Name :- Anand Gautam
Roll no :- 19010 4011
Branch :- CSE
Topic :- Operation Research Unit-1

Ans-1- Max $Z = 2x_1 + 5x_2 + 7x_3$

$$3x_1 + 2x_2 + 4x_3 + x_4 + 0x_5 + 0x_6 = 100$$
$$x_1 + 4x_2 + 2x_3 + 0x_4 + x_5 + 0x_6 = 100$$
$$x_1 + x_2 + 3x_3 + 0x_4 + 0x_5 + x_6 = 100$$

Now,

Max $Z = 2x_1 + 5x_2 + 7x_3 + 0x_4 + 0x_5 + 0x_6$

	C_j		2	5	7	0	0	0	
C_B	X_B	b	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio
0	x_4	100	3	2	$\boxed{4}$	1	0	0	$100/4 = 25 \leftarrow$
0	x_5	100	1	4	2	0	1	0	$100/2 = 50$
0	x_6	100	1	1	3	0	0	1	$100/3 = 33.33$
	$Z_j - C_j$		-2	-5	-7	0	0	0	
					$\uparrow$				

Date
Page

		C_j	2	5	7	0	0	0	
C_B	X_B	b	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio
7	x_3	25	3/4	1/2	1	1/4	0	0	$25/\frac{1}{2}=50$
0	x_5	50	-1/2	$\boxed{3}$	0	-1/2	1	0	$50/3=16.66$
0	x_6	25	-5/4	-1/2	0	-3/4	0	1	~~25/-1/2~~
		$z_j - c_j =$	13/4	-3/2	0	7/4	0	0	
				↑					

		C_j	2	5	7	0	0	0	
C_B	X_B	b	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio
7	x_3	50/3	5/6	0	1	1/3	-1/6	0	-
5	x_2	50/3	-1/6	1	0	-1/6	1/3	0	-
0	x_6	100/3	-8/6	0	0	-5/6	1/6	1	-
		$z_j - c_j =$	~~13/4~~	~~-3/2~~	~~0~~	~~7/4~~	~~0~~	~~0~~	
			3	0	0	3/2	1/2	0	

$\because \Delta_j \geq 0$

Optimal sol^n $x_1 = 0$

$x_2 = 50/3$

$x_3 = 50/3$

$\text{Max } Z = 2x_1 + 5x_2 + 7x_3$

$= 0 \times 2 + 5 \times (50/3) + 7 \times (50/3)$

$= \boxed{200}$

Date
Page

Sol^n- 2→ let no. of products for $A = x$
$B = y$
$C = z$

$$z' = 3x + 2y + 4z$$

& we have to maximize then all constraints

for G → $4x + 3y + 5z \leq 2000$
$3x + 2y + 4z \leq 2500$

∵ no. of units will change be +ve
so, $x, y, z \geq 0$
also given
$100 \leq x \leq 150$
$y \geq 200$
$z \geq 50$

→ 5 constraints

finally we can write like this
maximize $z' = 3x + 2y + 4z$
s.t.
$4x + 3y + 5z \leq 2000$
$2x + 2y + 4z \leq 2500$
$x, y, z \geq 0$

Date
Page

Sol^n-3→ first we will convert it into standard primal form

$$\max z' = -x_1 - 2x_2 + 0x_3$$

$$\text{s.t.} \quad -x_1 + x_2 + 3x_3 \leq -4$$
$$-x_1 + 2x_2 + 2x_3 \leq -3$$

now converting into dual form

$$\min Z_D' = -4w_1 - 3w_2$$
$$-w_1 - w_2 \geq -1$$
$$w_1 + 2w_2 \geq -2$$
$$3w_1 + 2w_2 \geq 0$$
$$\boxed{w_1, w_2 \geq 0}$$

Sol^n-4→ let x_1 = quantity of scrap which is bought from supplier A

x_2 = quantity of scrap which is bought from supplier B.

