Regular Grammar

Raju Hazari

Department of Computer Science and Engineering
National Institute of Technology Calicut

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Regular Grammar

- A language is regular if and only if there exists some deterministic finite automata.
- Another way of describing regular language is by means of certain grammars.
- Grammars are often an alternative way of specifying languages.
- So, regular grammars are associated with regular languages and that for every regular language there is regular grammar.

Regular Grammar

- A grammar $G = (N, T, S, P)$ is said to be **right-linear** if all productions are of the form
$$A \to xB$$
$$A \to x$$
where $A, B \in N$, and $x \in T^*$.

- A grammar is said to be **left-linear** if all the productions are of the form
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- A **regular grammar** is one that is either right-linear or left linear.
  - In a regular grammar, at most one non-terminal appears on the right side of any production.
  - The non-terminal must consistently be either the rightmost or leftmost symbol of the right side of any production.

Regular Grammar

Example 1:

- The grammar $G_1 = (\{S\}, \{a, b\}, S, P_1)$, with P_1 given as

  $$S \to abS|a$$

  So, it is a right-linear grammar.

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- The grammar $G_2 = (\{S, S_1, S_2\}, \{a, b\}, S, P_2)$, with P_2 given as

$$\begin{aligned} S &\rightarrow S_1 ab, \\ S_1 &\rightarrow S_1 ab | S_2, \\ S_2 &\rightarrow a \end{aligned}$$

So, it is a left-linear grammar.

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- The sequence $S \Rightarrow S_1ab \Rightarrow S_1abab \Rightarrow S_1ababab \Rightarrow S_2ababab \Rightarrow aababab$ is a derivation with G_2. So, the language generated by the grammar $L = \{aab(ab)^n | n \geq 0\}$

Regular Grammar

Example 2:

- The grammar $G = (\{S, A, B\}, \{a, b\}, S, P)$ with productions

$$\begin{aligned} S &\rightarrow A, \\ A &\rightarrow aB|a, \\ B &\rightarrow Ab \end{aligned}$$

Regular Grammar

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- The grammar $G = (\{S, A, B\}, \{a, b\}, S, P)$ with productions

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So, it is not regular.

Regular Grammar

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  So, it is not regular.

- Although every production is either in right-linear or left-linear form, the grammar itself is neither right-linear nor left-linear, and therefore is not regular.

Equivalence between Finite Automaton and Regular Grammar

Given a regular grammar $G = (N, T, S, P)$. Construct an NFA $M = (Q, \Sigma, \delta, q_0, F)$, such that $L(M) = L(G)$.

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The construction is like this-

- $Q = N \cup \{q_f\}$
  - Each non-terminal will correspond to a state in Q, apart from that there is one more final state.

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- $F = \{q_f\}$
  - F consists of just q_f.

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- $q_0 = S$
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- $F = \{q_f\}$
  - F consists of just q_f.
- δ is define as follows:
  - If $A \to aB$ is a rule in P, then $\delta(A, a)$ contains B.
  - If $A \to a$ is in P, then $\delta(A, a)$ contains q_f.

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- Consider the grammar $G = (\{S, A\}, \{a, b\}, S, P)$ with productions

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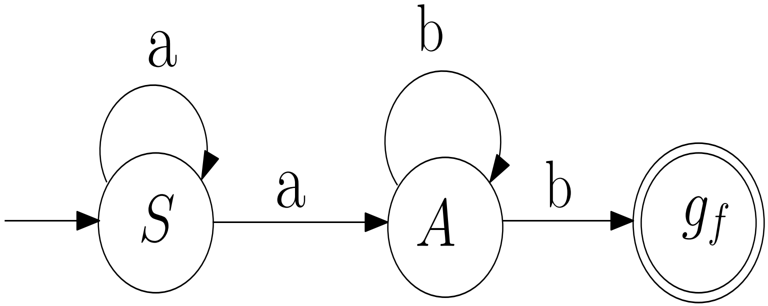
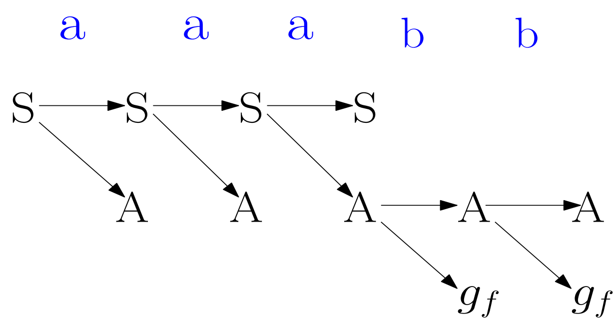


It is also accept the same language L.

