LINEAR ALGEBRA AND ITS APPLICATIONS

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MATRICES AND
GAUSSIAN ELIMINATION

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Course Content: Prerequisites

What?

Why?

PREREQUISITES:

- **What** is Linear Algebra?

- Linear Algebra is the study of systems of Linear equations.
- It is concerned with vector spaces and linear mappings between such spaces.
- It includes the study of lines, planes and subspaces and is also concerned with properties common to all vector spaces.
- Linear Algebra is the geometry of n-dimensional space and its linear transformations. Thus Linear Algebra helps us to develop our geometric instinct to visualize the concepts in higher dimensions.

PREREQUISITES:

Why do we need to study Linear Algebra?

- Linear Algebra is used in the everyday world to solve problems in Mathematics, Physics, Biology, Chemistry, Engineering, Statistics, Economics, Finance, Psycology, Sociology, etc.,
- Applications that use Linear Algebra include the transmission of information, the development of special effects in film and video, recording of sound, Web search engines on the Internet and economic analyses.
- Linear Algebra is a study of Linear Transformations. It is used in Graph Theory, Networks, Signal Processing, Probability Theory, Real Analysis, Communication.
- Linear Algebra is used in coding theory, Artificial Intelligence, Machine Learning, Image Processing, Computer Graphics, Numerical Analysis, Control Systems, Networking, Ordinary/Partial Differential Equations.

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Course Content of Linear Algebra:

- **Module 1:** Matrices and Gaussian Elimination
- **Module 2:** Vector Spaces
- **Module 3:** Linear Transformations and Orthogonality
- **Module 4:** Orthogonalization , Eigen Values and Eigen Vectors
- **Module 5:** Singular Value Decomposition

MATRICES AND GAUSSIAN ELIMINATION:

Course Content of Module 1:

❖Introduction

❖The Geometry of Linear Equations

❖Gaussian Elimination

❖Singular Cases

❖Elimination Matrices

❖Triangular Factors and Row Exchanges

❖Inverses and Transposes

❖Inverse by Gauss Jordan Method

INTRODUCTION:

Course Content: Introduction

What is a linear Equation?

- A linear equation in n variables is of the form $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$

where $x_1, x_2, \cdots, x_n$ are **unknowns or variables,** $a_i \in \square \ (i = 1, 2, \cdots, n)$

are known as the **Coefficients** of the variables $x_i \ (i = 1, 2, \cdots, n)$

and $b \in \square$ are constants.

Examples of **Linear equations** are: $2x_1 - 3x_2 = 7 \ ; \left(\sqrt{2}\right)x_1 - \left(\sqrt{5}\right)x_2 = 1$

Examples of **Non-Linear equations** are: $x_1x_2 - 3x_3 = 2; y = \log x + \sin x;$

$$2x + 3y^{1/3} = 5$$

INTRODUCTION:

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What is a system of linear Equations?

- A **system of linear equation** is a set of linear equations involving the same **unknowns or variables .**

- A system of **two** equations with **two** unknowns is of the form

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

- A system of **three** equations with **three** unknowns is of the form

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

INTRODUCTION:

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Matrix Notation:

❖A system of **m** equations with **n** unknowns is given by

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
\vdots \qquad\quad \vdots \qquad\qquad\quad \vdots \quad &\quad \vdots \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m
\end{aligned}
$$

This can be represented in matrix form as **Ax=b** where

$$
A = \begin{pmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} \cdots & a_{mn} \end{pmatrix}_{mxn} ; \quad x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{nx1} ; \quad b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}_{mx1}
$$

Here **A** is a matrix of order mxn known as **The Co-efficient Matrix**,

x_{nx1} is a nx1 column matrix(or vector) known as **The Vector of unknowns or variables**

b_{mx1} is a column vector of order mx1 known as **The Constant Vector**

INTRODUCTION:

- a_{ij} is the component in the i^{th} **row** and j^{th} **column** of A.
- **If m=n,** A will be a **Square matrix of order n** and the system will have **n** equations with **n** variables.
- **If all b_i 's are zero,** the system is known as **Homogeneous system of Equations.** If at least one of **b_i 's is not zero** then the system is said to be **Non-Homogeneous system of Equations.**
- **Matrix** $[A:b] = \begin{pmatrix} a_{11} & a_{12} \cdots & a_{1n} & : b_1 \\ a_{21} & a_{22} \cdots & a_{2n} & : b_2 \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} \cdots & a_{mn} & : b_m \end{pmatrix}_{mx(n+1)}$ Is called the **Augmented Matrix.**

INTRODUCTION:

- **Solution to the System of Equations:** A set of values or numbers assigned for variables $x_1, x_2, \cdots, x_n$ which satisfy all the equations simultaneously is defined as the **Solution** to the system of equations.

