LINEAR ALGEBRA

UE19MA251

APARNA B S
Department of Science and Humanities

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Agenda

- Semi Definite Matrices

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Semi Definite Matrices

If A is a symmetric matrix the it is said to be positive semi-definite if

1) $x^T A x \geq 0 \quad \forall x$

2) All eigenvalues of A are greater than or equal to zero i.e $\lambda_i \geq 0$.

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Semi Definite Matrices

3) None of the principal submatrices have negative determinants.

4) Pivots are non-negative (≥ 0)

5) There exists a matrix R, possibly with dependent columns such that $A = R^T R$

1. Test the matrix for positive semi-definiteness.

$$A = \begin{bmatrix} 5 & 2 & 1 \\ 2 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix}$$

Eigenvalues of A are $\lambda = 4, 4+2\sqrt{3}, 4-2\sqrt{3} > 0$

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Example on semi-definite matrix

$$A \sim \begin{bmatrix} 5 & 2 & 1 \\ 0 & 6/5 & 8/5 \\ 0 & 8/5 & 24/5 \end{bmatrix} \sim \begin{bmatrix} 5 & 2 & 1 \\ 0 & 6/5 & 8/5 \\ 0 & 0 & 8/3 \end{bmatrix}$$

1) The pivots are: $0, \frac{6}{5}, \frac{8}{3} > 0$

2) Determinants of submatrices: $|A_1| = 5 > 0$

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Example on semi-definite matrix

3) $|A_2| = \begin{vmatrix} 5 & 2 \\ 2 & 2 \end{vmatrix} = 6 > 0$

4) $|A_3| = 6 > 0$

The above submatrices have positive determinants.

$\therefore$ The given matrix is positive definite.

Example on semi-definite matrix

2) Test the positive semi-definiteness of the matrix

$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

$R_2 \to R_2 + \frac{1}{2} R_1$

$R_3 \to R_3 + \frac{1}{2} R_1$

Example on semi-definite matrix

Solution: Characteristic Equation is $|A-\lambda I| = 0$

$$\Rightarrow \quad \lambda^3 - 6\lambda^2 + 9\lambda - 18 = 0$$

$$\Rightarrow \quad \lambda = 0, 3, 3 \quad (\geq 0)$$

$$A \sim \begin{bmatrix} 2 & -1 & -1 \\ 0 & 3/2 & -3/2 \\ 0 & -3/2 & 3/2 \end{bmatrix} \xrightarrow{R_3 \to R_3 + R_2} \begin{bmatrix} 2 & -1 & -1 \\ 0 & 3/2 & -3/2 \\ 0 & 0 & 0 \end{bmatrix}$$

Example on semi-definite matrix

Pivots : $2, 3/2, 0$ are non-negative

Determinants of the sub-matrices are:

$|A_1| = 2 \geq 0$; $|A_3| = 0 \geq 0$

$|A_2| = 2 \geq 0$

Hence the given matrix is positive semi-definite.

THANK YOU

Aparna B. S
Department of Science & Humanities
aparnabs@pes.edu