SC

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1) Fuzzy logic - Assign ①

2) Fuzzy sets -

→ Fuzzy set theory introduced to deal with unreliable, incomplete and imprecise info which cannot be handled by the classical (crisp) sets.

→ Fuzzy set theory is an extension to classical set theory where elements have degree of membership.

→ Fuzzy logic uses the whole interval b/w 0 (false) and 1 (true) to describe human reasoning.

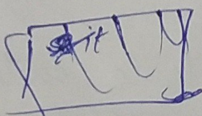
→ Classical set - A classical or crisp set has a crisp boundary.
For eg. $A = \{x \mid x > 6\}$
a classical set A of real numbers greater than 6

→ Classical sets don't reflect the nature of human concepts & thoughts.
Eg. $A = \{x \mid x > 6\}$
where A = "tall person" and x = "height"
mathematically expressing the set of all tall persons as a collection of people whose height is more than 6 ft.

Person with 6.001 ft ✓ tall
5.999 ft ✗ not tall
} Classical set
⇒ unreasonable

→ Fuzzy sets theory permits membership function valued in the interval [0,1], whereas classical set theory allows the membership of elements in the set in binary terms only.

Classical set Theory	Fuzzy Set Theory
① Classes of objects with sharp boundaries.	Classes of objects with unsharp boundaries
② defined by crisp (exact) boundaries, i.e no uncertainty about location of set boundaries.	defined by its ambiguous boundaries i.e uncertainty about location of the set boundaries
③ Used in digital system design	Used in Fuzzy Controllers.



→ A membership function $\mu_A(x)$ is associated with a fuzzy set $\tilde{A}$ such that the function maps every element of universe of discourse X to the interval [0,1].

$$\mu_A(x) : X \to [0,1]$$

→ Membership function:- Fully defines the fuzzy set. Provides a measure of the degree of similarity of an element to a fuzzy set.

- Either chosen by the user arbitrarily based on user's experience
- Or designed using MC methods.

Definition: Fuzzy Set

If X is a collection of objects, denoted generally by x, then a fuzzy set $\tilde{A}$ in X is defined as a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$$

where $\mu_{\tilde{A}}(x)$ is called Membership function for the Fuzzy set $\tilde{A}$.

$X \rightarrow$ Universe of Discourse < discrete or continuous

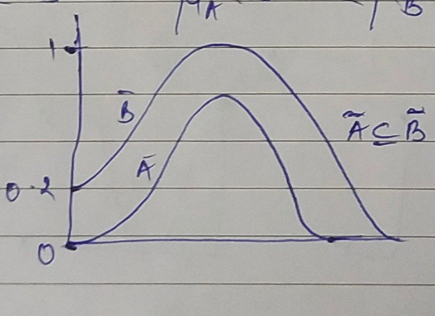
Operations of Crisp sets - Union, intersection, complement, difference

3) Operations on Fuzzy Sets

① Containment or Subset -

FS $\tilde{A}$ is contained in FS $\tilde{B}$ if and only if $\mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)$ for all x.

$$\tilde{A} \subseteq \tilde{B} \Leftrightarrow \mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)$$

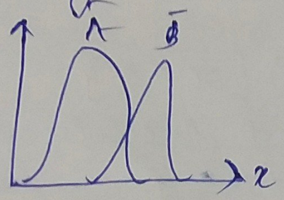


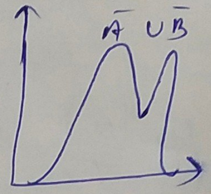
② Union (disjunction)

A union of two fuzzy sets $\tilde{A}$ and $\bar{B}$ is a fuzzy set $\bar{C}$, such that whose MF is

$$\mu_{\bar{C}}(x) = \max(\mu_{\bar{A}}(x), \mu_{\bar{B}}(x))$$

denoted by $(\tilde{A} \cup \bar{B})$ or $(\bar{A} \text{ or } \bar{B})$

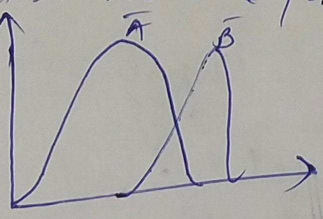


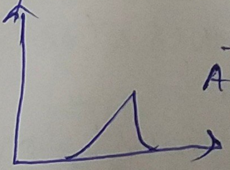


③ Intersection (conjunction)

