LINEAR ALGEBRA AND ITS APPLICATIONS

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MATRICES AND
GAUSSIAN ELIMINATION

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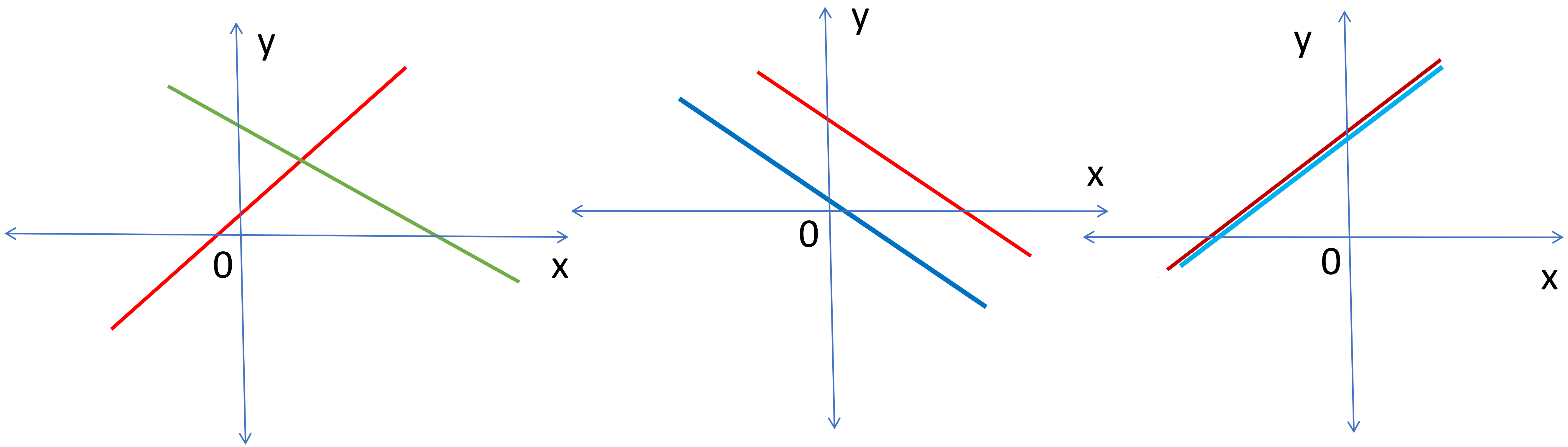
Course Content: The Geometry of Linear Equations

- SYSTEM OF 2 EQUATIONS WITH 2 VARIABLES:

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$



Intersecting lines
Unique solution

Parallel lines
No Solution

Overlapping lines
Infinite no. of solutions

System of m linear equations with n unknowns have either a unique solution or infinite solutions or no solution.

