



LINEAR ALGEBRA AND ITS APPLICATIONS

Renna Sultana

Department of Science and Humanities

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MATRICES AND GAUSSIAN ELIMINATION

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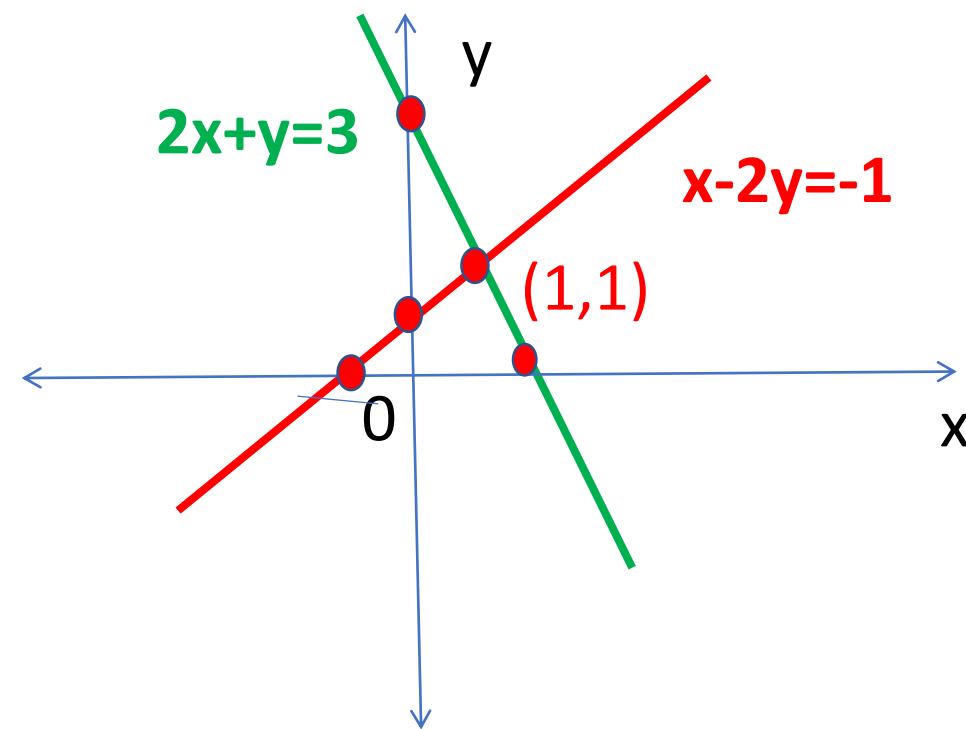
LINEAR ALGEBRA AND ITS APPLICATIONS

THE GEOMETRY OF LINEAR EQUATIONS:

Course Content: The Geometry of Linear Equations

1. Solve the system of equations $2x+y=3$; $x-2y=-1$ and draw the Row picture and Column picture.

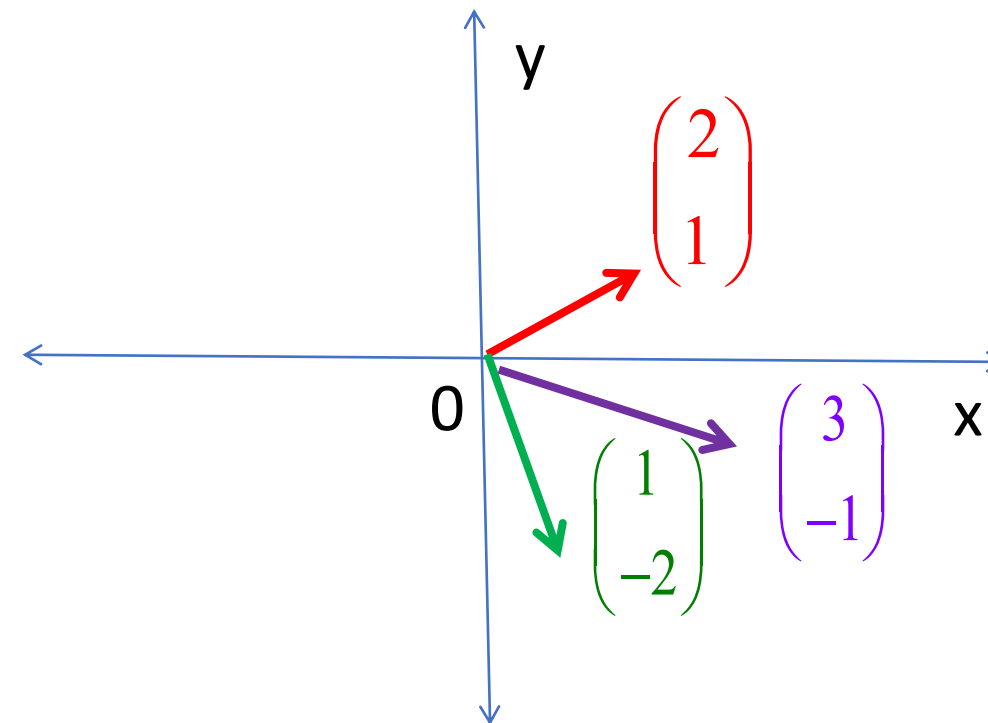
Row picture



Intersecting lines
Unique solution (1,1)

Column picture

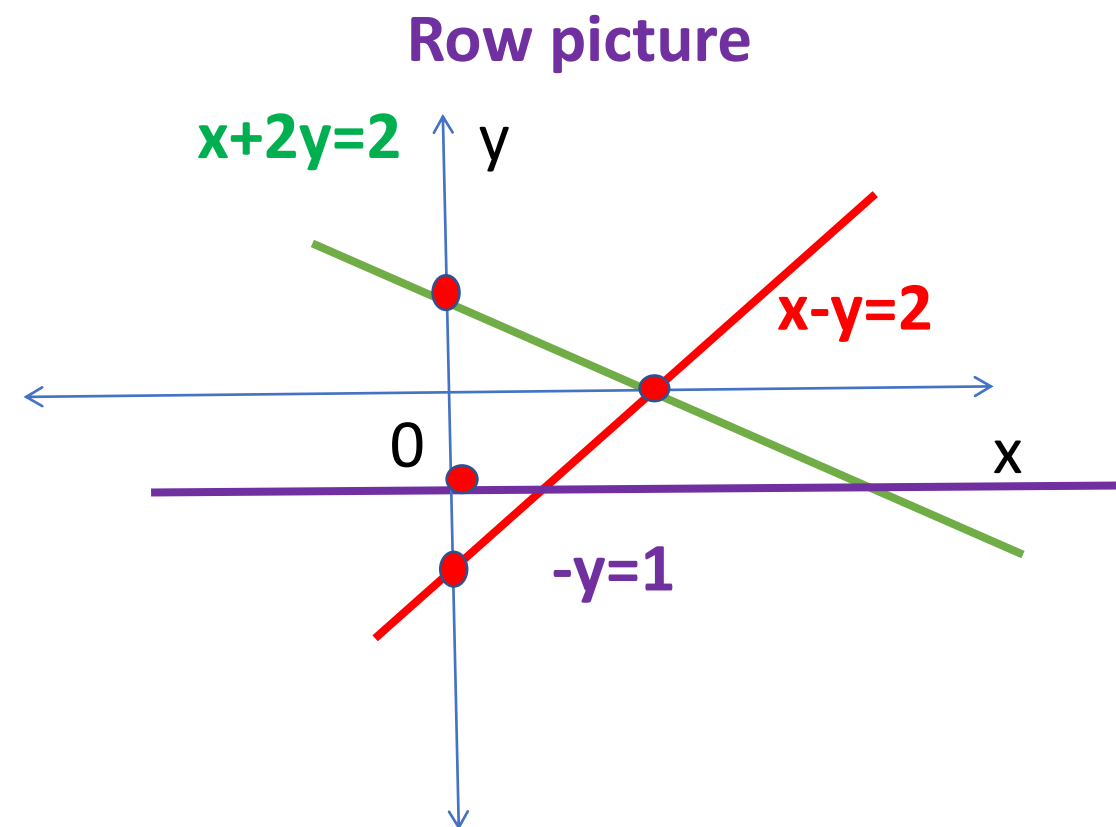
$$x \begin{pmatrix} 2 \\ 1 \end{pmatrix} + y \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \Rightarrow 1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + 1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$



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THE GEOMETRY OF LINEAR EQUATIONS:

2. Sketch the three lines $x+2y=2$; $x-y=2$; $-y=1$ (row picture only) and decide if the three equations are solvable. What happens if all right hand sides are zero? Is there any non-zero choice of right hand sides that allow the three lines to intersect at a common point of intersection?

