LINEAR ALGEBRA AND ITS APPLICATIONS
UE19MA251

Unit 3. Linear Transformations and Orthogonality

Transformations Represented by Matrices

The matrix of a linear transformation is a matrix for which T(x)=Ax , for a vector ‘x’ in the domain T. Such matrix is called standard matrix for the transformation.

Note:

1.Such matrix can be found for any Linear transformation T from R^n to R^m.

2. Standard basis for R^n = $\left\{ e_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \cdots e_n = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} \right\}$

e_i's are columns of Identity matrix of order 'n'.

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The standard matrix of transformation $T: \mathbb{R}^n \to \mathbb{R}^m$ has columns $T(e_1), T(e_2), \cdots T(e_n)$ where $e_1, e_2, \cdots e_n$ represents the standard basis, i.e

$$T(x) = Ax \iff A = [T(e_1), T(e_2) \cdots T(e_n)]$$

Transformations Represented by Matrices

Examples: $T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{pmatrix} x_1 - x_2 \\ 2x_3 \end{pmatrix}$

Here $T : \mathbb{R}^3 \to \mathbb{R}^2$.

Basis for $\mathbb{R}^3 = \left\{ e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \ e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \ e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

$$T(e_1) = T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1-0 \\ 2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$T(e_2) = T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0-1 \\ 2(0) \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$T(e_3) = T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0-0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

Transformations Represented by Matrices

The standard matrix for T is

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Verification!.

$$Ax = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_2 \\ 2x_3 \end{bmatrix}$$

Compare this to the rule for T from the problem

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 \\ 2x_3 \end{pmatrix}.$$

Transformations Represented by Matrices

Matrix Representation of Differentiation:

- Consider differentiation that goes from P_3 to P_2.

$$P_3 = \{ p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3,\ a_0, a_1, a_2, a_3 \in R \}$$

Basis is $\{ v_1 = 1, v_2 = t, v_3 = t^2, v_4 = t^3 \}$

$$P_2 = \{ q(t) = b_0 + b_1 t + b_2 t^2,\ b_0, b_1, b_2 \in R \}$$

Basis is $\{ u_1 = 1, u_2 = t, u_3 = t^2 \}$

$$\frac{d}{dt} = A_{diff} : P_3 \to P_2$$

Transformations Represented by Matrices

Matrix Representation of Differentiation:

To find A_{diff}.

$$\frac{d}{dt}(v_1) = \frac{d}{dt}(1) = 0 = 0\cdot u_1 + 0u_2 + 0u_3 \rightarrow (0,0,0)$$

$$\frac{d}{dt}(v_2) = \frac{d}{dt}(t) = 1 = 1\cdot u_1 + 0\cdot u_2 + 0\cdot u_3 \rightarrow (1,0,0)$$

$$\frac{d}{dt}(v_3) = \frac{d}{dt}(t^2) = 2t = 0\cdot u_1 + 2\cdot u_2 + 0\cdot u_3 \rightarrow (0,2,0)$$

$$\frac{d}{dt}(v_4) = \frac{d}{dt}(t^3) = 3t^2 = 0\cdot u_1 + 0\cdot u_2 + 3u_3 \rightarrow (0,0,3)$$

Transformations Represented by Matrices

We thus get the matrix of differentiation as

$$A_{diff} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}_{3x4}$$

Transformations Represented by Matrices

Verification: Let $p(t) = 3 + 6t - 7t^2 + 2t^3$

$$x = \begin{pmatrix} 3 \\ 6 \\ -7 \\ 2 \end{pmatrix}$$

$$A_{diff}(x) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \begin{pmatrix} 3 \\ 6 \\ -7 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -14 \\ 6 \end{pmatrix}$$

$$\frac{d}{dt}\left[p(t)\right] = 6 - 7\cdot 2\cdot t - 2\cdot 3\, t^2 \rightarrow \begin{pmatrix} 6 \\ -14 \\ 6 \end{pmatrix}$$

THANK YOU