

PYL556 Quantum Mechanics II

Problem No. 3 [Group 6]

# LANDAU-ZENER PROBLEM WITH HARMONIC COUPLING

## Group Members

- 2023PHS7217 ANURAG CHOUDHARY
- 2023PHS7215 ROSHAN KUMAR KUMAWAT
- 2023PHS7193 SHAKIL KAVID MULLA
- 2023PHS7173 MOHIT KUMAR JANGID
- 2023PHS7220 ANJALI

Instructor: Prof. Sankalpa Ghosh

# The Problem

We have been given a two level quantum system in which hamiltonian of the system is varying with time, such that the energy separation of the two states is a function of time. Dynamics of such time-independent system is given by time-dependent Schrodinger equation of the form

$$i\hbar \frac{\partial}{\partial t} |\Psi\rangle = \hat{H}(t) |\Psi\rangle$$

It has a similar description of that standard Landau Zener problem but with different perturbation term. Hamilton of the system is given as

$$\hat{H}(t) = A(t)\hat{\sigma}_z + B(t)\hat{\sigma}_x$$

(where  $\sigma_z$  and  $\sigma_x$  are pauli matrices)

$$A(t) = \mathbf{v}t/2$$

$$B(t) = \mathbf{g} \cos(\Omega t + \Phi)$$

( $\mathbf{g}$ ,  $\Omega$  and  $\mathbf{v}$  are the variable quantities)

It is given that the system starts at  $t=-\infty$  in the lower energy eigenstate.

Now we numerically integrate the time dependent schrodinger equation by converting it into a difference-equation and solving it using the Runge-Kutta method in the large time limit and calculated transition probability of remaining in the same (ground state ) state after long time.

# ROADMAP

## STEP 3

Normalising the  
wavefunction  $\Psi$

## STEP 1

Finding & Plotting the  
Energy Eigenvalues

## STEP 4

Plotting the transition  
probability of ground  
state

## STEP 2

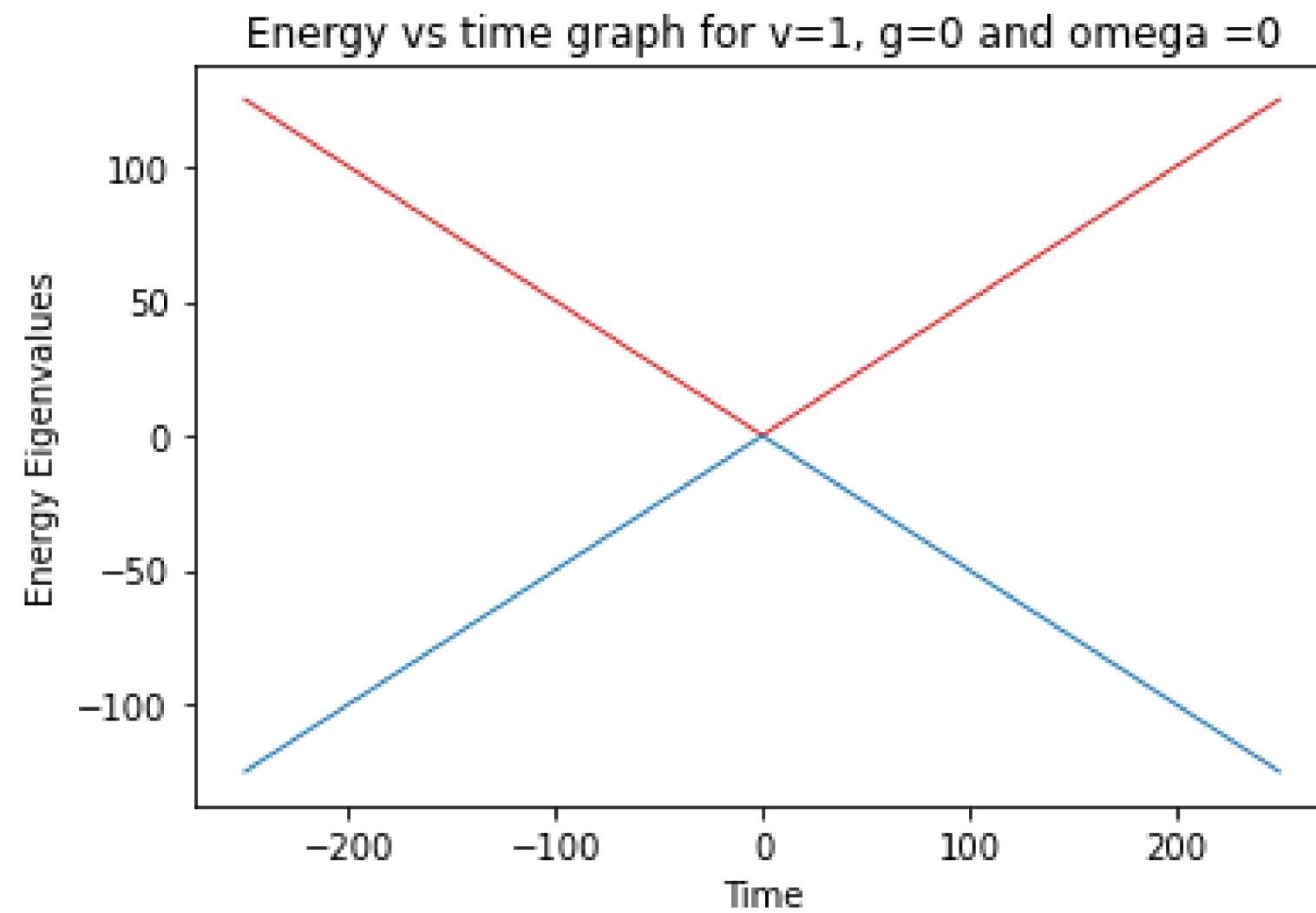
Invoking the RK-4  
Method to find the time  
evolutoin of  $\Psi$