The intersection of two fuzzy sets $\bar{A}$ and $\bar{B}$ is a fuzzy set $\bar{C}$, such that whose MF is

$$\mu_{\bar{C}}(x) = \min(\mu_{\bar{A}}(x), \mu_{\bar{B}}(x))$$

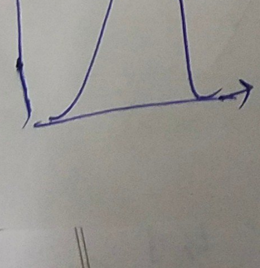


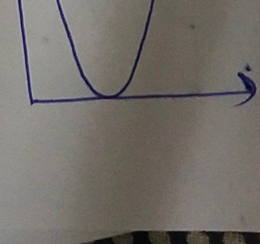


④ Complement (negation)

The complement of a fuzzy set $\tilde{A}$, denoted is defined as -

$$\mu_{\bar{\bar{A}}}(x) = 1 - \mu_{\bar{A}}(x)$$





More operations -

① Algebraic sum

$$\mu_{\tilde{A}+\tilde{B}}(x) = \mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x) - \mu_{\tilde{A}}(x).\mu_{\tilde{B}}(x)$$

② Algebraic product

$$\mu_{\tilde{A}.\tilde{B}}(x) = \mu_{\tilde{A}}(x) . \mu_{\tilde{B}}(x)$$

③ Bounded Sum

$$\mu_{\tilde{A}\oplus\tilde{B}}(x) = \min[1, \mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x)]$$

④ Bounded Difference

$$\mu_{\tilde{A}\odot\tilde{B}}(x) = \max[0, \mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x)]$$

Properties of Fuzzy Sets -

For FS, $A \cup \bar{\tilde{A}} \neq U$; $\tilde{A} \cap \bar{\tilde{A}} \neq \phi$

① Commutativity $\tilde{A} \cup \tilde{B} = \tilde{B} \cup \tilde{A}$ & $\tilde{A} \cap \tilde{B} = \tilde{B} \cap \tilde{A}$

② Associativity $\tilde{A} \cup (\tilde{B} \cup \tilde{C}) = (\tilde{A} \cup \tilde{B}) \cup \tilde{C}$

$A \cap (\tilde{B} \cap \tilde{C}) = (\tilde{A} \cap \tilde{B}) \cap \tilde{C}$

③ Distributivity $\tilde{A} \cup (\tilde{B} \cap \tilde{C}) = (\tilde{A} \cup \tilde{B}) \cap (\tilde{A} \cup \tilde{C})$

$\tilde{A} \cap (\tilde{B} \cup \tilde{C}) = (\tilde{A} \cap \tilde{B}) \cup (\tilde{A} \cap \tilde{C})$

④ Identity $\tilde{A} \cup \phi = \tilde{A}$; $\tilde{A} \cup U = U$

$\tilde{A} \cap \phi = \phi$; $\tilde{A} \cap U = \tilde{A}$

relations are very imp in fuzzy controller
they can describe interaction b/w variables

(5) Involution $\bar{\bar{A}} = \bar{A}$

(6) Transitivity If $\bar{A} \subset \bar{B} \subset \bar{C}$, then $\bar{A} \subset \bar{C}$

(7) De-Morgan's Law $\overline{\bar{A} \cup \bar{B}} = \bar{\bar{A}} \cap \bar{\bar{B}}$

$\overline{\bar{A} \cap \bar{B}} = \bar{\bar{A}} \cup \bar{\bar{B}}$

(Read crisp relation)

4) Fuzzy Relations

A relation can be considered as a set of tuples where a tuple is an ordered pair.

A fuzzy relation is a fuzzy set of tuples, i.e. each tuple has a membership degree b/w 0 & 1.

def:⇒ Let U & V be continuous universe, and $\mu_R : U \times V$ [0, 1], then

$$R = \int_{U \times V} \mu_R(u, v)/(u, v)$$

is a binary fuzzy relation on U × V.