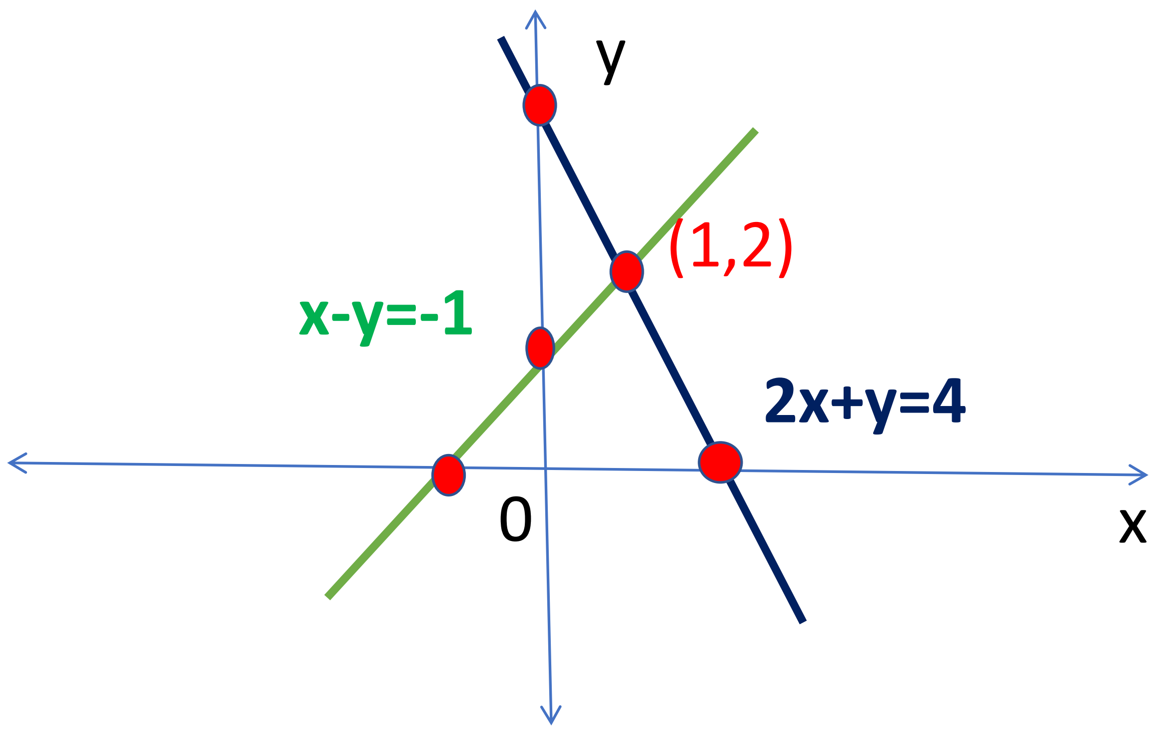
THE GEOMETRY OF LINEAR EQUATIONS:

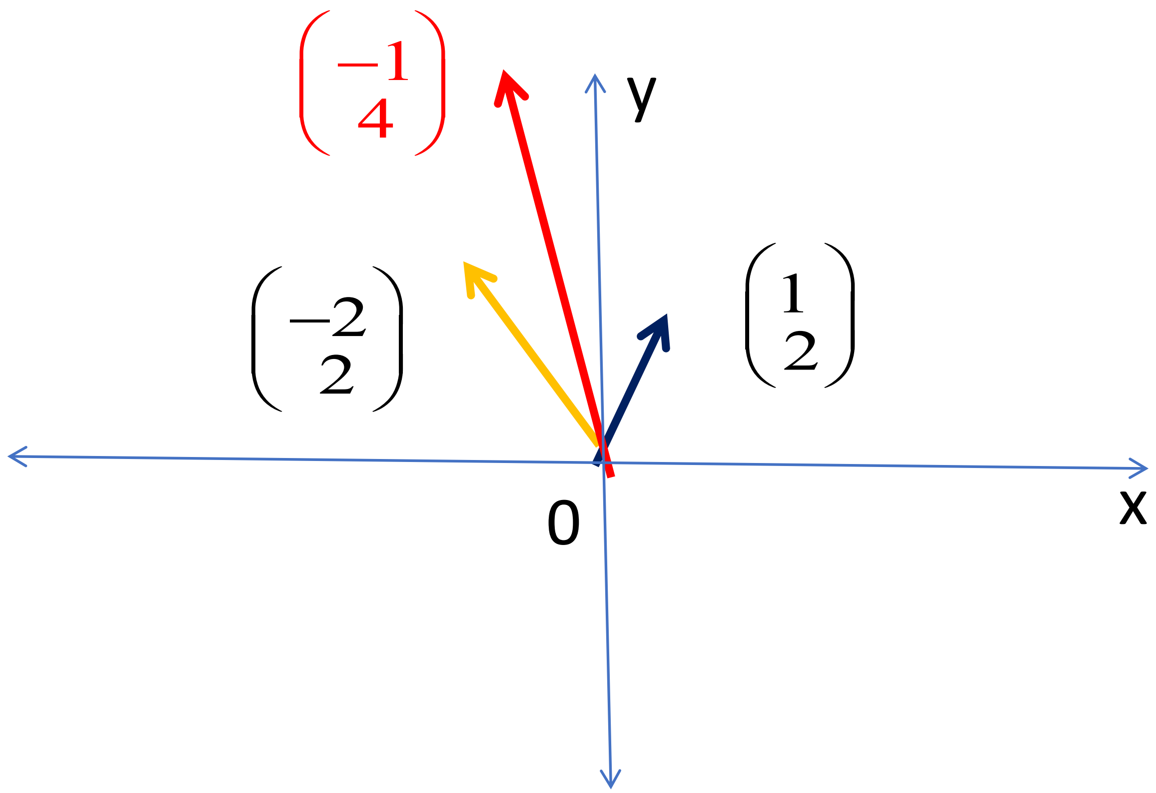
- (i) Consider a system of 2 equations with 2 variables $\begin{aligned} x - y &= -1 \\ 2x + y &= 4 \end{aligned}$
- **Row Picture:** Solving these 2 equations we get (1,2) as the point of intersection of the 2 lines. Hence this system has **a unique solution**.

