



# LINEAR ALGEBRA AND ITS APPLICATIONS

## UE19MA251

---

## Unit 3. Linear Transformations and Orthogonality

### *Orthogonal Vectors & Subspaces*

---



#### *Definition:*

The **norm or length** of a n-dimensional vector  $x = (x_1, x_2, \dots, x_n)$  is written as  $\|x\|$  and is defined as

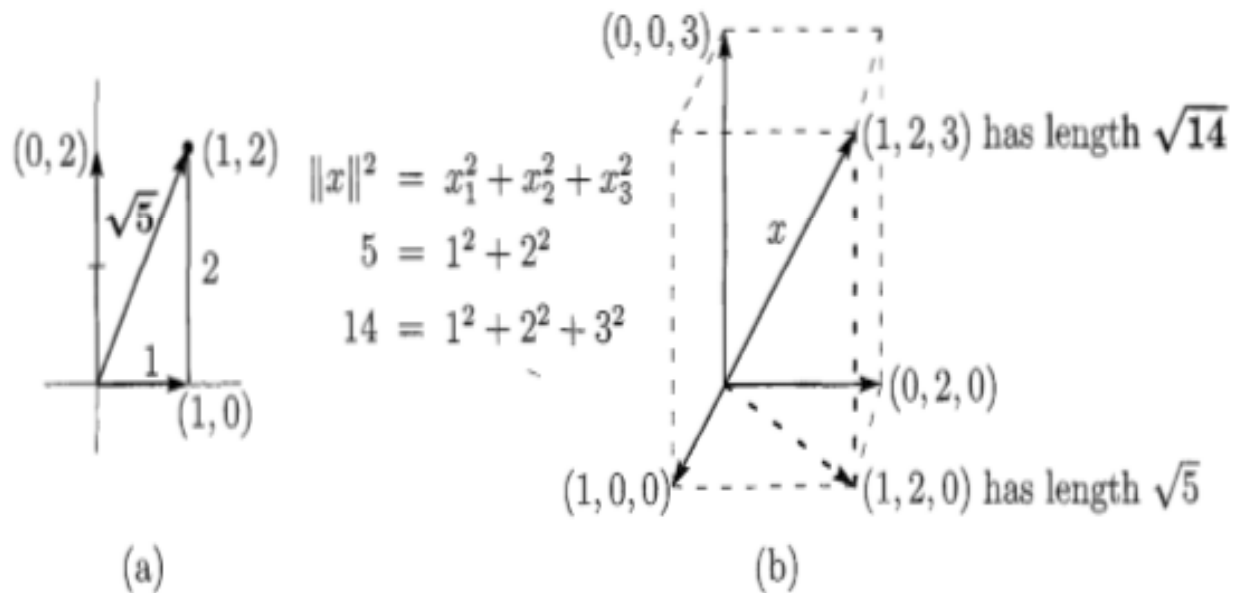
$$\|x\| = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$

We can also write  $\|x\|^2 = x^T x$

**Note** : Zero is the only vector whose norm is 0.

## Unit 3. Linear Transformations and Orthogonality

### *Orthogonal Vectors & Subspaces*



## Unit 3. Linear Transformations and Orthogonality

### *Orthogonal Vectors & Subspaces*

---



#### *Definition:*

The **inner product** or dot product or scalar product of two vectors  $x = (x_1, x_2, \dots, x_n)$  and  $y = (y_1, y_2, \dots, y_n)$  is denoted by

$$x^T y \text{ or } x \circ y \text{ or } \langle x, y \rangle$$

and is defined by

$$x^T y = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$$

Note that

$$x^T y = y^T x$$

## Unit 3. Linear Transformations and Orthogonality

### *Orthogonal Vectors & Subspaces*

---



#### *Definition :*

Two vectors  $x = (x_1, x_2, \dots, x_n)$  and  $y = (y_1, y_2, \dots, y_n)$  are said to be orthogonal if

