Elastic Structure Analysis of 2-D Structures

Direct Stiffness Method

Project Report

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Objective- To develop a generic Python code that takes geometry and loading details as input and performs elastic structural analysis using the direct stiffness method.

Problem Statement-

Perform structural analysis of the given structures shown using the direct stiffness method.

Introduction-

The direct stiffness method, also known as the matrix stiffness method, is particularly suited for computer-automated analysis of complex structures including the statically indeterminate frames. The direct stiffness method is the most common implementation of the finite element (FEM). When using the approach, the system must be represented as a collection of simpler, idealized elements connected at the nodes. The material stiffness properties of these elements are then, through matrix mathematics, compiled into a single matrix equation that governs the behavior of the entire idealized structure.

Methodology(Programming)-

1. First, we declared all the variables and assigned them the given values. Eg: the dimensions of the members of the portal frame, its elasticity modulus, etc.
2. We defined a function to generate our transformation matrix to convert the matrices in local coordinates to matrices in global coordinates.
3. Then we prepared our stiffness matrix.
4. Note the transformation matrix and the stiffness matrix are made using the standard formulae.

5. After this, we prepared the fixed end force vector (F_{FL1}) for the first member, then multiplied it with the transformation matrix to get it into global coordinate space (F_{Fg1}).

6. Then we calculated the stiffness matrix (K_{L1}) for the member and then by doing the following operation: T.K.T, we got the K_{g1} matrix which is a 6x6 matrix.

7. We followed the same steps for member 2 and member 3.

8. Now, we have F_{Fg1}, F_{Fg2}, F_{Fg3} and K_{g1}, K_{g2}, K_{g3} .Using F_{Fg1}, F_{Fg2} and F_{Fg3}, we found the fixed end force vector for the whole structure i.e. F_{Fga} (12x1 vector), and using K_{g1}, K_{g2} and K_{g3}, we found the global stiffness matrix for the whole structure i.e. K_{ga}, (12x12 matrix).

9. For this, we overlapped the matrices as shown below:

$$
\mathbf{F}fga = \begin{bmatrix} 0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix}
=
\mathbf{F}fga = \begin{bmatrix} 0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix}
=
\mathbf{F}fga = \begin{bmatrix} 0\\0\\0\\0\\ \frac{wL2}{2}\\ \frac{wL_2^2}{12}\\ 0\\ \frac{wL2}{2}\\ -\frac{wL_2^2}{12}\\ 0\\0\\0 \end{bmatrix}
=
\mathbf{F}fga = \begin{bmatrix} 0\\0\\0\\0\\ \frac{wL_2}{2}\\ \frac{wL_2^2}{12}\\ 0\\ \frac{wL_2}{2}\\ -\frac{wL_2^2}{12}\\ 0\\0\\0 \end{bmatrix}
$$

