LINEAR ALGEBRA AND ITS APPLICATIONS

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CLASS-8

PROBLEMS ON DIAGONALIZATION OF A MATRIX

1. Find the matrix A whose Eigen values are 2, 5 and Eigen vectors are $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ & $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Ans: $S = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow S^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ & $\Lambda = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$

$\therefore A = S \Lambda S^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$\therefore A = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$

Diagonalization of a matrix

2. Find the Eigen values and Eigen vectors of $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ and write 2 different diagonalizing matrices.

Ans: $|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0$

$$(1-\lambda)\left[(1-\lambda)^2 - 1\right] - 1\left[1-\lambda-1\right] + 1\left(1-(1-\lambda)\right) = 0$$

$$\Rightarrow \lambda^2(\lambda - 3) = 0$$

$$\Rightarrow \lambda = 0, 0, 3 \Rightarrow \text{Eigen values}$$

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Consider $[A - \lambda I]X = 0$

$$(1-\lambda)x + y + z = 0$$
$$x + (1-\lambda)y + z = 0$$
$$x + y + (1-\lambda)z = 0$$

Case 1:- When $\lambda = 0$ $\Rightarrow$

$$x + y + z = 0$$
$$x + y + z = 0$$
$$x + y + z = 0$$

$x = -y - z$. Let $y = k_1$ & $z = k_2$ $\therefore x = -k_1 - k_2$

For $\lambda = 0$ Let $k_1 = k_2 = 1$ $\Rightarrow x = -2, y = 1, z = 1 \Rightarrow X_1 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

$\lambda = 0$, let $k_1 = 0, k_2 = 1 \Rightarrow y = 0, z = 1, x = -1 \Rightarrow X_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

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Case 2: When $\lambda = 3$

$-2x + y + z = 0$

$x - 2y + z = 0$

$x + y - 2z = 0$

$\Rightarrow \dfrac{x}{1+2} = \dfrac{y}{-2-1} = \dfrac{z}{4-1}$

$\Rightarrow X_3 = \begin{bmatrix} 3 \\ -3 \\ 3 \end{bmatrix}$

$\therefore S_1 = \begin{bmatrix} -2 & -1 & 3 \\ 1 & 1 & -3 \\ 1 & 0 & 3 \end{bmatrix}$

$S_2 = \begin{bmatrix} -2 & -1 & 3 \\ 1 & 0 & -3 \\ 1 & 1 & 3 \end{bmatrix}$ (taking $K_2 = 0$, $K_1 = 1$)

THANK YOU

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