

Renna Sultana

Department of Science and Humanities



MATRICES AND GAUSSIAN ELIMINATION

Renna Sultana

Department of Science and Humanities

MATRICES AND GAUSSIAN ELIMINATION:

PES UNIVERSITY ONLINE

Course Content: Triangular Factors

❖ Consider the system of equations Ax=b. Using elementary row transformations we reduce A to U an Upper triangular matrix. While doing so Ax=b reduces to Ux=c(i.e [b] reduces to [c]) which can be solved by back substitution to obtain x. The steps involved are $Ax = b \implies E_{32}E_{31}E_{21}Ax = Ux = c$ (∴ $E_{32}E_{31}E_{21}A = U$) To undo the steps of elimination we must trace back the steps instead of subtracting we must add . For this we need inverses of Elementary Matrices i.e E_{21}^{-1} , E_{31}^{-1} , E_{32}^{-1} can be obtained by changing −l to l in the transformation. Similarly E_{31}^{-1} and E_{32}^{-1} can also be obtained. But while going from U to A the inverses should be in reverse direction i.e., $E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}U = A$

MATRICES AND GAUSSIAN ELIMINATION:

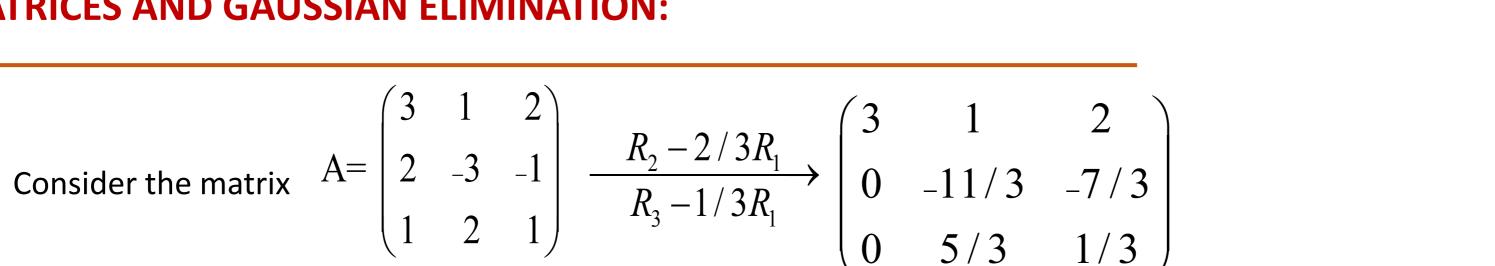
PES UNIVERSITY ONLINE

Triangular Factorization A=LU:

- Any square matrix A can be factored as A=LU where
- **L** is a Lower Triangular Matrix
- **▶** With 1's on the diagonal
- \succ Having multiplier's l_{ij} below the diagonal in their respective positions.
- >And
- U is an Upper Triangular Matrix
- Having pivots on the diagonal (u₁₁,u₂₂.u₃₃)

$$\mathbf{L} = \begin{pmatrix} 1 & 0 & 0 \\ \mathbf{l}_{21} & 1 & 0 \\ \mathbf{l}_{31} & \mathbf{l}_{32} & 1 \end{pmatrix} U = \begin{pmatrix} \mathbf{u}_{11} & \mathbf{u}_{12} & \mathbf{u}_{13} \\ 0 & \mathbf{u}_{22} & \mathbf{u}_{23} \\ 0 & 0 & \mathbf{u}_{33} \end{pmatrix} = \begin{pmatrix} \mathbf{d}_{1} & \mathbf{u}_{12} & \mathbf{u}_{13} \\ 0 & \mathbf{d}_{2} & \mathbf{u}_{23} \\ 0 & 0 & \mathbf{d}_{3} \end{pmatrix}$$

MATRICES AND GAUSSIAN ELIMINATION:



$$\frac{R_3 + (5/11)R_2}{0} \xrightarrow{-11/3} \begin{bmatrix} 3 & 1 & 2 \\ 0 & -11/3 & -7/3 \\ 0 & 0 & -8/11 \end{bmatrix} = UL = \begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1/3 & -5/11 & 1 \end{bmatrix}$$

$$A = LU = \begin{pmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1/3 & -5/11 & 1 \end{pmatrix} \begin{pmatrix} 3 & 1 & 2 \\ 0 & -11/3 & -7/3 \\ 0 & 0 & -8/11 \end{pmatrix}$$



