- Polynomial-time 'Reductions':

- If we knew that a problem A does not have an efficient algorithm (i.e, if $A \notin P$), how can we use this fact to show another problem B is also hard?

- i.e., if $A \notin P$, then $B \notin P$.

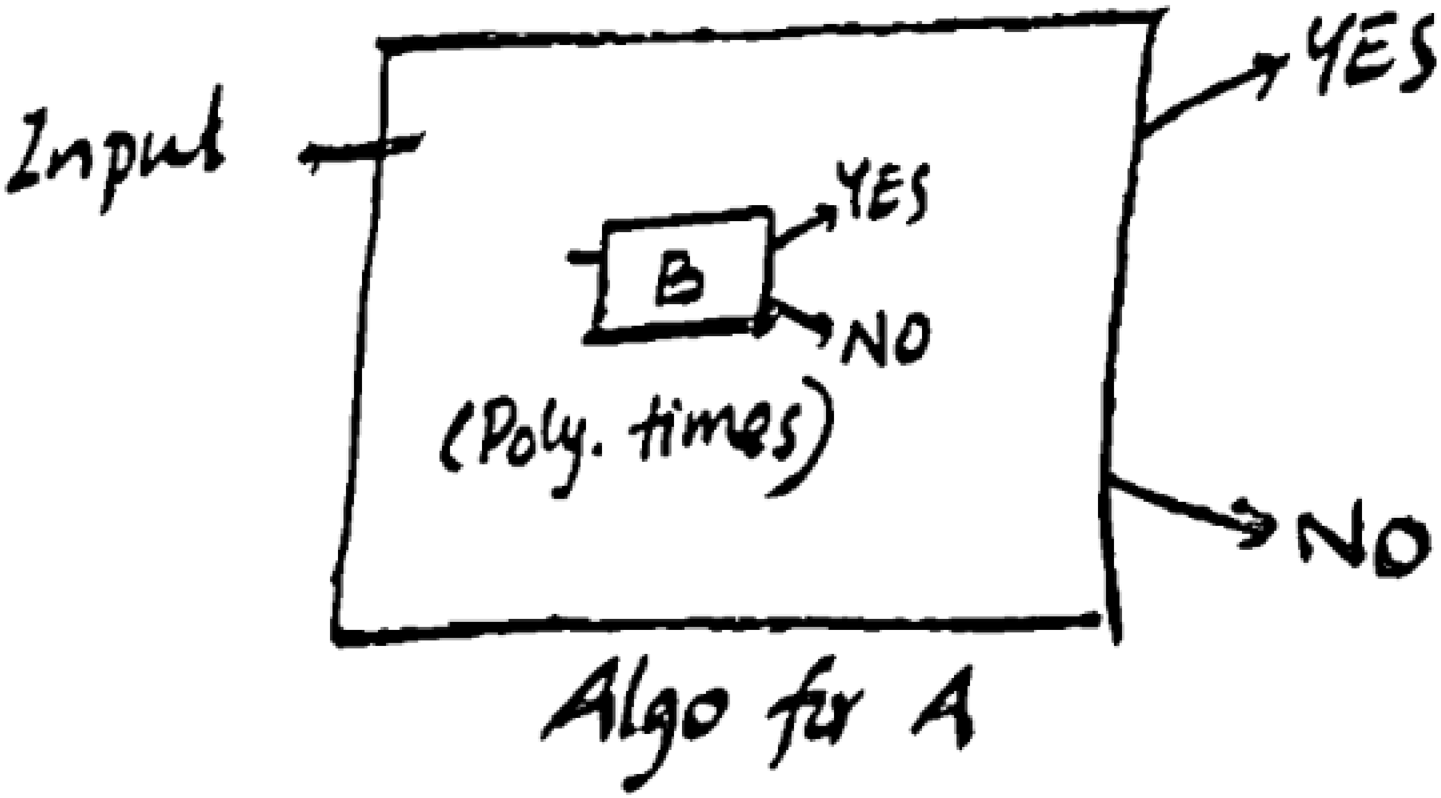
- It is not so difficult.

