Sum of Subspaces

The Sum of two subspaces U and W of a vector space V is defined as

$$U + W = \{u \in U, w \in W\}$$

Definition: Let U, W be subspaces of V . Then V is said to be the direct sum of U and W, and we write $V = U \oplus W$,
if $V = U + W$ and $U \cap W = \{0\}$.
Let U, W be subspaces of V . Then $V = U \oplus W$ if and only if for every $v \in V$ there exist unique vectors $u \in U$ and $w \in W$ such that $v = u + w$.

Properties :

1. The zero vector ‘0’ of V is in $U + W$.
2. For any $u, w \in U + W$, we have $u + v \in U + W$.
3. For any $v \in U + W$ and $\alpha \in R$, we have $\alpha v \in V \in U + W$
4. $v = u + w$ must be unique .

Sum of subspaces

Example : Consider $U = \{ (a, 0, 0) / a \in R \}$
$W = \{ (0, b, c) / b, c \in R \}$

Thus $V = U + W = \{ (a, b, c) / a, b, c \in R \}$

Hence the direct sum of subspaces U and W results into vector space R^3