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**Department of Science and Humanities** 



## **CLASS-2**

# RECTANGULAR MATRICES WITH ORTHONORMAL COLUMNS



### Rectangular matrices with orthonormal columns

- If Q has orthonormal columns, the leastsquares problem becomes easy.
- Q<sup>T</sup> Qx = Q<sup>T</sup> b are the normal equations for the best solution -in which Q<sup>T</sup> Q = I.
- $x = Q^{T}b$
- p=Qx the projection of b is  $(q_1^Tb)q_1+....+(q_n^Tb)q_n$
- $p = QQ^Tb$ , the projection matrix is  $P = QQ^T$ .



# Special cases:

I It B is a square matrix, 
$$\hat{\chi}_{=}^{2} = 8^{1}b$$
  
 $\hat{\chi}_{=}^{2} = 8^{1}b \left[8^{7} = 8^{1}\right]$ 

#### **Problems:**

1. Papiet 
$$b = (0,3,0)$$
 onto each of the orthonormal between  $a_1 = \left(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\right)$ ,  $a_2 = \left(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$  & then find its

$$P_1 = \left(\frac{\alpha_1^T b}{\alpha_1^T a_1}\right) \alpha_1$$
 but  $\alpha_1^T \alpha_1 = ||\alpha_1|| = \sqrt{\frac{1}{5} + \frac{1}{5} + \frac{1}{5}} = 1$ 

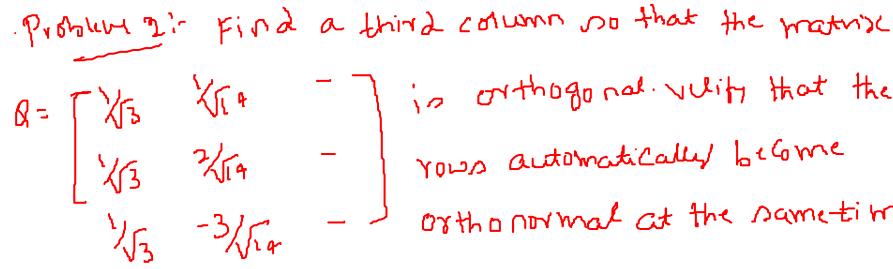
$$P_{1} = \left( \begin{bmatrix} \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} \begin{bmatrix} \frac{2}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix} = \sum_{i=1}^{3} P_{i} = \begin{bmatrix} \frac{4}{3} \\ \frac{4}{3} \\ -\frac{2}{3} \end{bmatrix}$$



#### **Problems:**

Similar, 
$$P_2 = (a_2^T b)a_2 = \begin{bmatrix} -\frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$





R= [X3 X14 - ] is orthogonal. With that the Yours automatically become orthonormal at the same-time.

Art By the definition of orthogonal madrix, first and second columns are orthogonal to the Sequence third convenior hat the third column be (x)



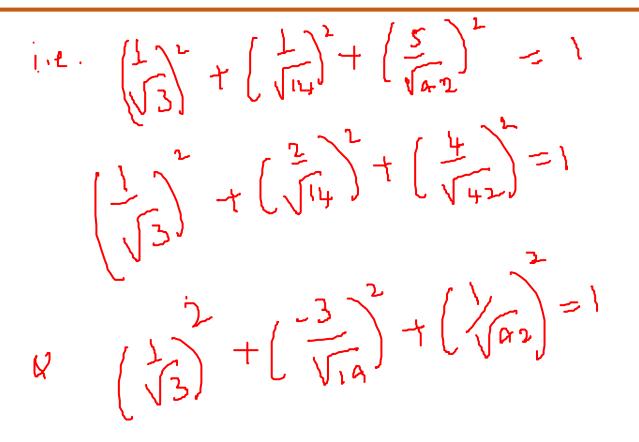


$$8 = \begin{bmatrix} \sqrt{3} & \sqrt{14} & 3 \\ \sqrt{3} & 2\sqrt{14} & 4 \\ \sqrt{3} & -3\sqrt{14} & 3 \end{bmatrix} \implies a^{7} (= 0 \Rightarrow) \begin{bmatrix} \sqrt{3} & \sqrt{3} & \sqrt{3} \\ \sqrt{3} & \sqrt{3} & \sqrt{3} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\Rightarrow \begin{array}{c} x \\ \sqrt{3} & \sqrt{3} & \sqrt{3} \\ \sqrt{3} & \sqrt{3} & \sqrt{3} \end{array} = 0 \Rightarrow x + y + 3 = 0 \implies x + y + 3 = 0 \implies x + y + 3 = 0 \implies x + y + 3 = 0$$
Similarly,  $b^{7} (= 0 \Rightarrow) \begin{bmatrix} \sqrt{14} & 2\sqrt{14} & \sqrt{3} & \sqrt{3} \\ y \\ 3 & 3 & 3 \end{pmatrix} = 0$ 

For the rows to be orthonormal, the norm must be equal to 1.









# THANK YOU

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