Objective $\min Z' = 200x_1 + 400x_2$

Cost $x_1 + x_2 \geq 200$

$0.25x_1 + 0.75x_2 \geq 100$

$0.10x_1 + 0.20x_2 \leq 35$

Solve by graphical method:-

$25x_1 + 75x_2 \geq 10000$ →

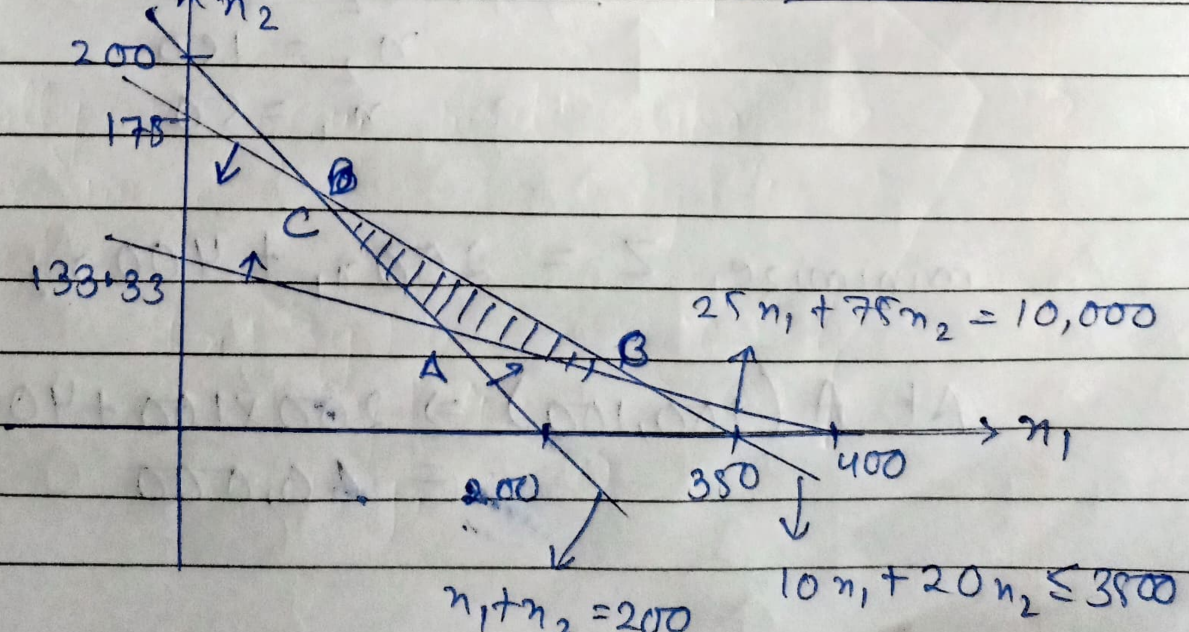
x_1	0	400
x_2	133.37	0

$10x_1 + 20x_2 \leq 3500$ →

x_1	0	350
x_2	175	0

$(x_1 + x_2 \geq 200)$ →

x_1	0	200
x_2	200	0



for A $x_1 + x_2 = 200$

$25x_1 + 75x_2 = 10000$

$\cancel{x_1} + 3x_2 = 400$

$\cancel{x_1} + x_2 = 200$

$2x_2 = 200$

$x_2 = 100, \quad x_1 = 100$

for B $10x_1 + 20x_2 = 3500$

$25x_1 + 75x_2 = 10,000$

$x_1 + 3x_2 = 400$

$x_1 + 2x_2 = 350$

$x_2 = 450$

$x_1 = 250$

for C $\cancel{x_1} + 2x_2 = 350$

$\cancel{x_1} + x_2 = 200$

$x_2 = 150$

$x_1 = 50$

minimize $Z = 200x_1 + 400x_2$

At A $(100,100) \Rightarrow 200 \times 100 + 400 \times 100$

$= 60,000$

At B $(250, 450) \Rightarrow 200 \times 250 + 400 \times 450$
$= 230,000$

At C $(50, 150) \Rightarrow 200 \times 50 + 150 \times 400$
$= 72500$

minimum at pt. A
i.e. $x_1 = 100$
$x_2 = 100$

Sol^{n} ⇒ 5 → $z' = -z = -\frac{15}{2}x_1 + 3x_2$

$3x_1 - x_2 - x_3 - x_4 + x_{a_1} = 3$

$x_1 - x_2 + x_3 - x_5 + x_{a_2} = 2$

$x_1, x_2, x_3, x_4, x_5, x_{a_1}, x_{a_2} \geq 0.$

Phase I :- Assigning costs in artificial variable & cost 0 to all other variables

