



# LINEAR ALGEBRA AND ITS APPLICATIONS

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## MATRICES AND GAUSSIAN ELIMINATION

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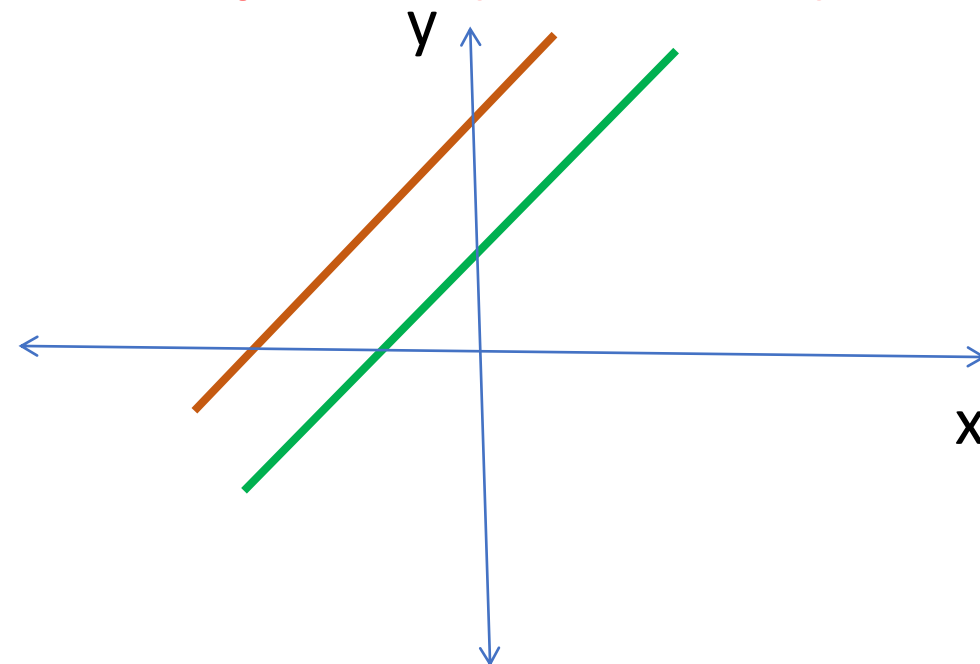
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### Course Content: Singular Cases

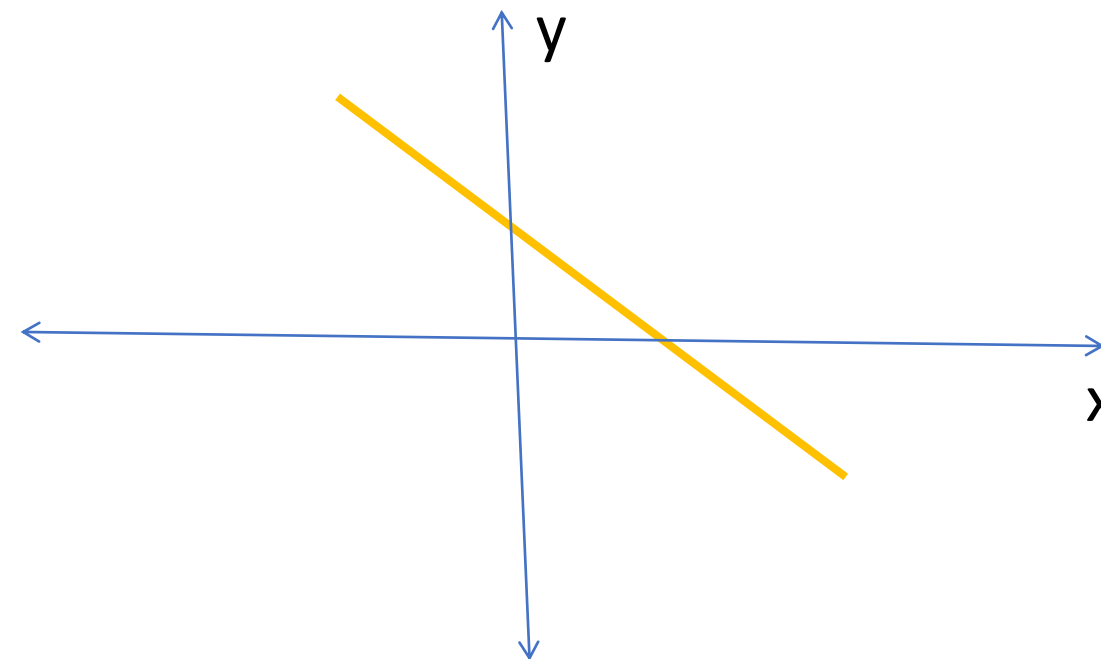
- ❖ **SINGULAR CASES in Two dimensions:** A system of linear equations is said to be singular ( $|A| = 0$ ) if it has no solution or has infinite number of solutions.
- **(i) ROW PICTURE (Two dimensions):** In two dimensions the lines are parallel if they have no solution and coincident if they have infinite number of solutions. In such a case the matrix A will have dependent row /column and  $\det(A)=0$ . Such a matrix is called Singular Matrix.