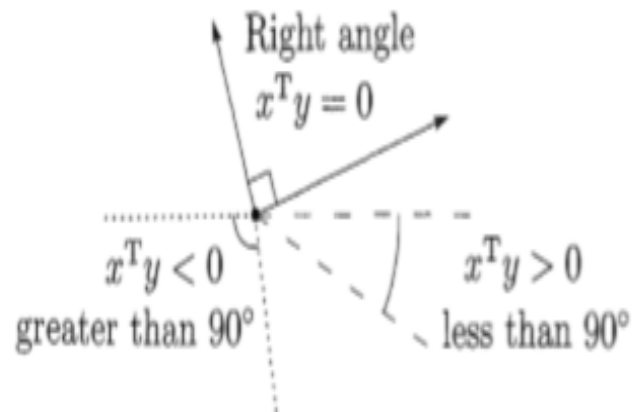
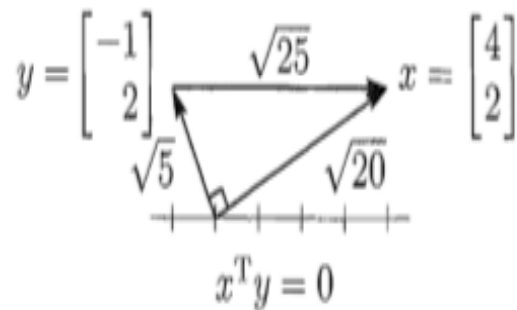
$$x^T y = y^T x = 0$$

#### *Note :*

1. Zero is the only vector that is orthogonal to itself.
2. Zero is the only vector that is orthogonal to every other vector.

## Unit 3. Linear Transformations and Orthogonality

### *Orthogonal Vectors & Subspaces*



## Unit 3. Linear Transformations and Orthogonality

### *Orthogonal Vectors & Subspaces*

---



#### Examples

1. The coordinate vectors  $(1, 0, \dots, 0)$ ,  $(0, 1, 0, \dots, 0)$ , ...,  $(0, 0, \dots, 0, 1)$  are mutually orthogonal in  $\mathbb{R}^n$ .
2. The vectors  $(c, s)$ ,  $(-s, c)$  are orthogonal in  $\mathbb{R}^2$ .
3. The vectors  $(2, 1, 0)$ ,  $(-1, 2, 0)$  are orthogonal in  $\mathbb{R}^3$ .

## Unit 3. Linear Transformations and Orthogonality

### *Orthogonal Vectors & Subspaces*

---



**Theorem :** If the non-zero vectors  $v_1, v_2, \dots, v_k$  are mutually orthogonal then these vectors are linearly independent but convex need not be true.

**Example :** Vectors  $(2, 1)$  and  $(1, 2)$  are linearly independent but they are not mutually orthogonal



## Unit 3. Linear Transformations and Orthogonality

### *Orthogonal Vectors & Subspaces*

---



*Definition :*

Two subspaces  $S$  and  $T$  of a vector space  $V$  are **orthogonal** if every vector  $x$  in  $S$  is orthogonal to every vector  $y$  in  $T$ . Thus,

$$x^T y = 0$$

for all  $x \in S$  and  $y \in T$ .

## Unit 3. Linear Transformations and Orthogonality

### *Orthogonal Vectors & Subspaces*

---



#### *Examples*

1.  $Z = \{0\}$  is orthogonal to all subspaces.
2. In  $\mathbb{R}^2$ , a line can be orthogonal to another line.
3. In  $\mathbb{R}^3$ , a line can be orthogonal to another line or a plane. But, a plane cannot be orthogonal to another plane.

#### *Note* :

If  $S$  and  $T$  are orthogonal in  $V$  then  
$$\dim S + \dim T \leq \dim V$$

## Unit 3. Linear Transformations and Orthogonality

### *Orthogonal Vectors & Subspaces*

---

#### *Fundamental theorem of Orthogonality :*

*Let  $A$  be an  $m \times n$  matrix then row space of  $A$  is orthogonal to its null space in  $R^n$  and the column space is orthogonal to left null space in  $R^m$  .*





THANK YOU

---