Finite State Automata with Output

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FSA with Output

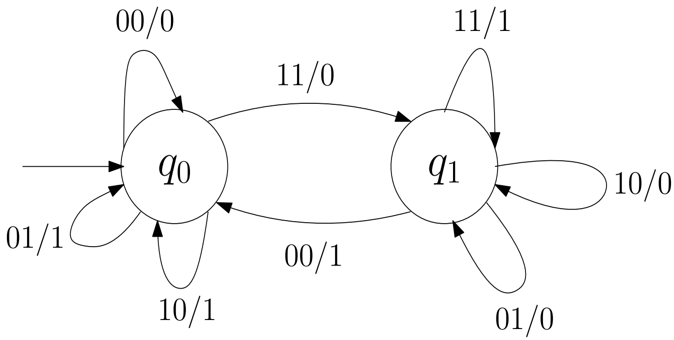
- The finite automata which we considered earlier have binary output, i.e., either they accept the string or they do not accept the string.

- This acceptability was decided on the basis of reachability of the final state by the initial state.

- Now, we remove this restriction and consider the model where the outputs can be chosen from some other alphabet.

- When we consider finite state automata with output, there are no final states associated with it.

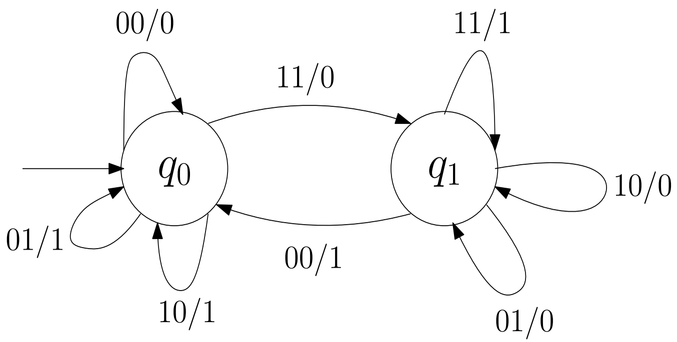
FSA with Output



- This is the state diagram of a serial adder and the inputs are two binary digits 00, 01, 10, 11.
- Input symbols are $\Sigma = \{00, 01, 10, 11\}$ and the output symbols $\Delta = \{0, 1\}$
- Consider two binary number and the addition of these two numbers

$$\begin{array}{c} 1\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 1 \\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 1\ 0 \\ \hline 1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1 \end{array}$$

FSA with Output



- This automata works in such a way that when it receives some input, it outputs some things.
- We can easily see that this output depends on the state and the input symbol. We call this kind of machine as **Mealy Machine**.
- Some times, the output may depend just on the state alone. We call this kind of machine as **Moore Machine**.

Mealy Machine

- A Mealy machine is defined by the six-tuple

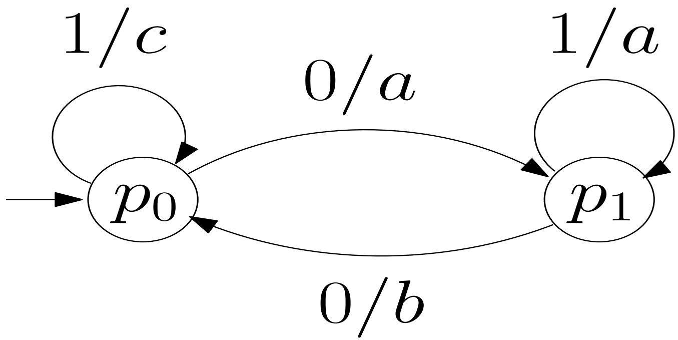
$$M = (Q, \Sigma, \Delta, \delta, \lambda, q_0) \text{ where,}$$

  - Q is a finite set of internal states,
  - Σ is the input alphabet,
  - Δ is the output alphabet,
  - $\delta : Q \times \Sigma \to Q$ is the transition function
  - $\lambda : Q \times \Sigma \to \Delta$ is the output function
  - $q_0 \in Q$ is the initial state of M

Mealy Machine

- **Example 1:** Consider the following Mealy machine where $Q = \{p_0, p_1\}$, $\Sigma = \{0, 1\}$, $\Delta = \{a, b, c\}$, initial state p_0 and