Equivalence between Finite Automaton and Regular Grammar

- Let us consider a string $aaabb$. How this string is generated by the grammar G and accept by the NFA M ?
- The sequence $S \Rightarrow aS \Rightarrow aaS \Rightarrow aaaA \Rightarrow aaabA \Rightarrow aaabb$
- The state sequence for NFA



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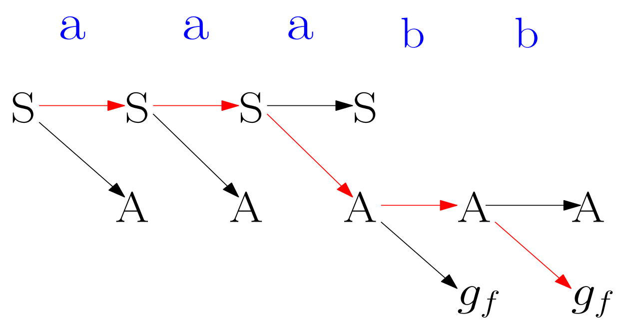

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Construct an NFA $M = (Q, \Sigma, \delta, q_0, F)$, such that $L(M) = L(G)$.
- The language generated by the grammar: $L = \{w | w \in \{a, b\}^*$, length of w is multiple of 3$\}$

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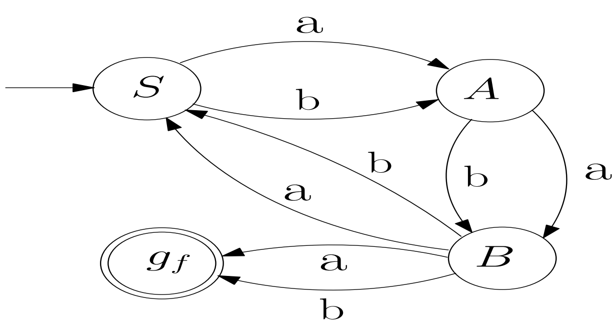

From this, it is clear that the language generated by the grammar and the language accepted by the NFA are same.

Equivalence between Finite Automaton and Regular Grammar

Given a DFA $M = (Q, \Sigma, \delta, q_0, F)$. Construct a regular grammar $G = (N, T, S, P)$, such that $L(G) = L(M)$

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The construction is like this-

- $N = Q$
  - For each state, there is a non-terminal.

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- P is define as follows:
  - If $\delta(A, a) = B$, then $A \to aB \in P$.
  - If $\delta(A, a) = B \wedge B \in F$, then $A \to a$.

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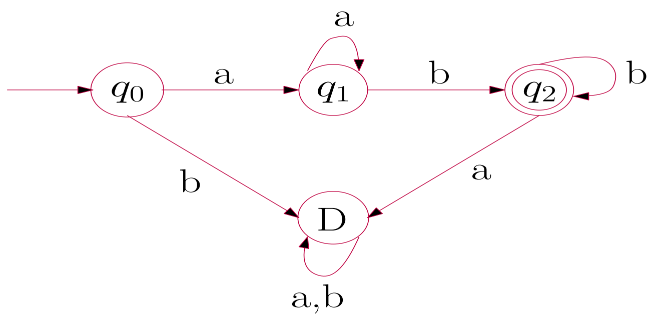

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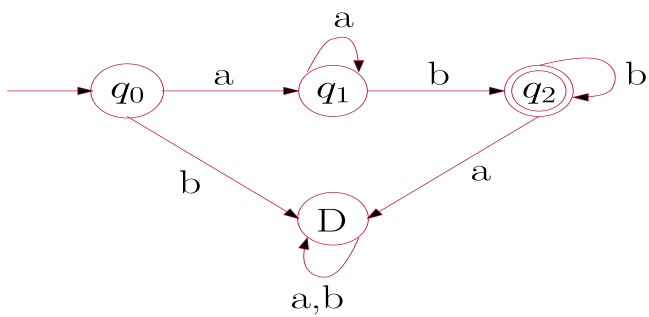

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- The language accepted by the DFA $L = \{a^n b^m | n, m \geq 1\}$

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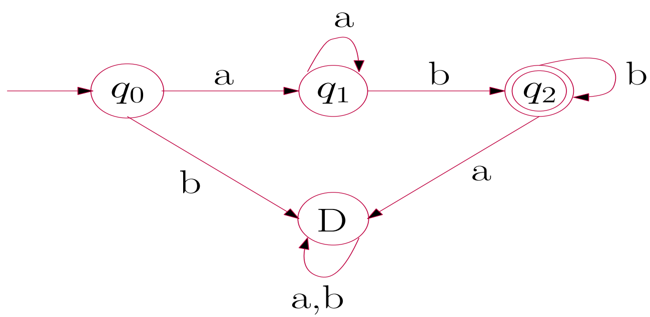

Construct a regular grammar $G = (N, T, S, P)$, such that $L(G) = L(M)$.

