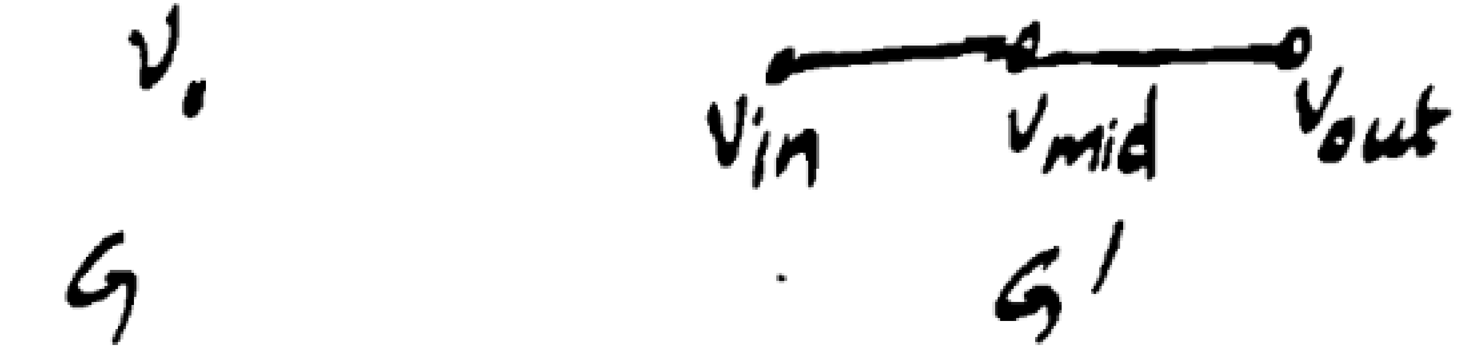
• If directed HAMPATH (in a directed graph G, is there a HAMPATH from s to t) is NP-hard, then HAMPATH is also NP-hard.

- Show: if undirected HAMPATH has a poly. time algorithm, it can be used to solve directed HAMPATH also.

- Constructing the undirected graph G':

For every vertex other than the 'source' s and 'sink' t:

Replace vertex v with 3 vertices,



For source s:

s $\qquad$ s_{out}

Sink $\quad t$ $\qquad$ t_{in}

Only one vertex each in G'.

Edges:

G (directed edge) $\qquad$ G' (undirected)

- <u>Claim</u>: G' has a HAMPATH from s_{out} to t_{in}
  $\iff$ G has a (directed) HAMPATH from s to t.

- Cook-Levin Theorem:

  SAT is NP-complete.

  - If SAT can be solved in poly. time, any problem A in NP has a poly. time algorithm

- Proof Idea:

  A 'generic' reduction. $A \leq_p SAT$, for any $A \in NP$.

- All we know is that $A \in NP$.
- We don't know the details of the language/problem A.
- So what to do? How to do the reduction?
- Idea: We know there is a non-determ. TM N that decides A in poly-time.

• Let us assume this TM N decides A (goes to q_{accept} if input $w \in A$) in at most n^k moves.

