Q4. $\oint_C \frac{\sin z}{(z-2)^2(z-3)} dz$

We see that function has 2 singularities ~~~~ at the points $z=2$ & $z=3$

$f(z) = \sin z$

$f'(z) = \cos z$

Now by Cauchy integral formula as curve goes around twice in z we will break curve in C_1 & C_2.

So $\oint_C \frac{f(z)}{(z-2)^2(z-1)} = -\oint_{C_1} \frac{f(z)}{(z-2)^2} - \oint_{C_2} \frac{f(z)}{(z-2)^2}$

$- \oint_{C_1} \frac{f(z)}{(z-3)}$

[-ve sign, since they go clockwise]

So $-\oint_{C_1} \frac{f(z)}{(z-2)^2} = -\oint_{C_2} \frac{f'(z)}{(z-2)^2}$

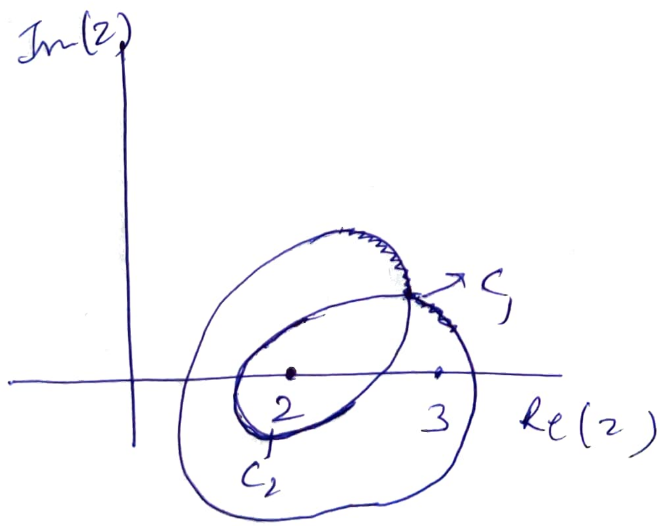
$$= \frac{-2\pi i}{2!}\left[\cos(2)\right]$$

$$= -\pi i \cos(2)$$

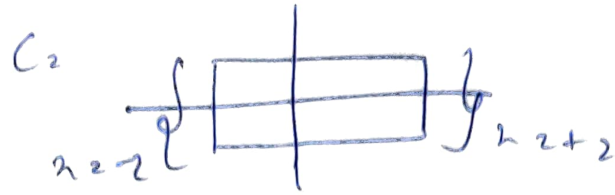
$$-\oint_{C_1} \frac{f(z)}{z-3} = -2\pi i\, f(3) = -2\pi i \sin(3)$$

so
$$\oint_{C} \frac{\sin z}{(z-2)^2(z-3)} = -\pi i \cos(2) - \pi i \cos(2) - 2\pi i \sin(3)$$

$$= -2\pi i\left(\cos(2) + \sin(3)\right)$$



(b)



$$\oint_C \frac{\sinh z}{(z-4)^2(z-5)}\, dz$$

S.P $\Rightarrow$ $z=4$, $z=5$ are exterior to curve

Hence

$$\oint_C \frac{\sinh z \; dz}{(z-4)^2(z-5)} = 0 \rightarrow \text{By Cauchy Theorem}$$