- Some decidable (or recursive) languages:
  - Strings of the form $0^n 1^n$
  - Prime numbers (Is a number Prime? YES/NO question)
  - Many such mathematical questions about numbers, graphs etc.

- Is a given graph 3-colourable?
- Given a directed graph, and 2 vertices u & v, Is there a path from u to v in this graph G?

• Consider the language

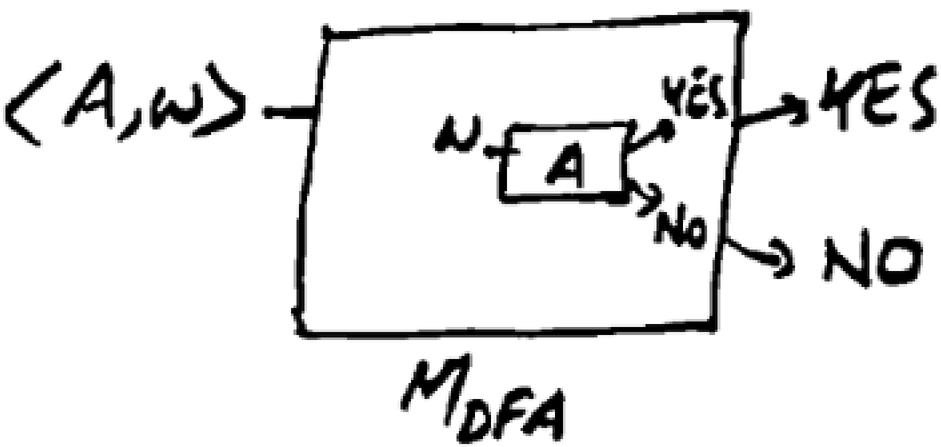
$$\{\langle A, w\rangle : \text{Given DFA } A \text{ accepts } w\}$$

- As a YES/NO question:

  Given a DFA A and a string w,

  Does A accept the string w?

- Claim: This is decidable.



- A turing machine M_{DFA} can be defined, which "runs" the DFA A with input w.
- If it ends in a final (accept) state, answer YES (go to q_{accept}).
- Else answer NO.

- This works for an NFA also.
  - Either we can convert it to a DFA and then run it with input w,
  - Or we can make a non-deterministic TM. (Need to track moves till end of w).

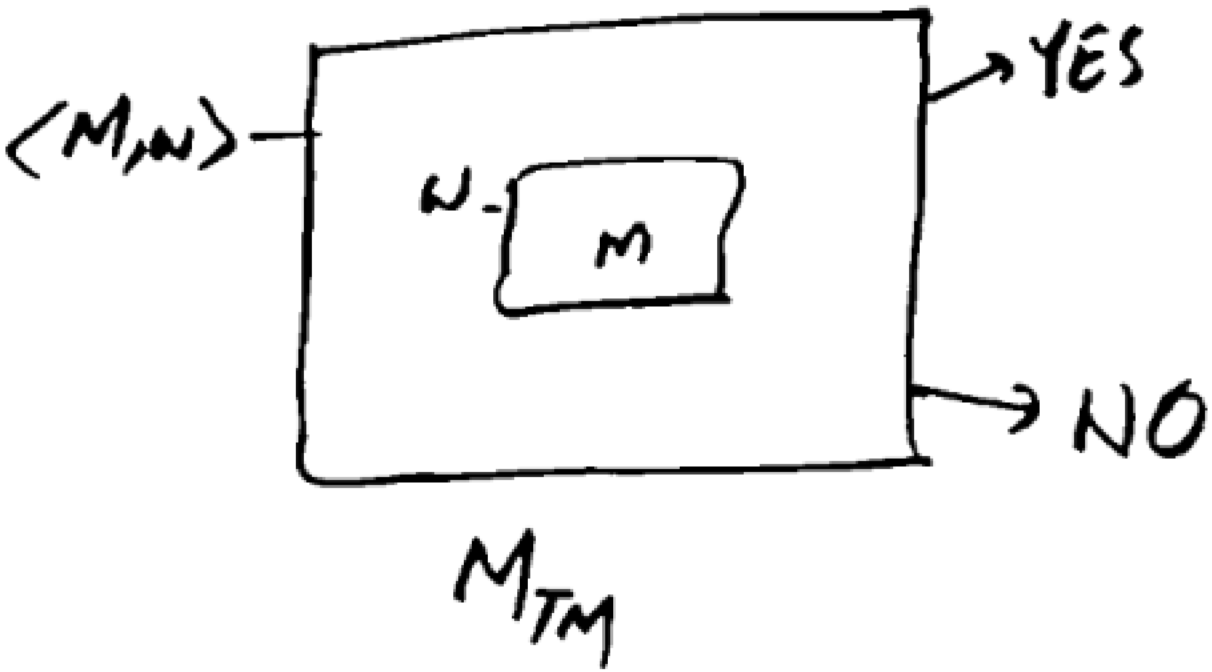
Either way, we can answer YES/NO correctly.