(2 lines):

**Lines parallel (no solution)**



**Lines coincident (infinite no. of solutions)**

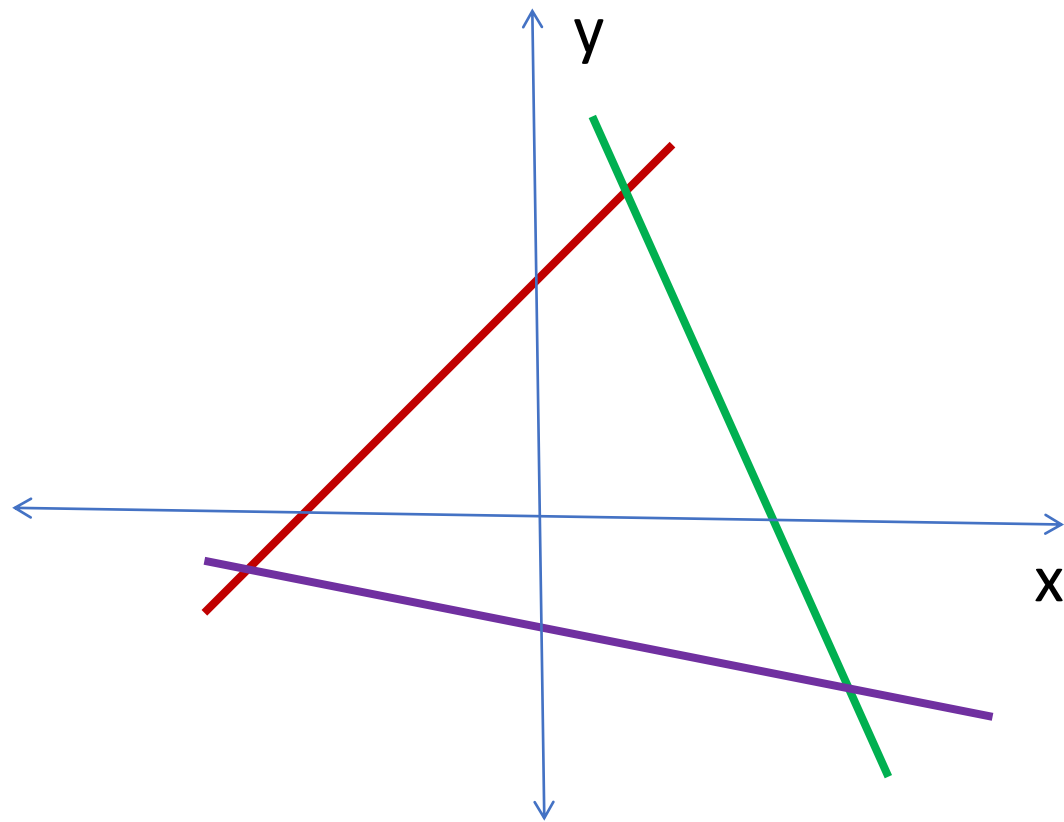


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