UNIVERSITY OF SOUTHAMPTON ELEC6213W1

SEMESTER 2 EXAMINATIONS 2017-18

IMAGE PROCESSING

DURATION 120 MINS (2 Hours)

This paper contains 4 questions

Answer **BOTH** questions from section **A.**
Answer **ONE** question from section **B.**

An outline marking scheme is shown in brackets to the right of each question.

Only University approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Section A

Q1)

a) **Describe** what *Aliasing* is and **calculate** the minimum *sampling frequency* if we want to reconstruct a low pass signal whose maximum frequency is 2.8 MHz in continuous domain.

[6 marks]

b) **Describe** what *Quantisation* is and **explain** why lower bits are noisier than higher bits of bytes (pixels) in images after quantisation.

[7 marks]

c) A common measure of transmission for digital data is the *baud rate,* defined as the number of bits transmitted per second. Generally, transmission is accomplished in packets consisting of a start bit, a byte (8 bits) of information and a stop bit. **Calculate** how many minutes it would take to transmit a 1024 X 1024 image with 256 intensity levels using 56K baud modem.

[10 marks]

d) **Calculate** what would be the time in (Q1-c) at 3000K baud, a representative medium speed of a phone DSL (Digital Subscriber Line) connection

[10 marks]

Q2)

a) **Describe** what are meant by *Fourier transforms* by writing its formula for 2D forward and inverse continuous Fourier transforms. **Explain** what Fourier transforms actually do.

[6 marks]

b) A certain X-ray imaging geometry produces a blurring degradation that can be modelled as the convolution of the sensed image with the spatial, circularly symmetric function

$$h(x,y)=\frac{x^2+y^2-2\sigma^2}{\sigma^4}\exp\left(\frac{-(x^2+y^2)}{2\sigma^2}\right)$$

Assuming continuous variables, **show** that the degradation in the frequency domain is given by the expression:

$$H(u,v)=-2\pi\sigma^2(u^2+v^2)\exp\left(-\sigma^2(u^2+v^2)/2\right)$$

Hint: Fourier transform of $\exp(-x^2/2\sigma^2)$ is $\sigma\sqrt{2\pi}\exp(\frac{-\sigma^2u^2}{2})$

[13 marks]

c) Suppose that you filter an image $f(x,y)$ with a spatial filter mask, $w(x,y)$ using convolution where the mask is smaller than the image in both spatial directions. **Show** the important property that, if the coefficients of the mask sum to zero, then the sum of all the elements in the resulting convolution array (filtered image) will be zero too. Also, you may assume that the border of the image has been padded with the appropriate number of zeros.

[14 marks]

Section B

Q3)

a) **Explain** what a *Wiener* filter is by writing its transfer function in frequency domain and describing all the terms in the transfer function.

[4 marks]

b) **Explain** what the Wiener filter changes to and why, if there is no noise in the system. If there is a small amount of noise in the system, **describe** what the Wiener filter becomes and why.

[10 marks]

c) Consider the problem of image blurring caused by uniform speed in the *x*-direction. If there is a technical problem in the shutter of the camera in a way that in order to capture a picture, the shutter immediately opens and stays open for T_1 seconds and then it suddenly closes and stays closed for T_2 seconds and finally it immediately opens again and stays open for T_3 seconds before it completely closes, **find** the blurring function $h(x,y)$, and its Fourier Transform $H(u,v)$ for $T_1 = T_2 = T_3 = T$ if the camera moves with a constant speed V_s along the *x*-direction.

[9 marks]

d) Using the transfer function you have found in section (c) of this question, **provide** an expression for a Wiener filter, assuming the ratio of the power spectra of the noise and un-degraded signal is a constant.

[10 marks]

Q4)

a) Describe how the *Laplacian of a Gaussian* (*LoG*) filter is used to detect edges in images.

[3 marks]

b) Show that the *LoG* filter can be derived from a *Gaussian* filter $G(x, y) = \exp\left(\frac{-(x^2 + y^2)}{2\sigma^2}\right)$ as follows:

$$\nabla^2 G(x, y) = \left(\frac{x^2 + y^2 - 2\sigma^2}{\sigma^4}\right)\exp\left(\frac{-(x^2 + y^2)}{2\sigma^2}\right) \qquad \text{(Q4-1)}$$

[12 marks]

c) *The Difference of a Gaussian (DoG)* filter is defined as:

$$DoG(x, y) = \frac{1}{2\pi\sigma_1^2}\exp\left(\frac{-(x^2 + y^2)}{2\sigma_1^2}\right) - \frac{1}{2\pi\sigma_2^2}\exp\left(\frac{-(x^2 + y^2)}{2\sigma_2^2}\right) \qquad \text{(Q4-2)}$$

In order to approximate an *LoG* filter with a *DoG* filter, the zero crossing of a *DoG* filter should be adjusted to be in the same place as the zero crossing of a *LoG* filter. **Show** that the relationship between the standard deviation, σ, of the *LoG* filter (shown in equation (Q4-1)) and the standard deviations σ_1 and σ_2 of the Difference of two Gaussians in the *DoG* filter (shown in equation (Q4-2)), is derived as shown in equation (Q4-3), if the *DoG* filter in equation (Q4-2) approximates the *LoG* filter in equation (Q4-1).

$$\sigma = \frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 - \sigma_2^2} Ln\left(\frac{\sigma_1^2}{\sigma_2^2}\right) \qquad \text{(Q4-3)}$$

[12 marks]

d) By using equation (Q4-3), **show** $\sigma \to \sigma_1$ (or $\sigma \to \sigma_2$) if $\sigma_1 \to \sigma_2$.

[6 marks]

END OF PAPER

