LINEAR ALGEBRA AND ITS APPLICATIONS

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CLASS-11

Supplementary Problems

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Q) $a_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad a_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad a_3 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

Let us convert L.I.V $\longrightarrow$ O.N.V using Gram-Schmidt Process.

$q_1 = \dfrac{a_1}{\|a_1\|}$ $\quad \|a_1\| = \sqrt{1^2+0+1^2} = \sqrt{2}$ $\quad a_2 \longrightarrow q_2$

$$\boxed{q_1 = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)}$$

$q_2 = \dfrac{c_2}{\|c_2\|}$

$$e_2 = (b) - (q_1^T a_2)\, q_1$$

$$= \begin{bmatrix}1\\0\\0\end{bmatrix} - \begin{bmatrix}\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\0\\\frac{1}{\sqrt{2}}\end{bmatrix}$$

$$= \begin{bmatrix}1\\0\\0\end{bmatrix} - \left(\frac{1}{\sqrt{2}}\right)\begin{pmatrix}\frac{1}{\sqrt{2}}\\0\\\frac{1}{\sqrt{2}}\end{pmatrix}$$

$$= \begin{bmatrix}1\\0\\0\end{bmatrix} - \begin{bmatrix}\frac{1}{2}\\0\\\frac{1}{2}\end{bmatrix} = \begin{bmatrix}\frac{1}{2}\\0\\-\frac{1}{2}\end{bmatrix}$$

$$\|e_2\| = \sqrt{\left(\tfrac{1}{2}\right)^2 + 0 + \left(-\tfrac{1}{2}\right)^2}$$

$$= \frac{1}{\sqrt{2}}$$

$$q_2 = \frac{e_2}{\|e_2\|}$$

$$= \left(\frac{1}{\sqrt{2}},\ 0,\ \frac{1}{\sqrt{2}}\right)$$

$$q_3 = \frac{e_3}{\|e_3\|} \quad ; \quad e_3 = a_3 - (q_1^T a_3) q_1 - (q_2^T a_3) q_2$$

$$= \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} - \sqrt{2} \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix} - \sqrt{2} \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$e_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

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$$\|e_3\| = \sqrt{0+1^2+0} = 1$$

$$q_3 = \frac{e_3}{\|e_3\|} = (0, 1, 0)$$

$$(a, b, c) \to (q_1, q_2, q_3)$$

$$q_1 = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$$

$$q_2 = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{pmatrix}$$

$$q_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

orthonormal columns

$$\begin{cases} q_1^T q_2 = q_2^T q_3 = q_3^T q_1 = 0 \\ q_1^T q_1 = q_2^T q_2 = q_3^T q_3 = 1 \end{cases}$$

$(a, b, c) \longrightarrow (q_1, q_2, q_3)$

Gram-Schmidt orthogonalization

Q) Check the matrix are orthogonally diagonalizable.

If not then orthogonally diagonalize it as $A = S\Lambda S^{-1} = Q\Lambda Q^{-1}$

$= Q\Lambda Q^T$

where Q is an orthogonal matrix.

Soln:- $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow |A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0$$

$\lambda^3 - 3\lambda^2 = 0$

$\lambda^2(\lambda - 3) = 0$

$\lambda = 0, 0, 3$

Consider $[A - \lambda I]x = 0$

$(1-\lambda)x + y + z = 0$; $x + (1-\lambda)y + z = 0$; $x + y + (1-\lambda)z = 0$

$x = y = z = 0 \quad X \neq 0$

Case 1: $\lambda = 0 \Rightarrow$

$$\left.\begin{aligned} x+y+z&=0 \\ x+y+z&=0 \\ x+y+z&=0 \end{aligned}\right\}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right]$$

$R_2' = R_2 - R_1$

$R_3' = R_3 - R_1$

$$\begin{array}{cccc} * & FV & FV & \\ 1 & 1 & 1 & : 0 \\ 0 & 0 & 0 & : 0 \\ 0 & 0 & 0 & : 0 \end{array}$$

$y = k_1$

$z = k_2$

$x = -y - z$

$x = -k_1 - k_2$

$\lambda = 0, \; X_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \quad [k_1 = 1, k_2 = 0]$

$\lambda = 0 \; X_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad k_1 = 0, k_2 = 1$

Case 3: $\lambda = 3$

$$\left.\begin{aligned} -2x + y + z &= 0 \\ x - 2y + z &= 0 \\ x + y - 2z &= 0 \end{aligned}\right\}$$

$$\begin{array}{cccccc} & x & & y & & z \\ -2 & 1 & 1 & -2 & 1 & 1 \\ 1 & -2 & 1 & 1 & -2 & 1 \end{array}$$

$$\frac{x}{3} = \frac{y}{3} = \frac{z}{3}$$

$$X_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$S = \begin{bmatrix} -1 & -1 & 3 \\ 1 & 0 & 3 \\ 0 & 1 & 3 \end{bmatrix}$$

or

$$S = \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$a^T b = b^T c$$

$$= c^T a \neq 0$$

$$a^T c = 0 = b^T c$$

$$a^T b = \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = 1 \neq 0$$

$q_1 = \frac{a}{\|a\|} \Rightarrow \left(-1/\sqrt{2}, 1/\sqrt{2}, 0\right)$

$q_2 = \frac{e_2}{\|e_2\|}$ $\quad e_2 = b - (q_1^T a) q_1$ $\quad q_2 = \left(-1/\sqrt{6}, -1/\sqrt{6}, 2/\sqrt{6}\right)$

$q_3 = \frac{e_3}{\|e_3\|}$ $\quad : e_3 = \left(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}\right)$

$$Q = \begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\ 0 & 2/\sqrt{6} & 1/\sqrt{3} \end{bmatrix}$$

$q_1^T q_2 = 0$

$q_2^T q_1 = 0$

$q_1^T q_3 = 0$

$A = Q \Lambda Q^{-1} \rightarrow Q^{-1} = Q^T$

$$\Lambda = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$A = Q \Lambda Q^{-1}$

THANK YOU

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