Soft Computing

Evolutionary computing -

Evolutionary computing is subfield of AI and is used extensively in optimization problems and for continous optimastion problem. It is used to solve problem that have too many variables for traditional algorithms.

Evolutionary computation generally involves techniques which are inspired by biological evolution such as reproduction, mutation, natural selection, survival of the fittest.

Genetic Algorithm - Genetic algorithm is a search based optimization technique based on principal of genetics and natural selection frequently used to find optimal solution to difficult problem which otherwise would take life time to solve.

Genetic algorithm simulate process simulate process of nature selection which means who can adopt to change are able to survive and reproduce to next generation. Each generation consist of population of individuals and each individual represent a point in search space and possible solution. Individual is represented in string/integer etc.

Foundation of genetic Algorithm

- Genetic Algorithm are based on analogy with genetic structure and behaviour of chromosomes of population.

Following is foundation of GA analogy -

- individual in population compete for resource, mate
- mate to create more
- one from fittest move to next generation.
- next generation is more suited to environment

Search Space

Population of individual are maintained with search space. Each individual represent solution in search space. Individual is coded a finite length vector of component. Component is similar to gene.

Fitness score - shows the ability of an individual to "compete".

Operator

1) Selection operator - The idea is to give preference to individual having high-fitness score.
2) Crossover operator - represent mating b/w individual. two individual are selected using selection operator and cross site are choosen randomly. The genes at crossover site are exchanged to create new individual.

mutation operator- The key idea is to insert random genes in offspring to maintain the diversity in population.

Algo

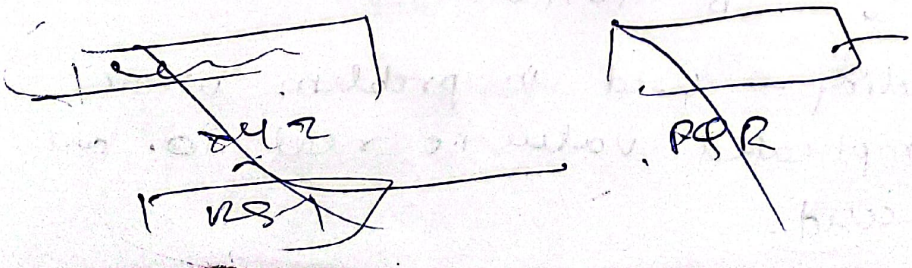
1) Random initialise population p
2) Determine fitness
3) until convergence repeat.
   -) Select parent from population
   -) Crossover and generate new p.
   -) Perform mutation
   -) calculate fitness for new p.

feature of GA

-) Robust
-) Provide optimisation over large space.
-) They do not break on slight change in input or noise.

Application

- RNN (Recurrent neural network)
- Mutation testing
- code breaking
- filtering and signal processing
- learning fuzzy rule base.



Advantages of GA

- Easy to understand
- support multi-objective optimisation
- suitable for noisy env.
- robust with respect to local minima/maxima
- work well on mixed discrete function.

Limitation

- GA implementation is still an art.
- designing objective function and getting representation is difficult.
- computationally expensive
- problem of identifying fitness function.

Encoding - In genetic algorithm an encoding function use to represent mapping of object variable to string code and opposite is decoding.

Encoding is first step in solving the problem. which depends upon the problem.

Encoding method -

Binary encoding - chromosomes are string of 1 and 0. each position represent characteristics of problem.

Chromosome A 10110100

" B 10110111

Value encoding - Used in problem when complicated value i.e real no. are used..

C A 1.23 1.32 5.23
C B left right back

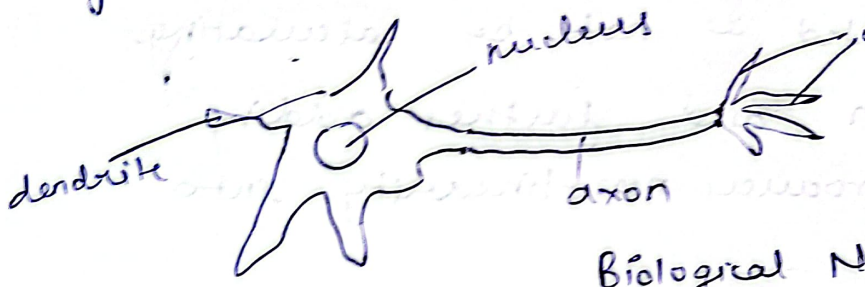
Permutation encoding - Useful for ordering problem.

C.A 1 5 3 2 4 6
C.B 1 3 2 4 5 6

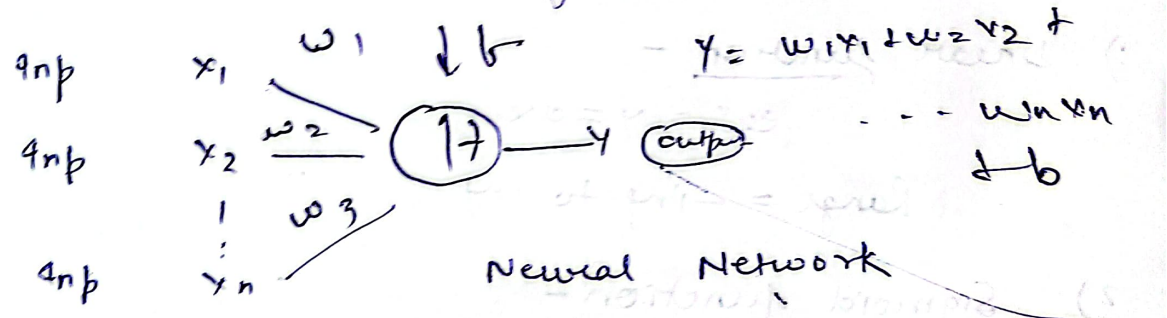
Neuro computing

It is the information processing performed by network of nuron called neural network.

