VECTOR SPACES

Deepthi Rao

Department of Science & Humanities

CLASS 12 : CONTENT

- Problems on Uniqueness, Existence of right and left inverse

  Matrix of rank 1

RIGHT INVERSE

Problem 1: Find the left or Right Inverse for the following matrices, whichever exists:-

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

Solution:- Step I: Apply Gauss elimination and obtain rank

Step II: check if rank 'ρ' of the matrix is equal to 'm' or 'n' i.e. number of rows or columns of the matrix A.

Step III: if $\rho(A) = m$ then (Right Inverse exists) $A_{m\times n} B_{n\times m} = I_{m\times m}$

if $\rho(A) = n$ then (left inverse exists) $C_{n\times m} A_{m\times n} = I_{n\times n}$

RIGHT INVERSE

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$S(A) = 2 = m$ $\therefore$ Right inverse exists

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

Best Right Inverse is $A^T(AA^T)^{-1}$

$$AA^T = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}; \quad (AA^T)^{-1} = \frac{1}{3}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$

LEFT INVERSE

$$A^T(AA^T)^{-1} = \frac{1}{3}\begin{pmatrix}1 & 0\\ 1 & 1\\ 0 & 1\end{pmatrix}\begin{pmatrix}2 & -1\\ -1 & 2\end{pmatrix} = \begin{pmatrix}2/3 & -1/3\\ 1/3 & 1/3\\ -1/3 & 2/3\end{pmatrix}$$ Ans

2) Find inverse for the matrix $A = \begin{pmatrix}1 & 0\\ 1 & 1\\ 0 & 1\end{pmatrix}$

Solution :

$$\begin{pmatrix}1 & 0\\ 1 & 1\\ 0 & 1\end{pmatrix} \sim \begin{pmatrix}1 & 0\\ 0 & 1\\ 0 & 1\end{pmatrix} \sim \begin{pmatrix}1 & 0\\ 0 & 1\\ 0 & 0\end{pmatrix}$$

$S(A) = 2 = n =$ number of Columns

A has Left inverse ; Best left inverse is $(A^TA)^{-1}A^T$

$$(A^TA)^{-1} = \frac{1}{3}\begin{pmatrix}2 & -1\\ -1 & 2\end{pmatrix} ; \quad (A^TA)^{-1}A^T = \begin{pmatrix}2/3 & 1/3 & -1/3\\ -1/3 & 1/3 & 2/3\end{pmatrix}$$ Ans

VECTOR SPACES

MATRIX OF RANK 1

Matrices Of Rank One:

When the rank of a matrix is as small as possible,

a complicated system of equations can be broken into simple pieces.

Those simple pieces are matrices of rank one.

The matrix has rank r = 1.

We can write such matrices as a column times row.

That is $\begin{bmatrix} 2 & 1 & 1 \\ 4 & 2 & 2 \\ 6 & 3 & 3 \\ 8 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \end{bmatrix}$ $\Rightarrow A = uv^T$

$u = (1,2,3,4)$ $v = (2,1,1)$

THANK YOU

Deepthi Rao

Department of Science & Humanities