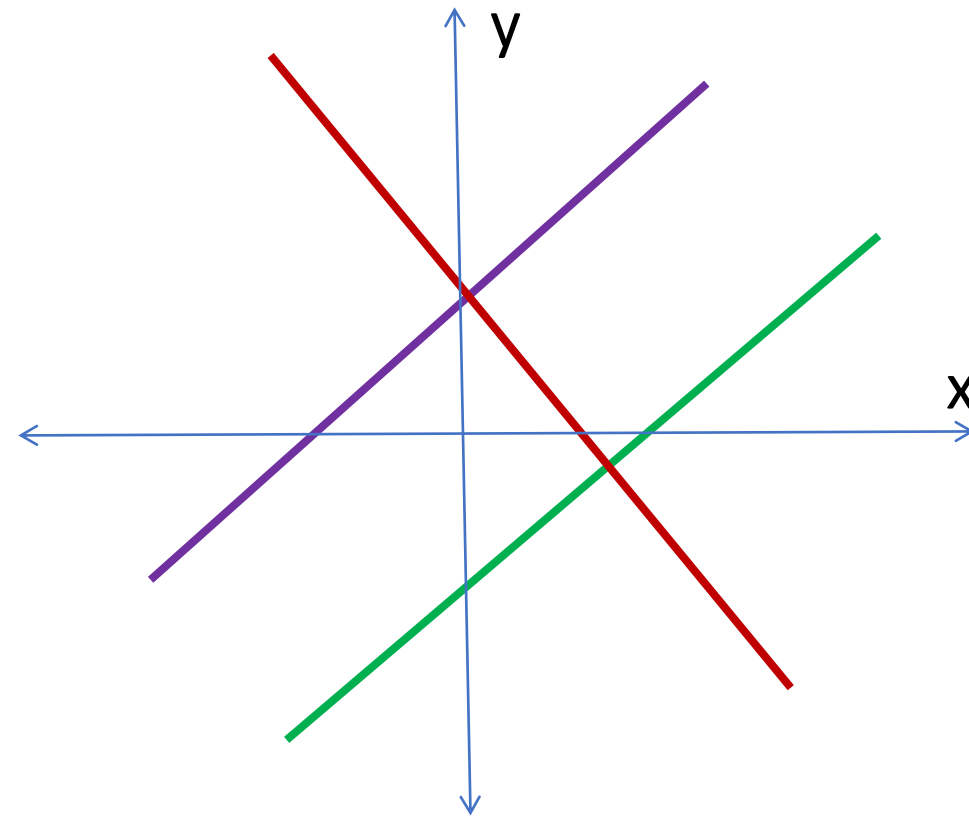
## THE GEOMETRY OF LINEAR EQUATIONS:

(3 lines):

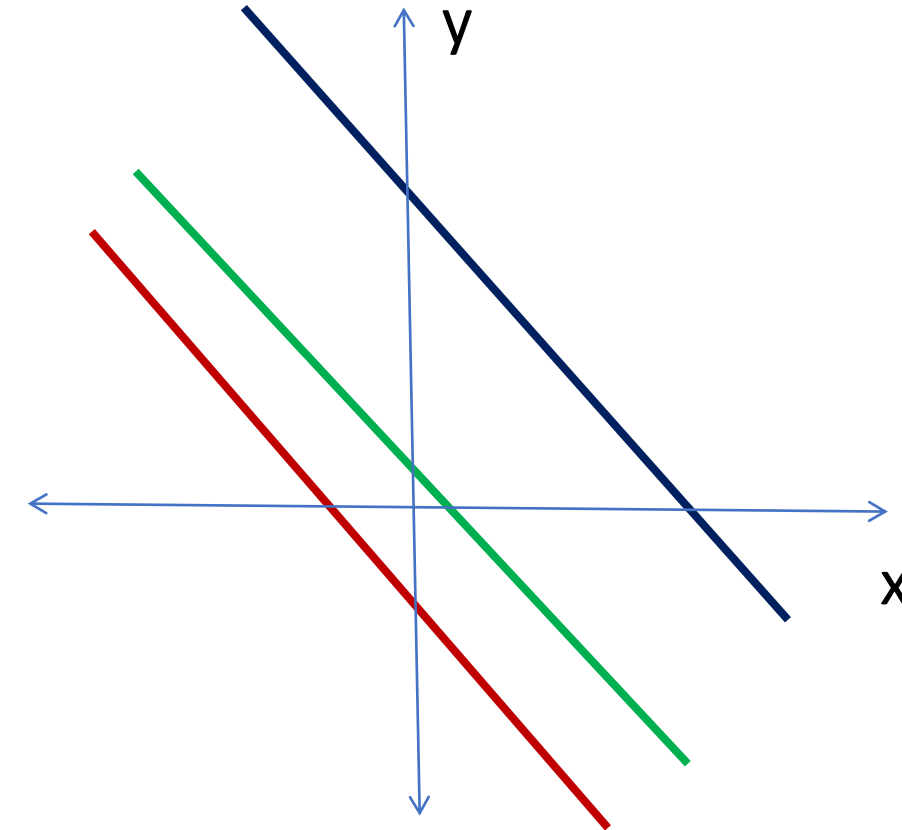
Lines intersecting in pairs  
(no solution)