Neural Network are derived from biological neural network that develop structure of human brain.



Biological Neuron



Neural Network

Similarity

Biological Neural Network	Artificial Neural Network
Dendrite	→ Input
Nucleus	→ Node
Axon	→ Output.

x y z
1 1 1

feature of Neural Networks

- It has ability to learn, recall and generalize from given data.
- Extremely powerful computational model.
- NN have parallelism which make them very efficient.
- They can learn and generalise from training data so no need of enormous programming.

Activation function

- Activation function decide whether Neuron will be activated or not by calculating weighted sum and further adding bias. It introduces non-linearity into output.

1) Linear function -

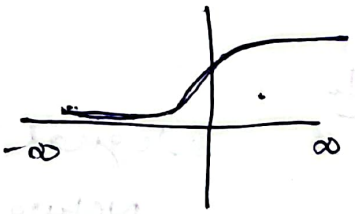
eqn = $y = ax$

Range = -inf to inf.

2) Sigmoid function -

Eqn - $\frac{1}{1+e^{-x}}$

Nature - non-linear



Value - 0 to 1

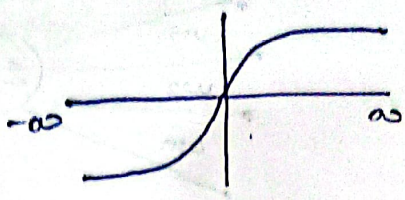
Used in binary classification.

3) Tanh function - Tangent hyperbolic function

It's actually shifted version of sigmoid function.

tanh = $2/1+e^{-2x} - 1$

or $2 * sigmoid(2x) - 1$



range - -1 to 1

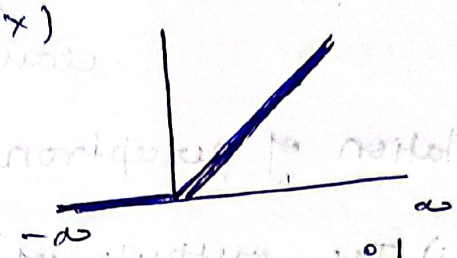
Nature - non-linear

4) Relu - Rectified linear unit implemented in hidden layer. more expensive than tanh and sigmoid. as it involve more mathematical calculation.

Eq" - $A(x) = max(0, x)$

value - $[0, inf)$

Nature - non-linear



5) softmax function - Also a type of sigmoid function but it handy to handle classification problem.

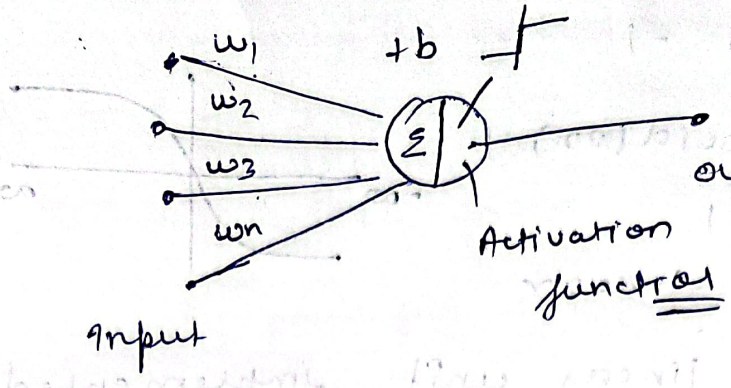
Nature - non-linear

. usually used in output layer.

Perceptron - Perceptron is Machine learning for supervised learning of various binary classification task. It is one of the simplest neural network. It is a single layer neural network with four parameter

-type - single layer
multi layer

-) Input
-) Bias
-) Sum
-) Activation function.



$\sum_{i=1}^{n} * w_i x_i$ — Activation f^n → output,

0 ↙ ↘ 1.

class 1 class-2

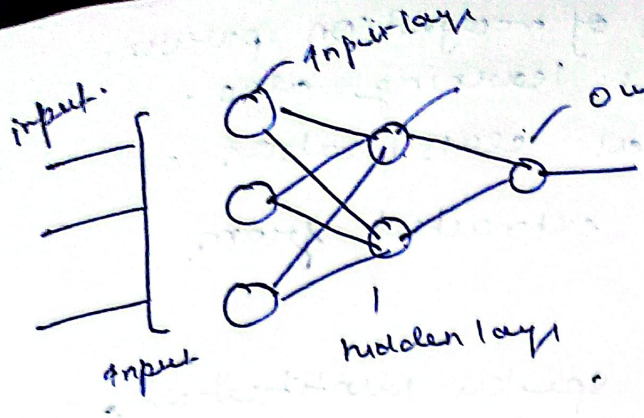
limitation of perceptron model -

1) The output of perceptron can only be a binary (0 or 1) due to hard limit transfer function

2) Perceptron can only be used to classify linearly separable set of input vector.

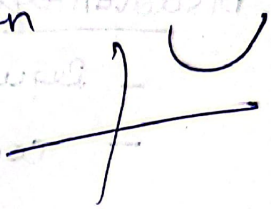
Feed forward neural Networks also called Multilayer perceptron

The model are called feedforward because information flows in one direction till output comes. There are no feedback connection in which output are fed back to itself.



