LINEAR ALGEBRA

UE19MA251

APARNA B S
Department of Science and Humanities

LINEAR ALGEBRA – UE19MA251

Agenda – Problems on SVD

- Example 1

LINEAR ALGEBRA – UE19MA251

Agenda – Problems on SVD

Example 1. Find the singular value decomposition for the matrix:

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 3 & 2 & -2 \end{bmatrix}$$

<u>Solution</u>: The singular value decomposition of A is given by $\underset{2\times3}{A} = \underset{2\times3}{U}\ \underset{2\times3}{\Sigma}\ \underset{2\times3}{U^T}$

where $U = AA^T = \begin{bmatrix} 17 & 8 \\ 8 & 17 \end{bmatrix}$, a square symmetric matrix.

Let $B = AA^T$. Eigenvalues of B can be found from: $|B - \lambda I| = 0$

$$\therefore \quad \lambda_1 = 25, \quad \lambda_2 = 9$$

Corresponding Eigenvectors are:

$$X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } X_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Note that x_1 and x_2 are orthogonal.

The orthonormal vectors are

$$u_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \quad \text{and} \quad u_2 = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$\therefore \quad U = [u_1 \ u_2] = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

Singular values are the square roots of the eigenvalues.

$$\therefore \quad \sigma_1 = \sqrt{\lambda_1} = \sqrt{25} = 5$$

and $$\sigma_2 = \sqrt{\lambda_2} = \sqrt{9} = 3$$

LINEAR ALGEBRA – UE19MA251

Agenda – Problems on SVD

Define $v_i = \dfrac{u_i^T A}{\sigma_i}$; $i = 1, 2$

$$\therefore \quad v_1 = \frac{u_1^T A}{\sigma_1} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}^T$$

$$v_2 = \frac{u_2^T A}{\sigma_2} = \begin{bmatrix} -1/\sqrt{32} & 1/\sqrt{32} & -4/\sqrt{32} \end{bmatrix}^T$$

LINEAR ALGEBRA – UE19MA251

Agenda – Problems on SVD

$$V = [v_1 \; v_2 \; v_3] = \begin{array}{c} \begin{array}{ccc} & & v_3 \end{array} \\ \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{32} & * \\ 1/\sqrt{2} & 1/\sqrt{32} & * \\ 0 & -4/\sqrt{32} & * \end{bmatrix} \end{array}$$

The 3rd vector v_3 must be orthogonal to both v_1 and v_2.

$$\therefore \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{32} & 1/\sqrt{32} & -4/\sqrt{32} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving, we get $\dfrac{v_3}{\|v_3\|} = \dfrac{1}{\|v_3\|} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2/3 \\ 2/3 \\ 1/3 \end{bmatrix}$

Thus $V^T = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{32} & 1/\sqrt{32} & -4/\sqrt{32} \\ -2/3 & 2/3 & 1/3 \end{bmatrix}$

The SVD is

$$A = U \Sigma V^T$$

where $\Sigma = \begin{bmatrix} \sqrt{25} & 0 & 0 \\ 0 & \sqrt{9} & 0 \end{bmatrix}_{2\times 3}$

THANK YOU

Aparna B. S
Department of Science & Humanities
aparnabs@pes.edu