- **Column Picture:** $x\begin{pmatrix}1\\2\end{pmatrix} + y\begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}-1\\4\end{pmatrix} \Rightarrow 1\begin{pmatrix}1\\2\end{pmatrix} + 2\begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}-1\\4\end{pmatrix}$

  x=1 and y=2 will satisfy this equation.

  Hence the linear combination of the column vectors on LHS produces the vector on RHS.





THE GEOMETRY OF LINEAR EQUATIONS:

- (ii) Consider the system

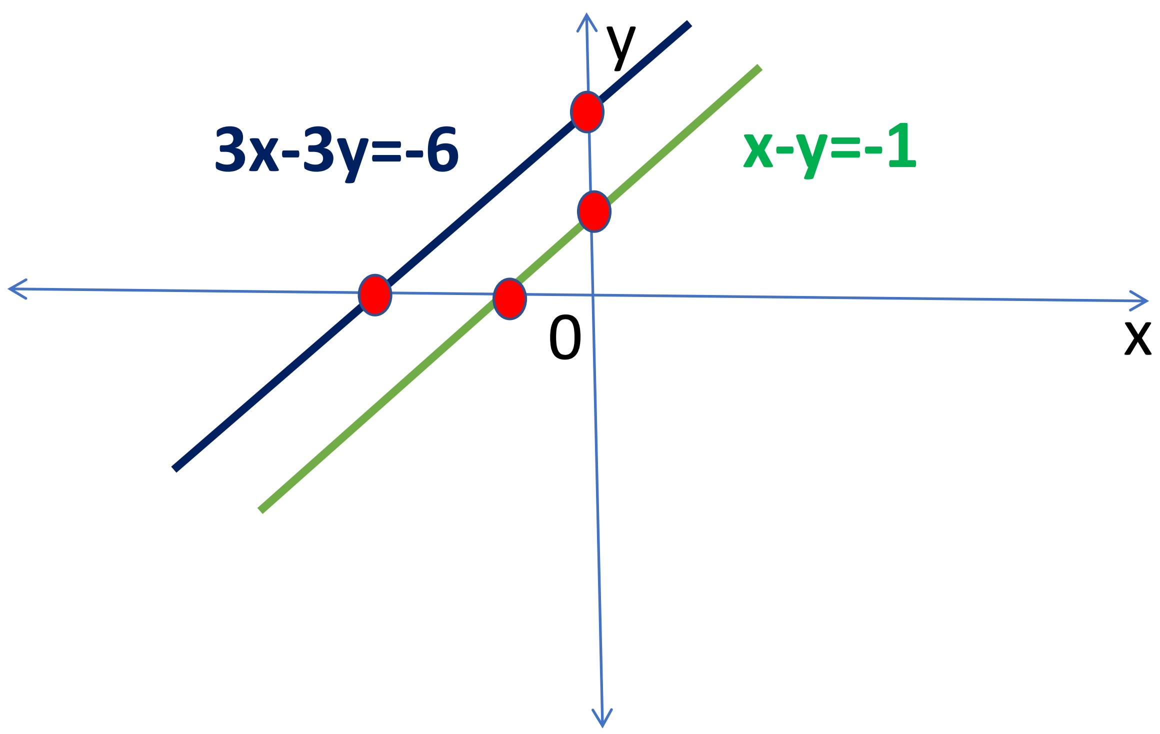
$$x - y = -1$$
$$3x - 3y = -6$$

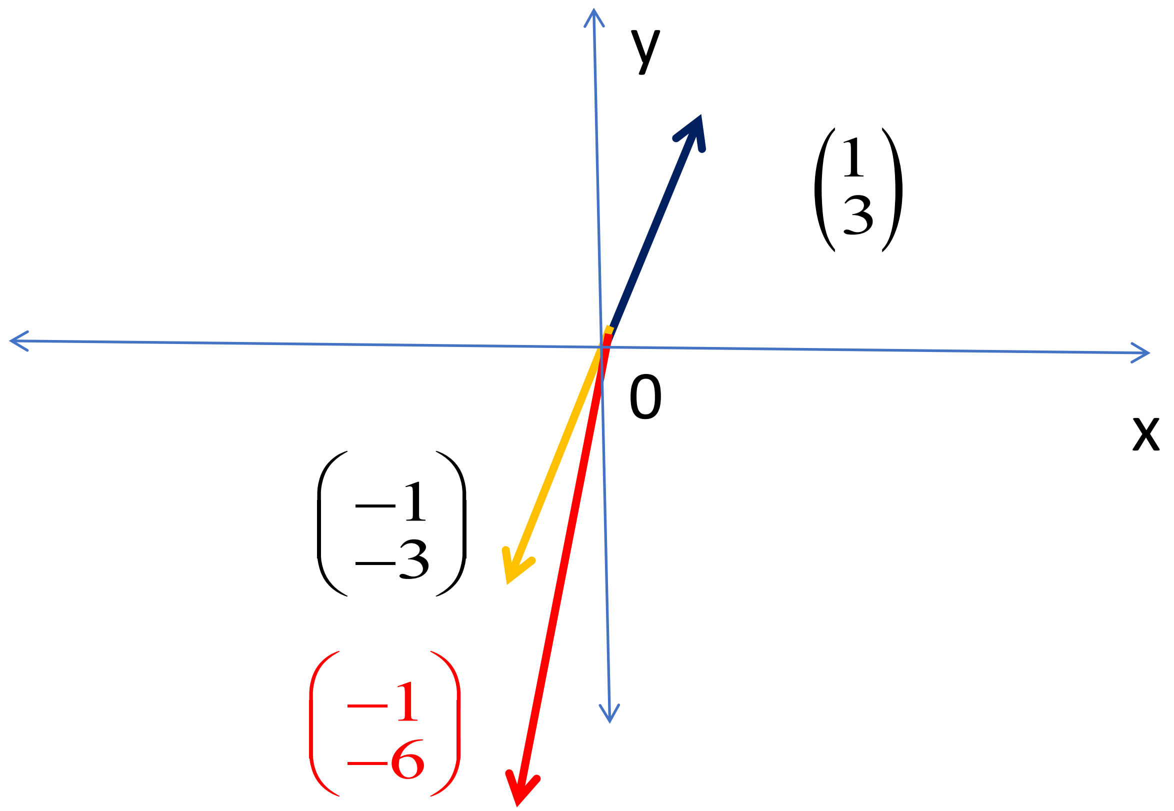
- **Row Picture:** These are parallel lines and so has no point of intersection. Hence this system has **no solution**.

$$x\begin{pmatrix}1\\3\end{pmatrix} + y\begin{pmatrix}-1\\-3\end{pmatrix} = \begin{pmatrix}-1\\-6\end{pmatrix}$$

- **Column Picture:**

There is no linear combination of the column vectors on LHS which produces the vector on RHS.