- What about the language

$$\{\langle M, w\rangle : \text{Turing Machine } M \text{ accepts the string } w\}\ ?$$

- Or, given a TM M and a string w, Does M accept w? (Give YES/NO)

- The earlier trick will not work.



- Why?

- Because the M may not be a deciding turing machine, so we can not be sure of a NO answer even if $w \notin L(M)$.

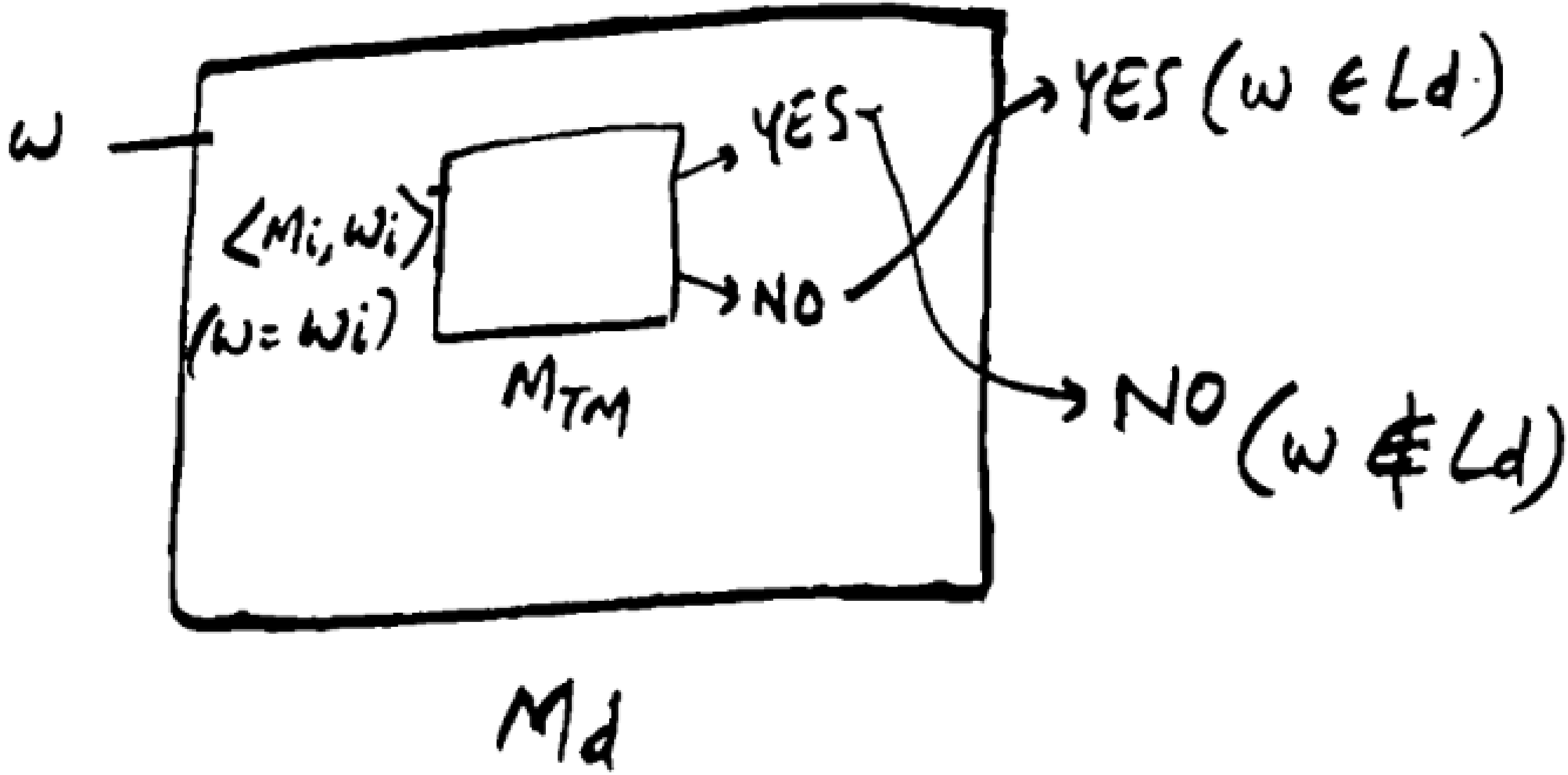
- $L(M)$ means the language of M. M accepts w if and only if $w \in L(M)$.

- But can we actually give a proof that this language, let us call it L_{TM}, is not decidable?

  - YES!
  - We can prove it with the help of L_d that we defined.

HOW?

- Input: $\langle M, w \rangle$.
- Let us assume there is some M_{TM} that decides this language.
  - i.e., it says YES if M accepts w;
    No otherwise.
- Proof idea:
  If such an M_{TM} exists, L_d is decidable.



- Find i such that $w = w_i$.
  Give $\langle M_i, w_i \rangle$ as input to M_{TM}.

- Go over it and convince yourself.

i.e, if L_{TM} is decidable,

L_d is also decidable.

- But we know L_d is not even recognizable. So we have a contradiction.
- It means L_{TM} is not decidable.