It is same as proving that

if B is in P, then A is also in P.

- For this, just give an algorithm for A, using an algorithm (TM) for B.
- Make sure 'B' is called at most n^k times, for some constant k.

. This is called
a Polynomial-time reduction.



- This concept can be applied for problems other than YES/NO questions also.
- We shall see some examples, to understand this kind of reductions

In some of them, we call B only once.

- Problem A:

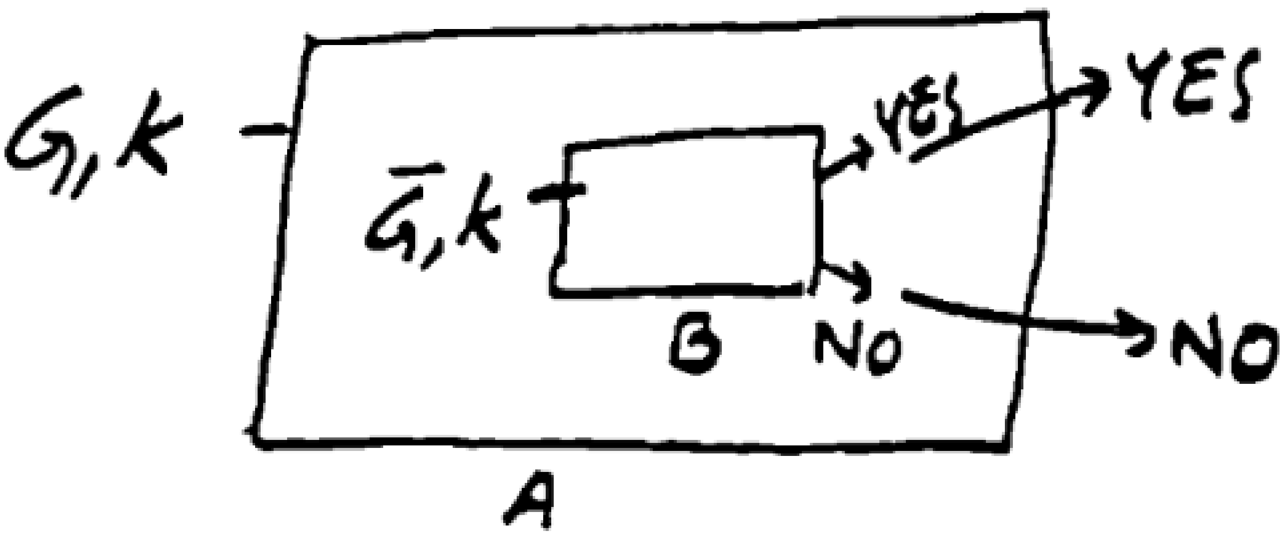
  Given a graph G and a positive integer k, Does G have a clique of size k?

- B: Given (G, k), Does G have an independent size of size k?

  (Show: If A is hard, so is B).

• Easy!

• If B had an algorithm that runs in polynomial time, we will show that A also has.



- Another Example:

When one is a YES/NO problem, the other one is NOT so.

- A: Same (Does G have a k-clique?)
- B: Find a clique of size k in G.

- This is also easy. If B finds a clique, answer for A is YES. Else NO.
- What if it was the other way round?
- If we knew B is hard, how to show A is hard?

