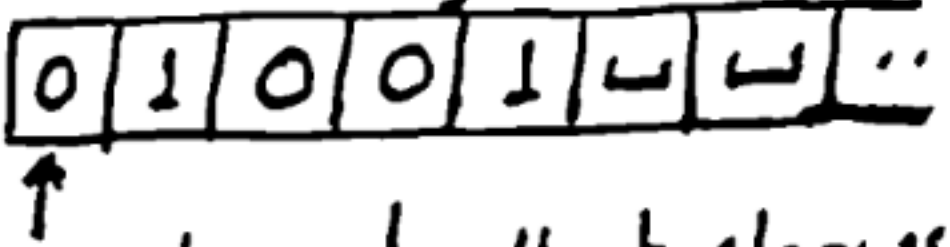
• Turing Machines

formally: It contains

- A tape (called input tape, but it can carry intermediate information and output also).

Initially it contains input, w.



- A tape head, that shows "current" symbol

- It can also have a finite number of states. (eg: Total no. of bits seen till now is odd or even)
- An "accept" or YES state: q_{accept}

  Machine goes to q_{accept} means the algorithm stops, with a YES answer.

- An initial state, q_0
  when the computation (algorithm) begins, the machine is in this state.
- Optional: A 'reject' or 'NO' state, q_{reject}. Machine goes to q_{reject} means it stops with a 'NO'.

- Input alphabet : Set of input symbols. Denoted as Σ.
- Tape alphabet : Σ + additional symbols needed. Denoted Γ. Γ contains 'blank' symbol, $\sqcup$.
- Moves, or how the machine/algorithm works. Denoted δ, transition function.

- A 'move' depends on
  - current symbol
    and
  - current state, q

- It dictates:
  - new symbol, b
  - new state, r
  - direction : Left or Right

(symbol and/or state may remain unchanged)

So, transition function δ looks like:

$$\delta(a, q) = (b, r, D)$$

where D is the direction: L or R.

eg: $\delta(0, q_1) = (0, q_2, R)$ means on seeing a 0 when in state q_1, machine goes to state q_2, symbol is not changed, and the head moves right.

i.e, δ is a function,

$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}.$$

The machine M can be defined completely with

$$(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, \underbrace{q_{reject}}_{\text{optional}})$$

We use 'machine' and 'algorithm', they mean the same.

- Algorithm for checking if a given string is of the form 0^n1^n (assume $n \geq 1$):

