LINEAR ALGEBRA

UE19MA251

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Department of Science and Humanities

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Agenda – Problems on SVD

- Example 2

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Example 2. Find the SVD for $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & 2 \end{bmatrix}$

Soln: SVD: $A = U \Sigma V^T$

$$AA^T = \begin{bmatrix} 9 & -1 \\ -1 & 9 \end{bmatrix}$$

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Eigenvalues of AA^T are

$\lambda_1 = 10$, & (arranged in increasing order)

Corresponding Eigenvectors are

$x_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

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Clearly X_1 and X_2 are orthogonal.

$$V = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} = V^T$$

$$\Sigma = \begin{bmatrix} \sqrt{10} & 0 \\ 0 & \sqrt{8} \\ 0 & 0 \end{bmatrix}$$

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$$u_i = \frac{A v_i}{\sigma_i} \quad ; \quad i \quad 1, 2, 3 \; ; \; \sigma_1 = \sqrt{10} \; , \; \sigma_2 = \sqrt{8}$$

$$u_1 = \begin{bmatrix} -1/\sqrt{5} \\ 2/\sqrt{5} \\ 0 \end{bmatrix} \qquad u_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$u_1^T u_2 = 0 \quad ; \quad u_1 \perp u_2 .$$

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Using orthogonality condition, u_3 can be obtained. Let $u_3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$\begin{bmatrix} -1/\sqrt{5} & 2/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$\therefore \quad u' = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \text{ normalizing, we get}$$

$$u_3 = \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{bmatrix} \; ; \quad U = \begin{bmatrix} -1/\sqrt{5} & 0 & 2/\sqrt{5} \\ 2/\sqrt{5} & 0 & 1/\sqrt{5} \\ 0 & 1 & 0 \end{bmatrix}$$

$$\therefore \quad A = U \Sigma V^T$$

THANK YOU

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