PES UNIVERSITY ONLINE

LINEAR ALGEBRA AND ITS APPLICATIONS

Renna Sultana
Department of Science and Humanities

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MATRICES AND GAUSSIAN ELIMINATION

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Course Content: Supplementary problems

1. Find PA=LU and PA=LDU for A= $\begin{pmatrix} 3 & -1 & 0 \\ 6 & -2 & 0 \\ 1 & 1 & 2 \end{pmatrix}$

$$A=\begin{pmatrix} 3 & -1 & 0 \\ 6 & -2 & 0 \\ 1 & 1 & 2 \end{pmatrix} \xrightarrow[R_3-1/3R_1]{R_2-2R_1} \begin{pmatrix} 3 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 4/3 & 2 \end{pmatrix} \xrightarrow{R_3 \leftrightarrow R_2} \begin{pmatrix} 3 & -1 & 0 \\ 0 & 4/3 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$PA=\begin{pmatrix} 3 & -1 & 0 \\ 1 & 1 & 2 \\ 6 & -2 & 0 \end{pmatrix} \xrightarrow[R_3-2R_1]{R_2-1/3R_1} \begin{pmatrix} 3 & -1 & 0 \\ 0 & 4/3 & 2 \\ 0 & 0 & 0 \end{pmatrix} = U$$

$$PA=\begin{pmatrix} 1 & 0 & 0 \\ 1/3 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}\begin{pmatrix} 3 & -1 & 0 \\ 0 & 4/3 & 2 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1/3 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 & 0 \\ 0 & 4/3 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & -1/3 & 0 \\ 0 & 1 & 3/2 \\ 0 & 0 & 0 \end{pmatrix}$$

MATRICES AND GAUSSIAN ELIMINATION:

2. For what values of a and b does the following system have (i)a trivial solution

(ii) Infinitely many solutions.

$$x+2y+3z=0$$
$$-x-2y+az=0$$
$$2x+by+6z=0$$

$$\begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & a \\ 2 & b & 6 \end{pmatrix} \xrightarrow[R_3-2R_1]{R_2+R_1} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & a+3 \\ 0 & b-4 & 0 \end{pmatrix} \xrightarrow{R_3 \leftrightarrow R_2} \begin{pmatrix} 1 & 2 & 3 \\ 0 & b-4 & 0 \\ 0 & 0 & a+3 \end{pmatrix}$$

This system will have only either a trivial solution or infinitely many solutions. It will have

(i) **a trivial solution** if $a \neq -3 \ \& \ b \neq 4$ then r(A)=3=n

(ii) **Infinitely many solutions** if $a=-3 \text{ or } b=4 \text{ or both}$ then r(A) will be 2 or 1 respectively.

GAUSSIAN ELIMINATION:

3. Check for consistency and solve the following system of equations if consistent. Also discuss its rank:

$$3x+y+2z=3$$
$$2x-3y-z=-3$$
$$x+2y+z=4$$

$$\begin{pmatrix} 3 & 1 & 2:3 \\ 2 & -3 & -1:-3 \\ 1 & 2 & 1:4 \end{pmatrix} \xrightarrow[R_3-\left(\frac{1}{3}\right)R_1]{R_2-\left(\frac{2}{3}\right)R_1} \begin{pmatrix} 3 & 1 & 2:3 \\ 0 & -11/3 & -7/3:-5 \\ 0 & 5/3 & -1/3:3 \end{pmatrix}$$

$$\xrightarrow{R_3+\left(\frac{5}{11}\right)R_2} \begin{pmatrix} 3 & 1 & 2:3 \\ 0 & -11/3 & -7/3:-5 \\ 0 & 0 & -24/33:8/11 \end{pmatrix} \Rightarrow \begin{cases} 3x+y+2z=3 \\ (-11/3)y - 7/3z=-5 \\ (-24/33)z=8/11 \end{cases}$$

r(A)=r(A:b)=3=n hence system is **consistent** and has **a unique solution**.

i.e **(x, y, z)=(1, 2,-1). Its rank is 3.**

4. Find an LU and LDU factorization for A. What is the rank of A?

$$A=\begin{pmatrix} 2 & 4 & -1 & 5 & -2 \\ -4 & -5 & 3 & -8 & 1 \\ 2 & -5 & -4 & 1 & 8 \\ -6 & 0 & 7 & -3 & 1 \end{pmatrix} \xrightarrow[\substack{R_3-R_1 \\ R_4+3R_1}]{R_2+2R_1} \begin{pmatrix} 2 & 4 & -1 & 5 & -2 \\ 0 & 3 & 1 & 2 & -3 \\ 0 & -9 & -3 & -4 & 10 \\ 0 & 12 & 4 & 12 & -5 \end{pmatrix} \xrightarrow[R_4-4R_2]{R_3+3R_2} \begin{pmatrix} 2 & 4 & -1 & 5 & -2 \\ 0 & 3 & 1 & 2 & -3 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 4 & 7 \end{pmatrix}$$

$$\xrightarrow{R_4-2R_3} \begin{pmatrix} 2 & 4 & -1 & 5 & -2 \\ 0 & 3 & 1 & 2 & -3 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 5 \end{pmatrix} \quad A = LU = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & -3 & 1 & 0 \\ -3 & 4 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 4 & -1 & 5 & -2 \\ 0 & 3 & 1 & 2 & -3 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 5 \end{pmatrix}$$

$$A = LDU = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & -3 & 1 & 0 \\ -3 & 4 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & -1/2 & 5/2 & -1 \\ 0 & 1 & 1/3 & 2/3 & -1 \\ 0 & 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Rank of A is 4.

MATRICES AND GAUSSIAN ELIMINATION:

5. Solve Ax=b for x if $A=\begin{pmatrix} 1 & 0 & -2 \\ 2 & 1 & 3 \\ 4 & 2 & 5 \end{pmatrix}$ and $b=\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

$$[A:I]=\begin{pmatrix} 1 & 0 & -2 & : & 1 & 0 & 0 \\ 2 & 1 & 3 & : & 0 & 1 & 0 \\ 4 & 2 & 5 & : & 0 & 0 & 1 \end{pmatrix} \xrightarrow[R_3-4R_1]{R_2-2R_1} \begin{pmatrix} 1 & 0 & -2 & : & 1 & 0 & 0 \\ 0 & 1 & 7 & : & -2 & 1 & 0 \\ 0 & 2 & 13 & : & -4 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_3-2R_2} \begin{pmatrix} 1 & 0 & -2 & : & 1 & 0 & 0 \\ 0 & 1 & 7 & : & -2 & 1 & 0 \\ 0 & 0 & -1 & : & 0 & -2 & 1 \end{pmatrix} \xrightarrow[R_2+7R_3]{R_1-2R_3} \begin{pmatrix} 1 & 0 & 0 & : & 1 & 4 & -2 \\ 0 & 1 & 0 & : & -2 & -13 & 7 \\ 0 & 0 & -1 & : & 0 & -2 & 1 \end{pmatrix}$$

$$\xrightarrow{-R_3} \begin{pmatrix} 1 & 0 & 0 & : & 1 & 4 & -2 \\ 0 & 1 & 0 & : & -2 & -13 & 7 \\ 0 & 0 & 1 & : & 0 & 2 & -1 \end{pmatrix} \quad A^{-1}=\begin{pmatrix} 1 & 4 & -2 \\ -2 & -13 & 7 \\ 0 & 2 & -1 \end{pmatrix} \quad x=\begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix}$$

THANK YOU

Renna Sultana
Department of Science and Humanities

rennasultana@pes.edu