2 Lines parallel & one  
intersecting (no solution)



All 3 parallel lines  
(no solution)



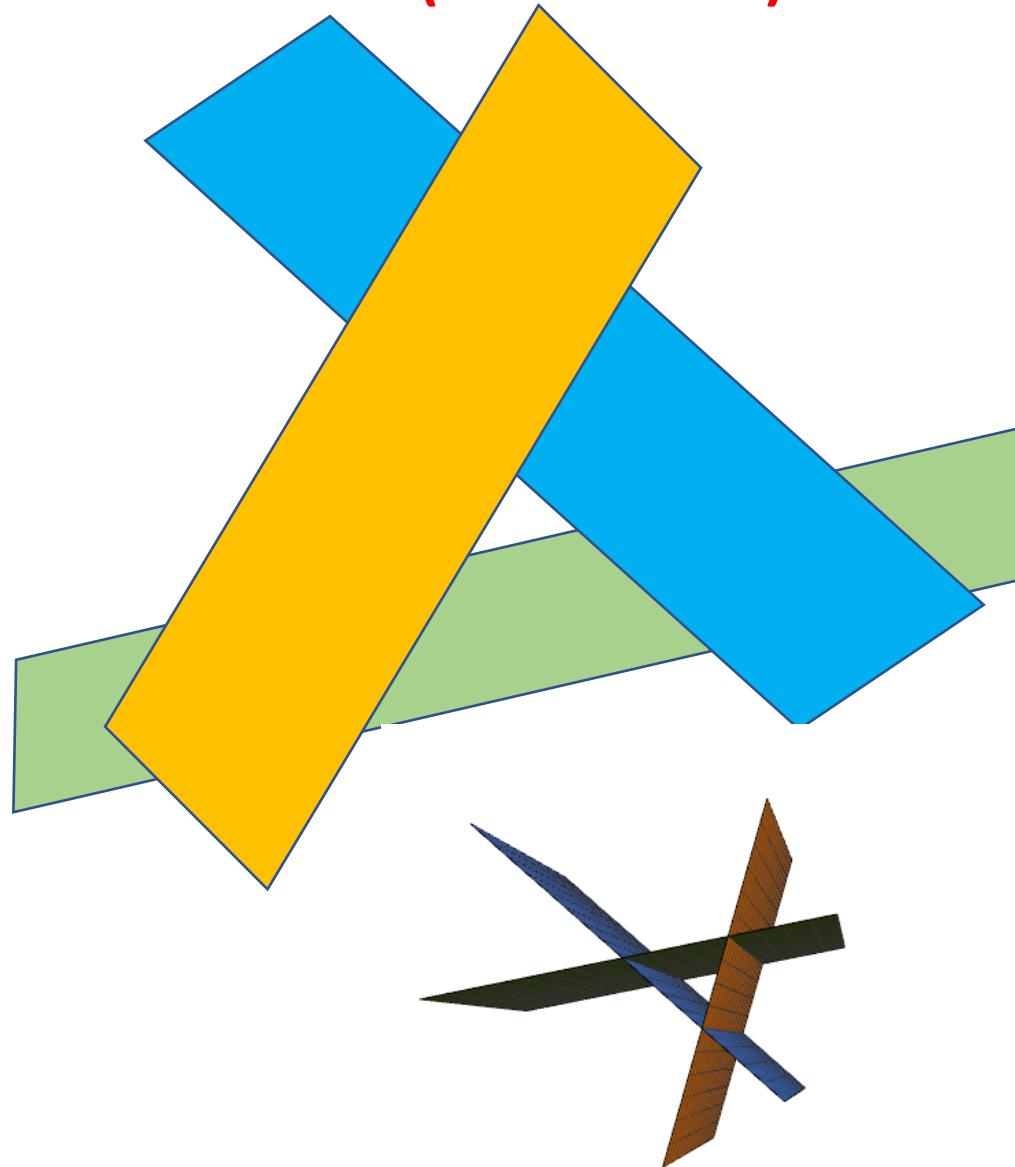
# LINEAR ALGEBRA AND ITS APPLICATIONS

## THE GEOMETRY