LINEAR ALGEBRA AND ITS APPLICATIONS

Algebra A(V) of Linear operators

Let V be a vector space over a field K. The linear mappings of the form $T: V \to V$ are called linear operators and linear transformations on V .

Note:

1. If dim V = n then dim A(V) = n^2.
2. For any mapping F ,G from A(V), the composition G.F exists and also belongs to A(V).

Algebra A(V) of Linear operators

Remark : An algebra a over a field K is a vector space over K in which an operation of multiplication is defined satisfying , for F, G , H ∈ A and every k ∈ K

1 . F (G+H) = FG + FH

2 . (G+H) F = GF + HF

3 . K(GF) = (kG)F = G(kF)

Invertible Maps and Isomorphism

A mapping $f : A \rightarrow B$ is said to be *One-to-one* or 1-1 or *injective* if different elements of A have distinct images; that is

IF $a \neq a', then\ f(a) \neq f(a')$.

Equivalently,

IF $a = a', then\ f(a) = f(a')$.

A mapping f: $A \rightarrow B$ is said to be *onto* or *surjective* if every $b \in B$ is the image of at least one $a \in A$.

A mapping $f : A \rightarrow B$ is said to be *One-to-one correspondence* between A and B or bijective if f is both *one-to-one and onto.*

Invertible Maps and Isomorphism

A mapping $f : A \to B$ is said to be invertible if f is one-to-one and onto.

Example: Let $f: R \to R$ be defined by $f(x) = 2x - 3$. Now f is one-to-one and onto.

Let y be the image of x under the mapping f, i.e., y =2x-3.

Interchange x and y to obtain $x = 2y - 3 \Rightarrow y = \frac{x+3}{2} \Rightarrow f^{-1} = \frac{x+3}{2}$.

Invertible Maps and Isomorphism

A mapping $F: V \rightarrow U$ is called isomorphism if F is linear and bijective, i.e., one-to-one and onto.

Example : The mapping $T: R^2 \rightarrow R^2$ is defined as $T(x, y) = (x + 4y, y - 3x)$ is isomorphism.