- The language accepted by the DFA $L = \{a^n b^m | n, m \geq 1\}$
- The corresponding grammar $N = \{q_0, q_1, q_2, D\}$, $T = \{a, b\}$, $S = q_0$ and P:

$$q_0 \to aq_1$$
$$q_0 \to bD$$
$$q_1 \to aq_1$$
$$q_1 \to bq_2$$
$$q_2 \to bq_2$$
$$q_2 \to aD$$
$$D \to aD$$
$$D \to bD$$

These are non-terminal rules. Now, whenever you have q_2, you have terminal rule. So, here $q_1 \to b$ and $q_2 \to b$ are the terminal rules.

Equivalence between Finite Automaton and Regular Grammar

- From D you cannot drive a terminal string. So, all these D is a useless non-terminal, all these rules can be removed. So we will end with only six rules.

$$\begin{aligned}
q_0 &\rightarrow aq_1 \\
q_1 &\rightarrow aq_1 \\
q_1 &\rightarrow bq_2 \\
q_2 &\rightarrow bq_2 \\
q_1 &\rightarrow b \\
q_2 &\rightarrow b
\end{aligned}$$

- You can generate one a by $q_0 \rightarrow aq_1$ and any number of a by $q_1 \rightarrow aq_1$. Similarly, you can generate one b by $q_1 \rightarrow bq_2$ and any number of b by $q_2 \rightarrow bq_2$.

- So, the language generated by the grammar is a same as the language accepted by the machine.

Equivalence between Finite Automaton and Regular Grammar

Now we consider ϵ, means $\epsilon \in L$

- For any grammar, to include ϵ in a language do the following:
  - The rule $S \rightarrow \epsilon$ can be applied in the first step only and make sure S does not appear on the right hand side of the any production.

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- For any string $L - \epsilon$, will be accepted the usual way we constructed earlier. For ϵ, make initial state a final state. So, ϵ will be accepted.

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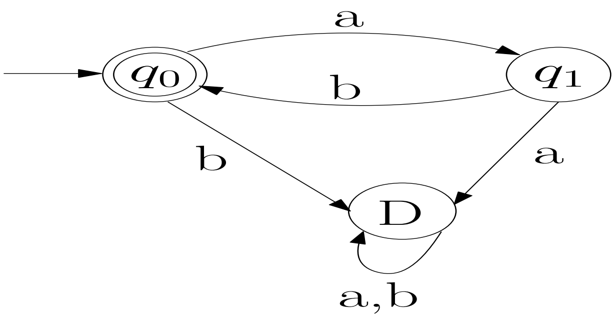
- For any string $L - \epsilon$, will be accepted the usual way we constructed earlier. For ϵ, make initial state a final state. So, ϵ will be accepted.

Given a DFA $M = (Q, \Sigma, \delta, q_0, F)$. Construct a regular grammar $G = (N, T, S, P)$, such that $L(G) = L(M)$

- You have just add the rule $q_0 \to \epsilon$. When you add this rule, you have to make sure that q_0 does not appear on the right hand side of any production. If it is happen, you have to make a slight adjustment.

Equivalence between Finite Automaton and Regular Grammar

Consider the DFA



- $N = \{q_0, q_1, D\},\ T = \{a, b\},\ S = q_0,\ P$:

$$
\begin{aligned}
q_0 &\to aq_1\\
q_0 &\to bD\\
q_1 &\to bq_0\\
q_1 &\to aD\\
D &\to aD\\
D &\to bD\\
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- From D we can not drive a terminal string. So, all these D is a useless non-terminal, all these rule can be removed.
- So, we get only four rules

$$
\begin{aligned}
&q_0 \rightarrow aq_1 \\
&q_1 \rightarrow bq_0 \\
&q_1 \rightarrow b \\
&q_0 \rightarrow \epsilon
\end{aligned}
$$

- Now, the start symbol q_0 and $q_0 \rightarrow \epsilon$, but it is occurring on the right hand side. In order to avoid that, add a new start symbol S and $S \rightarrow \epsilon$ and remove $q_0 \rightarrow \epsilon$. Then, whatever is there with q_0, also have with S.

$$
\begin{aligned}
&q_0 \rightarrow aq_1 \\
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