DESIGN AND ANALYSIS OF ALGORITHMS
UE19CS251

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Merge Sort

Major Slides Content: Anany Levitin

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Merge Sort - Idea

- Split array A[0..n-1] into about equal halves and make copies of each half in arrays B and C
- Sort arrays B and C recursively
- Merge sorted arrays B and C into array A as follows:
  - Repeat the following until no elements remain in one of the arrays:
    - compare the first elements in the remaining unprocessed portions of the arrays
    - copy the smaller of the two into A, while incrementing the index indicating the unprocessed portion of that array
  - Once all elements in one of the arrays are processed, copy the remaining unprocessed elements from the other array into A

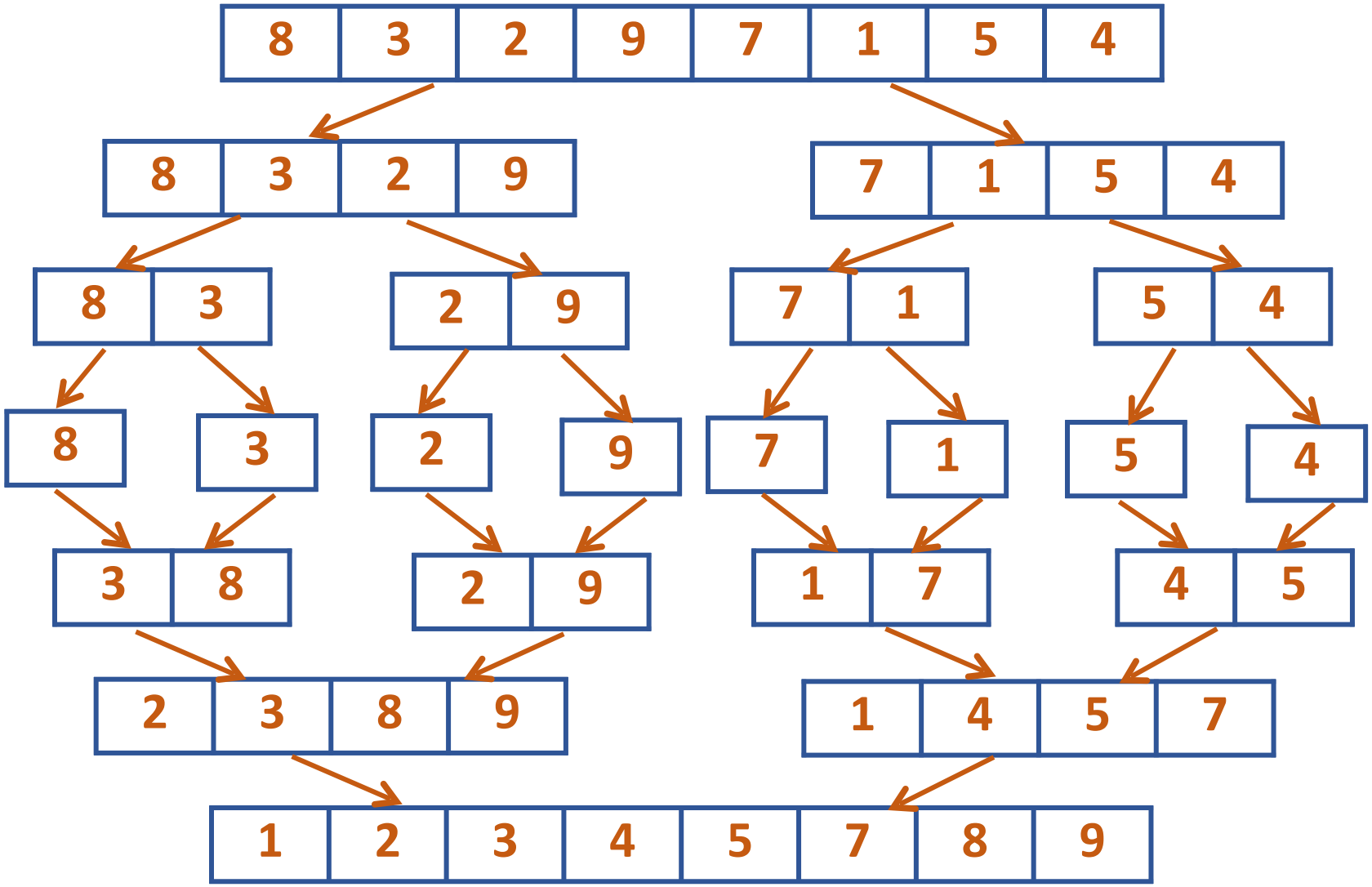
Merge Sort - Algorithm

ALGORITHM Mergesort(A[0 .. n-1])
//Sorts array A[0 .. n-1] by recursive mergesort
//Input: An array A[0 .. n-1] of orderable elements
//Output: Array A[0 .. n-1] sorted in non decreasing order
if n > 1
  copy A[0 .. $\lfloor n/2 \rfloor$ -1] to B[0 .. $\lfloor n/2 \rfloor$ -1]
  copy A[$\lfloor n/2 \rfloor$ .. n -1] to C[0..$\lceil n/2 \rceil$ -1]
  Mergesort(B[0 .. $\lfloor n/2 \rfloor$ - 1])
  Mergesort(C[0 ..$\lceil n/2 \rceil$ -1])
  Merge(B, C, A)

Merge Sort - Algorithm

```
ALGORITHM  Merge(B[0 .. p- 1],  C[0 .. q  -1], A[0 .. p  + q  -1])
//Merges two sorted arrays into one sorted array
//Input: Arrays B[0 .. p -1] and C[0 .. q -1] both sorted
//Output: Sorted array A[0 .. p  + q  -1] of the elements of B and
   C
i←0;  j←0;  k←0
while i < p and j < q do
   if B[i] ≤ C[j]
            A[k] ←    B[i];  i ←    i  + 1
   else  A[k] ←C[j];  j←    j  + 1
   k←k+1
if i =  p
   copy C[j .. q-1] to A[k .. p + q - 1]
else
   copy B[i .. p -1] to A[k .. p + q -1]
```

Merge Sort - Example



Merge Sort - Algorithm

- Assuming for simplicity that n is a power of 2, the recurrence relation for the number of key comparisons C(n) is:

$$C(n) = 2C(n/2) + C_{merge}(n) \text{ [for } n > 1], C(1) = 0$$

- The number of key comparisons performed during the merging stage in the worst case is:

$$C_{merge}(n) = n - 1$$

- Using the above equation:

$$C_{worst}(n) = 2C_{worst}(n/2) + n - 1 \text{ [for } n > 1], C_{worst}(1) = 0$$

- Applying Master Theorem to the above equation:

$$C_{worst}(n) \in \Theta(n \log n)$$

THANK YOU

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