$\max Z'' = 0x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5 - x_{a_1} - x_{a_2}$

b	C_B	X_B	x_1	x_2	x_3	x_4	x_5	x_{a_1}	x_{a_2}	min ratio
		C_j	0	0	0	0	0	-1	-1	
x_{a_1}	-1	3	$\boxed{3}$	-1	-1	-1	0	1	0	$\frac{3}{3}$ ←
x_{a_2}	-1	2	1	-1	1	0	-1	0	1	$\frac{2}{1}$
$z_j - c_j$ ⇒			-4 ↑	2	0	1	1	0	0	

b	C_B	X_B	x_1	x_2	x_3	x_4	x_5	x_{a_1}	min ratio
		C_j	0	0	0	0	0	-1	
x_1	0	1	1	$-1/3$	$-1/3$	$-1/3$	0	$1/3$	-ve
x_{a_2}	-1	1	0	$-2/3$	$\boxed{4/3}$	$1/3$	-1	$-1/3$	$3/4$ ←
$z_j - c_j$ ⇒			0	$2/3$	$-4/3$	$-1/3$	1	$4/3$	

C_j			0	0	0	0	0
b	C_B	X_B	x_1	x_2	x_3	x_4	x_5
x_1	0	5/4	1	-1/2	0	-1/4	-3/4
x_3	0	3/4	0	-1/2	1	1/4	-3/4
		$Z_j - C_j \Rightarrow$	0	0	0	0	0

$\because \Delta_j \geq 0$ we can proceed to Phase II.

Phase-II :- Assigning the actual costs to original variable, and cost zero to surplus variable, and the objective function becomes:-

$$\max Z' = -15/2\, x_1 + 3x_2 + 0x_3 + 0x_4 + 0x_5$$

C_j			-15/2	3	0	0	0
b	C_B	X_B	x_1	x_2	x_3	x_4	x_5
x_1	-15/2	5/4	1	-1/2	0	-1/4	-1/4
x_3	0	3/4	0	-1/2	1	1/4	-3/4
		$\Delta_j \Rightarrow Z_j - C_j =$	0	3/4	0	15/8	15/8

$\because$ all $Z_j - C_j \geq 0$. $\therefore$ sol^n is optimum.

so, $x_1 = 5/4$ & $x_2 = 0$, $x_3 = 3/4$

& min $Z = \frac{75}{8} = 9.375$

Date
Page

Sol^n-6 → Min $Z = 12x + 20y$

s.t. $6x + 8y \geq 100$

$7x + 12y \geq 120$

$6x + 8y - S_1 + 0S_2 + A_1 + 0 \cdot A_2 = 100$

$7x + 12y + 0 \cdot S_1 - S_2 + 0A_1 + A_2 = 120$

Using Big M method,

Max $Z' = -12x - 20y + 0s_1 + 0s_2 - MA_1 - MA_2$

	C_j		-12	-20	0	0	-M	-M	
C_B	X_B	b	x_1	x_2	S_1	S_2	A_1	A_2	min ratio
-M	A_1	100	6	8	-1	0	1	0	$100/8 = 12.5$
-M	A_2	120	7	$\boxed{12}$	0	-1	0	1	$120/12 = 10$ ←
$Z_j - C_j =$			$-13m+12$	$-20m+20$ ↓	M	M	0	0	

↑
incoming

	C_j		-12	-20	0	0	-M	
C_B	X_B	b	x_1	x_2	S_1	S_2	A_1	Min ratio
-m	A_1	20	$\boxed{4/3}$	0	-1	3/4	1	$20/(4/3)$ ←
20	x_2	10	7/12	1	0	-1/12	0	$10/(7/12)$
$Z_j - C_j =$			$\frac{-4m+1}{3}$	0	m	$\frac{-3m}{4}+\frac{5}{3}$	0	

↑

		C_j	-12	-20	0	0	
C_B	X_B	b	x_1	x_2	S_1	S_2	min ratio
-12	x_1	15	1	0	$-3/4$	$9/16$	$-$
20	x_2	$5/4$	0	1	$7/16$	$-79/192$	$-$
	$z_j - c_j$	$= 0$	0	0	$1/4$	$71/148$	

$\because z_j - c_j \geq 0$

$\therefore$ this is an optimum sol^n

& $x = 15$ gm & $y = 1.25$ gm

$$\min Z = 12x + 20y$$
$$= 12(15) + 20(1.25)$$
$$= 180 + 25$$
$$= \boxed{205 \text{ paise}}$$