$\mathbf{F}fg(member-1)$ $\mathbf{F}fg(member-2)$ $\mathbf{F}fg(member-3)$

$\mathbf{Kg}(member-1)$ $\mathbf{Kg}(member-2)$ $\mathbf{Kg}(member-3)$

$$
= \begin{bmatrix}
\frac{12EI}{L1^3} & 0 & -\frac{6EI}{L1^2} & -\frac{12EI}{L1^3} & 0 & -\frac{6EI}{L1^2} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \frac{AE}{L1} & 0 & 0 & -\frac{AE}{L1} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\frac{6EI}{L1^2} & 0 & \frac{4EI}{L1} & -\frac{6EI}{L1^2} & 0 & \frac{2EI}{L1} & 0 & 0 & 0 & 0 & 0 & 0\\
\frac{-12EI}{L1^3} & 0 & \frac{6EI}{L1^2} & \frac{12EI}{L1^3}+\frac{AE}{L2} & 0 & \frac{6EI}{L1^2} & -\frac{AE}{L2} & 0 & 0 & 0 & 0 & 0\\
0 & -\frac{AE}{L1} & 0 & 0 & -\frac{AE}{L1}+\frac{12EI}{L2^3} & \frac{6EI}{L2^2} & 0 & -\frac{12EI}{L2^3} & \frac{6EI}{L2^2} & 0 & 0 & 0\\
\frac{6EI}{L1^2} & 0 & \frac{2EI}{L1} & -\frac{6EI}{L1^2} & \frac{6EI}{L2^2} & \frac{4EI}{L1}+\frac{4EI}{L2} & 0 & -\frac{6EI}{L2^2} & \frac{2EI}{L2} & 0 & 0 & 0\\
0 & 0 & 0 & -\frac{AE}{L2} & 0 & 0 & \frac{AE}{L2}+\frac{12EI}{L1^3} & 0 & \frac{6EI}{L1^2} & -\frac{12EI}{L1^3} & 0 & \frac{6EI}{L1^2}\\
0 & 0 & 0 & 0 & -\frac{12EI}{L2^3} & -\frac{6EI}{L2^2} & 0 & \frac{12EI}{L2^3}+\frac{AE}{L1} & -\frac{6EI}{L2^2} & 0 & -\frac{AE}{L1} & 0\\
0 & 0 & 0 & 0 & \frac{6EI}{L2^2} & \frac{2EI}{L2} & \frac{6EI}{L1^2} & -\frac{6EI}{L2^2} & \frac{4EI}{L2}+\frac{4EI}{L1} & -\frac{6EI}{L1^2} & 0 & \frac{2EI}{L1}\\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{12EI}{L1^3} & 0 & -\frac{6EI}{L1^2} & \frac{12EI}{L1^3} & 0 & -\frac{6EI}{L1^2}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{AE}{L1} & 0 & 0 & \frac{AE}{L1} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & \frac{6EI}{L1^2} & 0 & \frac{2EI}{L1} & -\frac{6EI}{L1^2} & 0 & \frac{4EI}{L1}
\end{bmatrix}
$$

10. Using the following formula:

$$\mathbf{F}_g = \mathbf{F}_{\mathbf{F}g} + \mathbf{K}_g \mathbf{d}_g$$

We get:

$$d_{ga} = (K_{ga,aa})^{-1} . [(F_{ga,a} - F_{Fg,a}) - (K_{ga,ar}\, d_{gr})]$$

11. From this, we got the active/unrestrained displacement vector(active degrees of freedom x 1 vector). This is the vector that tells us the displacement and rotation at all the unconstrained/unrestrained nodes. Clubbing the restrained displacement vector with the evaluated unrestricted displacement vector, we get a complete displacement vector for the assembly.

12. Further using the following relation, we compute the reaction forces at the supports.

$$\mathbf{F}_g = \mathbf{F}_{\mathbf{F}g} + \mathbf{K}_g \mathbf{d}_g$$

13. For computing Member end forces we use the given relation.

$$F_l^{(i)} = F_{Fl}^{(i)} + k_l^{(i)} . d_l^{(i)}$$

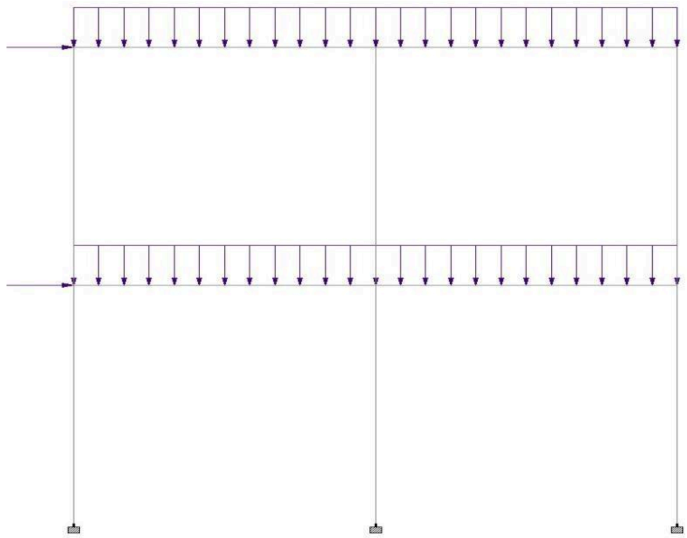
$$d_g^{(i)} = T^{(i)} . d_l^{(i)}$$

Procedure –

- The STAAD Pro files were created using the same procedures used in the previous Labs which were used to run structural analysis.
- Here we ran test cases on different structural designs and ran analysis on them in STAAD Pro, and compared it with the code which took the geometry and loading details as input and performed the analysis using the Direct Stiffness method.