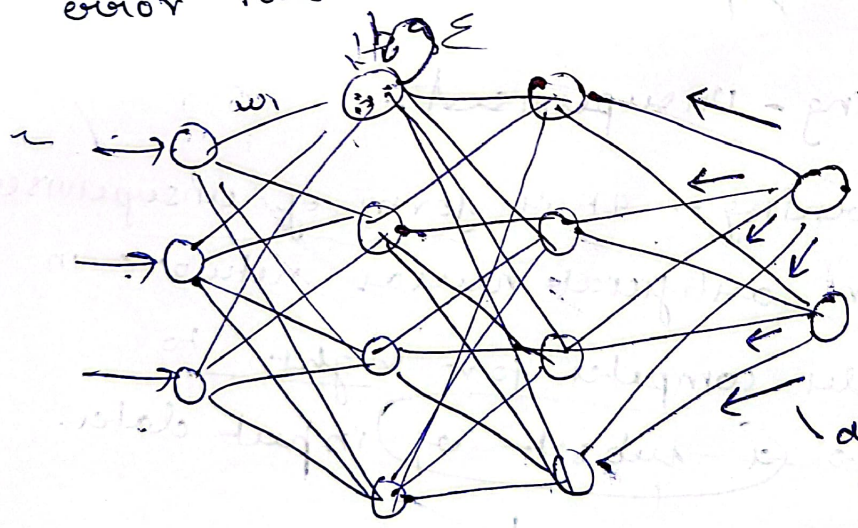
limitation of multilayer Perceptron

- computation are difficult and time consuming
- It is difficult to predict how ~~dep~~ dependent variable affect independent variable.
- The model function depend on quality of training.



Back propagation learning

- It is the method of fine tuning weights of neural network based on error rate obtained in previous pass.



working

- Input layer receive input
- Input is multiplied with wt and A.f is applied.
- Output is calculated.
- Evaluation of ~~g~~ error.
- To decrease error weight are adjusted by going back in hidden layer. Repeat till error is min.

Pattern recognition - Process of recognition patterns by using machine learning algo.
It can be defined as classification of data based on idea extracted from patterns.

Ex- speech recognition, speaker identification, finger print

Advantage
- Pattern recognition solve classification problem
- Solve biometric detection
- Useful for visually impaired

Disadvantage
- Require large dataset
- complex to implement.

Application Image processing, segmentation
Radar signal classification.
Speech recognition
fingerprint identification.

* Neuro computing - Unsupervised.

Competive learning - It is form of unsupervised learning in artificial neural network in which nodes compete for right to respond to ~~a subset of~~ input data.
learn from

Q1- It is winner take all strategy. In this type of learning when an input pattern is sent to network all neuron compete and winning neuron have weight adjustment.

Hebb learning rule

It is one of first and earliest learning rules in neural network. It used for pattern classification. It is a single layer neural network, input layer can have many units, output have one unit, Hebbian rule works by updating weights between neurons in Neural Network for each training sample.

Algo

-) Set all weight to 0 and bias to 0

-) for each input vector s and t (target vector) ~~repeat~~ repeat below 3 rule.

-) Set activation for input unit with vector $x_i = s_i$ for $i = 1$ to n

-) Set corresponding output value to output neuron, $y = t$

→ update weight and bias by hebb rule

for $i = 1$ to n

$$w_i(new) = w_i(old) + x_i y_i$$

$$b(new) = b(old) + y$$

Implementing and gate

IP	x_1	x_2	b
x_1	-1	-1	1
x_2	-1	1	1
x_3	1	-1	1
x_4	1	1	1

OP	y
y_1	-1
y_2	-1
y_3	-1
y_4	1

$w = [0\ 0\ 0]$

$x_1 = [-1\ -1\ 1]$ $\quad y_1 = [-1]$

$x_2 = [-1\ 1\ 1]$ $\quad y_2 = [-1]$

$x_3 = [1\ -1\ 1]$ $\quad y_3 = [-1]$

$x_4 = [1\ 1\ 1]$ $\quad y_4 = [1]$

Step = w_{ij} = wold + x.y.

$= [0\ 0\ 0]^T + [-1\ -1\ 1]^T [-1]$

$= [1\ 1\ -1]^T$

second =

$w_{new} = [1\ 1\ -1] + [-1\ -1\ 1]^T [-1]$

$= [2\ 0\ -2]^T$

third ~~Second~~ -

$w(new) = [2\ 0\ -2]^T + [1\ -1\ 1]^T \cdot [-$

$= [1\ 1\ -3]^T$

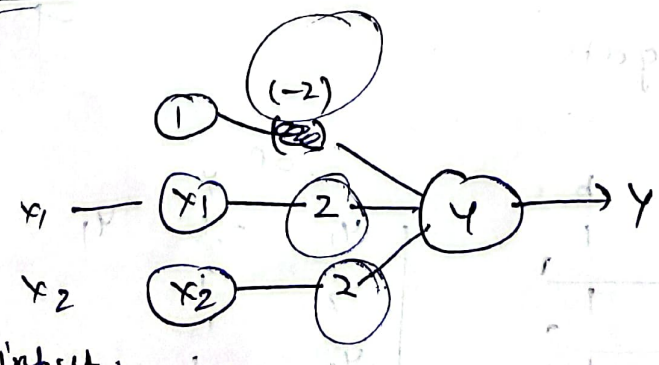
fourth iteration -

$[1\ 1\ -3] + [1\ 1\ 1]^T [1]$

$= [2\ 2\ -2]$

↳ final weights.