  ie, 01 or 0011 or 00001111 →

  Algorithm should stop with YES,

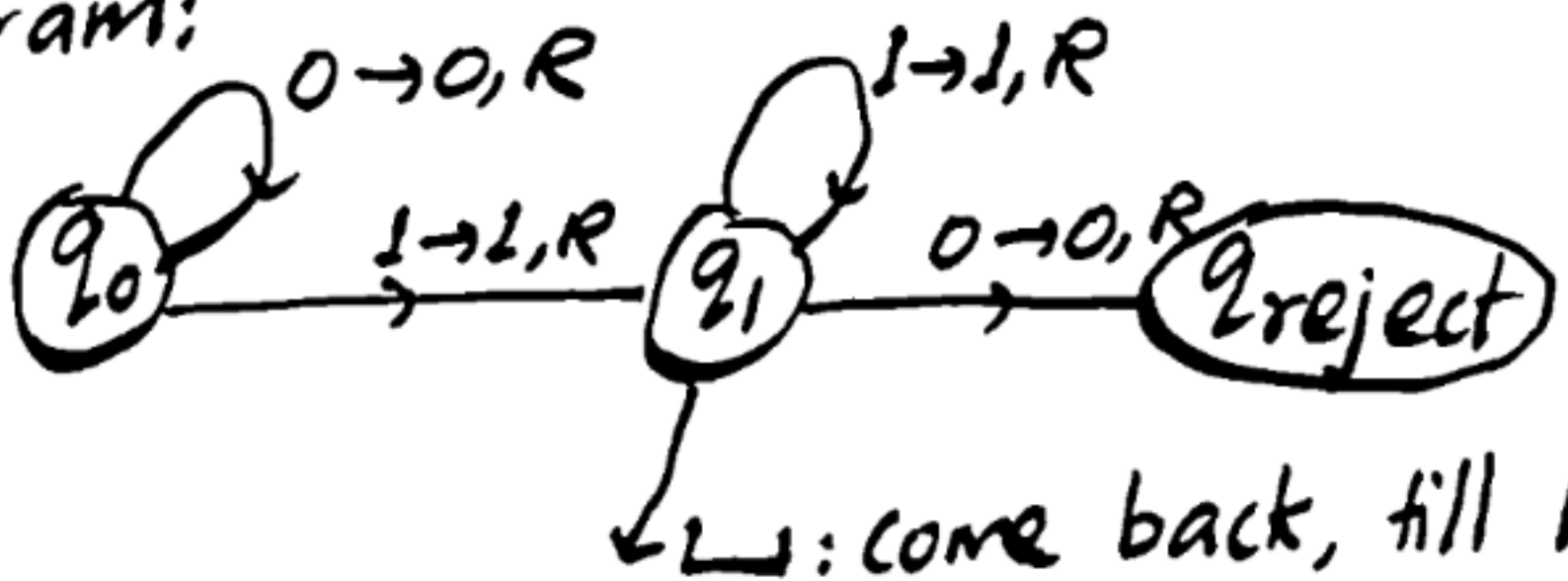
  0, 1, 10, 0110, empty string →

  It should stop with a 'NO' answer.

Step 1: Scan till end of the input string,
if we see a 0 after the first 1,
say NO and stop (ie, "reject").

Step 2: "Cross" the first 0, then go right to
find the first 1, cross that also.
Continue doing this. If no 1 to cross
or if 1's are left, reject. Blank → accept.

Trying to 'draw' step 2, using a state diagram:



↳ : come back, till left end of the tape.

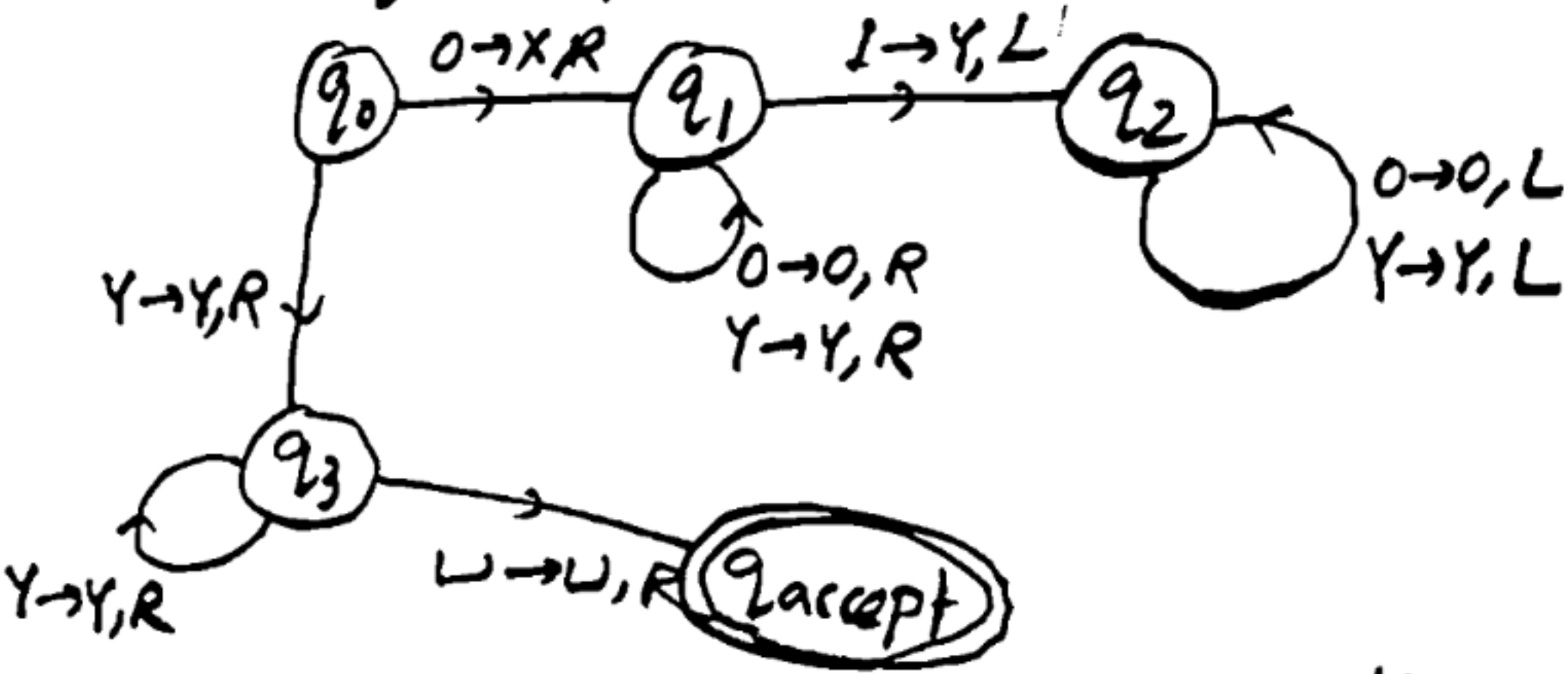
$\delta(q_0, 0) = (q_0, 0, R)$ $\quad$ $\delta(q_1, 1) = (q_1, 1, R)$

