- Turing machine (algorithm) for checking if a string (of zeroes) is of the form 0^{2^n} (i.e, 2^n zeroes, $n \geq 0$)

Language $A = \{0^{2^n} ; n \geq 0\}$

Algorithm outline: (idea)

- if total number of zeroes is odd, we can reject right away! (only 2 states needed)

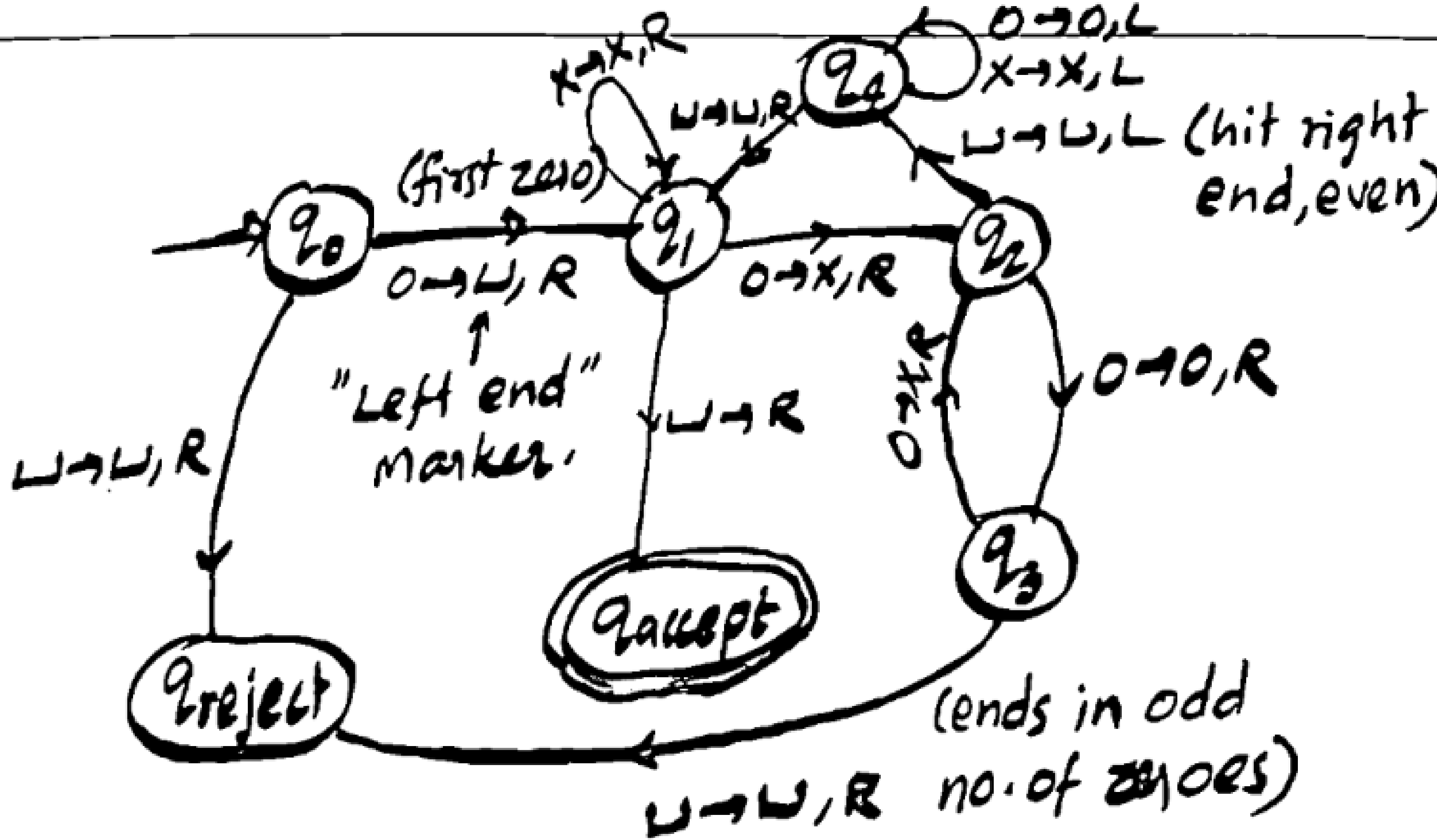
But that is not enough.

Can we extend this idea?

- If only one zero, accept.

What else?

- when it is not a single zero,
(in a loop)
cross off every alternative zero.
So half the no. of zeroes remain.
if this number is odd (and not 1),
reject! if it is 1 (single 0), accept!
- We have an algorithm!


q0
q1
q2
q3
q4
qaccept
qreject
(first zero)
0→⊔,R
"Left end" marker.
0→X,R
X→X,R
⊔→⊔,R
0→0,L
X→X,L
⊔→⊔,L (hit right end, even)
0→0,R
0→X,R
⊔→R
⊔→⊔,R
⊔→⊔,R (ends in odd no. of zeroes)

- Understand how it works:
  - If in any round, after crossing off alternative zeroes, we hit the right end at an odd length, it rejects.
  - when it is a valid string, what happens? Say, 0000.

q_0, tape: $\boxed{0}\boxed{0}\boxed{0}\boxed{0}\boxed{\sqcup}\boxed{\sqcup}\ \cdots$

q_1, tape: $\sqcup\ 0\ 0\ 0\ \sqcup$

q_2, tape: $\sqcup\ \times\ 0\ 0\ \sqcup$ $\quad(\sqcup \times q_2\, 0\, 0\, \sqcup)$

q_3, tape: $\sqcup\ \times\ 0\ 0\ \sqcup$ $\quad$ (jumps over 0)

q_2, $\quad \sqcup\ \times\ 0\ \times\ \sqcup$

Now, we start going backwards.

Goes to q_4: ␣ X 0 X ␣ (or ␣ X 0 q_4 X ␣)

It jumps over all zeroes and X's.

After 3 more moves: q_4

␣	X	0	X	␣	...

Goes right: q_1, ␣ X 0 X ␣

Jumps over: q_1, ␣ X 0 X ␣

0 → X, goes to q_2: ␣ X X X ␣

- No more zeroes remaining.
- On blank it goes to q_4, then goes to q_1.
- Jumps over X's, goes right,
- hits blank (right end), accepts.

- Multitape turing machines:
  - Makes things simple.
  - Think of the first example, $0^n 1^n$

On each tape, move should be clear.

i.e, $\delta : Q \times \Gamma^k \to Q \times \Gamma^k \times \{L, R\}^k$

or $Q \times \Gamma^k \to Q \times (\Gamma \times \{L, R\})^k$

What it means is that

- it has only one state at a time,
- k tapes (and k tape heads),
- so k "current" symbols.

eg: $\delta(q, a_1, a_2) = (r, (b_1, L), (b_2, R))$

or $(r, b_1, b_2, \underset{\substack{\uparrow \\ \text{tape 1}}}{L}, \underset{\substack{\uparrow \\ \text{tape 2}}}{R})$