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Department of Science and Humanities



MATRICES AND GAUSSIAN ELIMINATION

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PREREQUISITES:

PESUNIVERSITY

Course Content: Prerequisites

What?

Why?

PREREQUISITES:



- What is Linear Algebra?
- Linear Algebra is the study of systems of Linear equations.
- It is concerned with vector spaces and linear mappings between such spaces.
- It includes the study of lines, planes and subspaces and is also concerned with properties common to all vector spaces.
- Linear Algebra is the geometry of n-dimensional space and its linear transformations. Thus Linear Algebra helps us to develop our geometric instinct to visualize the concepts in higher dimensions.

PREREQUISITES:

Why do we need to study Linear Algebra?

- Linear Algebra is used in the everyday world to solve problems in Mathematics, Physics, Biology, Chemistry, Engineering, Statistics, Economics, Finance, Psycology, Sociology, etc.,
- ❖ Applications that use Linear Algebra include the transmission of information, the development of special effects in film and video, recording of sound, Web search engines on the Internet and economic analyses.
- Linear Algebra is a study of Linear Transformations. It is used in Graph Theory, Networks, Signal Processing, Probability Theory, Real Analysis, Communication.
- Linear Algebra is used in coding theory, Artificial Intelligence, Machine Learning, Image Processing, Computer Graphics, Numerical Analysis, Control Systems, Networking, Ordinary/Partial Differential Equations.



INTRODUCTION:

Course Content of Linear Algebra:

- * Module 1: Matrices and Gaussian Elimination
- Module 2: Vector Spaces
- Module 3: Linear Transformations and Orthogonality
- Module 4: Orthogonalization, Eigen Values and Eigen Vectors
- * Module 5: Singular Value Decomposition



MATRICES AND GAUSSIAN ELIMINATION:

Course Content of Module 1:

- Introduction
- The Geometry of Linear Equations
- Gaussian Elimination
- **❖** Singular Cases
- Elimination Matrices
- Triangular Factors and Row Exchanges
- Inverses and Transposes
- Inverse by Gauss Jordan Method



INTRODUCTION:

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Course Content: Introduction

What is a linear Equation?

 \clubsuit A linear equation in n variables is of the form $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$

where X_1, X_2, \dots, X_n are unknowns or variables, $a_i \in \square$ $(i = 1, 2, \dots, n)$

are known as the Coefficients of the variables $x_i \ (i=1,2,\cdots,n)$

and $b \in \square$ are constants.

Examples of Linear equations are: $2x_1-3x_2=7$; $\left(\sqrt{2}\right)x_1-\left(\sqrt{5}\right)x_2=1$

Examples of Non-Linear equations are: $x_1x_2 - 3x_3 = 2$; $y = \log x + \sin x$, $2x + 3y^{1/3} = 5$

INTRODUCTION:

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What is a system of linear Equations?

- A system of linear equation is a set of linear equations involving the same unknowns or variables.
- A system of two equations with two unknowns is $a_1x + b_1y = c_1$ of the form $a_2x + b_2y = c_2$
- \clubsuit A system of three equations with three $a_1x+b_1y+c_1z=d_1$ unknowns is of the form $a_2x+b_2y+c_2z=d_2$

$$a_3 x + b_3 y + c_3 z = d_3$$

INTRODUCTION:

Matrix Notation:

❖A system of m equations with n unknowns is given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_n$$

This can be represented in matrix form as Ax=b where

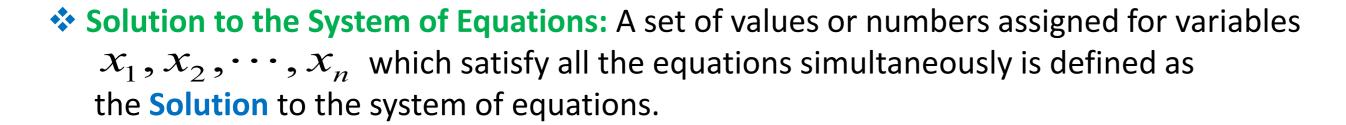
$$A = \begin{pmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} \cdots & a_{mn} \end{pmatrix}_{mxn} ; x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{nx1} ; b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}_{mx1}$$

Here A is a matrix of order mxn known as The Co-efficient Matrix, x_{nx1} is a nx1 column matrix(or vector) known as The Vector of unknowns or variables b_{mx1} is a column vector of order mx1 known as The Constant Vector





- \bullet a_{ij} is the component in the ith row and jth column of A.
- ❖ If m=n, A will be a Square matrix of order n and the system will have n equations with n variables.
- ❖ If all b_i 's are zero, the system is known as Homogeneous system of Equations.
 If at least one of b_i 's is not zero then the system is said to be Non-Homogeneous system of Equations.