Given below are a few of the structures and their comparison of the results in both STAAD and the code written:

Structure 1:



Shown below are the resultant displacement computed in the code and STAAD Pro

Displacement:

Displacements

Known Degrees of Freedom

Node	Ux(mm)	Vy(mm)	θz
1	0	0	0
4	0	0	0
7	0	0	0

UnKnown Degrees of Freedom

Node	Ux(mm)	Vy(mm)	θz
2	495.36	-0.1	-0.07097
3	905.56	-0.05	-0.07298
5	495.35	-0.77	-0.02306
6	905.48	-1.07	-0.01048
8	495.36	-0.19	-0.00571
9	905.42	-0.17	0.03586

Displacement STAAD:

Node	L/C	X mm	Y mm	Z mm	mm	rX rad	rY rad	rZ rad
1	1 LOAD CAS	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	1 LOAD CAS	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	1 LOAD CAS	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	1 LOAD CAS	501.842	-0.411	0.000	501.842	0.000	0.000	-0.071
5	1 LOAD CAS	501.837	-1.087	0.000	501.838	0.000	0.000	-0.023
6	1 LOAD CAS	501.846	-0.502	0.000	501.846	0.000	0.000	-0.006
7	1 LOAD CAS	917.415	-0.619	0.000	917.415	0.000	0.000	-0.073
8	1 LOAD CAS	917.334	-1.641	0.000	917.336	0.000	0.000	-0.011
9	1 LOAD CAS	917.278	-0.740	0.000	917.278	0.000	0.000	0.036

Moments :

```
Member End Forces

Known Degrees of Freedom
  Member     U1(KN)     V1(KN)     M1(KN-m)     U2(KN)     V2(mm)     M2(KN-m)
--------   --------   --------   ----------   --------   --------   ----------
       1      82.29       2.81        25.88     -82.29      -2.81         2.22
       2      41.67      -6.19       -30.62     -41.67       6.19       -31.29
       3       0.97      40.61        28.4       -0.97      59.39      -122.3
       4      16.16      41.67        31.29     -16.16      58.33      -114.55
       5     217.35       7.6         41.85    -217.35      -7.6         34.16
       6     110.85       4.85        22.14    -110.85      -4.85        26.34
       7      -1.85      47.11        66          1.85      52.89       -94.89
       8      11.25      52.52        88.21     -11.25      47.48       -63.01
       9     100.37       9.34        47.63    -100.37      -9.34        45.73
      10      47.49      11.22        49.15     -47.49     -11.22        63.01
```

Beam	L/C	Node	Fx kN	Fy kN	Fz kN	Mx kN-m	My kN-m	Mz kN-m
1	1 LOAD CAS	3	82.169	2.888	0.000	0.000	0.000	26.353
		4	-82.169	7.112	0.000	0.000	0.000	2.526
2	1 LOAD CAS	4	41.637	-6.158	0.000	0.000	0.000	-30.499
		7	-41.637	16.158	0.000	0.000	0.000	-31.084
3	1 LOAD CAS	7	16.158	41.637	0.000	0.000	0.000	31.084
		8	-16.158	58.363	0.000	0.000	0.000	-114.710
4	1 LOAD CAS	8	110.845	4.910	0.000	0.000	0.000	26.674
		5	-110.845	-4.910	0.000	0.000	0.000	22.425
5	1 LOAD CAS	5	217.346	7.698	0.000	0.000	0.000	34.595
		1	-217.346	-7.698	0.000	0.000	0.000	42.384
6	1 LOAD CAS	8	11.248	52.482	0.000	0.000	0.000	88.036
		9	-11.248	47.518	0.000	0.000	0.000	-63.214
7	1 LOAD CAS	9	47.518	11.248	0.000	0.000	0.000	63.214
		6	-47.518	-11.248	0.000	0.000	0.000	49.271
8	1 LOAD CAS	6	100.485	9.414	0.000	0.000	0.000	46.038
		2	-100.485	-9.414	0.000	0.000	0.000	48.104
9	1 LOAD CAS	4	0.954	40.532	0.000	0.000	0.000	27.973
		5	-0.954	59.468	0.000	0.000	0.000	-122.656
10	1 LOAD CAS	5	-1.834	47.033	0.000	0.000	0.000	65.636
		6	1.834	52.967	0.000	0.000	0.000	-95.309