- Proof idea:

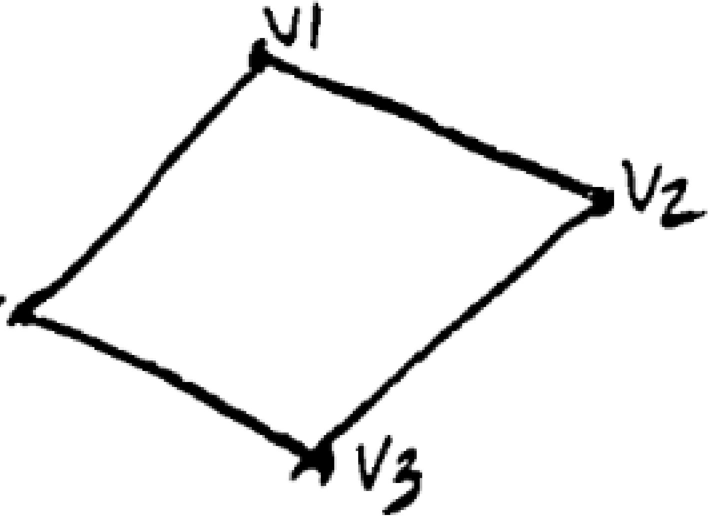
Keep removing vertices, and keep running the YES/NO algorithm for A.

- Do not remove a vertex if removal results in NO answer.
- We end with only k vertices.

- Vertex cover: A set of vertices that 'cover' all edges of the graph. (every edge has at least one endpoint in this set).

- Qn: Given G & k, Does G have a vertex cover of size k?

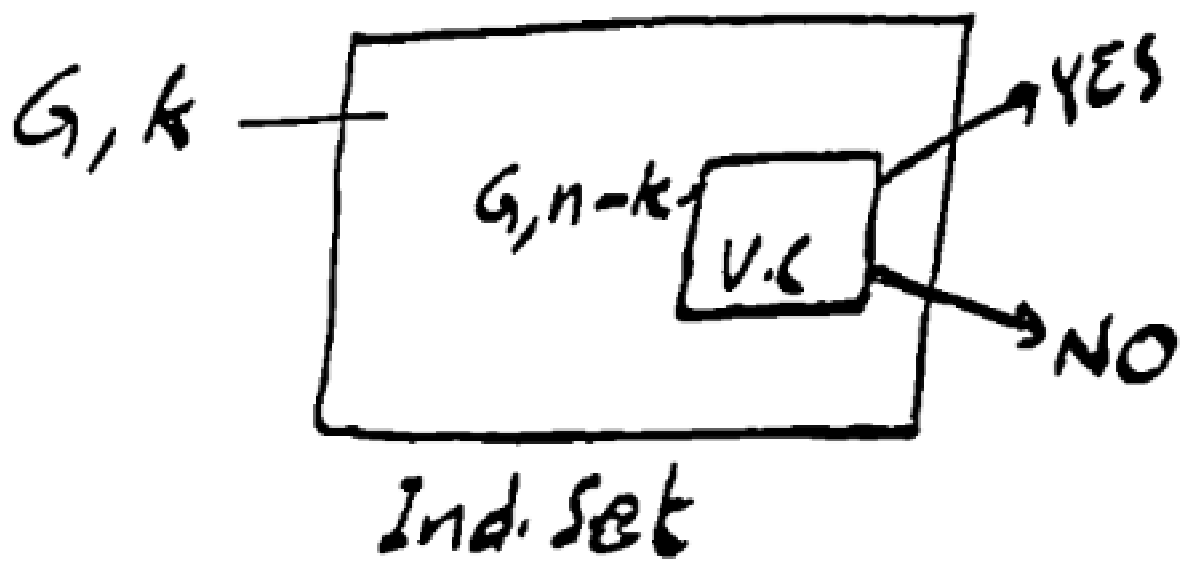
$\{v_1, v_3\}$
is a vertex cover.



$\{v_2, v_4\}$ is also a vertex cover.

- If independent set problem is hard, how to show vertex cover is also hard?

• Idea: If we remove a vertex cover (all vertices in a vertex cover) from G, what remains is an independent set. The other way also.



• Convince yourself that this is correct.

- YES (G has a vertex cover of size $n-k$), then G has an ind. set of size k. YES.

- NO. (if G has an ind. set of size k, it can not say NO. Because removal of the ind. set gives a vertex cover, size $n-k$). So, NO answer is also correct.