Sol^n-7 →

$$\min Z = 2x_1 + 3x_2 + 4x_3$$

s.t.
$$2x_1 + 3x_2 + 5x_3 \geq 2$$
$$3x_1 + x_2 + 7x_3 = 3$$
$$x_1 + 4x_2 + 6x_3 \leq 5$$

$$x_3 = x_3' - x_3''$$

$$\min Z = 2x_1 + 3x_2 + 4x_3' - 4x_3''$$

s.t.

$$2x_1 + 3x_2 + 5x_3' - 5x_3'' \geq 2$$
$$3x_1 + x_2 + 7x_3' - 7x_3'' \geq 3$$
$$3x_1 + x_2 + 7x_3' - 7x_3'' \leq 3$$
$$x_1 + 4x_2 + 6x_3' - 6x_3'' \leq 5$$

Standard primal :-

$$\max Z_p = -2x_1 - 3x_2 - 4x_3' + 4x_3''$$

s.t.

$$-2x_1 - 3x_2 - 5x_3' + 5x_3'' \leq -2$$
$$-3x_1 - x_2 - 7x_3' + 7x_2'' \leq -3$$
$$3x_1 + x_2 + 7x_3' - 7x_3'' \leq 3$$
$$x_1 + 4x_2 + 6x_3' - 6x_3'' \leq 5$$

dual :-

$$\min Z_D = -2y_1 - 3y_2 + 3y_3 + 5y_4$$

s.t.

$$-2y_1 - 3y_2 + 3y_3 + y_4 \geq -2$$
$$-3y_1 - y_2 + y_3 + 4y_4 \geq -3$$
$$-5y_1 - 7y_2 + 7y_3 + 6y_4 \geq 3$$
$$-5y_1 + 7y_2 - 7y_3 - 6y_4 \geq 5$$

$$y_1, y_2, y_3, y_4 \geq 0.$$

Ans-9→ Suppose primal is

$$\max Z = c^T x$$
$$\text{s.t.} \quad Ax \le b, \; x \ge 0$$

its dual is $\min W = b^T y$

$$\text{s.t.} \quad A^T y \ge c, \; y \ge 0.$$

which can be written in standard form

$$\max -W = (-b)^T y$$
$$\text{s.t.} \quad (-A)^T y \le -c, \; y \ge 0$$

The dual of the dual is therefore

$$\min \bar{Z} = (-c)^T x$$
$$\text{s.t} \quad ((-A)^T)^T x \ge -b, \; x \ge 0,$$

But this is equivalent to

$$\max Z = c^T x$$
$$\text{s.t.} \quad Ax \le b, \; x \ge 0$$

∴ dual of the dual of given primal is primal itself

Date
Page

Sol^n - 10 → Assume no. of doalls of type A is x
" " B is y

$$x + 2y \le 2000$$
$$x + y \le 1500$$
$$y \le 600$$
$$x, y \ge 0$$

$$\max Z = 3x + 5y$$

Using simplex method now,

$$x + 2y + s_1 + 0s_2 = 2000$$
$$x + y + 0s_1 + s_2 = 1500$$
$$\max Z = 3x + 5y + 0s_1 + 0s_2$$

		C_j	3	5	0	0	
C_B	X_B	b	x_1	x_2	s_1	s_2	min ratio
0	s_1	2000	1	$\boxed{2}$	1	0	1000 ←
0	s_2	1500	1	1	0	1	1800
	$z_j - c_j =$		−3	−5	0	0	
				↑ incoming			

		C_j	3	5	0	0	
C_B	X_B	b	x_1	x_2	s_1	s_2	min ratio
5	x_2	1000	$1/2$	1	$1/2$	0	2000
0	s_2	500	$\boxed{1/2}$	0	$-1/2$	1	1000 ←
	$z_j - c_j =$		$-1/2$	0	$5/2$	0	
			↑				

		C_j	3	5	0	0
C_B	X_B	b	x_1	x_2	s_1	s_2
5	x_2	500	0	1	1	-1
3	x_1	1000	1	0	-1	2
	$z_j - c_j =$		0	0	2	1

$\because z_j - c_j \geq 0$

$\therefore$ soln is optimal.