Hebb net



And

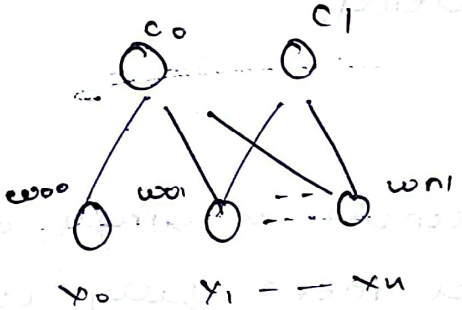
Kohonen self organising map (or SOM)

It is a type of ANN which is also inspired by biological ~~~~ models of neural network. It follows an unsupervised learning approach.

SOM is used for clustering and mapping technique to map multidimensional data onto lower dimensional which reduce complex problem for easy interpretation.

SOM has two layers - input, output

self organising map.



working - let say input data of size (m, n) where m is number of training example, n is feature. It initialise weights of size (n, c) where c is cluster.

- weight updation -

$$wij = wij(old) + alpha(t) * (xi^k - wij(old))$$

α is learning rate.

j, j denotes winning vector.

i is i^{th} feature of j^{th} training example

K denotes k^{th} training example.

Algorithm

1 - Weight initialization
2 - for 1 to N epocs
3 - select training example
4 - compute winning vector
5 - update winning vector
- Repeat 3, 4, 5
- checking test sample.

Application of SOM

- Analysis of financial stability
- Fault diagnosis of plant
- Atmospheric science

Hopfield Network - It consist of single layer which contain one or more fully connected recurrent neurons. Hopfield network is commonly used for auto association and optimisation task.

Discrete hopfield network - It is fully interconnected neural network network where each unit is connected to every other unit. It behaves in discrete manner give finite distinct output.

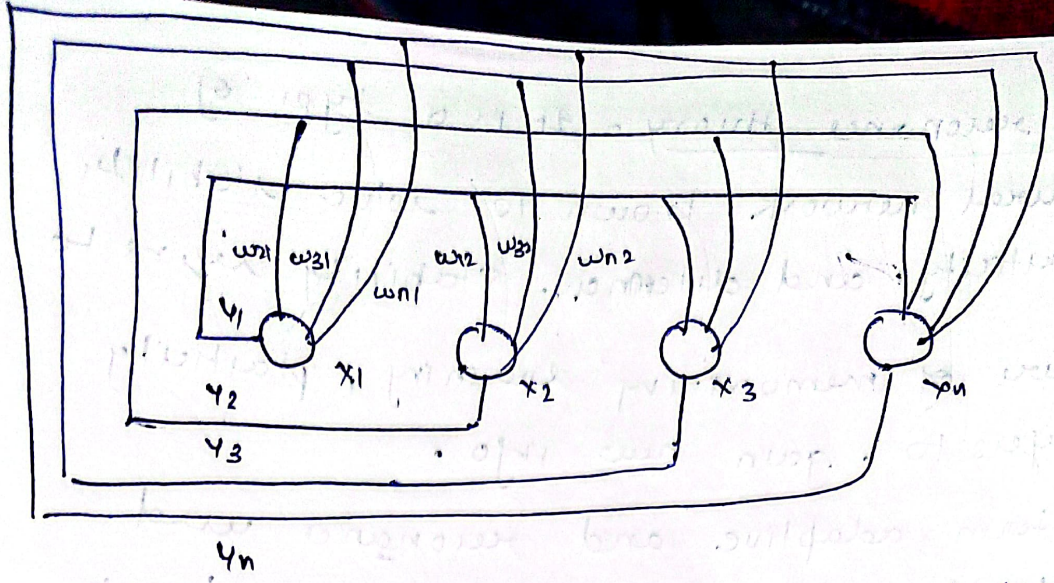
Binary (0,1)
or Bipolar (-1,1)

$x_1, x_2, \ldots x_n$ - Input to neuron
$y_1, y_2 \ldots y_n$ - output return
w_{ij} = weights

s(p) p=1 to P $s(p) = s_1(p) \ldots s_n(p)$

Binary → $w_{ij} = \sum_{i=1}^{P} [2s_i(p)-1][2s_j(p)-1]$

for storing input pattern weight is given by w_{ij} for all $i \neq j$



Boltzmann machine - It is unsupervised deep learning model in which every node is connected to every other node.

There are two type of nodes -

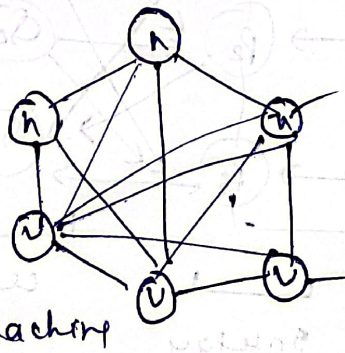
visible node - those nodes which we can and do measure.

hidden node - those nodes which we cannot or donot measure

Although node are different but machine consider them same and work as single system.

training data is fed into Boltzman m/c and wt. are adjusted accordingly.

It helps to understand abnormalities by learning about working of system in normal condition.



Types -

- Restricted BM
- Deep belief network
- Deep Boltzman Machine

Adaptive resonance theory - It is a type of neural network known to solve stability, plasticity, and dilemma. Stability refers to nature of memorizing learning, plasticity refers to gain new info.

The term adaptive and resonance used here suggest they are open to learn new without discarding previous old information.

