LINEAR ALGEBRA AND ITS APPLICATIONS

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MATRICES AND
GAUSSIAN ELIMINATION

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Course Content: Elementary Matrices

- Elementary Matrix E_{ij} is obtained from the Identity Matrix I by using transformation $R_i - l_{ij} R_j$ where l_{ij} is the multiplier

i.e

$$I \longrightarrow E_{ij}$$

Example: E_{32} is obtained as follows:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_3 - 2R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} = E_{32}$$

MATRICES AND GAUSSIAN ELIMINATION:

Example:

Consider the matrix $A=\begin{pmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{pmatrix} \xrightarrow[R_3-1/3R_1]{R_2-2/3R_1} \begin{pmatrix} 3 & 1 & 2 \\ 0 & -11/3 & -7/3 \\ 0 & 5/3 & 1/3 \end{pmatrix}$

$$\xrightarrow{R_3+(5/11)R_2} \begin{pmatrix} 3 & 1 & 2 \\ 0 & -11/3 & -7/3 \\ 0 & 0 & -8/11 \end{pmatrix} = U$$

$$E = E_{21} = R_2 - (2/3)R_1 = \begin{pmatrix} 1 & 0 & 0 \\ -2/3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$F = E_{31} = R_3 - (1/3)R_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/3 & 0 & 1 \end{pmatrix}$$

$$G = E_{32} = R_3 + (5/11)R_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5/11 & 1 \end{pmatrix}$$

$$E_{32}E_{31}E_{21}A = U$$

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Problems:

1. Which elimination matrices put A into upper triangular matrix U?

$$A=\begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix} \xrightarrow{R_2+(1/2)R_1} \begin{pmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix} \xrightarrow{R_3+(2/3)R_2} \begin{pmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

$$\xrightarrow{R_4+(3/4)R_3} \begin{pmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & 0 & 5/4 \end{pmatrix} = U$$

The elimination matrices E_{21}, E_{32} and E_{43} put A into upper triangular matrix i.e.,

$$E_{43}E_{32}E_{21}A = U$$

$$E_{43}E_{32}E_{21}A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 3/4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2/3 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} A = U$$

MATRICES AND GAUSSIAN ELIMINATION:

2. Which elementary matrices convert $A = \begin{pmatrix} 0 & 2 & -6 & -2 & 4 \\ 0 & -1 & 3 & 3 & 2 \\ 0 & -1 & 3 & 7 & 10 \end{pmatrix}$ into upper triangular matrix U?

$$A = \begin{pmatrix} 0 & 2 & -6 & -2 & 4 \\ 0 & -1 & 3 & 3 & 2 \\ 0 & -1 & 3 & 7 & 10 \end{pmatrix} \xrightarrow[R_3 + 1/2R_1]{R_2 + 1/2R_1} \begin{pmatrix} 0 & 2 & -6 & -2 & 4 \\ 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 6 & 12 \end{pmatrix}$$

$$\xrightarrow{R_3 - 3R_2} \begin{pmatrix} 0 & 2 & -6 & -2 & 4 \\ 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = U$$

Matrices E_{21}, E_{31}, E_{32} convert A into triangular form U.

$$E_{21} = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad E_{31} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1 \end{pmatrix} \qquad E_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{pmatrix}$$

THANK YOU

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