UE19CS251
Design and Analysis of Algorithms
Unit 5: Limitations of Algorithmic Power and Coping with the Limitations

Backtracking

PES University

Outline

Concepts covered

- Backtracking
  - Introduction
  - N Queens
  - Hamiltonian Circuit
  - Subset Sum
  - Algorithm

1 Tackling Difficult Combinatorial Problems

- There are two principal approaches to tackling difficult combinatorial problems (NP-hard problems):
  - Use a strategy that guarantees solving the problem exactly but doesn't guarantee to find a solution in polynomial time
  - Use an approximation algorithm that can find an approximate (sub-optimal) solution in polynomial time

2 Exact Solution Strategies

- Exhaustive search (brute force)

- useful only for small instances

- Dynamic programming
  - applicable to some problems (e.g., the knapsack problem)

- Backtracking
  - eliminates some unnecessary cases from consideration
  - yields solutions in reasonable time for many instances but worst case is still exponential

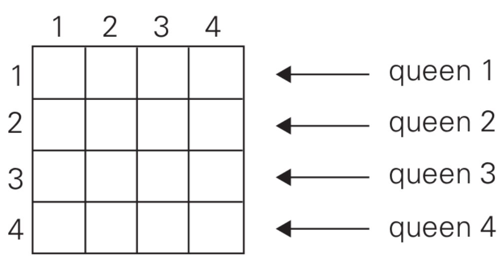
- Branch-and-bound
  - further refines the backtracking idea for optimization problems

3 Introduction

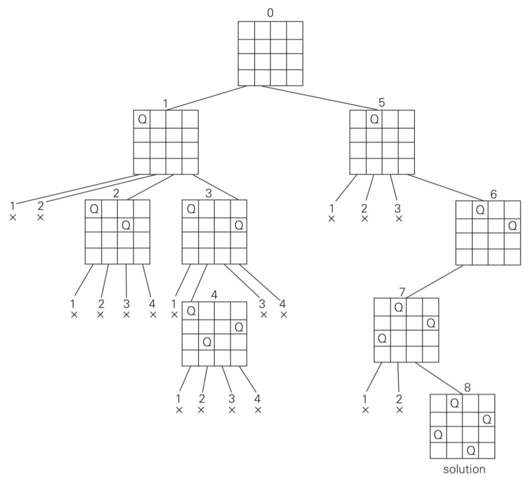
- Construct the *state-space tree*
  - nodes: partial solutions
  - edges: choices in extending partial solutions
- Explore the state space tree using depth-first search
- "Prune" *nonpromising nodes*
  - DFS stops exploring subtrees rooted at nodes that cannot lead to a solution and backtracks to such a node's parent to continue the search

4 Example: N-Queens Problem

- Place N queens on an $N \times N$ chess board so that no two of them are in the same row, column, or diagonal



5 State-Space Tree of the 4-Queens Problem

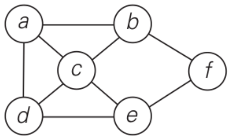


6 Example: Hamiltonian Circuit Problem

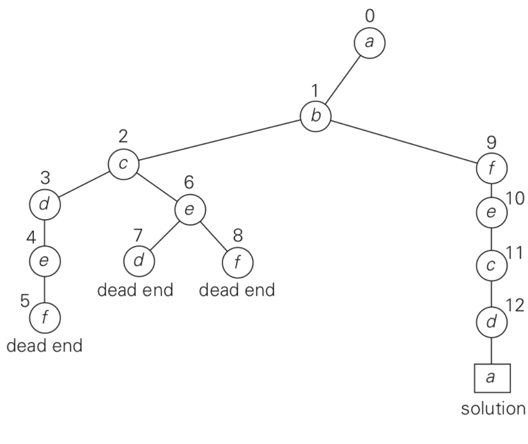
Hamiltonian Circuit

A Hamiltonian circuit is defined as a cycle that passes through all the vertices of the graph exactly once.

- Example graph:



- State-space tree for finding a Hamiltonian circuit (numbers above the nodes of indicate the order in which the nodes are generated):

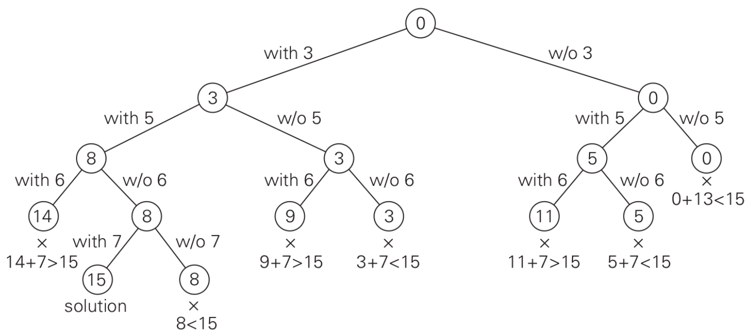


7 Example: Subset Sum Problem

Subset Sum Problem

Given set $A = \{a_1, \ldots, a_n\}$ of n positive integers, find a subset whose sum is equal to a given positive integer d

- State space tree for $A = \{3, 5, 6, 7\}$ and $d = 15$ (number in each node is the sum so far):



8 Algorithm

Backtrack Algorithm

procedure Backtrack($X[1 \ldots i]$)
  ▷ **Input:** $X[1 \ldots i]$ specifies first i promising components of a solution
  ▷ **Output:** All the tuples representing the problem's solutions
  if $X[1 \ldots i]$ is a solution **then**
    write $X[1 \ldots i]$
  else
    for each element $x \in S_{i+1}$consistent with $X[1 \ldots i]$ and the constraints **do**
      $X[i+1] \leftarrow x$
      Backtrack ($X[1 \ldots i+1]$)

- Output: n-tuples $(x_1, x_2, ..., x_n)$
- Each $x_i \in S_i$, some finite linearly ordered set

9 Think About It

- Continue the backtracking search for a solution to the four-queens problem, to find the second solution to the problem
- Explain how the board's symmetry can be used to find the second solution to the four-queens problem