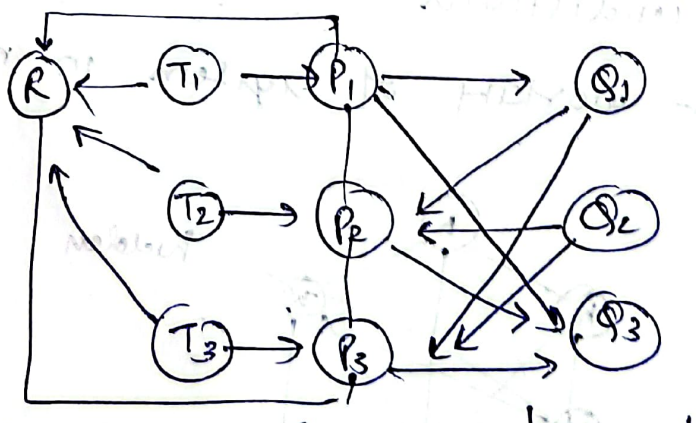
Advantage of ART

1) Target recognition
2) Medical diagnosis
3) Signature verification
4) Mobile control robot.

Types of ART

ART1 - Simplest ART capable of binary input value.

ART2 - Capable of clustering continous valued data.



- L1 Input layer
- Interface Portion
- Cluster unit.

- Application of evolutionary computing in image processing / computer vision.
- Soft computing in mobile-ad hoc network
- Application of fuzzy in pattern recognition.

Soft Computing

An intelligent machine is technologically advanced machine that respond to world. It can learn from its experience It includes AI-based software system such as chatbots.

characteristics

1) Real time simulation.
2) learning is based on ML
3) Action based on sensing data.

Knowledge based system is form of AI that aims to capture the human experts to support decision making.

Architecture of knowledge based system -

Interface - Enable user to query the knowledge based system

Inference Engine - Interact with knowledge base to get insights to make some decision.

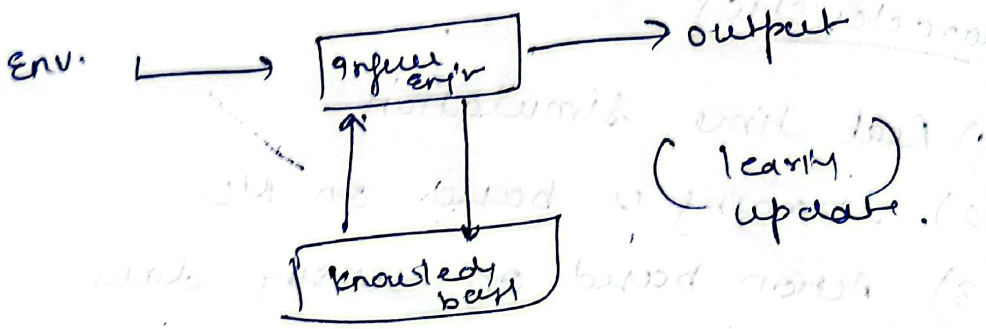
Knowledge base - Expert knowledge encoded rules
[solution to old problem are represented as cases)

Uses of KBS

Healthcare → clinical decision support
industrial equipment fault diagnosis
~~path~~ cash management.

Knowledge based system are slightly differ

AI.



A knowledge based agent must do following -

1) It must represent state
2) It should incorporate new percepts.
3) can update internal details.
4) can deduce internal details
5) can take Action.

Knowledge based representation and Processing.

1) Logical representation -
   - It is language with some concrete rules
   - It deals with proposition
   - logical representation mean drawing conclusion on various condition.
   - It consist syntax and semantic which found inference.

Each sentence can be translated to logic.

Syntax -
- Rules which decide how to construct sentence in logic.
- It define which symbol can be used
- How to write

Semantic - rules which we can interpret from logic.
- It means assigning meaning to sentence.

a) Propositional logics.
b) Predicate logics

Advantages -
- help in logical reasoning
- logical representation is basis of programming language.

Disadvantages -
- It have some restriction.
- It may not be very natural and efficient.

Semantic Network Representation -

It semantic network are alternative of predicate logic for knowledge representation

we can represent knowledge in the form of graphical network.

Network consist of nodes representing object and arc which describe relationship b/w object.

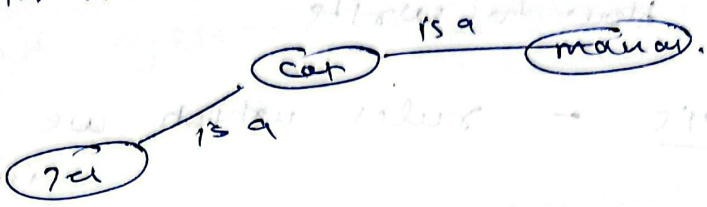
Type — is a relation

kind of relation.

Statements

a) Jerry is cat

Jerry is mammal



Drawback

- more computation power required.
- Try to immitate human memory which is not possible.
- Inadequate representation
- Not intelligent system.

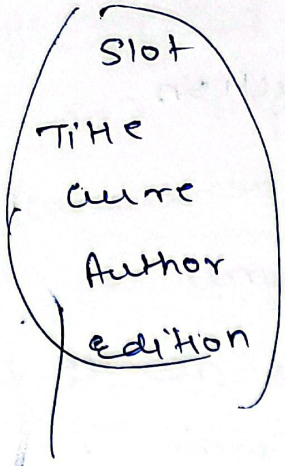
Adv - Semantic are natural representation of knowledge.

- It convey meaning
- simple & easy to understand.

Frame Representation

- Representation in the form of frame i.e into record like structure

It consist collect of slot & their values.



Filter.
Artificial on
Space
None
third

} multiple of more values

Facets.

Frame are derived from semantic network. and later evolved to modern class and object

Frame contain collection of frame, slot system

frame is not useful.

They are widely used in Natural language processing.

Advantage -

1) Frame Knowledge representation make programming easy.

2) flexible

3) Add new slot, attribute.

4) Easy to understand.

Disadv

- Inference mechanism is not easily processing.
- It can not be smoothly proceeded
- Much generalised approch.

