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MATRICES AND GAUSSIAN ELIMINATION

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GAUSSIAN ELIMINATION:

1. Check for consistency and solve the following system of equations if consistent:

(i)
$$x_1 + x_2 - 2x_3 + 3x_4 = 4$$

 $2x_1 + 3x_2 + 3x_3 - x_4 = 3$ $[A:b] =$

$$\begin{cases}
1 & 1 & -2 & 3:4 \\
2 & 3 & 3 & -1:3 \\
5 & 7 & 4 & 1:5
\end{cases}$$
 $\xrightarrow{R_3 - 5R_1}$

This gives 0=-5 which is not possible.

Also r(A)=2 and r[A:b]=3

System is inconsistent and has no solution



GAUSSIAN ELIMINATION:

(ii)
$$x_1 + 2 x_2 + x_3 = 3$$

 $2 x_1 + 5 x_2 - x_3 = -4$
 $3 x_1 - 2 x_2 - x_3 = 5$

$$\begin{bmatrix} A:b \end{bmatrix} = \begin{pmatrix} 1 & 2 & 1: & 3 \\ 2 & 5 & -1: & -4 \\ 3 & -2 & -1: & 5 \end{pmatrix}$$

$$\downarrow R_3 - 3R_1$$

$$\Rightarrow \begin{cases} x_1 + 2x_2 + x_3 = 3 \\ x_2 - 3x_3 = -10 \\ -28x_3 = -84 \end{cases}$$
 r(A)=r[A:b]=3=n. System is consistent a unique solution.

$$[A:b] = \begin{pmatrix} 1 & 2 & 1: & 3 \\ 2 & 5 & -1: -4 \\ 3 & -2 & -1:5 \end{pmatrix}$$

$$\downarrow \xrightarrow{R_2 - 2R} \xrightarrow{R_2 - 3R_1} \xrightarrow{R_1} \xrightarrow{R_2 - 3R_2} \xrightarrow{R_2 - 3R_1} \xrightarrow{R_2 - 3R_2} \xrightarrow{R_2 -$$

r(A)=r[A:b]=3=n. System is **consistent** and has

$$(x_1, x_2, x_3) = (2, -1, 3)$$



GAUSSIAN ELIMINATION:



(iii)
$$2x - 3y + 2z = 1$$

 $5x - 8y + 7z = 1$
 $y - 4z = 3$

$$\begin{pmatrix}
2 & -3 & 2:1 \\
5 & -8 & 7:1 \\
0 & 1 & -4:3
\end{pmatrix}$$

$$\begin{pmatrix}
2 & -3 & 2:1 \\
0 & -1/2 & 2:-3/2 \\
0 & 1 & -4:3
\end{pmatrix}$$

$$\begin{pmatrix}
2 & -3 & 2:1 \\
0 & -1/2 & 2:-3/2 \\
0 & 1 & -4:3
\end{pmatrix}$$

$$\begin{pmatrix}
2 & -3 & 2:1 \\
0 & -1/2 & 2:-3/2 \\
0 & 0:0
\end{pmatrix}$$

$$\Rightarrow \begin{cases} |2x - 3y + 2z = 1 \\ -(1/2)y + 2z = -3/2 \end{cases}$$

r(A)=r(A:b)=2< n(=3) hence system is **consistent** and has **infinite number of solutions**. i.e (x, y, z)=(5k+5, 4k-+3, k)

GAUSSIAN ELIMINATION:

2. Find all values of a for which the resulting linear system has (a)no solution (b)a unique solution and (c) infinitely many solutions:

$$x + y - z = 2$$

$$x + 2y + z = 3$$

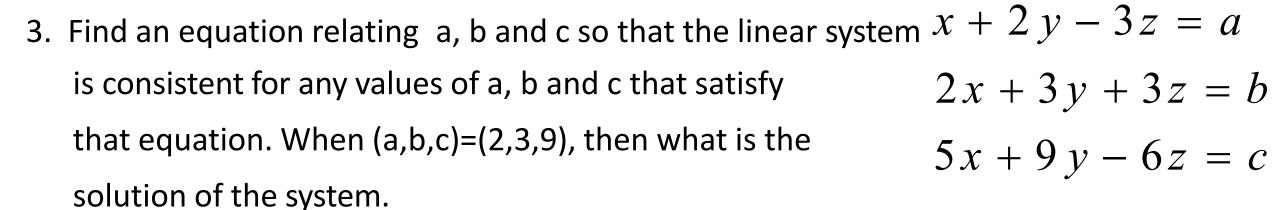
$$x + y + (a^{2} - 5)z = a$$

$$[A:b] = \begin{pmatrix} 1 & 1 & -1:2 \\ 1 & 2 & 1:-3 \\ 1 & 1 & a^{2}-5:a \end{pmatrix} \qquad \downarrow \begin{array}{c} R_{2} - R_{1} \\ R_{3} - R_{1} \end{array} \qquad \begin{pmatrix} 1 & 1 & -1:2 \\ 0 & 1 & 2:-5 \\ 0 & 0 & a^{2}-4:a-2 \end{pmatrix}$$

