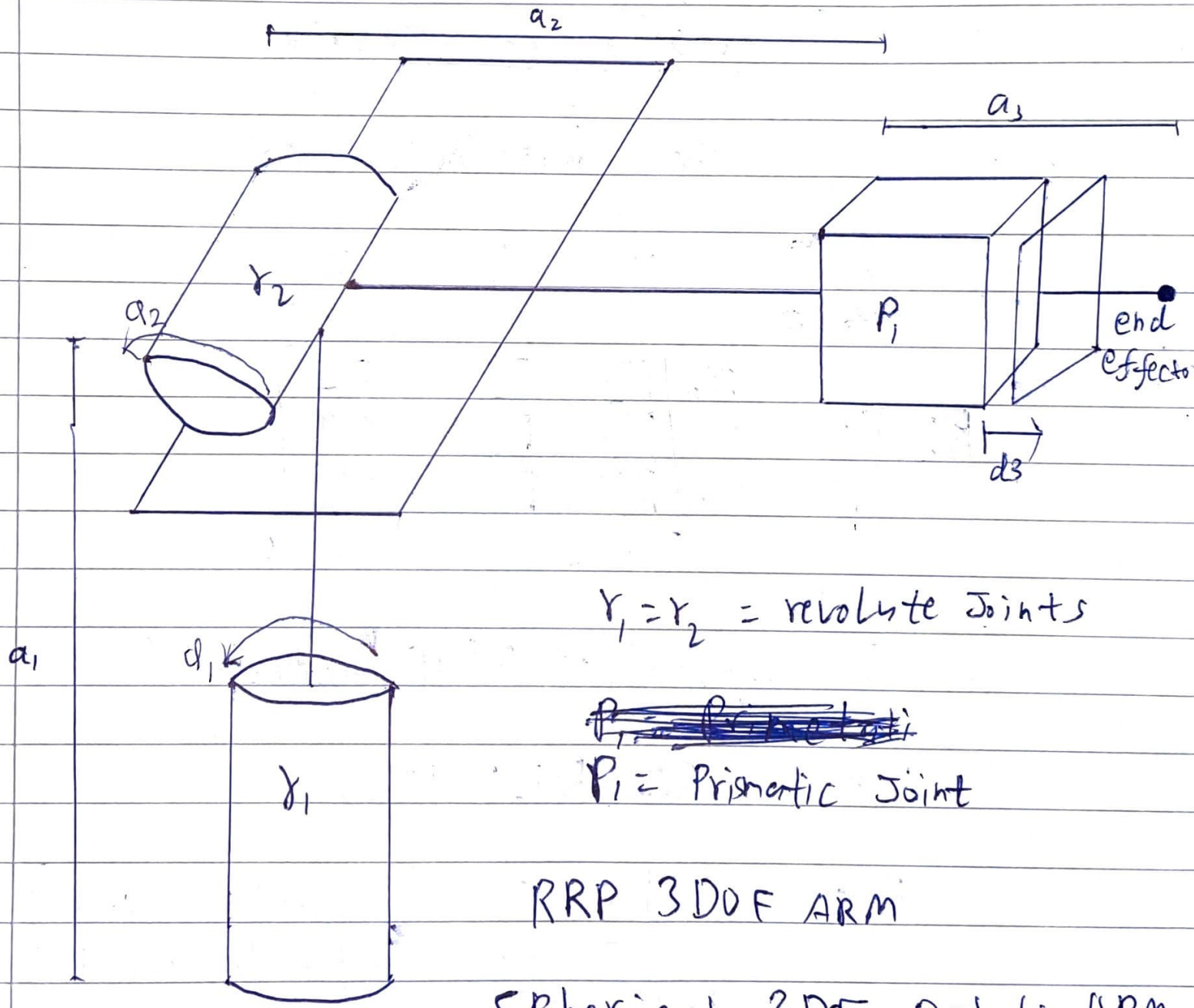


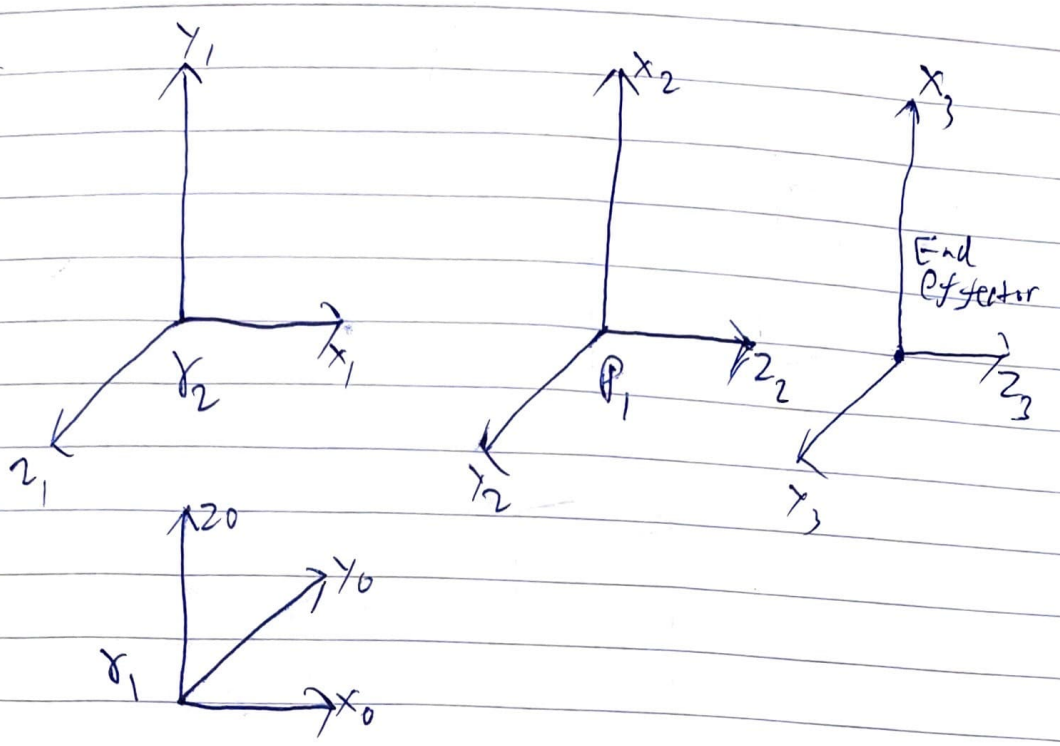
$r_1 = r_2$ = revolute Joints

~~P_1 = Prismatic Joint~~

P_1 = Prismatic Joint

RRP 3DOF ARM

Spherical 3DOF Robotic ARM



Rotation matrix

The Projection of

x_1 y_1 z_1

on

$$R0\_1 = \begin{bmatrix} x_0 & \square & \square & \square \\ y_0 & \square & \square & \square \\ z_0 & \square & \square & \square \end{bmatrix}$$

For spherical 3 DOF arm

$$R0\_3 = R0\_1 \, R1\_2 \, R2\_3$$

$$R0\_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{0\_1} = \begin{bmatrix} \cos q_1 & 0 & \sin q_1 \\ \sin q_1 & 0 & -\cos q_1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_{1\_2} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \cos q_2 & 0 & \sin q_2 \\ 0 & 1 & 0 \\ -\sin q_2 & 0 & \cos q_2 \end{bmatrix}$$

$$R_{1\_2} = \begin{bmatrix} -\sin q_2 & 0 & \cos q_2 \\ \cos q_2 & 0 & \sin q_2 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_{2\_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Displacement vectors

$$d_{0\_1} = \begin{bmatrix} 0 \\ 0 \\ q_1 \end{bmatrix} \quad d_{1\_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad d_{2\_3} = \begin{bmatrix} 0 \\ 0 \\ a_2 + a_3 + d_2 \end{bmatrix}$$

Homogenous Transformation Matrix

$$H_n^n = \begin{bmatrix} R_n^n & d_n^n \\ 0\ 0\ 0 & 1 \end{bmatrix} \Rightarrow H_{0\_1} = \begin{bmatrix} R_{0\_1} & d_{0\_1} \\ 0\ 0\ 0 & 1 \end{bmatrix}$$

Denavit - Hartenberg Method

number of rows = number of frames

Columns = θ, a, r, α

For spherical 3DOF Arm

D-H Parameters

	θ	α	r	d
1	θ_1	90	0	q_1
2	$\theta_2 + 90$	90	0	0
3	0	0	0	$a_2 + a_3 + d_3$

HTM using D-H Parameters can be written as

$$H_n^{n-1} = \begin{bmatrix} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & r_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & r_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For spherical 3Dof Arm

$$H0\_3 = H0\_1\ H1\_2\ H2\_3$$

$$H0\_3 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & P_x \\ r_{21} & r_{22} & r_{23} & P_y \\ r_{31} & r_{32} & r_3 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Here, P_x = x cordinate of end effector
P_y = y cordinate of end effector
P_z = z cordinate of end effector

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = d0\_3$$

P_x, P_y, P_z = location of end effector