LINEAR ALGEBRA AND ITS APPLICATIONS
UE19MA251

Rotation Matrices Q:

The linear system of equations $Ax = b$ can be represented as a Linear Transformation

$T_A(x) = Ax$, where $T_A : \mathbb{R}^n \to \mathbb{R}^m$

Rotation Matrices Q:

The matrix that rotates (left) every point in R^2 about origin through θ is given by Q_θ

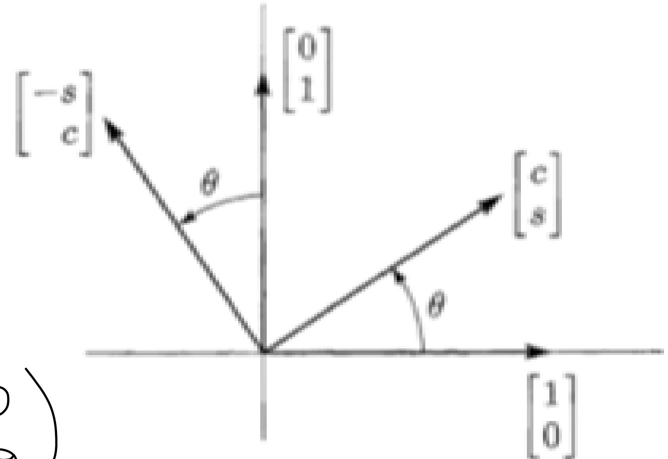
$Q_\theta : \mathbb{R}^2 \to \mathbb{R}^2$

Basis for $\mathbb{R}^2 = \left\{ e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$

$Q_\theta \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$

$Q_\theta \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(90+\theta) \\ \sin(90+\theta) \end{pmatrix} = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$

$Q_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$



Rotation Matrices Q:

Note :

- $Q_\theta \cdot Q_\psi = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}$

$$= \begin{bmatrix} \cos\theta\cos\psi - \sin\theta\sin\psi & -\cos\theta\sin\psi - \sin\theta\cos\psi \\ \sin\theta\cos\psi - \cos\theta\sin\psi & -\sin\theta\sin\psi + \cos\theta\cos\psi \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta+\psi) & -\sin(\theta+\psi) \\ \sin(\theta+\psi) & \cos(\theta+\psi) \end{bmatrix} = Q(\theta+\psi)$$

- $Q_\theta \cdot Q_\theta = Q(\theta+\theta) = Q_{2\theta} \Rightarrow Q_\theta^2 = Q_{2\theta}$

Rotation Matrices Q:

- $Q_\theta \cdot Q_{-\theta} = Q(\theta - \theta) = Q_0 = \begin{bmatrix} \cos 0 & -\sin 0 \\ \sin 0 & \cos 0 \end{bmatrix} = I$

$\Rightarrow \quad [Q_\theta]^{-1} = Q(-\theta)$

- Rotation preserves all angles between the vectors as well as their length. So it is reversible process.

Projection Matrices P

'P' is a matrix, that projects every vector in $\mathbb{R}^2$ onto any 'θ' line.

$$P[e_1] = P\begin{bmatrix}1\\0\end{bmatrix} = A' = \begin{bmatrix} OA'\cdot\cos\theta \\ OA'\cdot\sin\theta\end{bmatrix} = \begin{bmatrix}\cos\theta\cdot\cos\theta\\ \cos\theta\cdot\sin\theta\end{bmatrix}$$

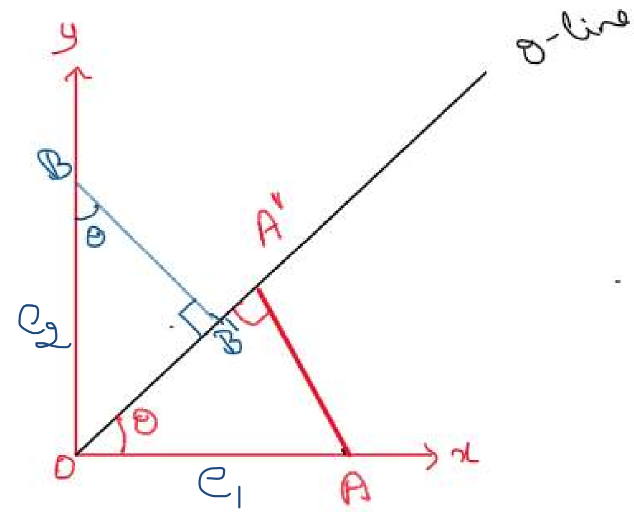
$$= \begin{bmatrix}\cos^2\theta\\ \cos\theta\sin\theta\end{bmatrix}$$

From right angle triangle $OA'A$, $OA' = \cos\theta$

$$P[e_2] = P\begin{bmatrix}0\\1\end{bmatrix} = B' = \begin{bmatrix}OB'\cos\theta\\ OB'\sin\theta\end{bmatrix} = \begin{bmatrix}\sin\theta\cos\theta\\ \sin\theta\sin\theta\end{bmatrix}$$

$$= \begin{bmatrix}\sin\theta\cos\theta\\ \sin^2\theta\end{bmatrix}$$

From right angle triangle $OB'B$, $OB' = \sin\theta$



Projection Matrices P

The matrix that projects every vector in $\mathbb{R}^2$ onto any θ line is given by

$$P = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}.$$

Note:

- This matrix has no inverse, because the transformation has no inverse.
- Projecting twice is the same as projection once i.e $P^2 = P$.

THANK YOU