- A multi-track turing machine and a single track turing machine are essentially same. We only have to redefine the alphabets properly.

- 2-track:

0	1	0	0	1	$\sqcup$	$\sqcup$	$\sqcup$
$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$

First cell contains: $\langle 0, \sqcup \rangle$

New Γ, or $\Gamma' = \Gamma \times \Gamma$ (symbols $\langle a, b \rangle$)

- How can we show that

A 2-way tape (blanks on both sides) TM and a 1-way tape TM are equivalent (in computing power)?

Idea:
We need to show the moves on a 2-way tape can be simulated on a 1-way tape turing machine.

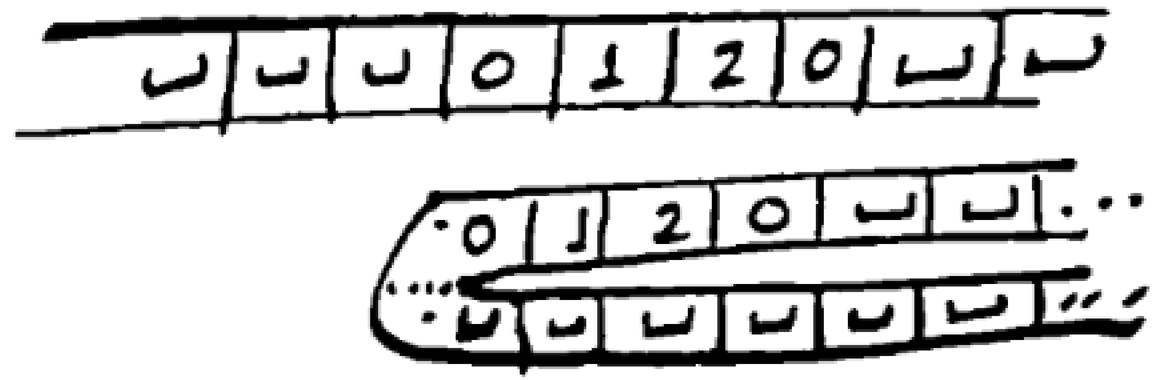
- How?

When the head goes to the 'left' of the left end, use a second track!

- Like 'folding' it.

M_1:

M_2:



Move on M_1: $\delta(q, 0) = (r, X, R)$

what is the move on M_2?

- The machine has to 'remember' whether it is reading the top symbol (right half of the tape) or the bottom symbol.
- It also needs to mark the left-most cell. For this, we add special symbols $\dot{a}$ for each $a \in \Gamma$.

• Now we have to take care of the 'jump over' moves also.

If there is a move

$\delta(q, a) = (r, c, L)$ that is expected when on the 'leftmost' end:

it becomes two moves, as it has to 'stay' on the same cell.

let $\dot{\Gamma} = \Gamma \cup \{\dot{a} \mid a \in \Gamma\}$

then the new tape alphabet,

$$\Gamma' = \dot{\Gamma} \times \dot{\Gamma} \times \{top, bottom\}$$

ie, one symbol looks like

$$\begin{array}{|c|}\hline a \\ \hline b \\ \hline \text{top} \\ \hline \end{array}$$

• In the leftmost cell, we use $\dot{a}$ instead of a.

ie, if it was

$\sqcup$	0	0	1	....

The new tape:

$\dot{0}$	0	1 ....
$\dot{\sqcup}$	$\sqcup$	.... ''
top	top	....

More on M_1: $\delta(q,0) = (r, X, R)$

- what are the corresponding moves on M_2?

Case 1: The machine is currently on the "top" half, or 'right' half:

$$\delta(q, \langle 0, b, top \rangle) = (r, \langle X, b, top \rangle, R)$$

for all $B \in \Gamma$

Case 2: Bottom (left) half:

$\delta(q, \langle b, o, bottom \rangle) = (r, \langle b, x, bot \rangle, L)$

i.e, it becomes a 'left' move.

Similarly, a 'left' move remains a Left move if it is on top half, becomes a right more if bottom.

- We need to take care of the special case of the moves on the leftmost cell.
- For example, what if this was a move that "jumps over" from left half to the right half?
  - It becomes two moves!

- Similarly, a multitape turing machine can also be simulated using a single tape turing machine.
- Idea: • Separate the 'tapes' by a #.
  • 'Mark' the tape head on each tape.