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UE19CS251: Design and Analysis of Algorithms (4-0-0-4-4)

Bubble Sort

Dr. Shylaja S S

Bubble Sort

Bubble Sort is a brute-force application to the sorting problem. In bubble sort, we compare the adjacent elements of the list and exchange them if they are out of order. By doing it repeatedly, we end up "bubbling up" the largest element to the last position on the list. The next pass bubbles up the second largest element, and so on until, after n - 1 passes, the list is sorted. Pass i ($0 \leq i \leq n-2$) of bubble sort can be represented by the following diagram:

$A[0], A[1], A[2], \ldots, A[j] \overset{?}{\leftrightarrow} A[j+1], \ldots, A[n-i-1] \mid A[n-i] \leq \ldots \leq A[n-1]$

in their final positions

Here is a pseudocode of this algorithm.

```
ALGORITHM BubbleSort(A[0 .. n - 1])
//Sorts a given array by bubble sort in their final positions
//Input: An array A[0 .. n - 1] of orderable elements
//Output: Array A[0 .. n- 1] sorted in ascending order
for i <-- 0 to n - 2 do
    for j <-- 0 to n - 2 - i do
        if A[ j + 1 ] < A[ j ] swap A[ j ] and A[ j + 1 ]
```

The action of the algorithm on the list 89, 45, 68, 90, 29, 34, 17 is illustrated as an example in Fig. 1.

89 ? 45 68 90 29 34 17
45 89 ? 68 90 29 34 17
45 68 89 ? 90 29 34 17
45 68 89 90 ? 29 34 17
45 68 89 29 90 ? 34 17
45 68 89 29 34 90 ? 17
45 68 89 29 34 17 | 90

45 ? 68 89 29 34 17 | 90
45 68 ? 89 29 34 17 | 90
45 68 89 ? 29 34 17 | 90
45 68 29 89 ? 34 17 | 90
45 68 29 34 89 ? 17 | 90
45 68 29 34 17 | 89 90

Fig. 1: First two passes of bubble sort on the list 89, 45, 68, 90, 29, 34, 17. The elements to the right of the vertical bar are in their final positions and are not considered in subsequent iterations of the algorithm.

The number of key comparisons for the bubble-sort version given above is the same for all arrays of size n; it is obtained by a sum that is almost identical to the sum for selection sort:

$$C(n) = \sum_{i=0}^{n-2} \sum_{j=0}^{n-2-i} 1 = \sum_{i=0}^{n-2} [(n-2-i) - 0 + 1]$$

$$= \sum_{i=0}^{n-2} (n-1-i) = \frac{(n-1)n}{2} \in \Theta(n^2)$$

Bubble Sort is a $\Theta(n^2)$ algorithm.