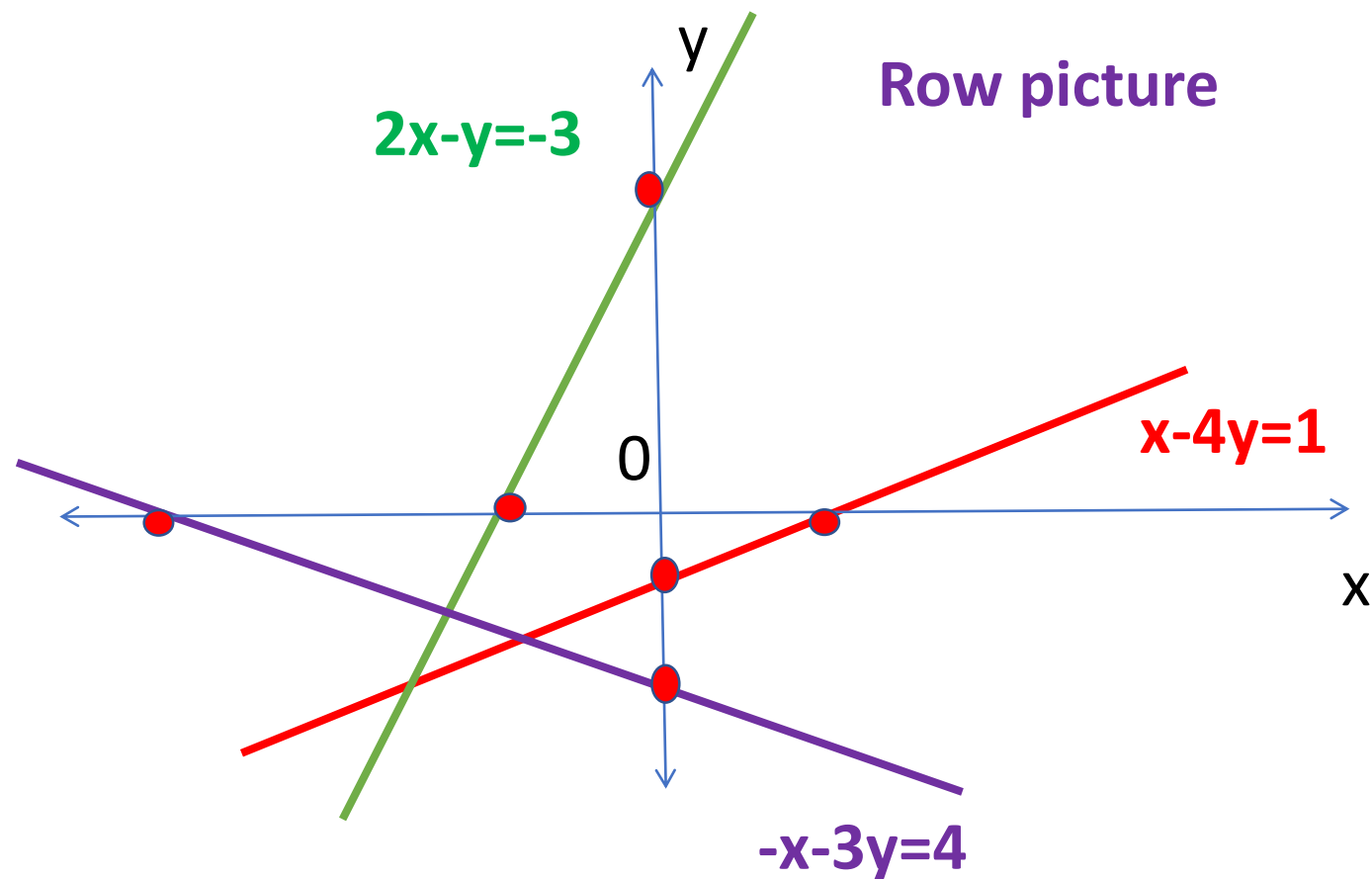


The given three equations are not solvable as they do not have a common point of intersection. If all right hand sides are zero we get a trivial(zero) solution i.e $x=0$, $y=0$. If RHS of first equation is -1 then the three lines $x+2y=-1$; $x-y=2$; $-y=1$ intersect at a common point of intersection $(1,-1)$

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THE GEOMETRY OF LINEAR EQUATIONS:

3. Sketch the three lines $x - 4y = 1$; $2x - y = -3$; $-x - 3y = 4$ and decide if the three equations are solvable. Do they have a common point of intersection? Explain.