OF LINEAR EQUATIONS:

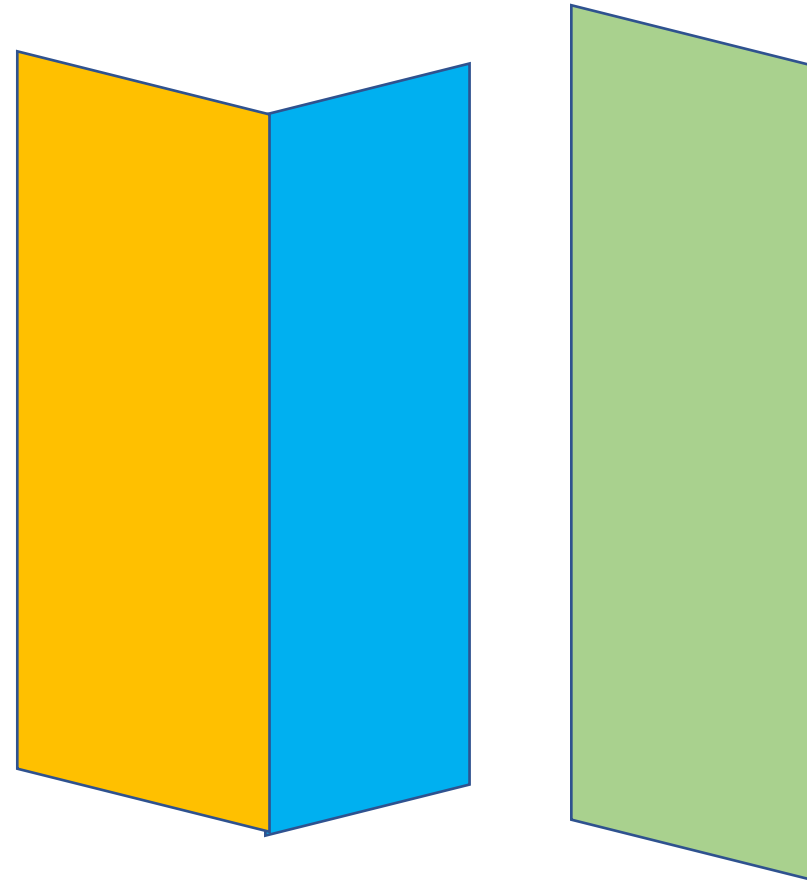
- **ROW PICTURE(Three dimensions):** In three dimensions if the 3 Planes do not intersect then we have the following cases :