## STEP 5

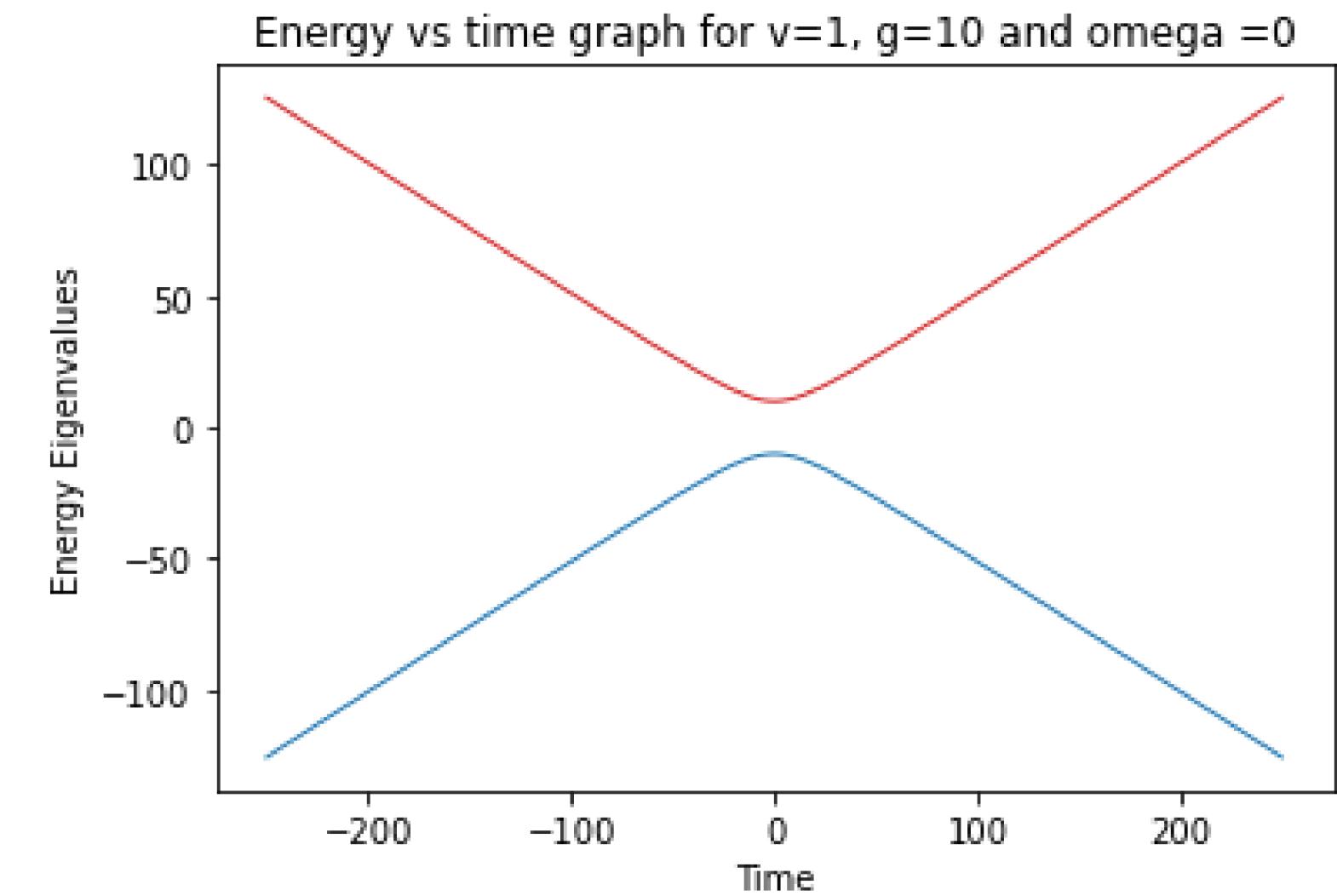
Comparison with  
standard Landau-Zener

# Expected Plots for Energy Eigenvalues

With No Perturbation



