mm is stretched between two clamps, calculate the tension in the wire when its temperature falls from 50°C to 40°C. Given α_l for steel is $1.1 \times 10^{-6}\ ^\circ C^{-1}$ and Young's modulus 21×10^{11} dyne cm^{-2}.

Sol. Given, Diameter $= 3$ mm $= 0.3$ cm, $r = \frac{0.3}{2} = 0.15$ cm

Change in temperature, $\Delta T = 50^\circ C - 40^\circ C = 10^\circ C$

Coefficient of linear expansion, $\alpha_l = 1.1 \times 10^{-6}\ ^\circ C^{-1}$

Young's modulus, $Y = 21 \times 10^{11}$ dyne cm^{-2}

Apply the formula for linear expansion.

$$\Delta l_1 = l_1 \alpha_l \Delta T = l_1 \times 1.1 \times 10^{-6} \times 10°C$$

$$\Rightarrow \quad \Delta l = 11 \times 10^{-6}\, l_1$$

$$\text{Young's modulus } (Y) = \frac{F \times l}{A \times \Delta l} = \frac{F \times l}{\pi r^2 \times \Delta l}$$

$$F = \frac{Y \times \pi r^2 \times \Delta l}{l_1} = \frac{21 \times 10^{11} \times \pi \times (0.15)^2 \times 11 \times 10^{-6}\, l_1}{l_1}$$

$$F = 16.32 \times 10^5 \text{ dyne}$$