$x = 1000, \quad y = 500$

& $\max Z = 3x + 5y$

$= 3 \times 1000 + 5 \times 500$

$= 3000 + 2500$

$= 5500$ Rs

Date
Page

Ans-11⇒ Degeneracy :-

The phenomenon of obtaining a degenrate Basic feasible solⁿ in a LPP is known as degeneracy. Degeneracy is in LPP may arise
(i) at the initial stage
(ii) at any subsequent iteration stage

However, one can reduce the no. of iterations for assuring at an optimal solⁿ by applying the following rules.

(i) Detect degeneracy and divide the coefficients of the slack variables by the corresponding positive numbers of the key column in the row starting from left to right.

(ii) Comparing from left to right the row that contains the smallest ratio b/w the key row.

Method to resolve degeneracy :-

Step-1 :- first find at the rows for which the minimum non-negative ratio is the same ; Suppose there is tie b/w first & third row.

Step-2 :- Now rearrange the columns of usual simplex table so that the columns forming the original unit matrix come first in proper order.

Step 3 :- find the minimum ration,

$$\left(\frac{\text{Elements of the first column of the unit matrix}}{\text{corresponding elements of key column}} \right)$$

only for the tied rows, i.e. for the first & 3rd rows.

(i) If the first row has the minimum ratio then this row will be the key row & the key element can be determined by intersecting the key row with key column.

(ii) if the minimum is also not unique then go to next step.

do this process until we get the unique answer of min ratio.

e.g. max $Z = 3x_1 + 9x_2$

$x_1 + 4x_2 \leq 8$

$x_1 + 2x_2 \leq 4$ $\quad x_1, x_2 \geq 0$

max $Z = 3x_1 + 9x_2 + 0s_1 + 0s_2$

$x_1 + 4x_2 + s_1 + 0s_2 = 8$

$x_1 + 2x_2 + 0s_1 + s_1 = 4$

now to find out the ~~in~~ IBFS put $x_1, x_2 = 0$ set $s_1 = 8, s_2 = 4$

		c_j	3	9	0	0	
x_B	C_B	B	x_1	x_2	s_1	s_2	min ratio
s_1	0	8	1	4	1	0	2 tie
s_2	0	4	1	2	0	1	2
$\Delta_j \Rightarrow (c_j - z_j)$			3	9 ↑	0	0	

next step

X_B	C_B	b	s_1	s_2	x_1	x_2	min ratio	
s_1	0	8	1	0	1	4	3/4	
s_2	0	4	0	1	1	$\boxed{2}$	0/2	← row
$\Delta_j = c_j - z_j$			0	0	3	9		
						↑		

now, we can solve forther.

Ans-12→ The duality in linear programming states that every linear programming problem has another linear programming problem related to it and thus can be derived from it. The original linear programming problem is called Primal while the derived linear problem is Dual.

Before solving for the duality, the original linear programming problem is to be formulated in its standard form. Standard form means, all the variables in the problem should be non-negative and "$\geq$", "$\leq$" sign is used in the minimization case & maximization case respectively.