**(3 planes):**

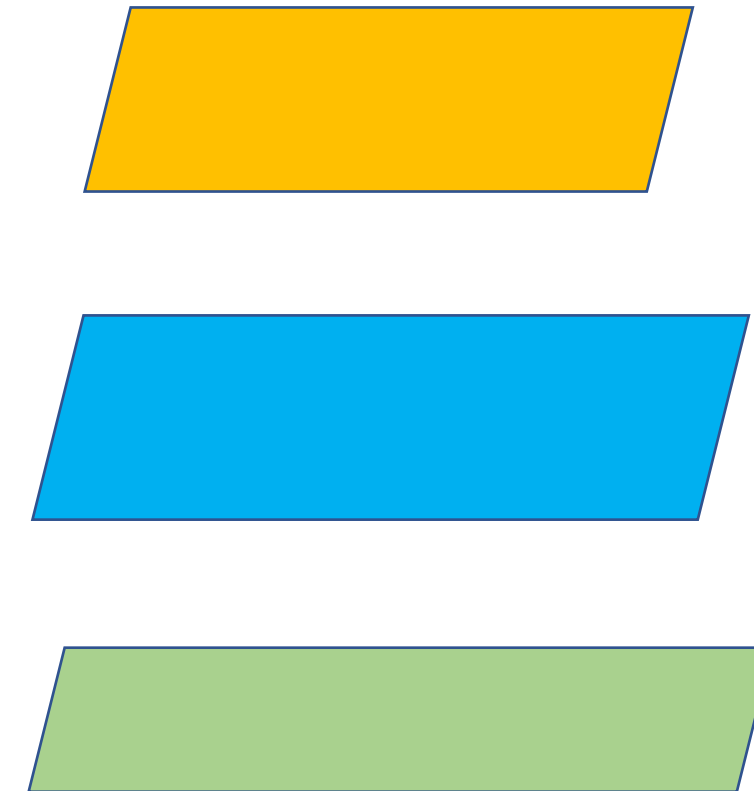
**Every pair of planes intersecting in a line (no solution)**