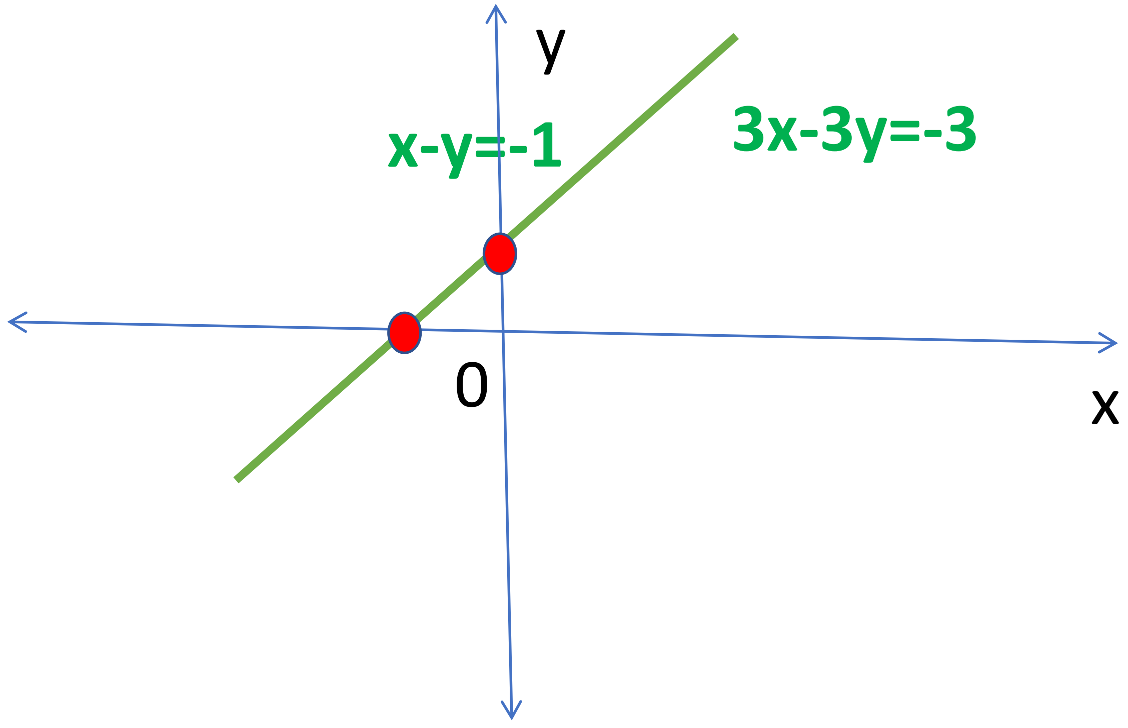


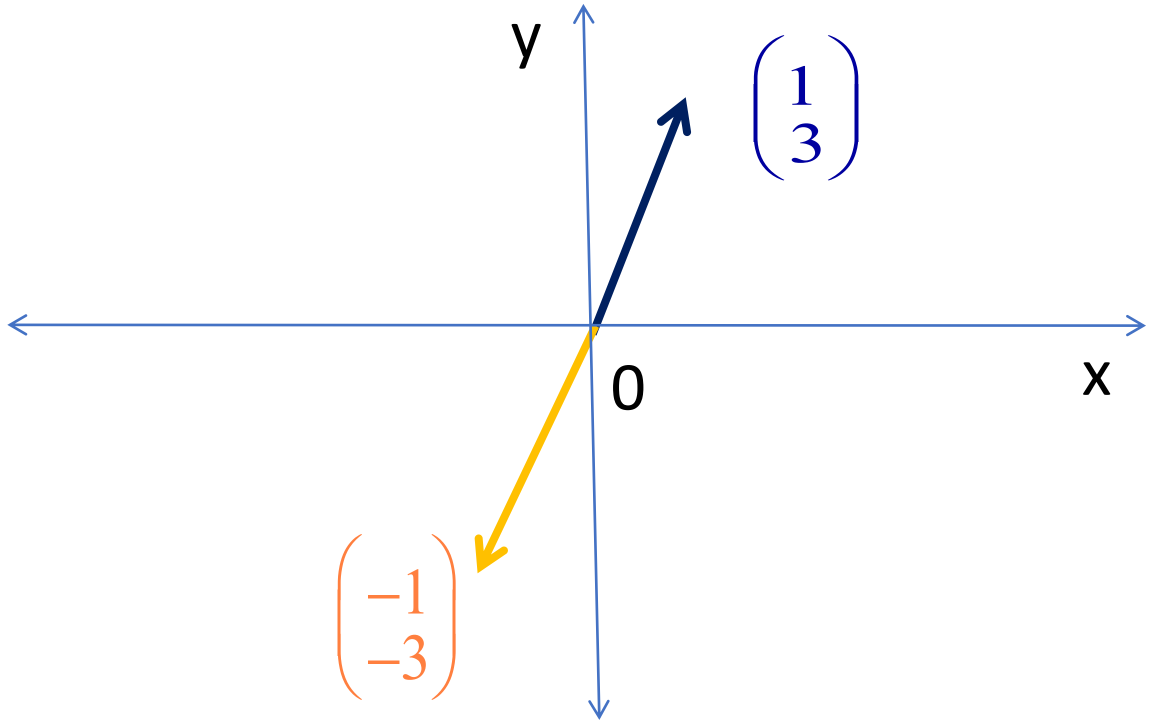
THE GEOMETRY OF LINEAR EQUATIONS:

- (iii)Consider the system $\begin{aligned} x - y &= -1 \\ 3x - 3y &= -3 \end{aligned}$
- **Row Picture:** These are coincident(overlapping) lines and so has infinite number of solutions. Hence this system has **infinitely many solutions**.

❖ **Column Picture:** $x\begin{pmatrix}1\\3\end{pmatrix} + y\begin{pmatrix}-1\\-3\end{pmatrix} = \begin{pmatrix}-1\\-3\end{pmatrix}$

There are infinite number of linear combinations of the column vectors on LHS which produces the vector on RHS.



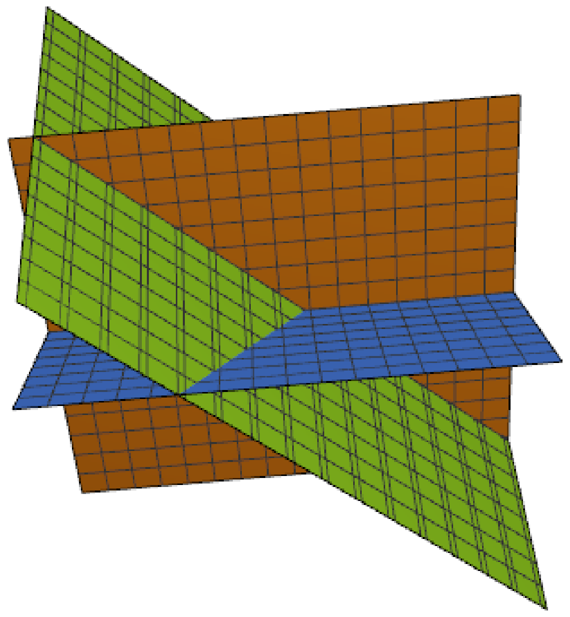