- (a) System has no solution if $\alpha = -2$ (when r(A) \neq r(A:b))
- (b) System has a unique solution if $\alpha \neq \pm 2$ (when r(A) = r(A:b)=3=n)
- (c) System has **infinitely many solutions** if $\alpha = 2$ (when r(A) = r(A:b)=2<n)



GAUSSIAN ELIMINATION:



$$\begin{pmatrix} 1 & 2 & -3: a \\ 2 & 3 & 3: b \\ 5 & 9 & -6: c \end{pmatrix} \qquad \downarrow \xrightarrow{R_2 - 2R} \qquad \begin{pmatrix} 1 & 2 & -3: a \\ 0 & -1 & 9: b-2a \\ 0 & -1 & 9: c-5a \end{pmatrix}$$

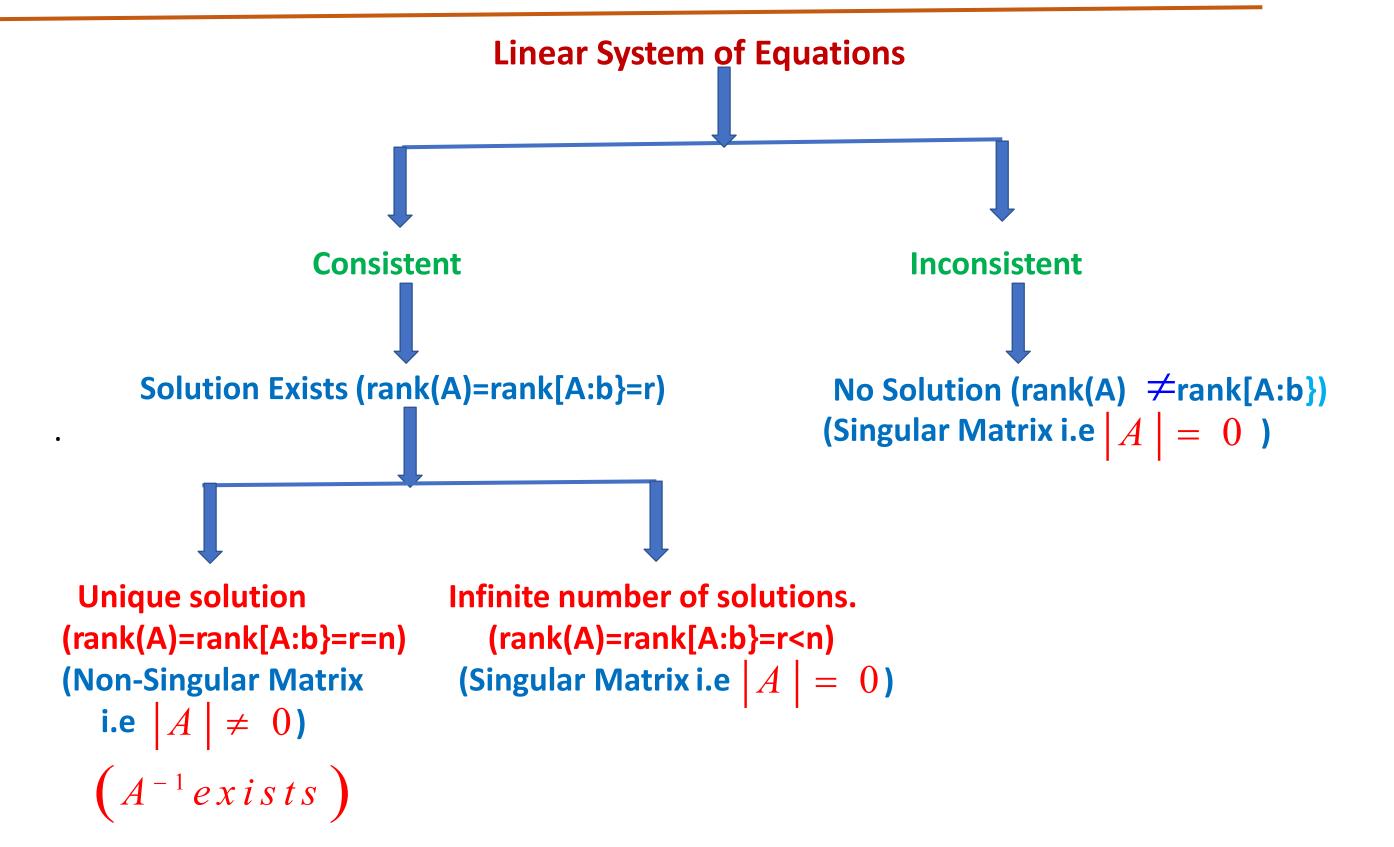
$$\downarrow R \downarrow R_2 \downarrow \qquad \begin{pmatrix} 1 & 2 & -3: & a \\ 0 & -1 & 9:b-2a \\ 0 & 0 & 0:c-b-3a \end{pmatrix}$$

The given linear system will be **consistent** if a,b,c satisfy the relation c-b-3a=0.

When (a, b, c)=(2, 3, 9), then the solution of the system is (x, y, z)=(-15k, 9k+1, k)

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GAUSSIAN ELIMINATION:







THANK YOU

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