⇒ If U and V are discrete universe then

$$R = \sum_{U \times V} \mu_R(u, v)/(u, v)$$

We can express fuzzy relation R = U × V in matrix form

$$R = \begin{bmatrix} \mu_R(u_1, v_1) & \mu_R(u_1, v_2) & \cdots & \mu_R(u_1, v_n) \\ \mu_R(u_2, v_1) & \mu_R(u_2, v_2) & \cdots & \mu_R(u_2, v_n) \\ \vdots & & & \\ \mu_R(u_m, v_1) & \mu_R(u_m, v_2) & \cdots & \mu_R(u_m, v_n) \end{bmatrix}$$

where $U = \{u_1, u_2, u_3 \ldots u_m\}$ and $V = \{v_1, v_2, v_3 \ldots v_n\}$ are universe of disclosure

Fuzzy relations are very imp in fuzzy controller because they can describe interaction b/w variables

Eg Pg 48 pdf

N-ary fuzzy relation
Fuzzy set of n tuples
In general it is a relation of pair
$\mu_R(x_1, x_2 \dots x_n)/(x_1, x_2 \dots x_n)$;

operations on Fuzzy Relation

4 types → Intersection, Union, Projection & Cylendrical extension

(1) Intersection -
Let R and S be binary relations defined on X × Y.
The intersection of R & S is defined by -

$\forall (x, y) \in X \times Y : \mu_{R \cap S}(x, y) = \min(\mu_R(x,y), \mu_S(x,y))$

Instead of min, any T-norm can be used.

Eg $T(x, y) = \frac{x \cdot y}{x + y - x \cdot y}$

(2) Union
Let R & S be binary relations defined on X × Y.
The union of R & S is given by

$\forall (x,y) \in X \times Y : \mu_{R \cup S}(x,y) = \max(\mu_R(x,y), \mu_S(x,y))$

Instead of max, any S-form can be used.

Eg A simple S-form

$$S(x,y) = x + y - x \cdot y$$

Eg Given 2 relations R & S

$$R = \begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ \hline x_1 & 0.3 & 0.2 & 0.1 \\ x_2 & 0.4 & 0.6 & 0.1 \\ x_3 & 0.2 & 0.3 & 0.5 \end{array} \qquad S = \begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ \hline x_1 & 0.4 & 0 & 0.1 \\ x_2 & 1 & 0.2 & 0.8 \\ x_3 & 0.3 & 0.2 & 0.4 \end{array}$$

① Using max operation,

$$R \cup S = \begin{bmatrix} 0.4 & 0.2 & 0.1 \\ 1 & 0.6 & 0.8 \\ 0.3 & 0.3 & 0.5 \end{bmatrix}$$

Using S-form

$$S(x,y) = x + y - x \cdot y,$$

then $R \cup S = \begin{bmatrix} 0.58 & 0.2 & 0.19 \\ 1 & 0.68 & 0.84 \\ 0.44 & 0.44 & 0.7 \end{bmatrix}$

More optimistic than max operation.
All membership degrees are atleast as high as in max.

② Using min operation

$$R \cap S = \begin{bmatrix} 0.3 & 0 & 0.1 \\ 0.4 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0.4 \end{bmatrix}$$

More optimistic than min ops.
All membership degrees are less than that in min operation

Using T-form

$$T(x,y) = \frac{x \cdot y}{x + y - x \cdot y},$$

$$R \cap S = \begin{bmatrix} 0.2 & 0 & 0.1 \\ 0.4 & 0.17 & 0.1 \\ 0.13 & 0.13 & 0.28 \end{bmatrix}$$

③ Projection

Projection relation brings a ternary relation back to a binary relation, or a binary relation to a fuzzy set, or a fuzzy set to a single crisp value.

eg Relation R =

	y_1	y_2	y_3	y_4
x_1	0.8	0.7	0.3	0.6
x_2	0	1	0	1
x_3	0.9	0.2	0.4	0.7

Then projection on X means that
x_1 is assigned the max of first row.
x_2 " " " " " 2nd " .
x_3 " " " " " 3rd " .

Thus, Proj. R on X = $\frac{0.8}{x_1} + \frac{1}{x_2} + \frac{0.9}{x_3}$

Similarly, Proj. R on Y = $\frac{0.9}{y_1} + \frac{1}{y_2} + \frac{0.4}{y_3} + \frac{1}{y_4}$

④ Cylindrical Extension

→ Projection operation is almost always used in combination with cylindrical extension.
→ More or less opposite of proj.
→ Converts a fuzzy set to a relation.

