LINEAR ALGEBRA AND ITS APPLICATIONS

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MATRICES AND
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MATRICES AND GAUSSIAN ELIMINATION:

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Course Content: Inverses and Transposes

- Let A be a square matrix of order n the **Inverse of A** is the matrix B such that **AB=I=BA**.

  Here $B=A^{-1}$.

- **Properties:** Inverse of a matrix is unique. i.e., AB=BA=I and AC=CA=I, then B=C
- Inverse of the product is the product of Inverses. $(ABCD)^{-1}=D^{-1}C^{-1}B^{-1}A^{-1}$
- If A=LU then $A^{-1}=U^{-1}L^{-1}$.
- Since $E_{32}E_{31}E_{21}A=U$ we have $E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}U=A \Rightarrow A^{-1}=U^{-1}E_{32}E_{31}E_{21}$

  $$\Rightarrow L=E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}$$

- A matrix A is invertible if and only if elimination produces n pivots with or without row exchanges. Elimination solves Ax=b without explicitly finding A^{-1} .
- If A is invertible, the one and only one solution to Ax=b is $x=A^{-1}b$

Gauss Jordan Method of computing A^{-1} :

- The inverse of an invertible matrix is obtained by a set of row operations that transforms A to I and I to A^{-1} . This process is known as **Gauss Jordan Method**.
- Consider the Augmented Matrix **[A:I].** Then perform row operations on it so that A reduces to Echelon form **U** and at the same time **I** reduces to **C.** Further reduce **U to I** using elementary row transformations which reduces **C to A^{-1}** .
- i.e., $$[A:I]\to[U:C]\to[I:A^{-1}]$$

Ex: Compute **A^{-1}** using Gauss Jordan Method given

$$A=\begin{pmatrix} 2 & 1 & 1 \\ 4 & 3 & 4 \\ -2 & 2 & 2 \end{pmatrix}$$

MATRICES AND GAUSSIAN ELIMINATION:

Ex: $$[A:I]=\begin{pmatrix} 2 & 1 & 1:1 & 0 & 0 \\ 4 & 3 & 4:0 & 1 & 0 \\ -2 & 2 & 2:0 & 0 & 1 \end{pmatrix} \xrightarrow[R_3+R_1]{R_2-2R_1} \begin{pmatrix} 2 & 1 & 1:1 & 0 & 0 \\ 0 & 1 & 2:-2 & 1 & 0 \\ 0 & 3 & 3:1 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_3-3R_2} \begin{pmatrix} 2 & 1 & 1:1 & 0 & 0 \\ 0 & 1 & 2:-2 & 1 & 0 \\ 0 & 0 & -3:7 & -3 & 1 \end{pmatrix} = [U:C] \xrightarrow[R_2+2/3R_3]{R_1+1/3R_3} \begin{pmatrix} 2 & 1 & 0:10/3 & -1 & 1/3 \\ 0 & 1 & 0:8/3 & -1 & 2/3 \\ 0 & 0 & -3:7 & -3 & 1 \end{pmatrix}$$

$$\xrightarrow{R_1-R_2} \begin{pmatrix} 2 & 0 & 0:2/3 & 0 & -1/3 \\ 0 & 1 & 0:8/3 & -1 & 2/3 \\ 0 & 0 & -3:7 & -3 & 1 \end{pmatrix} \xrightarrow[R_3=-1/3R_3]{R_1=1/2R_1} \begin{pmatrix} 1 & 0 & 0:1/3 & 0 & -1/6 \\ 0 & 1 & 0:8/3 & -1 & 2/3 \\ 0 & 0 & 1:-7/3 & 1 & -1/3 \end{pmatrix}$$

$$=[I:B]$$

$$B=A^{-1}=\begin{pmatrix} 1/3 & 0 & -1/6 \\ 8/3 & -1 & 2/3 \\ -7/3 & 1 & -1/3 \end{pmatrix}$$

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Transpose of a Matrix A^T :

- If $A = \left[a_{ij}\right]_{mxn}$ is an mxn matrix, then its transpose is obtained by interchanging its rows and columns and is denoted by $A^T = \left[a_{ji}\right]_{nxm}$.