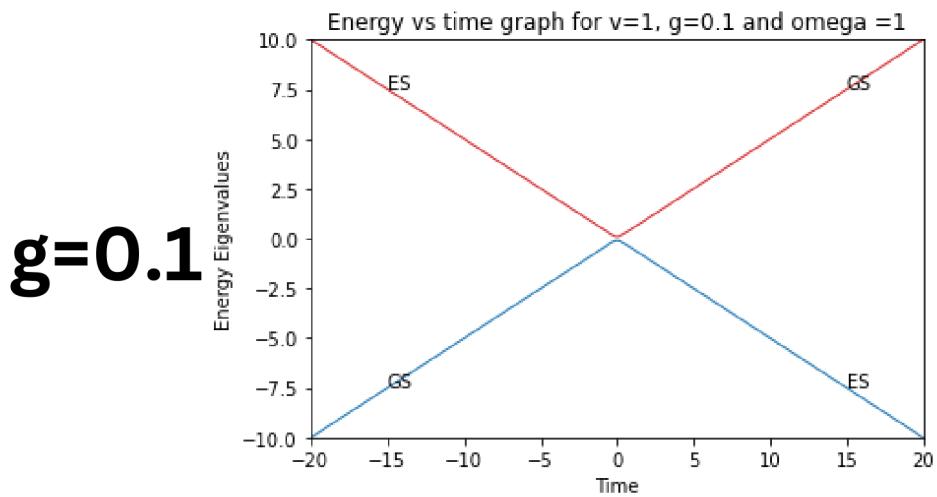
With Constant Perturbation



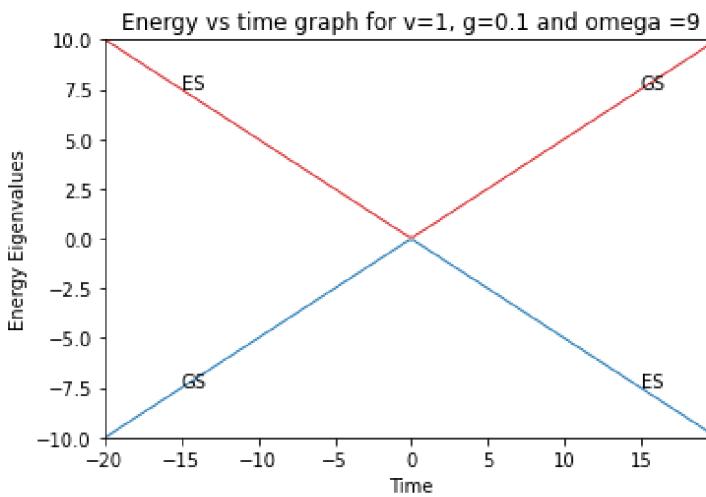
# Energy Eigenvalues

$v=1$

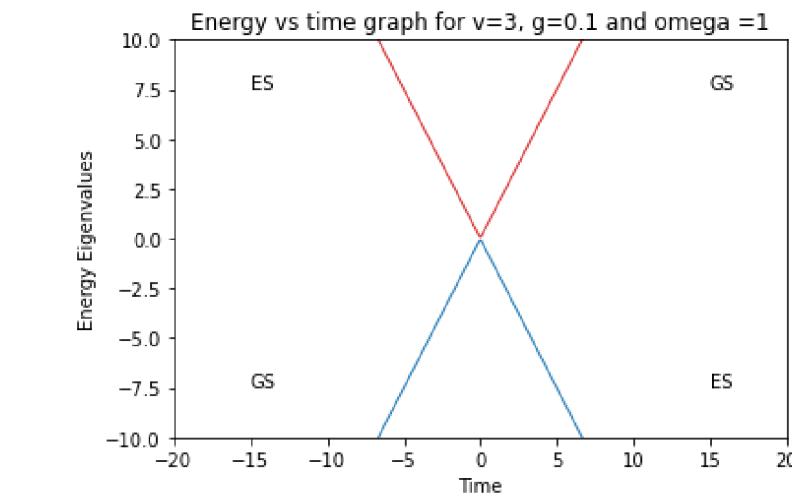
