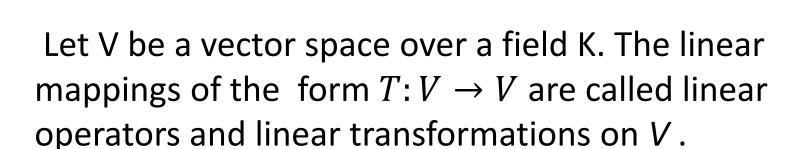


# LINEAR ALGEBRA AND ITS APPLICATIONS

### Algebra A(V) of Linear operators



#### Note:

- 1. If dim V = n then dim  $A(V) = n^2$ .
- 2. For any mapping F,G from A(V), the composition G.F exists and also belongs to A(V).



#### Algebra A(V) of Linear operators



Remark: An algebra a over a field K is a vector space over K in which an operation of multiplication is defined satisfying, for F,

- $G, H \in A$  and every  $k \in K$
- 1.F(G+H) = FG + FH
- 2.(G+H)F = GF + HF
- 3.K(GF) = (kG)F = G(kF)

# Invertible Maps and Isomorphism



A mapping  $f: A \rightarrow B$  is said to be *One-to-one* or 1-1 or *injective* if different elements of A have distinct images; that is

IF 
$$a \neq a'$$
, then  $f(a) \neq f(a')$ .

Equivalently,

IF 
$$a = a'$$
, then  $f(a) = f(a')$ .

A mapping  $f: A \to B$  is said to be *onto* or *surjective* if every  $b \in B$  is the image of at least one  $a \in A$ .

A mapping  $f: A \to B$  is said to be *One-to-one correspondence* between A and B or bijective if f is both *one-to-one and onto*.

# Invertible Maps and Isomorphism



A mapping  $f: A \to B$  is said to be invertible if f is one-to-one and onto.

**Example:** Let  $f: R \to R$  be defined by f(x) = 2x - 3. Now f is one-to-one and onto.

Let y be the image of x under the mapping f, i.e., y = 2x-3.

Interchange x and y to obtain 
$$x = 2y - 3 \Rightarrow y = \frac{x+3}{2} \Rightarrow f^{-1} = \frac{x+3}{2}$$
.

# Invertible Maps and Isomorphism



A mapping  $F: V \rightarrow U$  is called isomorphism if F is linear and bijective, i.e., one-to-one and onto.

Example: The mapping  $T: \mathbb{R}^2 \to \mathbb{R}^2$  is defined as T(x,y) = (x+4y,y-3x) is isomorphism.