Production Rules

It consist of condition and action.

If condition then action

└ 3 Parts

- Set of Production rule
- working memory
- Recognise act cycle.

Production check condition then perform corresponding action this cycle is called recognise act cycle.

working memory - contains the discription of current state of problem - solving.

If new situation generate multiple production can be fired called conflict set. In this agent need to slect rule called resolution of conflict

Adv - Expressed in Natural language.

Highly modular i.e can be changed easily.

Disadv -

- inefficient because of many active rules.
- No learning capabilities do not ~~result~~ store result.

Soft computing

Soft computing is use of approximate calculation to provide usable result to complex computational problem.

- It can solve problem which are too time consuming or hard to solve with hardware. It is also called computational intelligence.

It can map human mind and try to do calculation & computation like human.

Characteristic

- Provide approximate result
- algorithm are adaptive
- It learn from experimental data.
- It solve real world problem.
- based on fuzzy logic, genetic algo, ML, ANN. expert system.

Application – Image processing
Robotics.
Gaming product-
handwriting detection.

Eg Element

Soft computing

- Approximate reasoning
  - Probabilistic model
  - Fuzzy logic
- Functional approximation
  - Neural Network
  - Evolutionary Algo

Fuzzy logic - It nothing but mathematical logic which tries to solve problem with range of data.

ANN - immitation of human brain so that computer can think like as human does.

Genetic Algorithm - genetic algorithm are adaptive, heuristic search algorithm that belong to evolutionary algorithms.

- They are intelligent exploitation of random search provided with historical data.
- Provide high quality solution for optimisation problem.
- Simulate process of Natural selection.

Evolutionary computing - subfield of AI used to solve complex optimisation problem.
It is based on biological evolution theory.
It uses principle of inheritance i.e derive info from previous generation

imprecise result

Soft computing

takes less computation time.

depends on approximation and dispositional

Parallel computation

Approximate result

Neural Network

Precise result

Hard computing.

Control Acts

takes more computation time.

Based on binary logic and numerical system.

sequential computation.

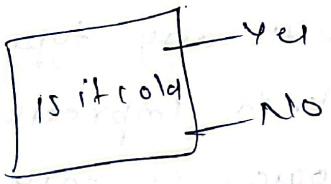
exact result.

Any traditional problem.

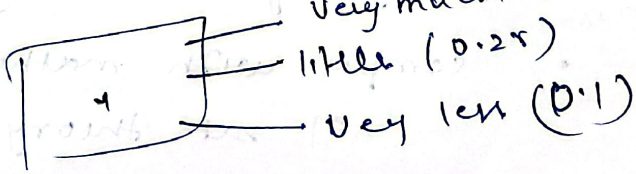
fuzzy logic -

when we can not determine situation in yes or no.

Boolean logic



fuzzy logic



Control aces of hard computing →

- unambigous and formally defined.

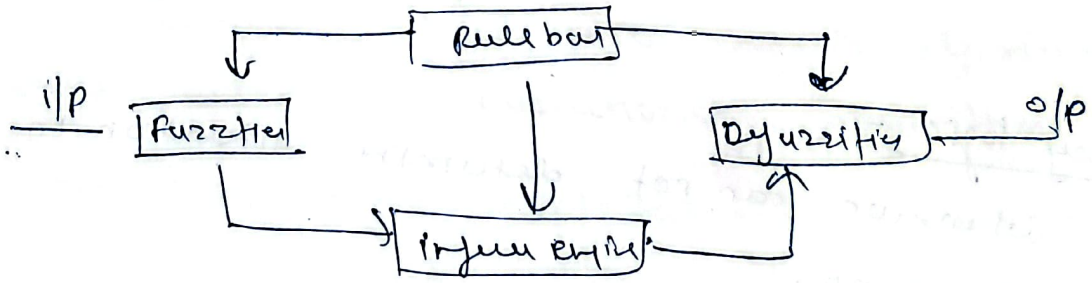
Architecture

Rule base - It contain set of rules if then condition.

fuzzyfication - It is used to convert crisp number to fuzzy, sets. crisp no. exact values measured by sensor such as temp.

Influence engine - It determine matching for the given current fuzzy input and decide which rules to be fired.

Defuzzification - It is used to convert fuzzy sets obtained by influence engine to crisp values.



Advantage -
- work with any type of input whether it is imprecise, noisy.
- fuzzy logic is easy understand.
- compex with mathematical concept of set theory.
- efficient solution to complex probl.

Disadv -
- Different way to solve problem then ambiguity
- work on precise as well as imprecise data to inaccurate.

Application

- NLP
- Expert System
- Decision making
- Neural Network.
- Automotive system.

Rough sets

Rough set is a new mathematical approach to imperfect knowledge.

Application
- Artificial intelligence
- Machine learning
- Knowledge discovery
- Data mining.
- Expert ~~discovery~~ system
- pattern recognition

Rough set are ~~are~~ generalization of classical set. It can used for feature slection feature extraction, data reduction.
Stored in form of table.

Indiscernibility - A way of reducing table size to store only one representative object for every set of same feature.

Decision System -

	Age	Lens	Walk
x_1	16-30	50	Yes
x_2	16-30	0	No
x_3	31-45	1-25	No
x_4	31-45	1-25	Yes
x_5	46-60	26-49	No
x_6	46-60	26-49	Yes
x_7	46-60	26-49	No

$IND\{Age\} = \{\{x_1, x_2, x_6\}\{x_3, x_4\}\{x_5, x_7\}\}$

$IND(Lens) = \{\{x_3, x_4\}\{x_5, x_6, x_7\}\{x_1\}\{x_2\}\}$.