**2 planes intersecting in a line and 3<sup>rd</sup> is parallel to this line (no solution)**



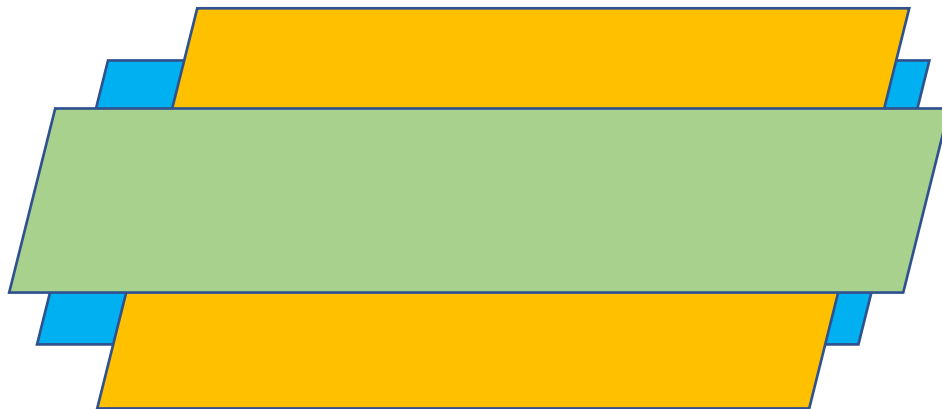
**All 3 parallel planes (no solution)**



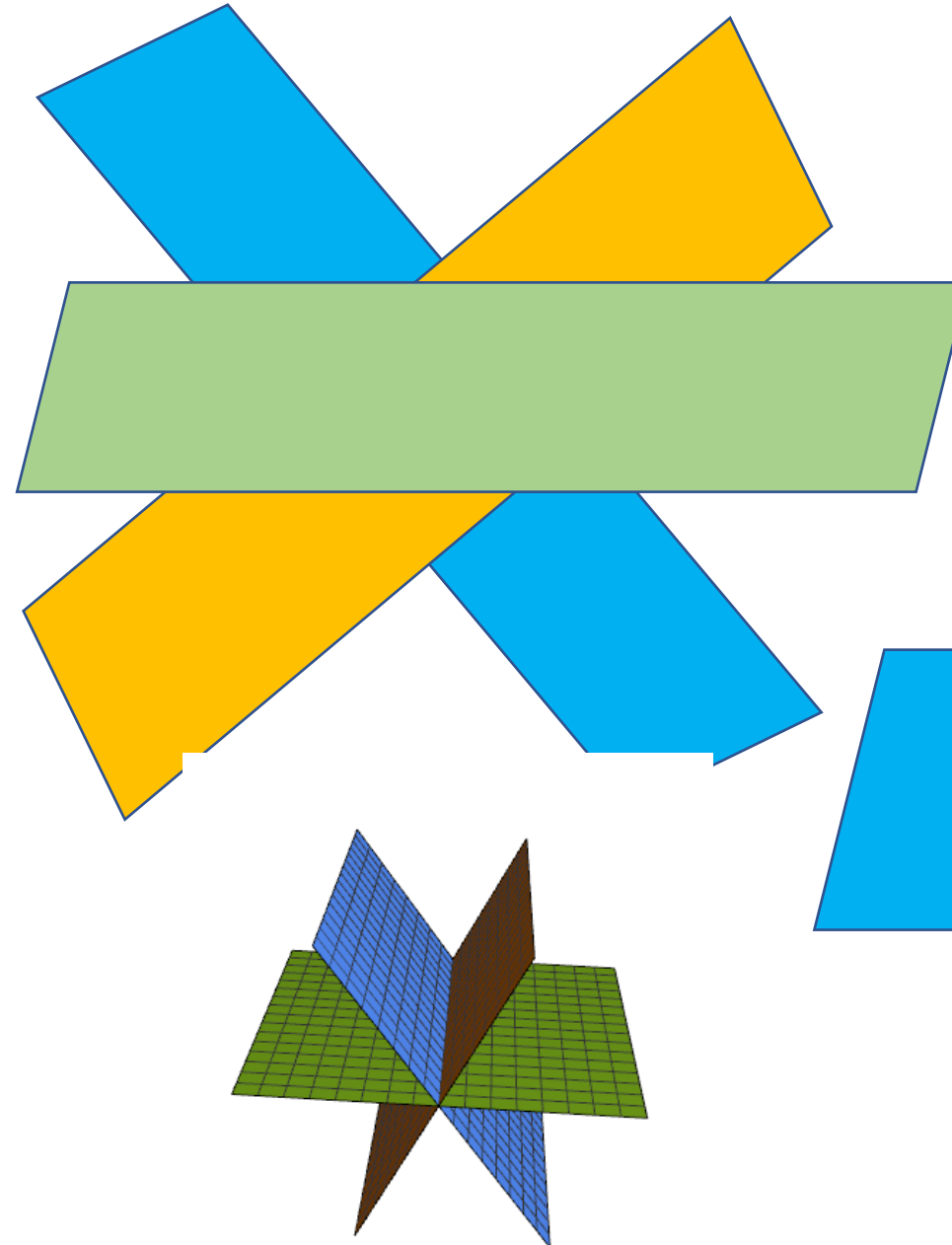
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## THE GEOMETRY OF LINEAR EQUATIONS:

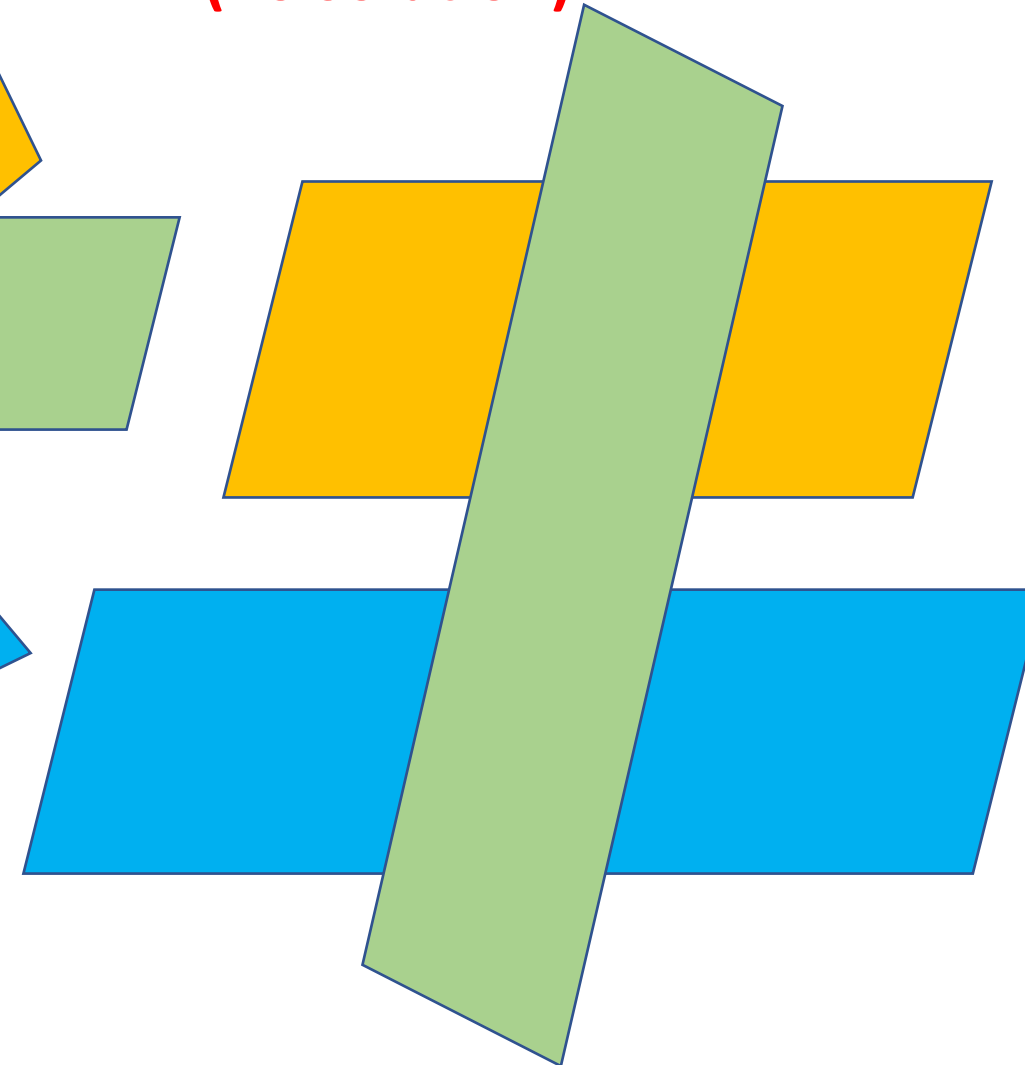
All 3 overlapping planes  
(infinite no. of solutions)



All 3 planes intersecting  
in a line (infinite no  
solutions)



2 parallel planes  
and third plane  
intersecting them  
(no solution)



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## THE GEOMETRY OF LINEAR EQUATIONS:

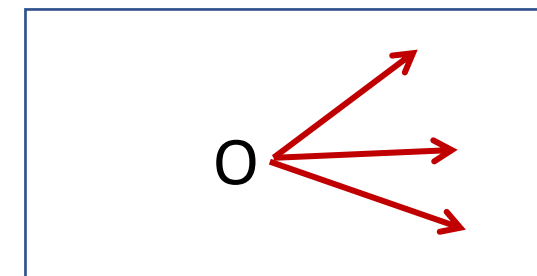
- (ii) **COLUMN PICTURE (Three dimensions)**: Consider the column picture for a system of 3 equations in 3 variables

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned} \Rightarrow x \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} + y \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix} + z \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

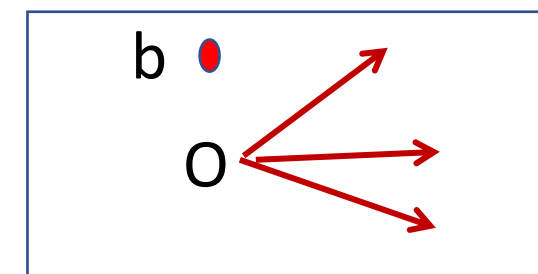
Each of these column vectors is a position vector with origin as a point, These column vectors lie in a plane (as they pass through the origin). Then every combination of these vectors on LHS lie in the same plane (3 vectors are coplanar).

• b

If the vector b is not in that plane, then solution is impossible. This system is singular and has no solution.



If the vector b lies in the plane, (i.e b is also coplanar), then there are too many solutions. The 3 columns combine in infinitely many ways to produce b. This system is singular and has infinite no. of solutions.



# ENGINEERING MATHEMATICS-III

## References/Links:

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[https://upload.wikimedia.org/wikipedia/commons/c/c0/Intersecting\\_Lines.svg](https://upload.wikimedia.org/wikipedia/commons/c/c0/Intersecting_Lines.svg)

Google search: Graphs of row and column picture for a system of linear equations







**THANK YOU**

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