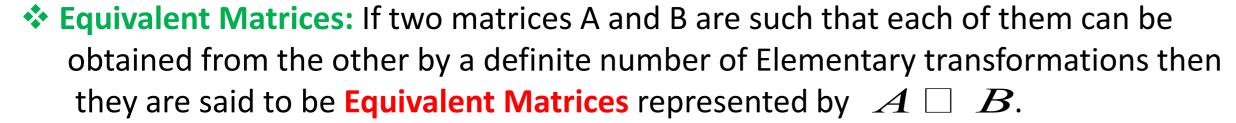
- The set of all possible solutions is called the **Solution Set** or **General Solution** of Linear system of equations.
- **Consistency:** A linear system of equations is said to be **Consistent** if it has a solution (unique solution or infinitely many solutions).
- ❖ A linear system of equations is said to be **Inconsistent** if it has no solution.



- Elementary Row Operations: In order to solve the system of linear equations using Matrix notation we define three basic operations called Elementary Row Operations or Elementary Row Transformations which are as follows:
- \triangleright Multiply the entries of a row by a non-zero scalar k i.e $R_i \longrightarrow kR_i \ (k \neq 0)$
- Replace one row by sum of itself and a non-zero scalar multiple k of another row i.e $R_i \to R_i + kR_i \, (k \neq 0)$.
- \triangleright Interchange of any two rows i.e $R_i \longleftrightarrow R_i$.
- **Elementary Matrix:** A square matrix of order n is called an **Elementary Matrix** E_n if it can be obtained from an Identity matrix I_n using a single row operation. i.e $I_n \to E_n$

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \xrightarrow{R_2 - 2R_1} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} = E_2$$





$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & -3 & 4 \end{pmatrix} \xrightarrow{R_2 + \left(\frac{1}{2}\right)R_1} \begin{pmatrix} 2 & -1 & 0 \\ 0 & 3/2 & 1 \\ 0 & -3 & 4 \end{pmatrix} \xrightarrow{R_3 + 2R_2} \begin{pmatrix} 2 & -1 & 0 \\ 0 & 3/2 & 1 \\ 0 & 0 & 6 \end{pmatrix} \square B \therefore A \square B$$



- Echelon Form Of A Matrix: A rectangular matrix A of order mxn is said to be in Echelon Form U if it satisfies the following conditions:
- The first non-zero element in every row is known as the pivot.
- > Below each pivot is a column of zeros obtained by Elementary row operations.
- Each pivot lies to the right of the pivot in the row above. This produces a Staircase pattern as shown below.
- Zero rows(if exist) appear at the bottom of the Matrix.

$$A \to \begin{pmatrix} a & b & c & d & e \\ 0 & 0 & \mathbf{f} & g & h \\ 0 & 0 & 0 & \mathbf{i} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}_{4x5} = U$$



INTRODUCTION:

- * Row Reduced Echelon Form Of A Matrix(RREF): A rectangular matrix A in Echelon Form must further undergo the following operations to reduce to RREF:
- > Every row in Echelon form must be divided by its pivot so that the first non-zero entry in every row is 1.
- Using the pivot rows produce zeros above the first non-zero entry (i.e 1).

This RREF is denoted by R.

$$A \to U = \begin{pmatrix} a & b & c & d & e \\ 0 & 0 & \mathbf{f} & g & h \\ 0 & 0 & 0 & \mathbf{i} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & b/a & 0 & 0 & e/a \\ 0 & 0 & 1 & 0 & h/f \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = R$$



INTRODUCTION:

Example:

$$A \to \begin{pmatrix} 1 & 1 & -2 & 3 & 4 \\ 2 & 3 & 3 & -1 & 3 \\ 5 & 7 & 4 & -1 & 5 \end{pmatrix}$$



ENGINEERING MATHEMATICS-III

References/Links:



https://en.wikipedia.org/wiki



THANK YOU

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INTRODUCTION:

Example:

$$A \rightarrow \begin{pmatrix} 1 & 1 & -2 & 3 & 4 \\ 2 & 3 & 3 & -1 & 3 \\ 5 & 7 & 4 & -1 & 5 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & -2 & 3 & 4 \\ 0 & 1 & 7 & -7 & -5 \\ 0 & 0 & 0 & -2 & -5 \end{pmatrix} = U \rightarrow \begin{pmatrix} 1 & 0 & -9 & 0 & -16 \\ 0 & 1 & 7 & 0 & -25/2 \\ 0 & 0 & 0 & 1 & 5/2 \end{pmatrix} = R$$