$IND(Lens, Age) = \{\{x_3, x_4\}\{x_5, x_7\}\{x_6\}\{x_1\}\{x_2\}\}$

Set approximation

- formal approximation of crisp set.

Upper approximation, Lower approximation.

Probability reasoning -

With knowledge representation we write $A \rightarrow B$ means A is true then B is true but when we are not sure about A then we can express this statement. then it uncertainity.

So, to represent uncertain knowledge we need to probabilistic reasoning.

Probabilistic reasoning - way of representation where we apply concept of probability.

we combine probability with logic to produce results.

Need
— when there is unpredictable outcomes.
— unknown error experiment

It involved two ways to solve problem

1) Bayi's rule
2) Bayesian statistics

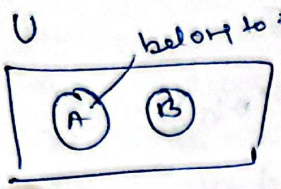
$$P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B)}$$

Crisp set

U = All student

A — 10H

B — 12H



Do not share boundary.

Fuzzy set

$G = \{ a, \mu(a) \}$ (μ) degree of goodness.

membership function

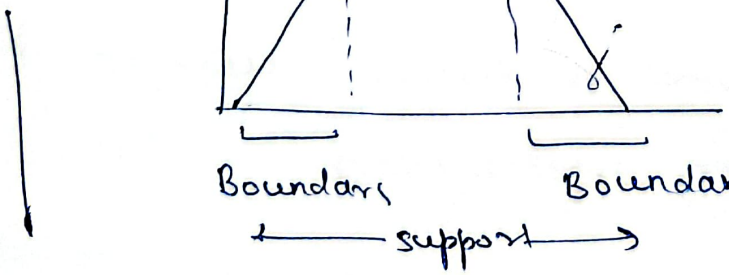
support - Set of all point such that $\mu_A(x) > 0$

support $A = \{ x \mid \mu_A(x) > 0 \}$.

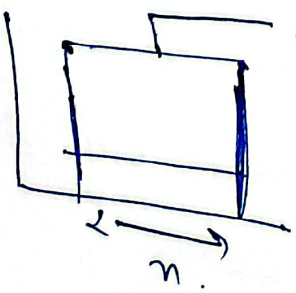
core $\mu_A(x) = 1$

$\text{core}(A) = \{ x = \mu_A(x) = 1 \}$.

Boundary -



$(1 > \mu_A > 0)$



operation

Union - $(A \cup B)$

$$\mu_{(A \cup B)}(x) = \max\left(\mu_A(x), \mu_B(x)\right)$$

Intersection = $(A \cap B)$

$$\mu_{(A \cap B)}(x) = \min\left(\mu_A(x), \mu_B(x)\right)$$

$A = \{(x_1, 0.6) \ (x_2, 0.7) \ (x_3, 0.4)\}$

$B = \{(x_1, 0.3) \ (x_2, 0.2) \ (x_3, 0.5)\}$

$A \cup B = \{(x_1, 0.6) \ (x_2, 0.7) \ (x_3, 0.5)\}$

$A \cap B = \{(x_1, 0.3) \ (x_2, 0.2) \ (x_3, 0.4)\}$

complement

$$\mu_{(A^c)}(x) = 1 - \mu_A(x)$$

$A = \{(x_1, 0.6) \ (x_2, 0.7) \ (x_3, 0.4)\}$

$A^c = \{(x_1, 0.4) \ (x_2, 0.3) \ (x_3, 0.6)\}$

Product $= A \cdot B$

$$\mu_{A \cdot B}(x) = \mu_A(x) \cdot \mu_B(x)$$

Scalar product $(\alpha \times A)$

$$\mu_{\alpha A}(x) = \alpha \times \mu_A(x)$$

Power (A^α) $\quad \mu_{A^\alpha}(x) = (\mu_A(x))^\alpha$

$A = \{(x_1, 0.6) \ (x_2, 0.7) \ (x_3, 0.4)\}$

$B = \{(x_1, 0.3) \ (x_2, 0.2) \ (x_3, 0.5)\}$

$A \cdot B = \{(x_1, 0.18) \ (x_2, 0.14) \ (x_3, 0.20)\}$

$x \rightarrow 0$ to 1 including 0 and 1

Sum = $(A+B)$

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x)\,\mu_B(x)$$

Difference = $(A-B)$

$$\mu_{A-B}(x) = \mu_{A \cap B^c}(x)$$

Disjunctive sum - $A \oplus B$

$$A - B \cup B - A$$

$$A \cap B' \cup B \cap A'$$

&

$$A' \cap B \cup A \cap B'$$

Cartesian product

$$\mu_{A \times B}(x,y) = \min\{\mu_A(x), \mu_B(y)\}$$

$A(X) = \{(x_1, 0.2)\ (x_2, 0.3)\ (x_3, 0.5)\}$

$A(Y) = \{(y_1, 0.8)\ (y_2, 0.6)\ (y_3, 0.3)\}$

	y_1	y_2	y_3
x_1	0.2	0.2	0.2
x_2	0.3	0.3	0.3
x_3	0.5	0.5	0.3

Fuzzy relation

$A = \{(x_1, 0.6)\ (x_2, 0.2)\ (x_3, 0.3)\}$

$B = \{(y_1, 0.7)\ (y_2, 0.3)\ (y_3, 0.4)\}$

	y_1	y_2	y_3
x_1	0.6	0.3	0.4
x_2	0.2	0.2	0.2
x_3	0.3	0.3	0.3

Fuzzy if then, implication

conditional statement.

if x is A y is B

R: A → B

if temp is high then pressure is low

$T_{High} = \{(25,0.1)(30,0.2)(35,0.5)(40,0.6)\}$

$T_{low} = \{(2,0.3)(5,0.5)(6,0.4)\}$

if $T_h \rightarrow T_{low}$

	2	5	6
25	0.1	0.1	0.1
30	0.2	0.2	0.2
35	0.3	0.5	0.4
40	0.3	0.5	0.4

Defuzzification - fuzzy to crisp.

