- Some other hard problems:

- HAMILTONIAN PATH:

  Given a graph G,
  Is there a path in G that goes thoough
  all the vertices of G?

- Variant: Given G, u, v:

  Is there such a path, from u to v?

(SUDEEP)

• HAMILTONIAN CYCLE:

Given a graph G,

Is there a cycle through all vertices?

- We will not get to the proof that they are NP-hard, but we show that if any one of these 3 problems is NP-hard, so are others.

- Between 2 variants of the path problem:

① Given G only ② Given $\langle G, u, v \rangle$

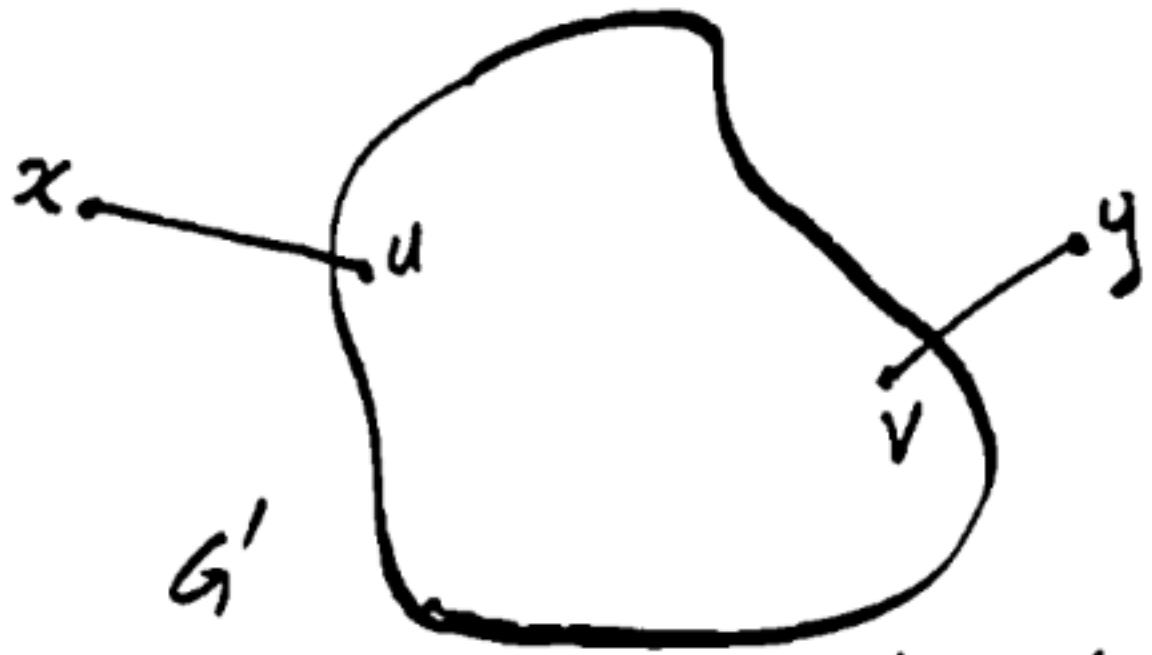
① $\leq_p$ ②:

If ② has a poly. time algorithm, we can call it with every pair of vertices u & v; to solve ①.

② $\leq_p$ ① : If we had an algorithm for ①, how to use it to check if there exists a HAM. path between a specific pair of vertices u and v?

- Note that ① takes only a graph G as its input.

Input G' for ①:



G' has a HAM. PATH if and only if G has a HAM. PATH from u to v. [Argue]

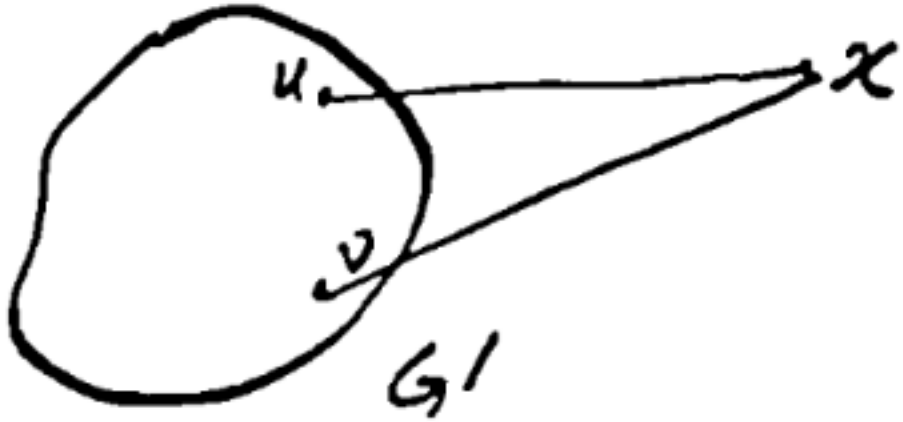
- Cycle to Path and Path to Cycle.
- Either version of the PATH is fine.
- Path to cycle reduction:

  Use algorithm for HAM. CYCLE,
  to design a poly. algorithm for
  HAM. PATH.

- Using the second version:

  Qn: $\langle G, u, v \rangle$: Is there a HAM. PATH from u to v in G?

- Input instance for CYCLE PROBLEM:



Proof:

(1) YES answer is correct.

G' has a HAM. CYCLE ⇒

G has a HAM. PATH from u to v.

(2) NO is correct.

G has a HAM. PATH from u to v

⇒ G' has a HAM. CYCLE.

• The other way?

HAM-CYCLE $\leq_p$ HAM-PATH.

It can be done in many ways.

- One way is to remove each edge (u,v) in G and ask if there is a HAM-PATH from u to v in the resulting graph.

-Can we do with a single call?

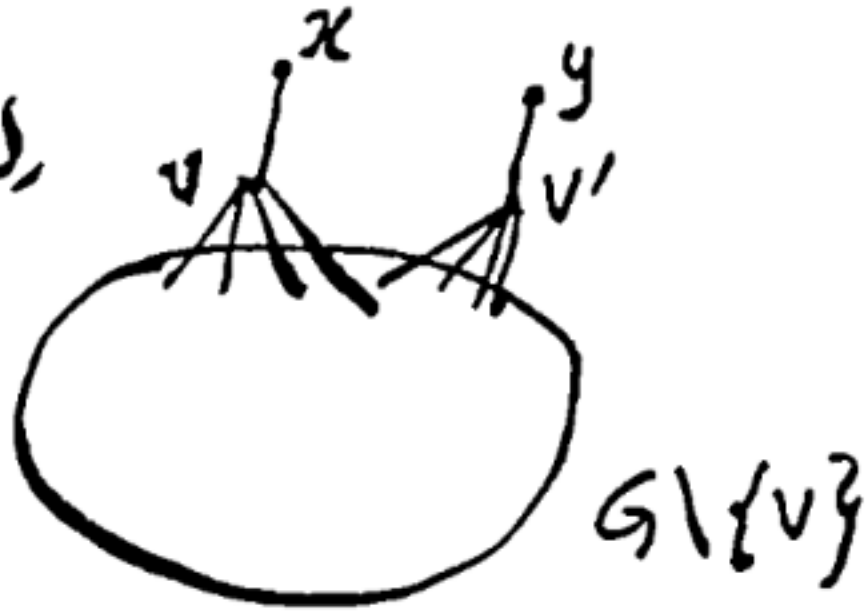
YES!

Take any vertex v, make a 'copy' of it, v'.

Add 2 new vertices,

x to v and

y to v'.



HAM-PATH in new graph (G') $\Leftrightarrow$ CYCLE in G.
G'

(SO DEEP)