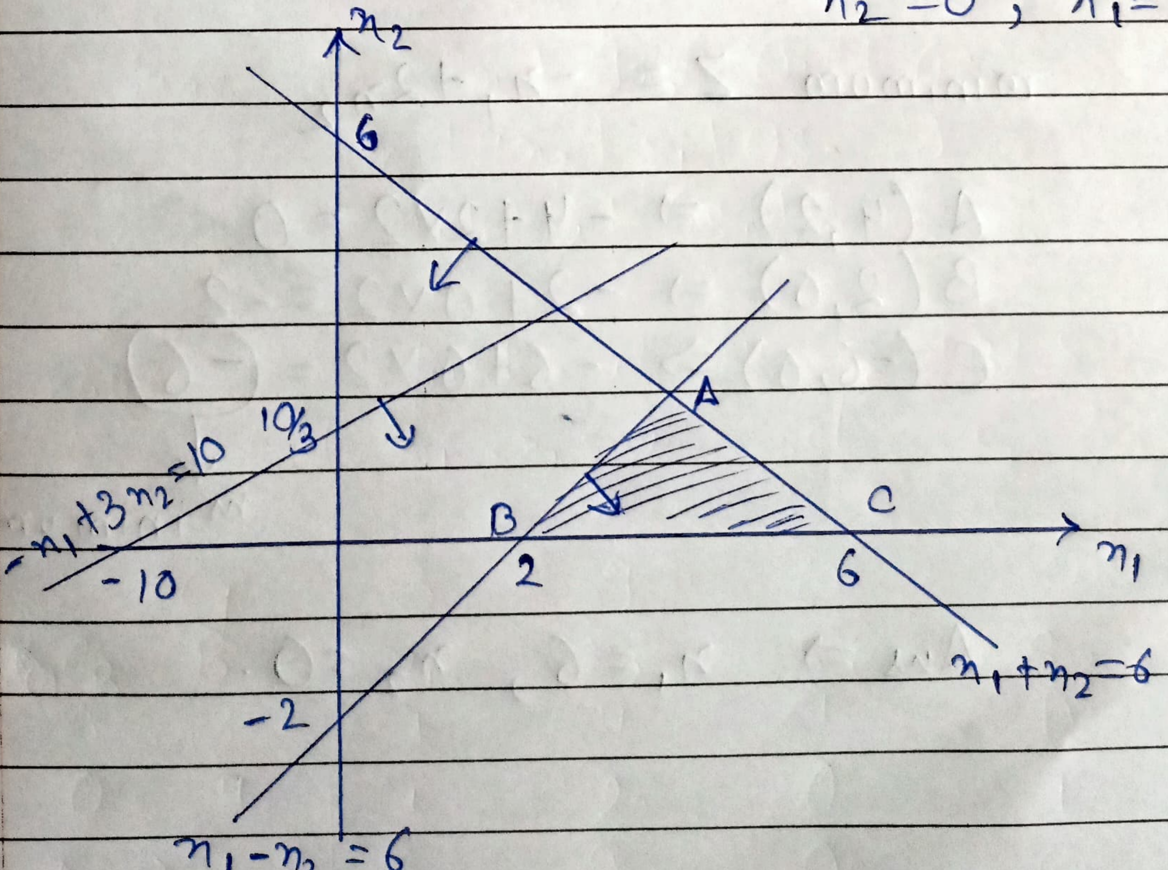
Date
Page

Ans-13→ Sol^n of 13 is covered in Sol^n of question 11.

Ans-14→ $-x_1 + 3x_2 \leq 10 \longrightarrow x_1 = 0,\ x_2 = 10/3$
$x_2 = 0,\ x_1 = -10$

$x_1 + x_2 \leq 6 \longrightarrow x_1 = 0,\ x_2 = 6$
$x_2 = 0,\ x_1 = 6$

$x_1 - x_2 \leq 2 \longrightarrow$ ~~$x_1 - x_2 = 2$~~
$x_1 = 0,\ x_2 = -2$
$x_2 = 0,\ x_1 = 2$



So, our condition is ~~this~~ satisfied in area ABC

$B(2,0)$, $C(6,0)$

for $A \longrightarrow$ $x_1 + x_2 = 6$

$x_1 - x_2 = 2$

$2x_1 = 8$

$x_1 = 4$

$x_2 = 2$

So $A(4,2)$

minimum $Z = -x_1 + 2x_2$

$A(4,2) \Rightarrow -4 + 2 \times 2 = 0$

$B(2,0) \Rightarrow -2 + 0 \times 2 = -2$

$C(-6,0) \Rightarrow -6 + 0 \times 2 = -6$

minimum

Ans $\Rightarrow$ $x_1 = 6$, $x_2 = 0$.

Ans-15→ According to HM wagner :-
OR is a scientific approach to problem solving for the executive management.

In other words we can say that OR is the systematic, method to oriented study of the basic structure, characteristic to provide the executive with a scientific & quantitative basis for decision making.

Characteristics ~~to~~ of OR

(i) Decision making→
OR is a decision science which helps management to make better decisions.

(ii) Quantitative solution→
OR provides the managers with a quantitative basis for decision making.

(iii) Scientific Approach→
OR is a scientific methods to solve the problem.

(iv) Inter-disciplinary team approach :-
OR is ~~an~~ interdisciplinary in nature.
Problem are multidimensional and approach needs team-work.

(v) System oriented :-
OR ~~is~~ provides systematic ways to solve problem.

(vi) Object-oriented approach :-
OR not only takes the overall view of problem but also endeavours to arrive at the best possible solⁿ to the problem in hands.