Eg consider 2 fuzzy sets

$A = \text{proj. of } R \text{ on } X = \frac{1}{x_1} + \frac{0.8}{x_2} + \frac{1}{x_3}$

CE on domain $X \times Y$ is

$ce(A) =$

	y_1	y_2	y_3	y_4
x_1	1	1	1	1
x_2	0.8	0.8	0.8	0.8
x_3	1	1	1	1

$B = \text{proj. of } R \text{ on } Y = \frac{0.9}{y_1} + \frac{0.8}{y_2} + \frac{0.7}{y_3} + \frac{0.8}{y_4}$

$ce(B) =$

	y_1	y_2	y_3	y_4
x_1	0.9	0.8	0.7	0.8
x_2	0.9	0.8	0.7	0.8
x_3	0.9	0.8	0.7	0.8

Properties of Fuzzy Relations

Let R, S & T be fuzzy relations defined on the universe $X \times Y$.

1. Commutativity $R \cup S = S \cup R$
   $R \cap S = S \cap R$
2. Associativity $R \cup (S \cup T) = (R \cup S) \cup T$
   $R \cap (S \cap T) = (R \cap S) \cap T$
3. Distributivity. $R \cup (S \cap T) = (R \cup S) \cap (R \cup T)$
   $R \cap (S \cup T) = (R \cap S) \cup (R \cap T)$
4. Idempotency $R \cup R = R$ $R \cap R = R$
5. Identity $R \cup \phi_R = R$, $R \cap \phi_R = \phi_R \rightarrow$ null relation (null matrix)
   $R \cup E_R = E_R$ $R \cap E_R = R$
   E_R ↓ complete relation (unit matrix of all 1s)

(6) Involution $\bar{\bar{R}} = R$

(7) De-Morgan's Law $\overline{R \cap S} = \bar{R} \cup \bar{S}$

$\overline{R \cup S} = \bar{R} \cap \bar{S}$

(8) $R \cup \bar{R} \neq E_R$

$R \cap \bar{R} \neq \phi$

Fuzzy Numbers

A fuzzy no. is a generalization of a regular, real no. such that it refers to a connected set of possible values rather than one single value. These values have their own weight b/w 0 and 1.

This weight is called membership function.

A fuzzy no. = A fuzzy interval

Arithmetic Operations on Fuzzy Nos.

Let A and B denote fuzzy nos. and * denote any of four basic arithmetic operators. Then a fuzzy set on R, $A * B$ is defined as

$$(A * B)(z) = \sup_{z = x * y} \min[A(x), B(y)] \quad \forall z \in R$$

i) $(A + B)(z) = \sup_{z = x + y} \min[A(x), B(y)]$

$(A - B)(z) = \sup_{z = x - y} \min[A(x), B(y)]$

$(A \cdot B)(z) = \sup_{z = x \cdot y} \min[A(x), B(y)]$

$(A / B)(z) = \sup_{z = x / y} \min[A(x), B(y)]$

Arithmetic Operations on Intervals —
* → any of the 4 arithmetic operations
enclosed intervals

1) $[a,b] + [d,e] = [a+d, b+e]$
2) $[a,b] - [d,e] = [a-e, b-d]$
3) $[a,b] * [d,e] = [\min(ad, ae, bd, be),$
$\max(ad, ae, bd, be)]$

4) $[a,b] / [d,e] = [a, b] * [1/d, 1/e]$

$= [\min(\frac{a}{d}, \frac{a}{e}, \frac{b}{d}, \frac{b}{e}), \max(\frac{a}{d}, \frac{a}{e}, \frac{b}{d}, \frac{b}{e})]$

eg $[1,3] * [2,-2] = [\min(2, -2, 6, -6), \max(2, -2, 6, -6)]$
$= [-6, 6]$

Fuzzy Composition Pg 54
to combine 2 fuzzy relations in different product spaces
(1) Max-min composition
(2) Max-product "

(1) Let $R_1 \rightarrow$ fuzzy relation on $X \times Y$
$R_2 \rightarrow$ " " on $Y \times Z$