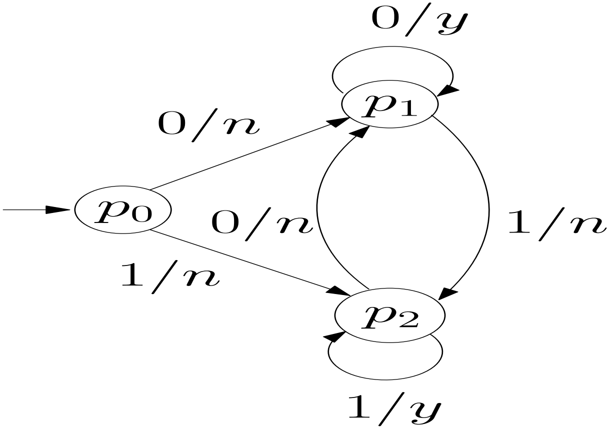
$$
\begin{array}{ll}
\delta(p_0, 0) = p_1 & \qquad \Delta(p_0, 0) = a \\
\delta(p_0, 1) = p_0 & \qquad \Delta(p_0, 1) = c \\
\delta(p_1, 0) = p_0 & \qquad \Delta(p_1, 0) = b \\
\delta(p_1, 1) = p_1 & \qquad \Delta(p_1, 1) = a
\end{array}
$$



This Mealy machine prints out *caab* when given the input string 1010.

Mealy Machine

- **Example 2:** Consider strings over the alphabet $\Sigma = \{0, 1\}$. The last two symbols read if they are 00 or 11, output symbol y. If they are not, output symbol n. So, the output alphabet $\Delta = \{y, n\}$



Mealy Machine

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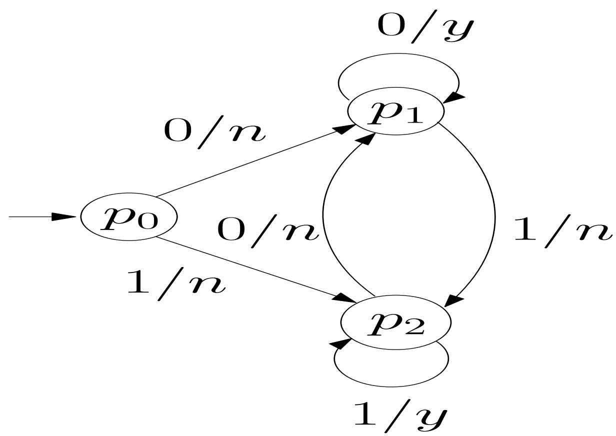
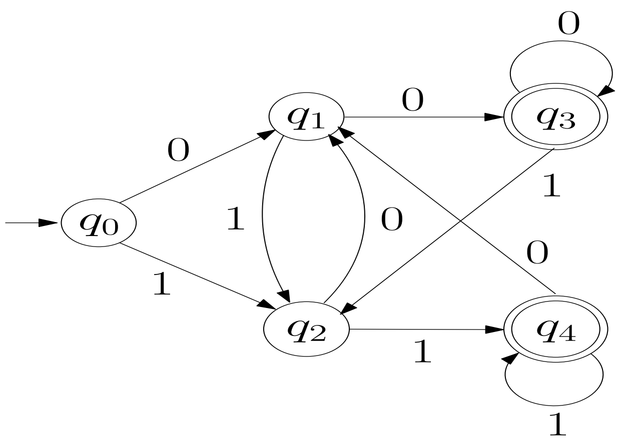


Input :		0	0	1	0	1	1	—— n
State Sequence :	p_0	p_1	p_1	p_2	p_1	p_2	p_2	—— $n+1$
Output :		n	y	n	n	n	y	—— n

Mealy Machine

- The corresponding regular expression : $(0+1)^*(00+11)$
- The corresponding DFA is



- In case of Mealy machine, to make a string accepted or come out with an output yes or no require only three states, but as an acceptor when we want to accept strings ending with 11 or 00 require five states. This is the advantage of having an automata with output.

