LINEAR ALGEBRA AND ITS APPLICATIONS

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Unit-4

Orthogonalization , Eigen Values and Eigen Vectors

CLASS-3

THE GRAM-SCHMIDT ORTHOGONALIZATION

The Gram-Schmidt process:

- It is a process of converting linearly independent vectors into orthonormal vectors.

- Consider any 3 independent vectors a, b, c. Then the first orthonormal q_1= a/norm(a).

- If 'b' is perpendicular to the vector 'a' then q_2=b/norm(b) otherwise $B=b-(q_1^Tb)q_1$ and q_2=B/norm(B).

The Gram-Schmidt process (Continued.....)

- If ‘c’ is perpendicular to the plane spanned by the vectors a and b then $q_3=c/\text{norm}(c)$ otherwise $C=c-(q_1^T c)q_1 - (q_2^T c)q_2$ and $q_3=C/\text{norm}(C)$.

This is the one idea of the whole Gram-Schmidt process, to subtract from every new vector its components in the directions that are already settled. That idea is used over and over again. When there is a fourth vector, we subtract away its components in the directions of q_1, q_2, q_3.

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Problems:-

1. From the vectors a, b, c find orthonormal vectors q_1, q_2 & q_3 given $a=\begin{pmatrix}1\\1\\0\end{pmatrix}, b=\begin{pmatrix}1\\0\\1\end{pmatrix}, c=\begin{pmatrix}0\\1\\1\end{pmatrix}$

Ans: From Gram-Schmidt Process $q_1 = \frac{a}{\|a\|} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

$q_2 = \frac{e_2}{\|e_2\|}$ where $e_2 = b - (q_1^T b) q_1$

$$= \begin{bmatrix}1\\0\\1\end{bmatrix} - \left[\left(1/\sqrt{2} \quad 1/\sqrt{2} \quad 0\right)\begin{pmatrix}1\\0\\1\end{pmatrix}\right]\begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\\0\end{pmatrix}$$

$$= \begin{pmatrix}1/2\\-1/2\\1\end{pmatrix}$$

$\therefore \; q_2 = \left(\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$

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$\cdot\ q_3 = \dfrac{e_3}{\|e_3\|}$, $e_3 = c - (q_1^T c) q_1 - (q_2^T c) q_2$

$$e_3 = \begin{pmatrix} -2/3 \\ 2/3 \\ 2/3 \end{pmatrix} , \quad \|e_3\| = \frac{2\sqrt{3}}{3}$$

$$\therefore q_3 = \begin{pmatrix} -1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix}$$

THANK YOU

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