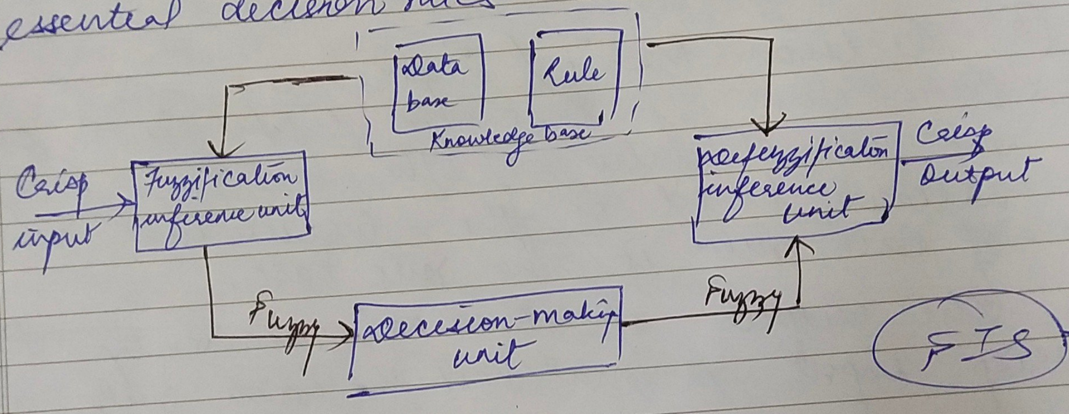
(1) Max Min
$$\mu_{R_1 \circ R_2}(x, z) = \max_{y \in Y} \{\min(\mu_{R_1}(x,y), \mu_{R_2}(y,z))\}$$

(2) Max Product
$$\mu_{R_1 \circ R_2}(x, z) = \max_{y \in Y} \{\mu_{R_1}(x,y) \cdot \mu_{R_2}(y,z)\}$$

See Pg 55

Inference in Fuzzy Logic

→ Fuzzy Inference System is a key unit of a fuzzy logic system.
→ Fuzzy inference (reasoning) is the actual process of mapping from a given input to an output using fuzzy logic.
→ It uses the "IF...THEN" rules along with connecters "OR" or "AND" for drawing essential decision rules.



Characteristics of FIS -
1. ~~The~~ Output from FIS is always a fuzzy set irrespective of its input which can be fuzzy or crisp
2. Output → fuzzy ⇒ used as a controller
3. defuzzification unit to convert fuzzy ~~have~~ into crisp variables

Modules in FIS

1. Fuzzification Module (FM) or Fuzzification Inference Unit
2. Decision Making Unit
3. Defuzzification Module or ~~Deffu~~ Defuzzification IU
4. Knowledge Base — Rule Base / Data Base

1. Fuzzification module or IU -

→ This block performs a fuzzification which converts a crisp input into a fuzzy set.

→ A proper fuzzification strategy is devised.

2. Decision Making / Inferencing :-

→ Basic func of inference engine is to compute the overall value of the control output variable based on the individual contribution of each rule in the rule base.

→ The output of the FM represents the crisp input is matched to each rule-antecedent.

→ The degree of match of each rule is established. Based on this degree of match, the value of the control output variable in the rule-consequent is modified. The result is the "clipped" fuzzy set representing the control of variable.

→ The set of all clipped control output values of the matched rules represent the overall fuzzy value of control output.

Defuzzification Module -
- Performs defuzzification which converts the overall control output into a single crisp value.

Knowledge base -
- Rule base and the database combined.
- Rule base - Contains fuzzy IF-THEN rules
- Database provides necessary info for proper functioning of the fuzzification module, the rule base and the defuzzification module.
- Info in db includes :-
① Fuzzy MFs for the input and output control variables
② The physical domains of the actual problems and their normalised values along with scaling factors.