Moore Machine

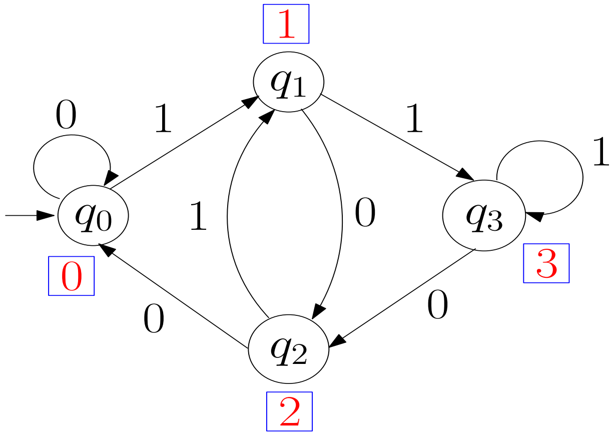
- A Moore machine is defined by the six-tuple

$$M = (Q, \Sigma, \Delta, \delta, \lambda, q_0) \text{ where,}$$

  - Q is a finite set of internal states,
  - Σ is the input alphabet,
  - Δ is the output alphabet,
  - $\delta : Q \times \Sigma \to Q$ is the transition function
  - $\lambda : Q \to \Delta$ is the output function
  - $q_0 \in Q$ is the initial state of M

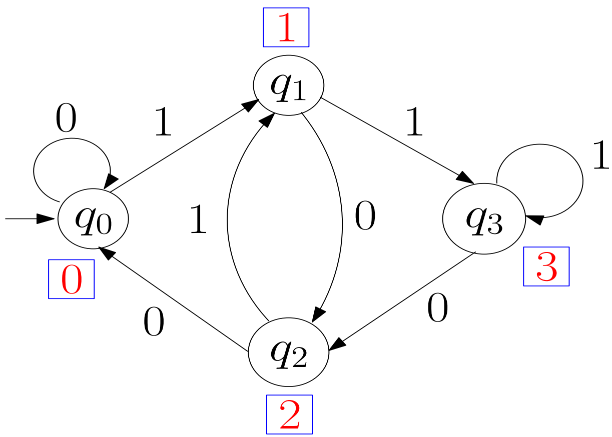
Moore Machine

- **Example 3:** Let us consider a finite state automata with input alphabet $\Sigma = \{0, 1\}$ and output alphabet $\Delta = \{0, 1, 2, 3\}$



Moore Machine

- **Example 3:** Let us consider a finite state automata with input alphabet $\Sigma = \{0, 1\}$ and output alphabet $\Delta = \{0, 1, 2, 3\}$



Input :	1	1	1	0	1		———— n
State Sequence :	q_0	q_1	q_3	q_3	q_2	q_1	———— $n+1$
Output :	0	1	3	3	2	1	———— $n+1$

Moore and Mealy Machine Equivalence

- Can you say a Moore machine is equivalent to a Mealy machine ?

Moore and Mealy Machine Equivalence

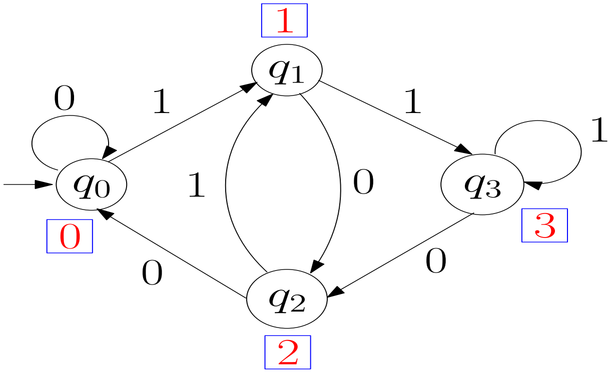
- Can you say a Moore machine is equivalent to a Mealy machine ?
  - If the input size of n, in the case of a Moore machine, the output will be of the length $n + 1$. But, in the case of Mealy machine, the output will be of length n.
  - So, there is no point in talking about the equivalence, because the length itself is different.
  - But, in the case of a Moore machine, at the initial state there is an output. So, if we ignore that output then they can be equivalent.

