VECTOR SPACES

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CLASS 7 : CONTENT

- Null Space

NULL SPACE

Definition :

Let A be a matrix of order m x n.

The ***null space of A*** is the set of all solutions of the homogeneous system of equations

Ax = 0 denoted by N (A) .

Thus,

$N(A) = \{ x \in R^n \,/\, Ax = 0 \}$

Note : N(A) is a subspace of R^n .

NULL SPACE

- Null Space of a matrix A is denoted by N(A)
- Null Space of A is spanned by Special solutions to Ax=0which is same as solving for Rx=0 where R is the row reduced echelon form of A
- Null space of A is a subspace of vector space R^n
- Dimension of Null space is ‘n-r’
- For a system of linear equations to be nonsingular and matrix A to be invertible N (A) = 0
- Special solutions are the basis of N(A) .

NULL SPACE

Example :

Let $A = \begin{bmatrix} 1 & 0 \\ 5 & 4 \\ 2 & 3 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 \\ 5 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(x, y both are pivot variables)

No Free variables.

Then gives x = y = 0 as the only solution.

The null space of this matrix thus contains only the zero vector (0, 0).

Null space of this matrix is 'origin' in R^2.

NULL SPACE

$$\begin{bmatrix} 1 & 0 & 1 \\ 5 & 4 & 9 \\ 2 & 3 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 4 & 4 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 0 \end{bmatrix}$$

$(x = -y = z)$ hence

Gives infinitely many solutions (c, -c c) all of which lie on a line that obviously passes through the origin.

The matrices $A = \begin{bmatrix} 1 & 0 & 1 \\ 5 & 4 & 9 \\ 2 & 3 & 5 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & 0 \\ 5 & 4 \\ 2 & 3 \end{bmatrix}$

have the same column space but different null space !!

THANK YOU

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