Given three equations are not solvable as they have no common point of intersection.

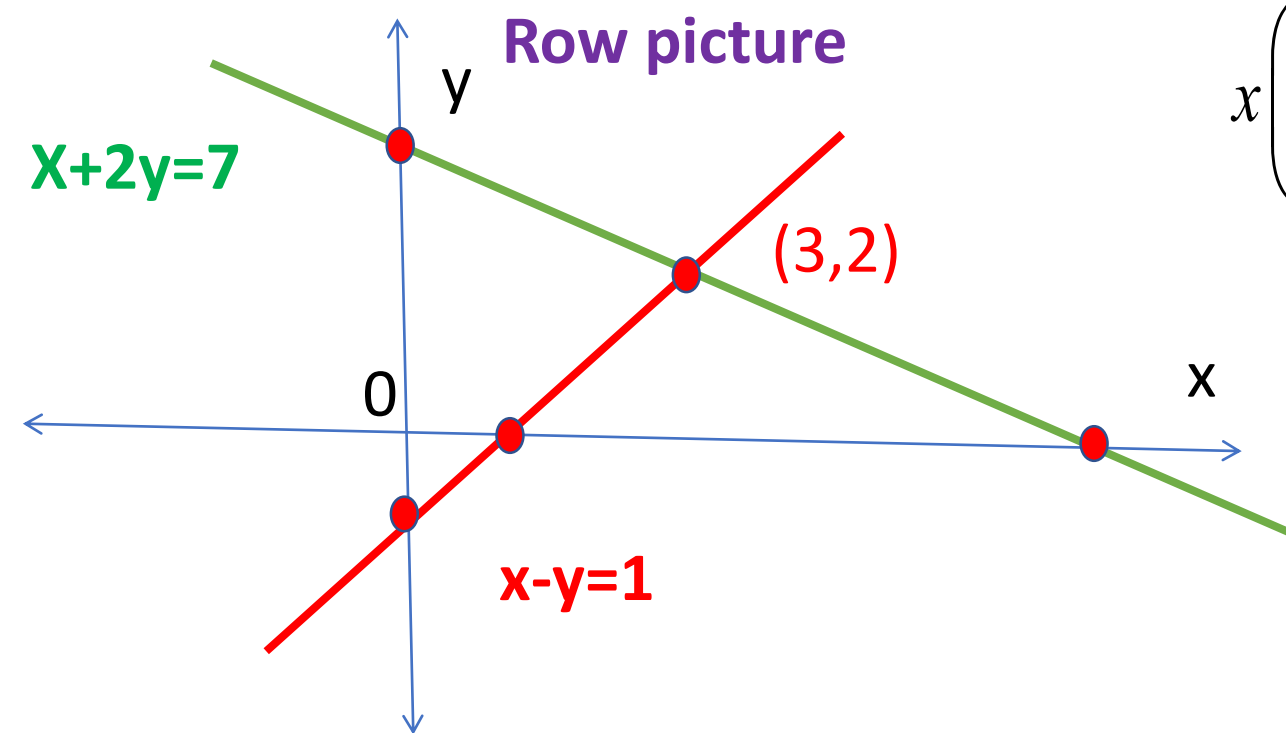
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THE GEOMETRY OF LINEAR EQUATIONS:

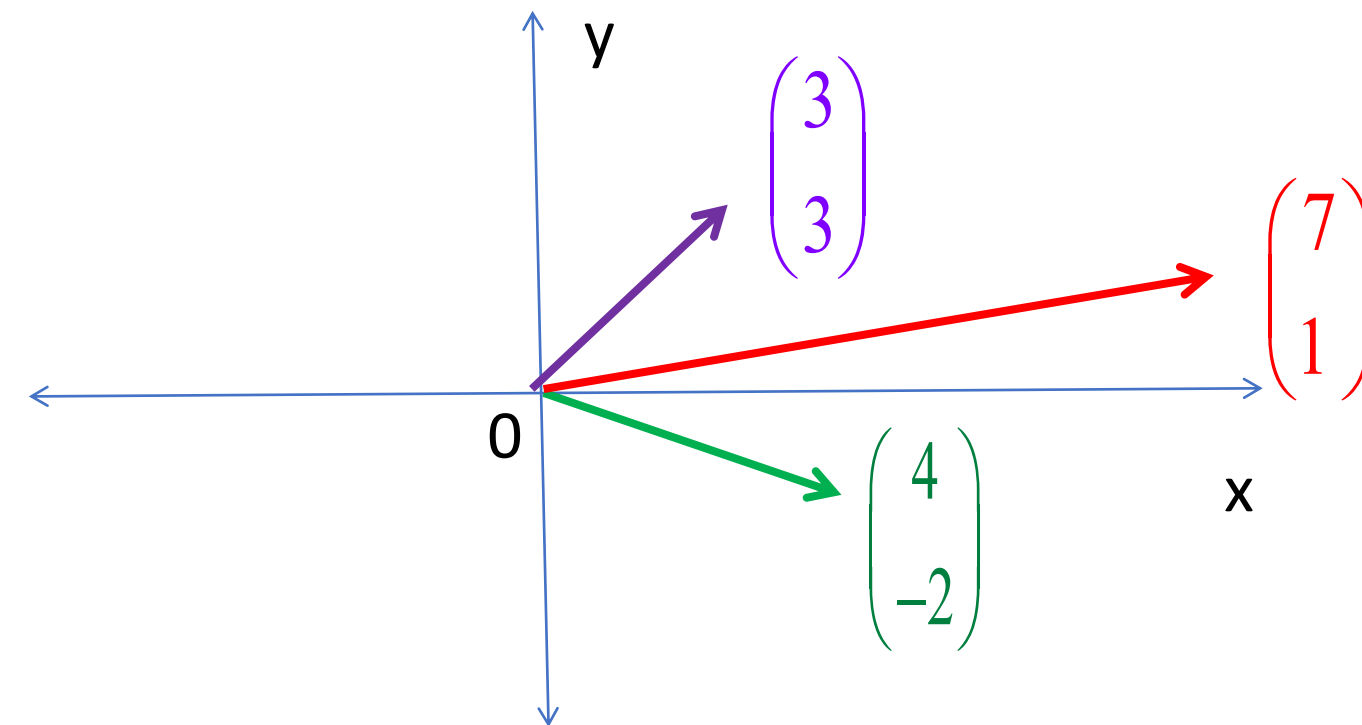
4. Draw the row picture and column picture for the following system of equations and discuss its consistency, singularity, and existence of the solution:

(i) $x + 2y = 7$; $x - y = 1$

Column picture



$$x \begin{pmatrix} 1 \\ 1 \end{pmatrix} + y \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \end{pmatrix} \Rightarrow 3 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$$



(ii) $2x + 3y = 6$; $4x + 6y = 12$ (iii) $x - 3y = 5$; $x - 3y = -5$

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INTRODUCTION:

Example:

$$A \rightarrow \left(\begin{array}{cccc|c} \textcircled{1} & 1 & -2 & 3 & 4 \\ 2 & 3 & 3 & -1 & 3 \\ 5 & 7 & 4 & -1 & 5 \end{array} \right) \xrightarrow[R_3 - 5R_1]{R_2 - 2R_1} \left(\begin{array}{cccc|c} 1 & 1 & -2 & 3 & 4 \\ 0 & 1 & 7 & -7 & -5 \\ 0 & 2 & 14 & -16 & -15 \end{array} \right) \xrightarrow{R_3 - 2R_2}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & -2 & 3 & 4 \\ 0 & 1 & 7 & -7 & -5 \\ 0 & 0 & 0 & -2 & -5 \end{array} \right) = U \xrightarrow{R_3 / (-2)} \left(\begin{array}{cccc|c} 1 & 1 & -2 & 3 & 4 \\ 0 & 1 & 7 & -7 & -5 \\ 0 & 0 & 0 & 1 & 5/2 \end{array} \right) \xrightarrow[R_2 + 7R_3]{R_1 - 3R_3}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & -7/2 \\ 0 & 1 & 7 & 0 & 25/2 \\ 0 & 0 & 0 & 1 & 5/2 \end{array} \right) \xrightarrow{R_1 - R_2} \left(\begin{array}{cccc|c} 1 & 0 & -9 & 0 & -16 \\ 0 & 1 & 7 & 0 & 25/2 \\ 0 & 0 & 0 & 1 & 5/2 \end{array} \right) = R$$

ENGINEERING MATHEMATICS-III

References/Links:

https://upload.wikimedia.org/wikipedia/commons/c/c0/Intersecting_Lines.svg

Google search: Graphs of row and column picture for a system of linear equations





THANK YOU

Renna Sultana

Department of Science and Humanities

rennasultana@pes.edu