Moore and Mealy Machine Equivalence

- Given a Moore machine, how to construct an equivalent Mealy machine ?
  - Consider the Moore machine $M = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$ and the equivalent Mealy machine $M^{'} = (Q, \Sigma, \Delta, \delta, \lambda^{'}, q_0)$.
  - Only the output function will be different. In M, it depends on the state alone, but in $M^{'}$ it depends on both the state and input symbol.
  - $\lambda^{'}(q, a) = \lambda(\delta(q, a))$
    - ★ In state q, if read symbol a goes to a particular state, say $q^{'}$. The output corresponding to that state is the output for the pair (q, a).

Moore and Mealy Machine Equivalence

Consider the following Moore machine



Moore and Mealy Machine Equivalence

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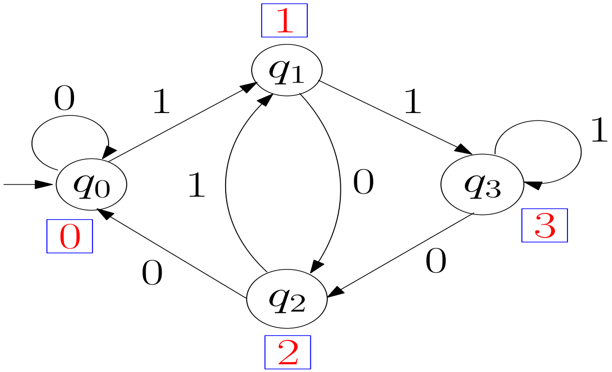
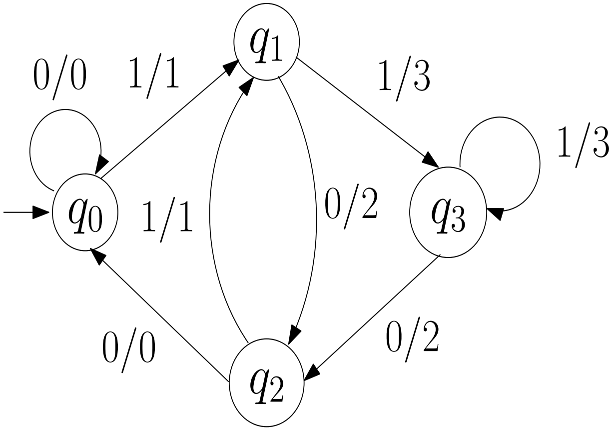


The equivalent Mealy machine is

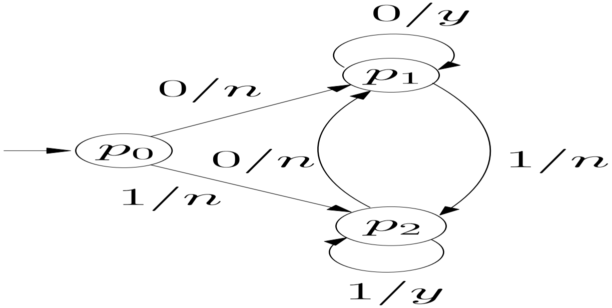


Moore and Mealy Machine Equivalence

- Given a Mealy machine, how to construct an equivalent Moore machine ?
    - Consider the Mealy machine $M = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$ and the equivalent Moore machine $M^{'} = (Q \times \Delta, \Sigma, \Delta, \delta^{'}, \lambda^{'}, [q_0, b])$.
    - Initial state is a pair, where first component from Q and second component from Δ.
    - $\delta^{'}([q, b], a) = [\delta(q, a), \lambda(q, a)]$
    - $\lambda^{'}([q, b])$, the output only depend on $[q, b]$ this pair, and it is given by the second component of the state.

Moore and Mealy Machine Equivalence

Consider the following Mealy machine



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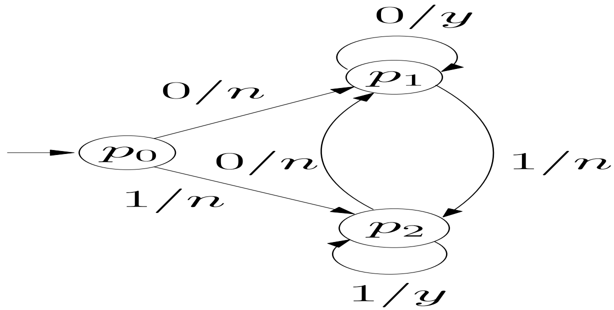

The equivalent Moore machine is