Ex: If $A = \begin{pmatrix} 2 & 1 & -3 \\ 4 & 2 & 0 \end{pmatrix}_{2x3}$ then $A^T = \begin{pmatrix} 2 & 4 \\ 1 & 2 \\ -3 & 0 \end{pmatrix}_{3x2}$

- **Properties:**
- The Transpose of a Lower Triangular Matrix is an Upper Triangular Matrix.

$$\left(A^T\right)^T = A \; ; \quad (AB)^T = B^T A^T \; ; \quad \left(A^{-1}\right)^T = \left(A^T\right)^{-1} \; ;$$

$$(A \pm B)^T = A^T \pm B^T \; ; \quad \left(A^{-1}\right)^T A^T = \left(AA^{-1}\right)^T = I$$

Symmetric Matrices:

- If A is a matrix of order n is said to be symmetric matrix of order if $A^T = A$

Ex: If $A = \begin{pmatrix} 2 & -3 \\ -3 & 5 \end{pmatrix}$ then $A^T = \begin{pmatrix} 2 & -3 \\ -3 & 5 \end{pmatrix}$

- **Properties:**
- If A is symmetric then A^{-1} may or may not exist.
- If for a symmetric matrix A^{-1} exists then A^{-1} is also symmetric.
- For a symmetric matrix A, we have $(A^{-1})^T = (A)^{-1} \; (\because A^T = A)$

Symmetric Products AA^T, A^TA, LDL^T:

- If A is a matrix of order mxn, then $AA^T \, and \, A^T A$ are both symmetric

Ex: If $A=\begin{pmatrix} 2 & 1 & -3 \\ 4 & 2 & 0 \end{pmatrix}_{2x3}$ then $AA^T=\begin{pmatrix} 14 & 10 \\ 10 & 20 \end{pmatrix}$ $A^TA=\begin{pmatrix} 20 & 10 & -6 \\ 10 & 5 & -3 \\ -6 & -3 & 9 \end{pmatrix}$

- If A is symmetric and if $A = LDU$ then,

$$A = A^T = LDL^T \left(\because U = L^T \;\&\; L = U^T\right)$$

$$A=\begin{pmatrix} 1 & 2 & -1 \\ 2 & 3 & 0 \\ -1 & 0 & 4 \end{pmatrix} \xrightarrow[R_3+R_1]{R_2-2R_1} \begin{pmatrix} 1 & 2 & -1 \\ 0 & -1 & 2 \\ 0 & 2 & 3 \end{pmatrix} \xrightarrow{R_3+2R_2} \begin{pmatrix} 1 & 2 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & 7 \end{pmatrix} = U$$

$$LDU = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 7 \end{pmatrix} \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U = L^T$$

MATRICES AND GAUSSIAN ELIMINATION:

- For which three numbers "c" is this matrix not invertible, and why not?

$$A=\begin{pmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{pmatrix} \xrightarrow[R_3-4R_1]{R_2-c/2R_1} \begin{pmatrix} 2 & c & c \\ 0 & c-\left(c^2/2\right) & c-\left(c^2/2\right) \\ 0 & 7-4c & -3c \end{pmatrix}$$

$$\xrightarrow{R_3-\left(\frac{7-4c}{2c-c^2}\right)2R_2} \begin{pmatrix} 2 & c & c \\ 0 & c-\left(c^2/2\right) & c-\left(c^2/2\right) \\ 0 & 0 & c-7 \end{pmatrix}$$

- Matrix A is not invertible for c=0,2,7
- For c=0.2,7 elimination gives one zero row ,hence A will be singular and so A will not be invertible.

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THANK YOU

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