$\delta(q_0, 1) = (q_1, 1, R)$ $\quad$ $\delta(q_1, 0) = (q_{reject}, 0, R)$

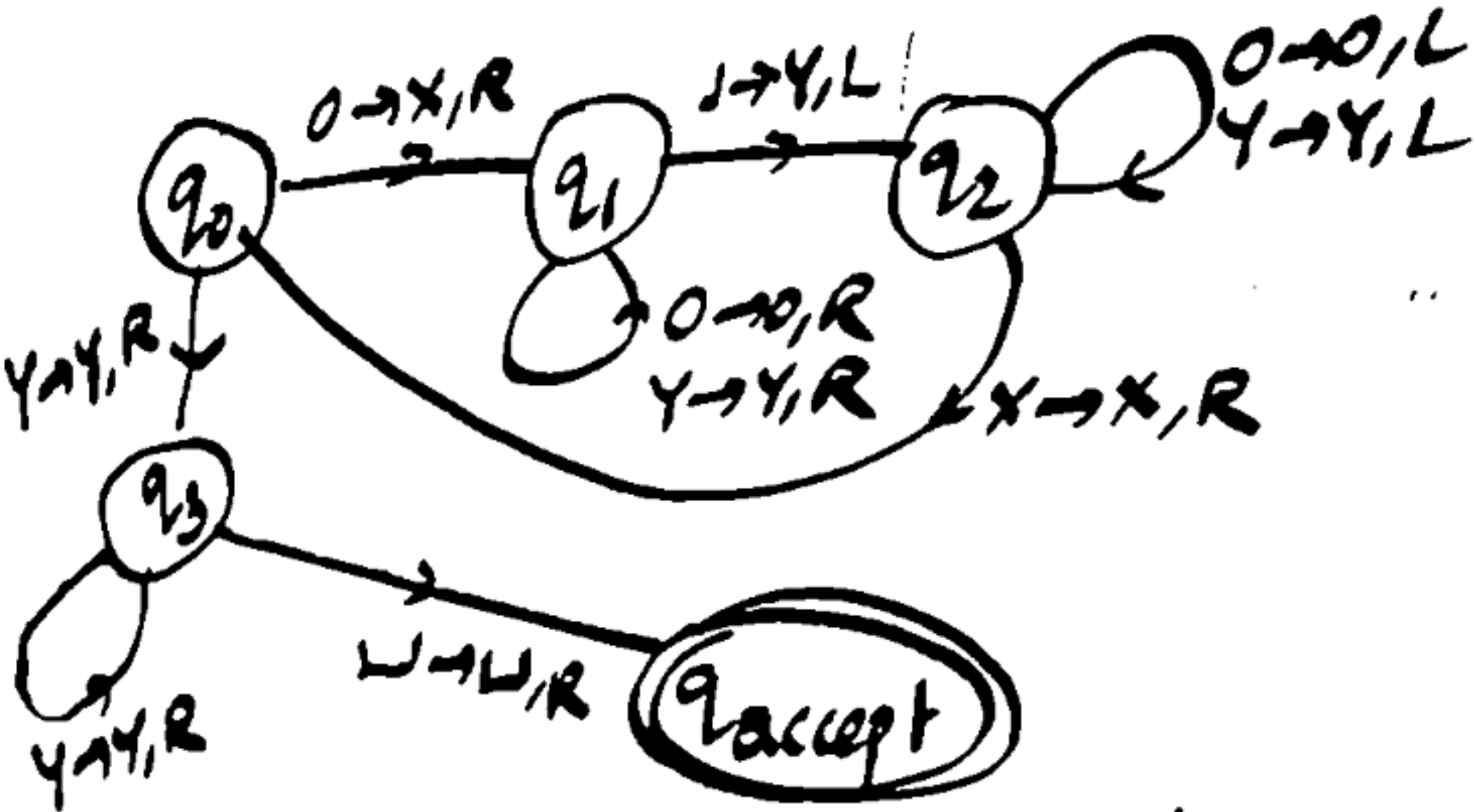
Some questions remain:

- How do we know we hit the leftmost end? (2 way tape it is easy, just look for the blank symbol)
- So we may have to start with marking the first 0 initially.

It turns out we can do all this in a single step also.



(Incomplete. One move more!)



This completes it.