MATRICES AND GAUSSIAN ELIMINATION:

PES UNIVERSITY ONLINE

Triangular Factorization A=LDU:

- ❖ A=LU factorization is Unsymmetric as L has 1's on the diagonal whereas U has pivots on the diagonal. In order to make this factorization symmetric we do A=LDU factorization
- \triangleright D is a Diagonal Matrix with pivots d_1, d_2, d_3 on the diagonal
- **L** is a Lower Triangular Matrix
- **→** With 1's on the diagonal
- \succ Having multiplier's l_{ij} below the diagonal in their respective positions.
- ▶ U is an Upper Triangular Matrix with 1's on the diagonal obtained by dividing each row by its pivot.

i.e
$$\mathbf{L} = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \mathbf{D} = \begin{pmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{pmatrix} U = \begin{pmatrix} 1 & u_{12}/u_{11} & u_{13}/u_{11} \\ 0 & 1 & u_{23}/u_{22} \\ 0 & 0 & 1 \end{pmatrix}$$

MATRICES AND GAUSSIAN ELIMINATION:

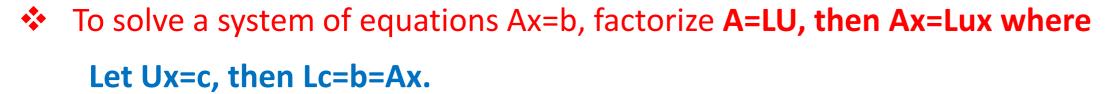


Factorise A=LU and hence A=LDU.

$$A = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix} \xrightarrow{R_2 + (1/2)R_1} \begin{pmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix} \xrightarrow{R_3 + (2/3)R_2} \begin{pmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

$$A = LDU = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & 0 \\ 0 & -2/3 & 1 & 0 \\ 0 & 0 & -3/4 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 3/2 & 0 & 0 \\ 0 & 0 & 4/3 & 0 \\ 0 & 0 & 0 & 5/4 \end{pmatrix} \begin{pmatrix} 1 & -1/2 & 0 & 0 \\ 0 & 1 & -2/3 & 0 \\ 0 & 0 & 1 & -3/4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

MATRICES AND GAUSSIAN ELIMINATION:



Solve Lc=b, using Forward elimination and then find x using Ux=c by Backward substitution which gives x.

Ex: Solve as triangular systems without multiplying LU to find A.

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 4 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$$
 Let $Lc = b \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$

$$\Rightarrow \begin{cases} c_1 = 2 \\ c_2 = -2 \text{ then } Ux = c \Rightarrow \begin{pmatrix} 2 & 4 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} 2x + 4y + 4z = 2 \\ y + 2z = -2 \\ z = 0 \end{cases}$$

$$(x, y, z) = (5, -2, 0)$$



MATRICES AND GAUSSIAN ELIMINATION:



$$A = \begin{pmatrix} 6 & -2 & -4 & 4 \\ 3 & -3 & -6 & 1 \\ -12 & 8 & 21 & -8 \\ -6 & 0 & -10 & 7 \end{pmatrix} \xrightarrow{R_2 - 1/2R_1} \begin{pmatrix} 6 & -2 & -4 & 4 \\ 0 & -2 & -4 & -1 \\ R_3 + 2R_1 \\ R_4 + R_1 \end{pmatrix} \xrightarrow{R_3 + 2R_1} \begin{pmatrix} 6 & -2 & -4 & 4 \\ 0 & -2 & -4 & -1 \\ 0 & 4 & 13 & 0 \\ 0 & -2 & -14 & 11 \end{pmatrix} \xrightarrow{R_3 + 2R_2} \begin{pmatrix} 6 & -2 & -4 & 4 \\ 0 & -2 & -4 & -1 \\ 0 & 0 & 5 & -2 \\ 0 & 0 & -10 & 12 \end{pmatrix}$$

$$A = LDU = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ -2 & -2 & 1 & 0 \\ -1 & 1 & 5/2 & 1 \end{pmatrix} \begin{pmatrix} 6 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 8 \end{pmatrix} \begin{pmatrix} 1 & -1/3 & -2/3 & 2/3 \\ 0 & 1 & 2 & 1/2 \\ 0 & 0 & 1 & -2/5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$





THANK YOU

Renna Sultana

Department of Science and Humanities

rennasultana@pes.edu