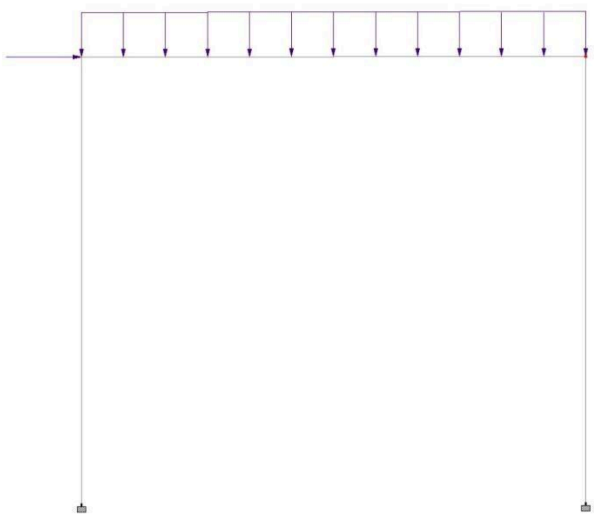
Reactions:

Support Reaction

Node	Hx(KN)	Vy(KN)	Mz(KN-m)
1	-2.86	82.28	25.88
4	-7.74	217.35	41.85
7	-9.4	100.37	47.63

		Horizontal	Vertical	Horizontal	Moment		
Node	L/C	Fx kN	Fy kN	Fz kN	Mx kN-m	My kN-m	Mz kN-m
1	1 LOAD CAS	-7.698	217.346	0.000	0.000	0.000	42.384
2	1 LOAD CAS	-9.414	100.485	0.000	0.000	0.000	48.104
3	1 LOAD CAS	-2.888	82.169	0.000	0.000	0.000	26.353

Structure 2:



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Displacements

Known Degrees of Freedom
  Node      Ux(mm)     Vy(mm)     θz
------  --------  --------  ----
     1         0         0     0
     4         0         0     0

UnKnown Degrees of Freedom
  Node      Ux(mm)     Vy(mm)          θz
------  --------  --------  --------
     2   2502.95      1.48  -0.23372
     3   2502.73     -1.98  -0.06703
```

		Horizontal	Vertical	Horizontal	Resultant	Rotational		
Node	L/C	X mm	Y mm	Z mm	mm	rX rad	rY rad	rZ rad
1	1 LIVE LOAD	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	1 LIVE LOAD	2500.951	-0.100	0.000	2500.951	0.000	0.000	-0.233
3	1 LIVE LOAD	2500.735	-0.400	0.000	2500.735	0.000	0.000	-0.067
4	1 LIVE LOAD	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Reactions :

Support Reaction

Node	Hx(KN)	Vy(KN)	Mz(KN-m)
1	-26.7	19.99	172.39
4	-43.3	80.01	227.93

		Horizontal	Vertical	Horizontal	Moment		
Node	L/C	Fx kN	Fy kN	Fz kN	Mx kN-m	My kN-m	Mz kN-m
1	1 LOAD CAS	-26.669	20.002	0.000	0.000	0.000	172.247
2	1 LOAD CAS	-43.331	79.998	0.000	0.000	0.000	227.773

Moment :

Member End Forces

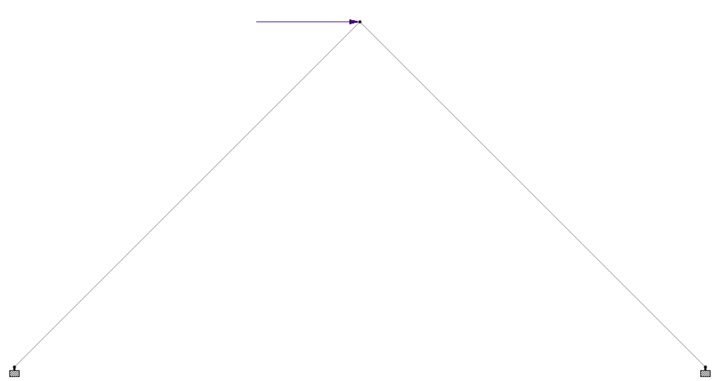
Known Degrees of Freedom

Member	U1(KN)	V1(KN)	M1(KN-m)	U2(KN)	V2(mm)	M2(KN-m)
1	20.01	26.69	172.39	-20.01	-26.69	94.48
2	43.3	19.99	-94.48	-43.3	80.01	-205.58
3	79.98	43.35	205.58	-79.98	-43.35	227.93

All / Summary / Envelope

Beam	L/C	Node	Fx kN	Fy kN	Fz kN	Mx kN-m	My kN-m	Mz kN-m
1	1 LOAD CAS	1	20.002	26.669	0.000	0.000	0.000	172.247
		4	-20.002	-26.669	0.000	0.000	0.000	94.442
2	1 LOAD CAS	4	-26.669	20.002	0.000	0.000	0.000	-94.442
		3	-43.331	79.998	0.000	0.000	0.000	-205.538
3	1 LOAD CAS	3	79.998	43.331	0.000	0.000	0.000	205.538
		2	-79.998	-43.331	0.000	0.000	0.000	227.773

Structure 3:



Displacements :