- The set of all possible solutions is called the **Solution Set** or **General Solution** of Linear system of equations.

- **Consistency:** A linear system of equations is said to be **Consistent** if it has a solution (unique solution or infinitely many solutions).

- A linear system of equations is said to be **Inconsistent** if it has no solution.

INTRODUCTION:

- **Elementary Row Operations:** In order to solve the system of linear equations using Matrix notation we define three basic operations called **Elementary Row Operations** or **Elementary Row Transformations** which are as follows:

- Multiply the entries of a row by a non-zero scalar k i.e $R_i \longrightarrow kR_i \,(k \neq 0)$
- Replace one row by sum of itself and a non-zero scalar multiple k of another row i.e $R_i \longrightarrow R_i + kR_j \,(k \neq 0)$.
- Interchange of any two rows i.e $R_i \longleftrightarrow R_j$.

- **Elementary Matrix:** A square matrix of order n is called an **Elementary Matrix** E_n if it can be obtained from an Identity matrix I_n using a single row operation. i.e $I_n \to E_n$

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \xrightarrow{R_2 - 2R_1} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} = E_2$$

INTRODUCTION:

- **Equivalent Matrices:** If two matrices A and B are such that each of them can be obtained from the other by a definite number of Elementary transformations then they are said to be **Equivalent Matrices** represented by $A \sim B$.

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & -3 & 4 \end{pmatrix} \xrightarrow{R_2 + \left(\frac{1}{2}\right) R_1} \begin{pmatrix} 2 & -1 & 0 \\ 0 & 3/2 & 1 \\ 0 & -3 & 4 \end{pmatrix} \xrightarrow{R_3 + 2R_2}$$

$$\begin{pmatrix} 2 & -1 & 0 \\ 0 & 3/2 & 1 \\ 0 & 0 & 6 \end{pmatrix} \sim B \quad \therefore A \sim B$$

INTRODUCTION:

- **Echelon Form Of A Matrix:** A rectangular matrix A of order mxn is said to be in **Echelon Form U** if it satisfies the following conditions:

- The first non-zero element in every row is known as the **pivot**.
- Below each pivot is a column of **zeros** obtained by Elementary row operations.
- Each pivot lies to the right of the pivot in the row above. This produces a **Staircase pattern** as shown below.
- Zero rows(if exist) appear at the bottom of the Matrix.

$$A \rightarrow \begin{pmatrix} a & b & c & d & e \\ 0 & 0 & f & g & h \\ 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}_{4x5} = U$$

INTRODUCTION:

❖ **Row Reduced Echelon Form Of A Matrix(RREF):** A rectangular matrix A in Echelon Form must further undergo the following operations to reduce to RREF :

- Every row in Echelon form must be divided by its pivot so that the first non-zero entry in every row is 1.
- Using the pivot rows produce zeros above the first non-zero entry (i.e 1).

This RREF is denoted by R.

$$A \to U = \begin{pmatrix} a & b & c & d & e \\ 0 & 0 & f & g & h \\ 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & b/a & 0 & 0 & e/a \\ 0 & 0 & 1 & 0 & h/f \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = R$$

INTRODUCTION:

Example:

$$A \rightarrow \begin{pmatrix} \color{red}{1} & 1 & -2 & 3: & 4 \\ 2 & 3 & 3 & -1: & 3 \\ 5 & 7 & 4 & -1: & 5 \end{pmatrix}$$

References/Links:

https://en.wikipedia.org/wiki

THANK YOU

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INTRODUCTION:

Example:

$$A \rightarrow \begin{pmatrix} 1 & 1 & -2 & 3: & 4 \\ 2 & 3 & 3 & -1: & 3 \\ 5 & 7 & 4 & -1: & 5 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & -2 & 3: & 4 \\ 0 & 1 & 7 & -7: & -5 \\ 0 & 0 & 0 & -2: & -5 \end{pmatrix} = U \rightarrow \begin{pmatrix} 1 & 0 & -9 & 0: & -16 \\ 0 & 1 & 7 & 0: & -25/2 \\ 0 & 0 & 0 & 1: & 5/2 \end{pmatrix} = R$$