

Renna Sultana

Department of Science and Humanities



MATRICES AND GAUSSIAN ELIMINATION

Renna Sultana

Department of Science and Humanities

MATRICES AND GAUSSIAN ELIMINATION:

PES UNIVERSITY ONLINE

Course Content: Row Exchanges and Permutation Matrices

Consider the system of equations Ax=b. While solving the system if a zero appears In the pivot position, it calls for a row exchange.

This row exchange is taken care by Permutation Matrices P.

Here A ≠ LU then PA=LU where P is a Permutation Matrix which is an Identity Matrix with rows in different order.

Ex:Consider the system $y=b_1$; $2x-3y=b_2$

$$Ax = b \Rightarrow \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$
 Here Gaussian elimination fails and so calls for a row exchange i.e., $\begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b_2 \\ b_1 \end{pmatrix}$

This is same as PAX=Pb

$$PA = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix} = U \text{ and } Pb = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} b_2 \\ b_1 \end{pmatrix}$$
$$\therefore PAx = PB \Rightarrow \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b_2 \\ b_1 \end{pmatrix}$$

MATRICES AND GAUSSIAN ELIMINATION:

PES UNIVERSITY ONLINE

Permutation Matrices:

- P is a Permutation Matrix which is an Identity Matrix with rows in different order.
- Product of two permutation matrices is also a Permutation Matrix.
- Inverse of a permutation matrices is also a Permutation Matrix.
- \triangleright P⁻¹ is always same as P^T.
- Permutation Matrices of order 2 are 2!=2 in number.

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad P_{21} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

MATRICES AND GAUSSIAN ELIMINATION:



❖ Permutation Matrices of order 3 are 3!=6 in number.

$$\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad ;$$

$$\mathbf{I} = egin{pmatrix} 1 & 0 & 0 \ 0 & 1 & 0 \ 0 & 0 & 1 \end{pmatrix} \hspace{0.2cm} ; \hspace{0.2cm} P_{21} = egin{pmatrix} 0 & 1 & 0 \ 1 & 0 & 0 \ 0 & 0 & 1 \end{pmatrix} = P_{21}^{-1} \hspace{0.2cm} ;$$

$$P_{31} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = P_{31}^{-1} \qquad P_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = P_{32}^{-1}$$

$$\mathbf{P}_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = P_{32}^{-1}$$

$$\mathbf{P}_{21}\mathbf{P}_{31} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} = (\mathbf{P}_{21}\mathbf{P}_{32})^{-i}$$

$$P_{21}P_{31} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} = (P_{21}P_{32})^{-i} \qquad P_{21}P_{32} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = (P_{21}P_{32})^{-1}$$

MATRICES AND GAUSSIAN ELIMINATION:



Ex:
$$A = \begin{pmatrix} 2 & 3 & 3 \\ 6 & 9 & 8 \\ 0 & 5 & 7 \end{pmatrix} \xrightarrow{R_2 - 3R_1} \begin{pmatrix} 2 & 3 & 3 \\ 0 & 0 & -1 \\ 0 & 5 & 7 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{pmatrix} = U$$

$$LU = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{pmatrix} \neq A$$

$$P_{23}A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 3 & 3 \\ 6 & 9 & 8 \\ 0 & 5 & 7 \end{pmatrix} = \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{pmatrix} \xrightarrow{R_3 - 3R_1} \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{pmatrix} = U$$

$$LU = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{pmatrix} = P_{23}A$$

MATRICES AND GAUSSIAN ELIMINATION:



Problem: Explain why A is not factorizable into LU? How can A be modified so that the new matrix can be factored into LU? Also obtain the factors L,D,U for the new matrix. What is the relation between L and U thus obtained? Explain.

$$A = \begin{pmatrix} 1 & -2 & 2 \\ 2 & -4 & 5 \\ -2 & 5 & -4 \end{pmatrix} \xrightarrow{R_2 - 2R_1} \begin{pmatrix} 1 & -2 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 1 & -2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

A is not factorizable into LU as Gaussian elimination fails and Row exchange is required. So A should be multiplied with permutation matrix P_{23} so that PA=LU.

$$\mathbf{P}_{23}\mathbf{A} = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 5 & -4 \\ 2 & -4 & 5 \end{pmatrix} \xrightarrow{R_2 + 2R_1 \atop R_3 - 2R_1} \begin{pmatrix} 1 & -2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = U \qquad L = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad U = \begin{pmatrix} 1 & -2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



THANK YOU

Renna Sultana

Department of Science and Humanities

rennasultana@pes.edu