```
Displacements

Known Degrees of Freedom
   Node     Ux(mm)     Vy(mm)    θz
------  --------  --------  ----
     1         0         0     0
     3         0         0     0

UnKnown Degrees of Freedom
   Node     Ux(mm)     Vy(mm)      θz
------  --------  --------  ------
     2      0.65         0  -5e-05
```

		Horizontal	Vertical	Horizontal	Resultant	Rotational		
Node	L/C	X mm	Y mm	Z mm	mm	rX rad	rY rad	rZ rad
1	1 LOAD CAS	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	1 LOAD CAS	0.651	-0.000	0.000	0.651	0.000	0.000	-0.000
3	1 LOAD CAS	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Reactions:

```
Support Reaction

  Node    Hx(KN)    Vy(KN)    Mz(KN-m)
------  --------  --------  ----------
     1        -5        -5           0
     3        -5         5           0
```

		Horizontal	Vertical	Horizontal	Moment		
Node	L/C	Fx kN	Fy kN	Fz kN	Mx kN-m	My kN-m	Mz kN-m
1	1 LOAD CAS	-5.000	-5.000	0.000	0.000	0.000	0.001
3	1 LOAD CAS	-5.000	5.000	0.000	0.000	0.000	0.001

Moments:

```
Member End Forces

Known Degrees of Freedom
  Member    U1(KN)    V1(KN)    M1(KN-m)    U2(KN)    V2(mm)    M2(KN-m)
--------  --------  --------  ----------  --------  --------  ----------
       1     -7.07         0           0      7.07        -0          -0
       2      7.07         0          -0     -7.07        -0           0
```

Beam	L/C	Node	Fx kN	Fy kN	Fz kN	Mx kN-m	My kN-m	Mz kN-m
1	1 LOAD CAS	1	-7.071	0.000	0.000	0.000	0.000	0.001
		2	-0.000	7.071	0.000	0.000	0.000	-0.000
2	1 LOAD CAS	2	7.071	0.000	0.000	0.000	0.000	0.000
		3	-7.071	-0.000	0.000	0.000	0.000	0.001

Sample Input files

Member Data

A	B	C	D	E	F	G	H	I	J	K	L
S. No.	N1	N2	L(m)	B(m)	D(m)	E(KN/m2)	w1(KN/m)	w2(KN/m)	P(KN)	a(m)	b(m)
1	1	2	10	0.1	0.1	21718000	10	10	0	0	0
2	2	3	10	0.1	0.1	21718000	10	20	0	0	0
3	3	4	10	0.1	0.1	21718000	0	0	5	5	5

Node Data

S. No.	X Coord(m	Y Coord(m	Nodal Loa	Nodal Loa	Nodal Loa	Ux	Vy	Theta
1	0	0	0	0	0	1	0	1
2	0	10	0	0	0	1	1	1
3	10	10	0	0	0	1	1	1
4	10	0	0	0	0	0	0	1

Conclusion:

In summary, the Python code developed to implement the Direct Stiffness Method (DSM) for structural analysis has shown promising results. A comparison with STAAD Pro yielded a marginal difference of approximately 2%, indicating the code's effectiveness.The marginal difference observed between the two methods can be attributed to various factors such as numerical approximation, element discretization, and computational precision. Additionally, the incorporation of support settlement considerations for the portal frame within the code further enhances its practical relevance, enabling a more comprehensive analysis of real-world structural behavior.