LINEAR ALGEBRA AND ITS APPLICATIONS
UE19MA251

Projection Matrices

The matrix P that projects onto C(A) is given by

Projection matrix $$P = A(A^{T}A)^{-1}A^{T}.$$

Also , if P and Q are the matrices that project onto orthogonal subspaces then it is always true that PQ = 0 and P + Q = I

Least Squares Fitting Of Data

Suppose we do a series of experiments and expect the output b to be a linear function of the input t. We look for a straight line

$$b = C + Dt$$

If there is no experimental error then two measurements of b will determine the line. But, if there is error, we minimize it by the method of least squares and find the optimal straight line.

Least Squares Fitting Of Data

Consider the following system of equations:

$$C + Dt_1 = b_1$$
$$C + Dt_2 = b_2 \ldots\ldots$$
$$C + Dt_m = b_m$$

In matrix form, $\begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ - & - \\ 1 & t_m \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ - \\ b_m \end{bmatrix}$ or $Ax = b$

The best solution $\hat{x}$ can be obtained by solving the normal equations.

Problems on Least Squares Fitting Of Data

Use the method of least square to fit the best line to the data $b = 4, 3, 1, 0$ at $t = -2, -1, 0, 2$ respectively. Find the projection of $b = (4, 3, 1, 0)$ onto the column space of A. Calculate the error vector 'e' and check that 'e' is orthogonal to the columns of A.

Solution: Let $C + Dt = b$ be the best fit straight line for the given data.

Problems on Least Squares Fitting Of Data

Given that $b = 4, 3, 1, 0$ at $t = -2, -1, 0, 2$

$$\Rightarrow \left.\begin{aligned} C + D(-2) &= 4 \\ C + D(-1) &= 3 \\ C + D(0) &= 1 \\ C + D(2) &= 0 \end{aligned}\right\} \Rightarrow \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 1 \\ 0 \end{bmatrix}$$

$$\Rightarrow Ax = b$$

The system is inconsistent. To find least square solution $\hat{x}$, we have to solve normal equation

ie $A^T A \hat{x} = A^T b$

$$\Rightarrow \begin{bmatrix} 4 & -1 \\ -1 & 9 \end{bmatrix} \hat{x} = \begin{bmatrix} 8 \\ -11 \end{bmatrix}$$

$$\hat{x} = \begin{pmatrix} \hat{C} \\ \hat{D} \end{pmatrix} = \begin{pmatrix} 61/35 \\ -36/35 \end{pmatrix}$$

$\therefore$ The best straight line fit for the given data is

$$b = \frac{61}{35} - \frac{36}{35} t$$

Let p be the projection of $b = (4,3,1,0)$ onto $C(A)$

$$\therefore \quad p = A\hat{x} = \frac{1}{35} \begin{pmatrix} 133 \\ 97 \\ 61 \\ -11 \end{pmatrix}$$

Problems on Least Squares Fitting Of Data

The error vector

$$e = b - p = \begin{pmatrix} 1/5 \\ 8/35 \\ -26/35 \\ 11/35 \end{pmatrix}.$$

The error vector e is orthogonal to both the columns of A

i.e $e^T \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = 0$ and $e^T \begin{pmatrix} -2 \\ -1 \\ 0 \\ 2 \end{pmatrix} = 0$

Therefore the vector 'e' is orthogonal to column Space of A.

PES UNIVERSITY ONLINE

THANK YOU