1) lambda cut
2) maxima method
3) weighted sum
4) centroid method.

lambda cut - $A\lambda = \{x \mid \mu_A(x) \geq \lambda\}$.

$0 \leq \lambda \leq 1$

$A = \{(x_1,0.2)(x_2,0.4)(x_3,0.6)\}$

$\lambda = 0.3$

$A = \{(x_1,0)(x_2,1)(x_3,1)\}$.

lambda cut

fuzzy relation

$$\begin{bmatrix} 0.2 & 1 \\ 0.3 & 0.6 \end{bmatrix}$$

$A = 1$

$$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

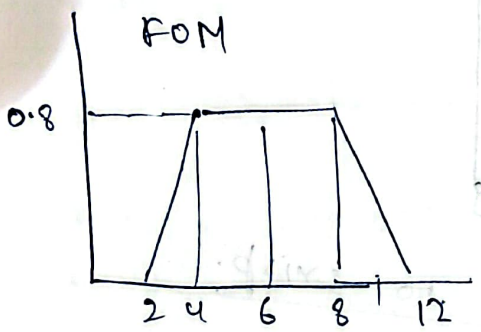
$A = 0.5$

$$\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}.$$

Maxima methods

First of maxima. last of maxima

Mean of maxima.



Last of Maxima

$\rightarrow$ 8

first element having max membership value

$x^* = 4$

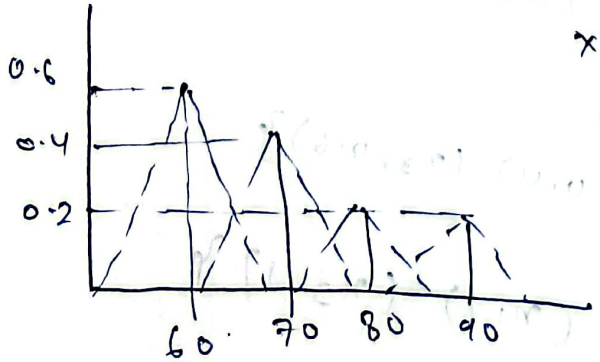
Mean of maxima

$$= \frac{4+6+8}{3}$$

$$= \frac{18}{3} = 6$$

$x^* = 6$

weighted Avege method



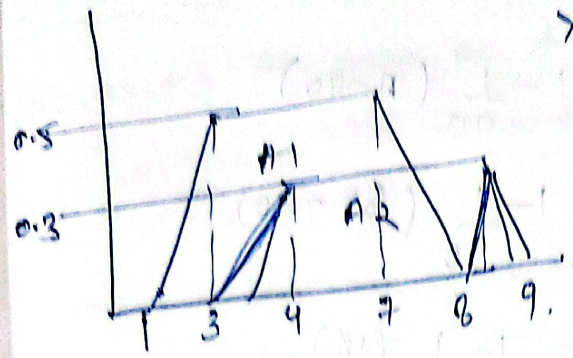
$$x^* = \frac{\sum \mu(x) \times x}{\sum \mu(x)}$$

$$= \frac{60 \times 0.6 + 70 \times 0.4 + 80 \times 0.2 + 90 \times 0.2}{0.6 + 0.4 + 0.2 + 0.2}$$

$$= 70$$

Centre of SOM

$x^* = \frac{A_1 \times \bar{x}_1 + A_2 \times \bar{x}_2}{A_1 + A_2}$ ← centre of each area.



$A_1 = \frac{1}{2}[7+4] \times 0.5 = 2.75$

$A_2 = \frac{1}{2}(9-3+8-4) \times 0.3 = 1.5$

$x^* = 2.75 \times \frac{5 + 1.50 \times 6}{2.75 + 1.5}$

~~Centre of~~

Triangle -

R = Right Δ

I = Isosceles Δ

E = equilateral Δ

IR = Isoceles Right Δ

T - combination of all Δ

membership value of Right Δ = $1 - \frac{1}{90}|A - 90°|$

Isoceles Δ = $1 - \frac{1}{60} \min\{A-B, B-C\}$

~~Right~~ Equilateral = $1 - \frac{1}{180}|A - C|$

Isoceles Right = $I \cap R$.

membership of all = $T = (R \cup I \cup E)^c$

$= R^c \cap I^c \cap E^c$

A, B, C = 80, 65, 35

membership of right = $1 - \frac{1}{90}(A - 90)$

$= 1 - \frac{1}{90}(80 - 90)$

$= 1 - \frac{1}{90}(10)$

$= 8/9$

Isosceles = $1 - \frac{1}{60}\{(80-65)(65-35)\}$

$= 1 - \frac{1}{60}(15 \times 30)$

$= 3/4$

Equilateral = $1 - \frac{1}{180}(80 - 35)$

$= 1 - \frac{1}{180}(45)$

$= 1 - 1/4 = 3/4$

$IR = I \cap R$

$= 3/4$

all = $\left(\frac{8}{9}, \frac{3}{4}, \frac{3}{4}\right)$

$= \left(1/9, 1/4, 1/4\right)^U$

$= \underline{1/9}$