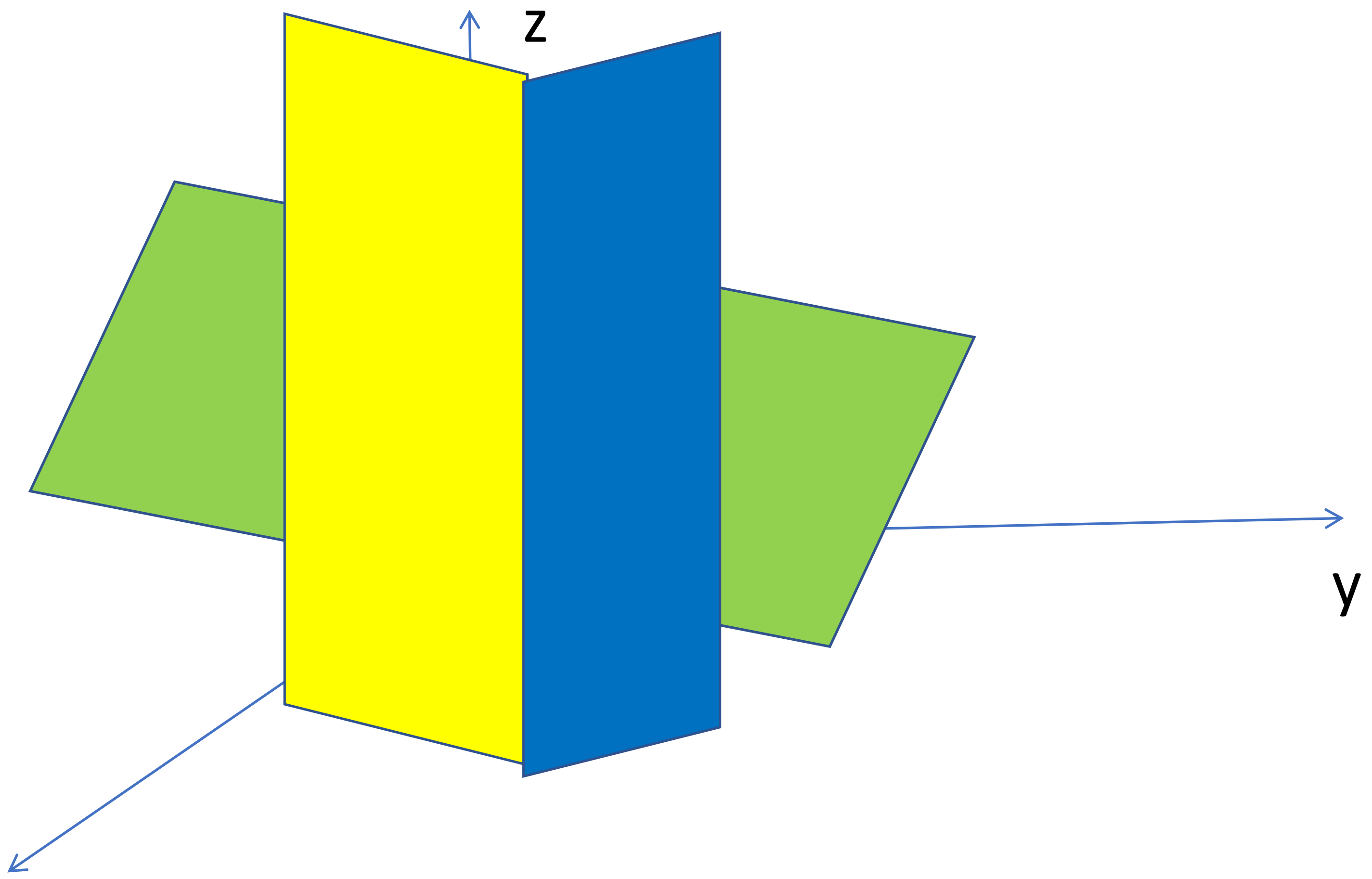
THE GEOMETRY OF LINEAR EQUATIONS:

- **SYSTEM OF 3 EQUATIONS WITH 3 VARIABLES:** Consider the system

$$\begin{aligned} x + y + 2z &= 1 \\ x + 2y - z &= -2 \\ x + 3y + z &= 5 \end{aligned}$$

- **Row Picture:** Each equation describes a 2-dimensional plane in R^3 . The first 2 planes intersect along a line and this line intersects with the third plane to produce a point (-6. 3, 2) which is the unique solution (point of intersection of the three planes) to the system of equations.





