LINEAR ALGEBRA AND ITS APPLICATIONS

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MATRICES AND GAUSSIAN ELIMINATION

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GAUSSIAN ELIMINATION:

1.Check for consistency and solve the following system of equations if consistent:

(i)
$$\begin{aligned} x_1 + x_2 - 2x_3 + 3x_4 &= 4 \\ 2x_1 + 3x_2 + 3x_3 - x_4 &= 3 \\ 5x_1 + 7x_2 + 4x_3 + x_4 &= 5 \end{aligned}$$

$$[A:b] = \begin{pmatrix} 1 & 1 & -2 & 3:4 \\ 2 & 3 & 3 & -1:3 \\ 5 & 7 & 4 & 1:5 \end{pmatrix} \xrightarrow{\substack{R_2 - 2R_1 \\ R_3 - 5R_1}}$$

$$\begin{bmatrix} 1 & 1 & -2 & 3 & : 4 \\ 0 & 1 & 7 & -7 & : -5 \\ 0 & 2 & 14 & -14 & : -15 \end{bmatrix} \xrightarrow{R_3 - 2R_2} \begin{bmatrix} 1 & 1 & -2 & 3 & : 4 \\ 0 & 1 & 7 & -7 & : -5 \\ 0 & 0 & 0 & 0 & : -5 \end{bmatrix}$$

This gives 0=-5 which is not possible.
Also r(A)=2 and r[A:b]=3
System is **inconsistent** and has **no solution**

GAUSSIAN ELIMINATION:

(ii)
$$\begin{aligned} x_1 + 2x_2 + x_3 &= 3 \\ 2x_1 + 5x_2 - x_3 &= -4 \\ 3x_1 - 2x_2 - x_3 &= 5 \end{aligned}$$

$$[A:b] = \begin{pmatrix} 1 & 2 & 1: & 3 \\ 2 & 5 & -1: & -4 \\ 3 & -2 & -1: & 5 \end{pmatrix}$$

$$\xrightarrow[R_3-3R_1]{R_2-2R_1} \begin{bmatrix} 1 & 2 & 1: & 3 \\ 0 & 1 & -3: & -10 \\ 0 & -8 & -4: & -4 \end{bmatrix} \xrightarrow{R_3+8R_2} \begin{bmatrix} 1 & 2 & 1: & 3 \\ 0 & 1 & -3: & -10 \\ 0 & 0 & -28: & -84 \end{bmatrix}$$

$$\Rightarrow \begin{cases} x_1 + 2x_2 + x_3 = 3 \\ x_2 - 3x_3 = -10 \\ -28x_3 = -84 \end{cases}$$

r(A)=r[A:b]=3=n. System is consistent and has a unique solution.

$$(x_1, x_2, x_3) = (2, -1, 3)$$

GAUSSIAN ELIMINATION:

(iii)
$$2x - 3y + 2z = 1$$
$$5x - 8y + 7z = 1$$
$$y - 4z = 3$$

$$\begin{pmatrix} 2 & -3 & 2:1 \\ 5 & -8 & 7:1 \\ 0 & 1 & -4:3 \end{pmatrix} \xrightarrow{R_2 - \left(\frac{5}{2}\right)R_1} \begin{pmatrix} 2 & -3 & 2:\ 1 \\ 0 & -1/2 & 2:-3/2 \\ 0 & 1 & -4:\ 3 \end{pmatrix} \xrightarrow{R_3 + 2R_2} \begin{pmatrix} 2 & -3 & 2:\ 1 \\ 0 & -1/2 & 2:-3/2 \\ 0 & 0 & 0:\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} 2x - 3y + 2z = 1 \\ -(1/2)y + 2z = -3/2 \end{cases}$$

r(A)=r(A:b)=2<n(=3) hence system is **consistent** and has **infinite number of solutions**.
i.e **(x, y, z)=(5k+5, 4k-+3, k)**

GAUSSIAN ELIMINATION:

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2. Find all values of a for which the resulting linear system has (a)no solution (b)a unique solution and (c) infinitely many solutions:

$$x + y - z = 2$$
$$x + 2y + z = 3$$
$$x + y + (a^2 - 5)z = a$$

$$[A:b] = \begin{pmatrix} 1 & 1 & -1 : 2 \\ 1 & 2 & 1 : -3 \\ 1 & 1 & a^2 - 5 : a \end{pmatrix} \xrightarrow{R_2 - R_1,\ R_3 - R_1} \begin{pmatrix} 1 & 1 & -1 : 2 \\ 0 & 1 & 2 : -5 \\ 0 & 0 & a^2 - 4 : a - 2 \end{pmatrix}$$

(a) System has **no solution** if $a = -2$ (when r(A) $\neq$ r(A:b))

(b) System has **a unique solution** if $a \neq \pm 2$ (when r(A) = r(A:b)=3=n)

(c) System has **infinitely many solutions** if $a = 2$ (when r(A) = r(A:b)=2<n)

GAUSSIAN ELIMINATION:

3. Find an equation relating a, b and c so that the linear system
$$\begin{aligned} x + 2y - 3z &= a \\ 2x + 3y + 3z &= b \\ 5x + 9y - 6z &= c \end{aligned}$$
is consistent for any values of a, b and c that satisfy that equation. When (a,b,c)=(2,3,9), then what is the solution of the system.

$$\left(\begin{array}{ccc:c} 1 & 2 & -3 & a \\ 2 & 3 & 3 & b \\ 5 & 9 & -6 & c \end{array}\right) \xrightarrow[R_3-5R_1]{R_2-2R_1} \left(\begin{array}{ccc:c} 1 & 2 & -3 & a \\ 0 & -1 & 9 & b-2a \\ 0 & -1 & 9 & c-5a \end{array}\right)$$

$$\xrightarrow{R_3-R_2} \left(\begin{array}{ccc:c} 1 & 2 & -3 & a \\ 0 & -1 & 9 & b-2a \\ 0 & 0 & 0 & c-b-3a \end{array}\right)$$

The given linear system will be consistent if a,b,c satisfy the relation c-b-3a=0.

When (a, b, c)=(2, 3, 9), then the solution of the system is **(x, y, z)=(-15k, 9k+1, k)**

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GAUSSIAN ELIMINATION:

Linear System of Equations

Consistent

Solution Exists (rank(A)=rank[A:b}=r)

Unique solution (rank(A)=rank[A:b}=r=n)
(Non-Singular Matrix i.e $|A| \neq 0$)
$\left(A^{-1} exists\right)$

Infinite number of solutions. (rank(A)=rank[A:b}=r<n)
(Singular Matrix i.e $|A| = 0$)

Inconsistent

No Solution (rank(A) $\neq$rank[A:b})
(Singular Matrix i.e $|A| = 0$)

THANK YOU

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