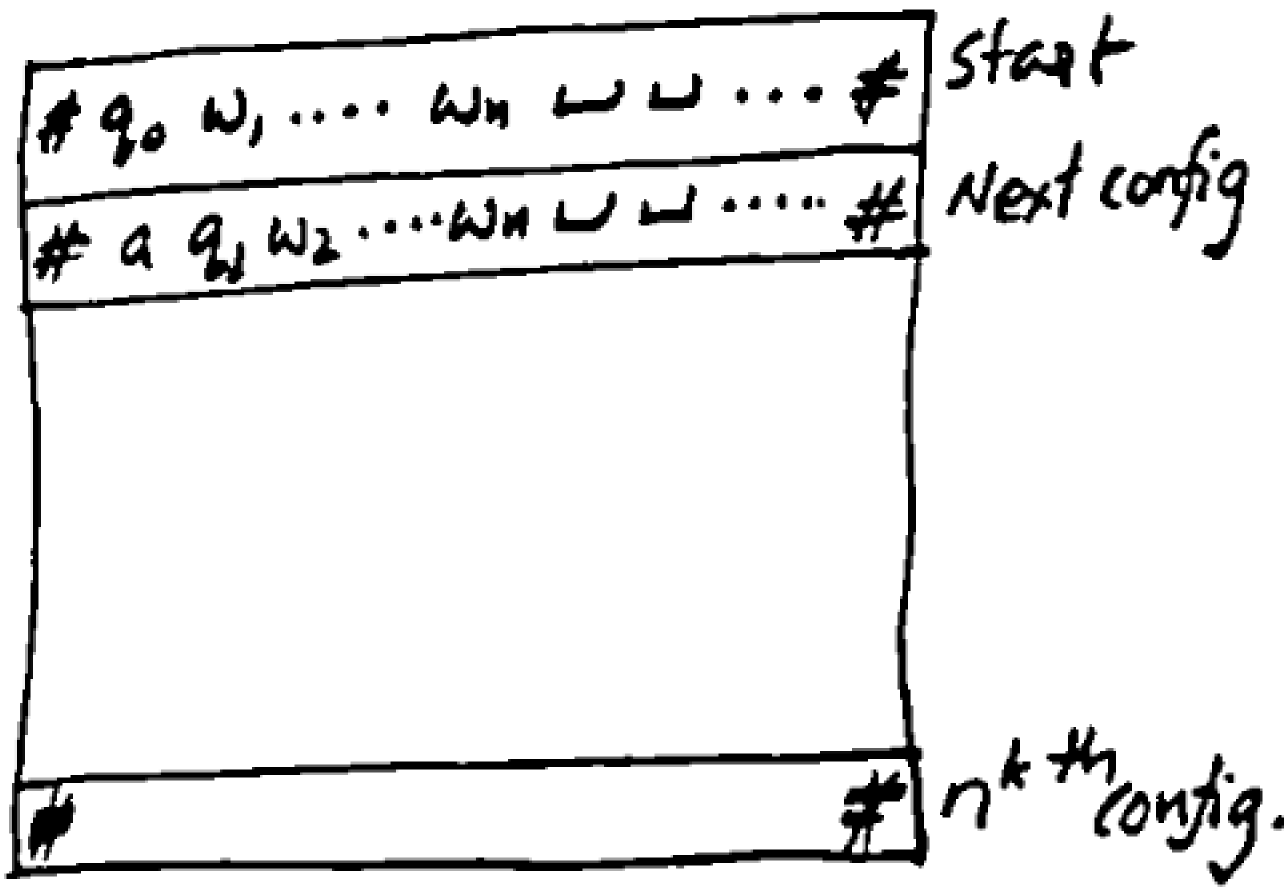
[Precisely, we assume $n^k - 3$ moves].

- We construct a boolean formula ϕ (instances of SAT) using the moves of N.

• ϕ is built based on "branches" of moves on N, and input w.

$w = w_1 w_2 \ldots\ldots w_n$

$|w| = n$



(SUDEEP)

- First row shows the start configuration of the non-det TM N, with ω as input.
- Subsequent rows:
  'Next' configuration, after one move in N.
- This block (or tableau, or matrix) corresponds to $n^k - 1$ moves.

- Symbols that appear in the matrix:
  - Tape symbols, Γ
  - States, Q
  - Special symbol #
- variables of ϕ:

Variables should be able to tell us the entries in this matrix.

- For a symbol $x \in \Gamma \cup Q \cup \{\#\}$,

  x_{ij} = 'true' means

  entry $(i,j) = x$ in the matrix.

- Thus we have $n^k \times n^k$ variables for each symbol x in the set.

- ϕ is to be defined such that...

- ϕ is satisfiable if and only if
  There is some branch of N,
  that, starting with w as input,
  takes N to q_{accept} within n^k-3 moves.

- Or, ϕ is satisfiable
  if and only if there is such a tableau
  that starts with w and goes to q_{accept}.