



LINEAR ALGEBRA AND ITS APPLICATIONS

UE19MA251

Unit 3. Linear Transformations and Orthogonality

Linear Transformations



Definition :

A transformation T is said to be linear if it satisfy the rule of linearity.

i.e., $A(cx+dy) = c A(x) + d A(y)$ for any scalar c, d are real constants.

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Example: In a linear system of equations $Ax = b$, Matrix A is a transformation from R^n to R^m .

Note: Consider a transformation $T : A \rightarrow B$
where A and B are subspaces.

1. A is the domain of the transformation.
2. B is the co domain of the transformation.
3. For any x in A , there exist Tx in B , here Tx is the image of T and x is the pre image of Tx

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4. The set of all images is the subset of B is called Range of the transformation.
5. For all x in A such that $Tx = 0$ is called the Kernel of the transformation.
6. Dimension of the range is called rank and dimension of Kernel is called nullity.

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Definition :

The space of all polynomials in t of degree n is a vector space called the *polynomial space* denoted by P_n .

$P_n = \{$ Its basis is $1, t, t^2, \dots, t^n$ and dimension is $n+1$.

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Example 1 : The operation of differentiation is linear. It takes P_{n+1} to P_n . The column space is the whole of P_n and the null space is P_0 , the 1-dimensional space of all constants.

Example 2: The operation of integration is linear. It takes P_n to P_{n+1} . The column space is a subspace of P_{n+1} and the null space is just the zero vector.

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Example 3 :

Multiplication by a fixed polynomial , say $3 + 4t$ is also a linear transformation.

Let $p(t) = a_0 + a_1t + a_2t^2 + \dots + a_nt^n$ then

$$Ap(t) = (3+4t)p(t) = 3a_0 + \dots + 4a_nt^{n+1}.$$

This A sends P_n to P_{n+1} .



THANK YOU