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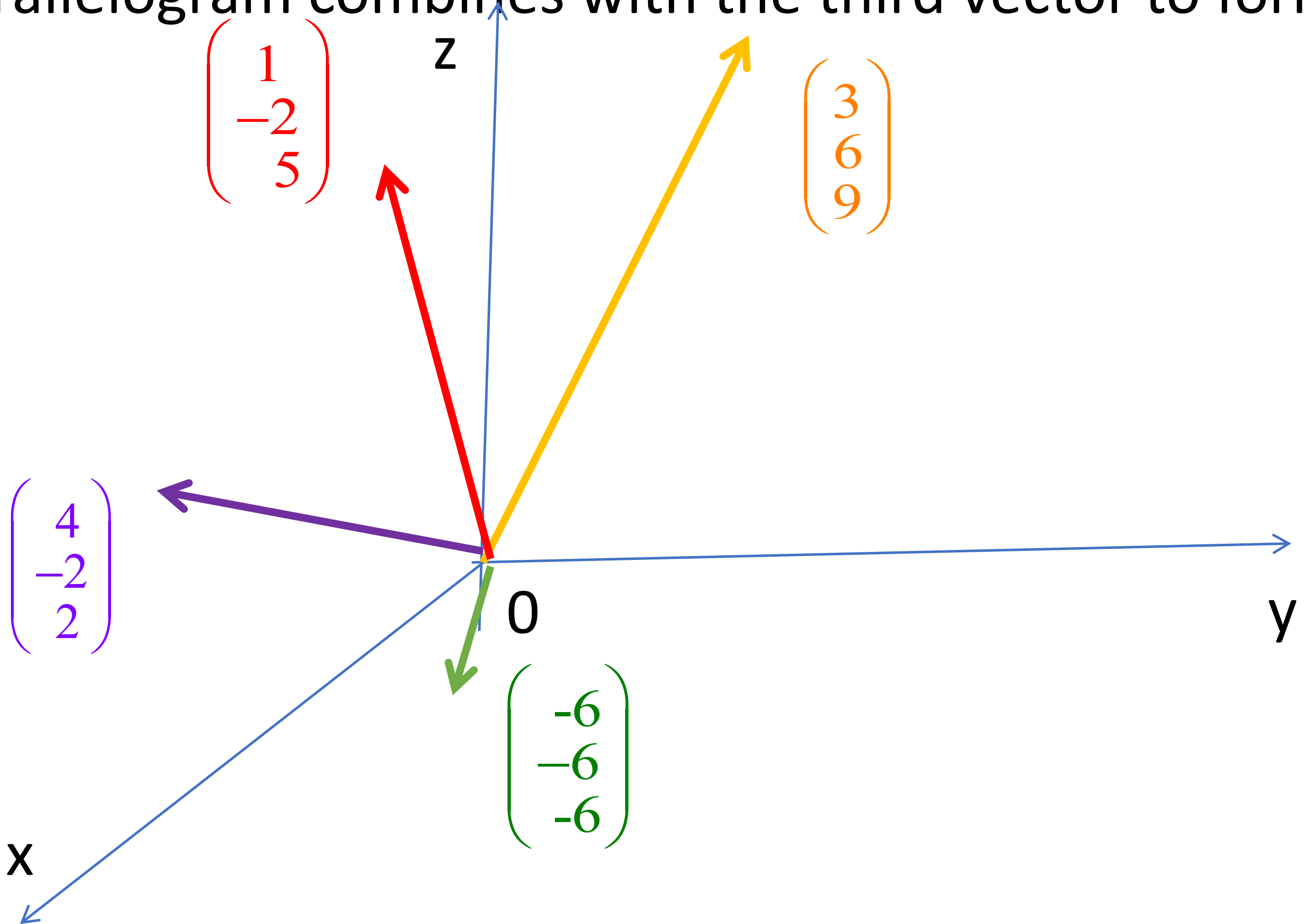
THE GEOMETRY OF LINEAR EQUATIONS:

❖ **Column Picture:** The vector equation for the given system can be written as

$$x\begin{pmatrix}1\\1\\1\end{pmatrix}+y\begin{pmatrix}1\\2\\3\end{pmatrix}+z\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}1\\-2\\5\end{pmatrix}\quad\Rightarrow -6\begin{pmatrix}1\\1\\1\end{pmatrix}+3\begin{pmatrix}1\\2\\3\end{pmatrix}=\begin{pmatrix}-3\\0\\3\end{pmatrix}+2\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}1\\-2\\5\end{pmatrix}$$

The first two vectors on the LHS combine to form a Parallelogram and the diagonal of this parallelogram combines with the third vector to form a Parallelopiped.



THE GEOMETRY OF LINEAR EQUATIONS:

- **Row Picture:** Intersection of Lines/Planes.
- **Column Picture:** Combination of Columns.

- **Row Picture:** A Line requires 2 equations in 3-dimensional space. Similarly a Line requires 3 equations in 4-dimensional space. Hence a Line requires (n-1) equations in n-dimensional space.
  The first equation represents a (n-1)dimensional plane in n dimensions. The second plane(equation) intersects it in a smaller set of dimension (n-2). Thus every new plane reduces the dimension by one. At the end when all the ‘n’ planes are accounted for the intersection has dimension zero. It is a point, which lies on all the planes and its co-ordinates satisfy all ‘n’ equations. This point is the solution.

References/Links:

https://upload.wikimedia.org/wikipedia/commons/c/c0/Intersecting_Lines.svg

Google search: Graphs of row and column pictiure for a system of linear equations

THANK YOU

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