3NF
Example

Ques 1. Consider a relation A with attributes A(P,Q,R,S,T,U,V,W) and given FDs:

P→RS
PRU→V
PS→QTU
QRV→S
RU→PW
RW→U
S→Q
W→STV

1) Compute all keys for A.
2) Compute a minimal cover.
3) Using minimal cover, apply 3NF synthesis algorithm to obtain a lossless and dependency-preserving decomposition of relation A.

Solution

1) Compute all keys for A.

- Examining all subsets of the attributes would be very time-consuming because there are 2^8 of them.
- By inspection, we can see that P^+ = PQRSTUVW, which means that P is a key and no superset of P can be a key.

- Also, RU+ = RUPWSTVQ, which means that RU is a key and no superset of RU can be a key. (That means that R alone or U alone could be part of a key, but RU cannot.)
- But there is no key that has R but not U. We can determine this because even if we use every other attribute except P (which we know can't be in any other key), we don't have a key: $QSTRVW^+ = QSTRVW$
- Similarly, there is no key that has U but not R. We know this because even if we use every other attribute except P (which we know can't be in any other key), we don't have a key: $QSTUVW^+ = QSTUVW$
- Therefore, the only keys are P and RU.

2) Minimal cover

Step1: Single attribute @ RHS

1. P→R
2. P→S
3. PRU→V
4. PS→Q
5. PS→T
6. PS→U
7. QRV→S
8. RU→P
9. RU→W
10. RW→U
11. S→Q
12. W→S
13. W→T
14. W→V

Step 2: Reduce LHS

			Final FDs (after reduction)
1	P→R	Cannot be reduced	P→R
2	P→S	Cannot be reduced	P→S
3	PRU→V	Can be reduced as P→R	PU→V
4	PS→Q	Can be reduced as P→S	P→Q
5	PS→T	Can be reduced as P→S	P→T
6	PS→U	Can be reduced as P→S	P→U
7	QRV→S	Cannot be reduced	QRV→S
8	RU→P	Cannot be reduced	RU→P
9	RU→W	Cannot be reduced	RU→W
10	RW→U	Cannot be reduced	RW→U
11	S→Q	Cannot be reduced	S→Q
12	W→S	Cannot be reduced	W→S
13	W→T	Cannot be reduced	W→T
14	W→V	Cannot be reduced	W→V

Step 3: Eliminate redundant FDs

		Closure (considering FD)	Closure without considering FD	Result (Keep/ Discard)	Final set
1	P→R	P^+= PRSQTUW	P^+= PSQTU There is no way to get R without this FD	Keep	P→R
2	P→S	P^+= PSRUWQTV	P^+= PRQUWSTV	Discard	
3	~~PU→V~~	PU^+ = PUVRSQTW	PU^+ = PURSQTWV	Discard	
4	~~P→Q~~	P^+ = PSQRTUW	P^+ = PRQTUW	Discard	
5	~~P→T~~	P^+ = PSQRTUW	P^+ = PSRUWQT	Discard	
6	P→U	P^+ = PSQRTUW	P^+ =PSRQ	Keep	P→U
7	QRV→S	QRV^+= QRVS	QRV^+= QRV	Keep	QRV→S
8	RU→P	RU^+=RUPSWQT	RU^+= RUWSTQ There is no way to get P without this FD	Keep	RU→P
9	RU→W	RU^+=RUPSWQT	RU^+=RUPSQ	Keep	RU→W
10	~~RW→U~~	RW^+ = RWUSTPQ	RW^+ = RWSTUPQ	Discard	
11	S→Q	S^+ = SQ	S^+ = S	Keep	S→Q
12	W→S	W^+ = WSTVQ	W^+ = WTV	Keep	W→S
13	W→T	W^+ = WSTVQ	W^+ = WSVQ	Keep	W→T
14	W→V	W^+ = WVSTQ	W^+ = WTSQ	Keep	W→V

Minimal cover

1. P→R
2. P→U
3. QRV→S
4. RU→P
5. RU→W
6. S→Q
7. W→S
8. W→T
9. W→V

3) Decomposition using 3NF synthesis

Following the 3NF synthesis algorithm, we would get one relation for each FD. However, we can merge the right-hand sides before doing so. This will yield a smaller set of relations and they will still form a lossless and dependency-preserving decomposition of relation R into a collection of relations that are in 3NF. So the final minimal set will be as follows:

1. P→RU
2. QRV→S
3. RU→PW
4. S→Q
5. W→STV

The set of relations that would result would have these attributes:

R1(PRU);
R2(QRVS);
R3(RUPW);
R4(SQ);
R5(WSTV)

Since the attributes, SQ occur within R2, we don't need to keep the relation R4. Similarly, since the attributes PRU occur in R3, we don't need to keep the relation R1. So, the resultant relations are:

R1(QRVS);
R2(RUPW);
R3(WSTV)

P is a key of R, so there is no need to add another relation that includes a key.

Therefore, the final set of relations is:
R1(QRVS);
R2(RUPW);
R3(WSTV)