- Complexity Theory

  - Measuring time and space needed for the computation.

  - Involves estimating the number of steps required, or number of cells on tape, as a function of input size.

- The 'running time' or 'time complexity' of a turing machine M (or of an algorithm) is the maximum number of moves it makes on any input of length n (before M accepts or rejects).
- As a function of n. Eg: $2n^2+5n+3$

- M runs in time f(n)
- M is an f(n) time turing machine.

- We will often skip the fine details about this function f(n). A rough estimate suffices most of the time.

- Big O notation: Refresh

when $f(n) = 2n^2 + 5n + 3$,

we often say $f(n) = O(n^2)$.

- What does it mean?
- In general, what is meant by

$f(n) = O(g(n))$?

- We may assume f and g are functions on non-negative integers, and the values they can have are non-negative.
- Reasonable assumptions, as n is the input size (in number of bits, or number of symbols on tape), f is time or space.

So, let f and g be two functions,

$$f, g : \mathbb{N} \to \mathbb{R}^+$$

we say $f(n) = O(g(n))$ if $\exists c > 0$

such that $f(n) \leq c\, g(n)$

for all $n \geq 1$. [or $n \geq n_0$,
n_0 a +ve integer]

- It means that
  g(n) is some kind of an upper bound for f(n), if we ignore constant factors.

"g(n) is an asymptotic upper bound".

- $2n^2 + 5 = O(n^2)$, and $2n^2 + 5 = O(n^3)$

- When it comes to $\log n$, the base does not matter.

  Because $\log_b n = \frac{\log_a n}{\log_a b}$,

  we can ignore the constant $\log_a b$.

- When $f(n) = O(n^k)$, it is a "polynomial".

. Small o notation:

$f(n) = o(g(n))$ means

$f(n) < c\, g(n)$ for all $n \geq n_0$.

. n^2 is NOT $o(n^2)$.

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0.$$

- Analysis of running time:
  - The first turing machine: 0^n1^n
  - Easy to see that it runs in $O(n^2)$ time, or $O(n^2)$ moves in the worst case.
  - 2-tapes, it runs in $O(n)$.
  - A faster algorithm on single tape?

- Idea: Cross off alternative zeroes, and then alternative ones. It reduces the input size.
- Total no. of bits odd - reject!