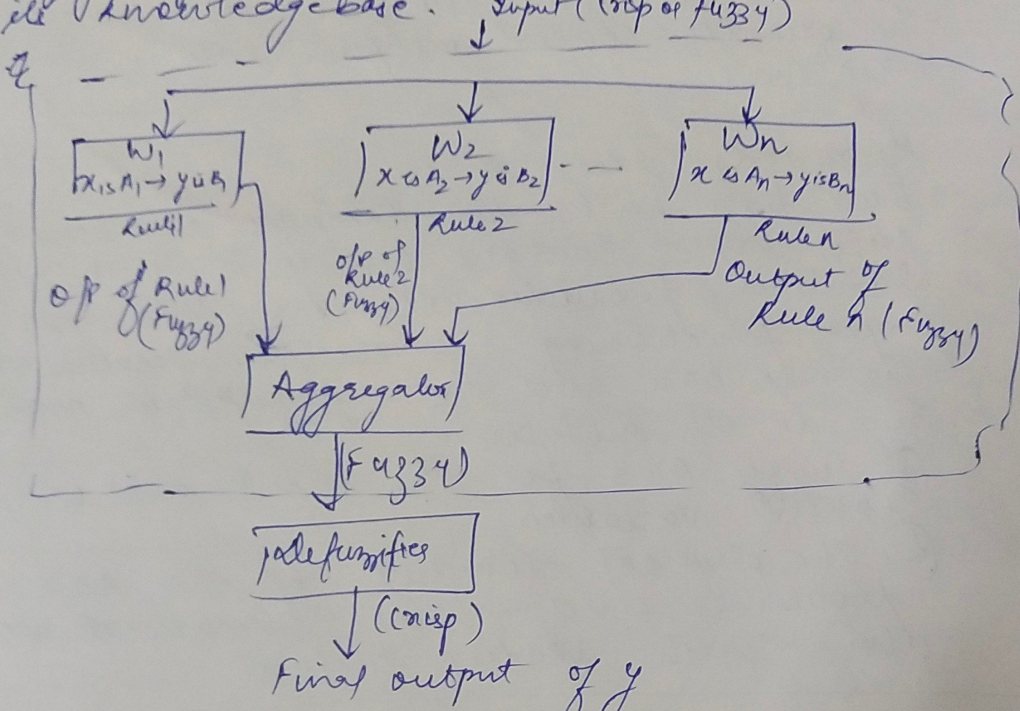
Methods of ~~Fuzzy (IF THEN rules~~ FIS
→ Mamdani FIS
→ Takagi - Sugeno FS

→

Fuzzy If-then Rules

1→ Fuzzy inference is the process of obtaining a new knowledge from an existing knowledge.

2→ Fuzzy logic uses IF-THEN rules to represent its knowledge base.



FIS using If then Rules

3→ The basic rule of inference in traditional two-valued logic is modus ponens, acc to which we can infer the truth of a proposition B from truth of A and implication A → B.

Eg If A ⇒ "tomato is red" and
B ⇒ "tomato is ripe" then
if A true then B true.

Read Pg 72 - 75

Fuzzy Modelling & Decision Making -

→ The attempt to utilize imprecise information in mathematical models led to the development of fuzzy models.

→ Fuzzy Modelling enables to transform a linguistic description into an algo whose result is an action.

→ Fuzzy model variables may represent fuzzy subsets of the universe ~~and may represent partitioning into 2~~

→ The main theories applied in fuzzy modelling are fuzzy logic and the fuzzy set theory.

→ Applications - Fuzzy control

Decision Making -

→ It is a powerful paradigm for dealing with human expert knowledge during designing of fuzzy model-based controllers.

→ Fuzzy Decision Making enhances fuzzy modelling by selecting the values of important parameters in fuzzy modelling algorithms

→ Combination of decision making with fuzzy control can be used to improve existing controllers.

Steps

(1) Determining the Set of Alternatives :- In this step, the alternatives from which the decision has to be taken must be determined.

(2) Evaluating Alternative :- Here, the alternatives must be evaluated so that the decision can be taken about one of the alternatives.

(3) Comparison b/w Alternatives :- In this step, a comparison b/w the evaluated alternatives is done.

Types of Decision Making -

(1) Individual Decision Making :- Single person

(2) Multi-person " " :- several person so that expert knowledge from them is utilized.

(3) Multi attribute D M :- Based on several attr of the object. Attr can be numerical data, linguistic data & qualitative data.

Neuro Fuzzy Modelling -

→ A neuro-fuzzy system is based on a fuzzy system which is trained by a learning algo derived from neural network theory. The heuristical learning ~~theory~~ procedure operates on local info and causes only local modifica in underlying fuzzy system.

→ A neuro fuzzy system can be viewed as 3-layer feedforward neural network.
First layer ⇒ input variables
middle (hidden) ⇒ fuzzy rules
Third layer ⇒ output variables

→ A NFS ⇒ interpreted as a system of fuzzy rules.

→ crisp values are not possible to apply, then fuzzy values are used.

→ ~~Ifaith~~ & learning help neural networks perform better in unexpected situation. ~~At~~ that time fuzzy values would be more applicable than crisp values.

Applications of FL –
Aerospace, Automotive, Business, defense, Electr
Medical, Transportation.
Read from page