$\Omega=1$



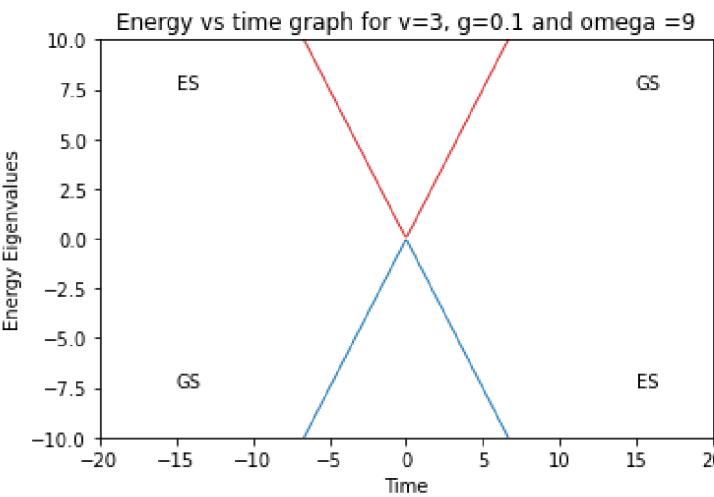
$\Omega=9$



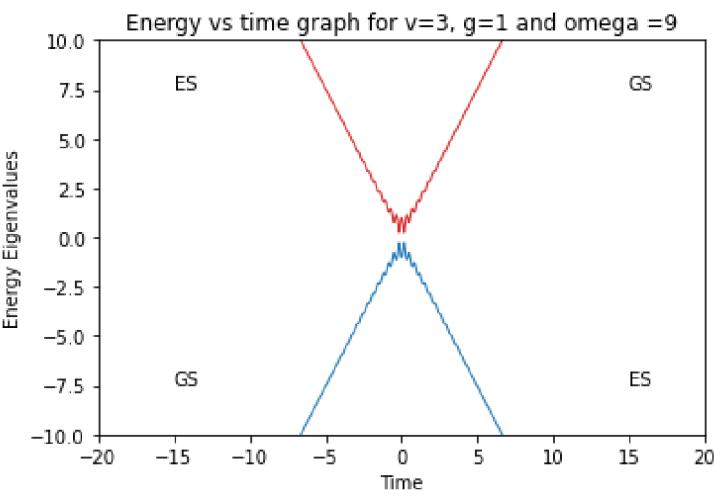
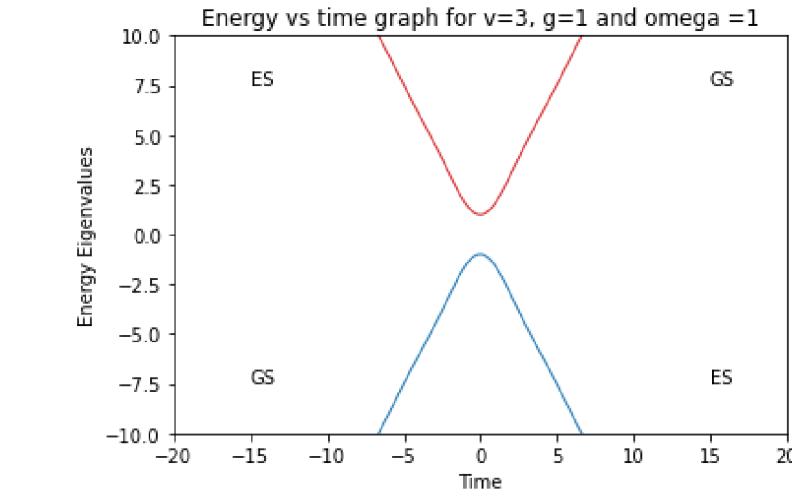
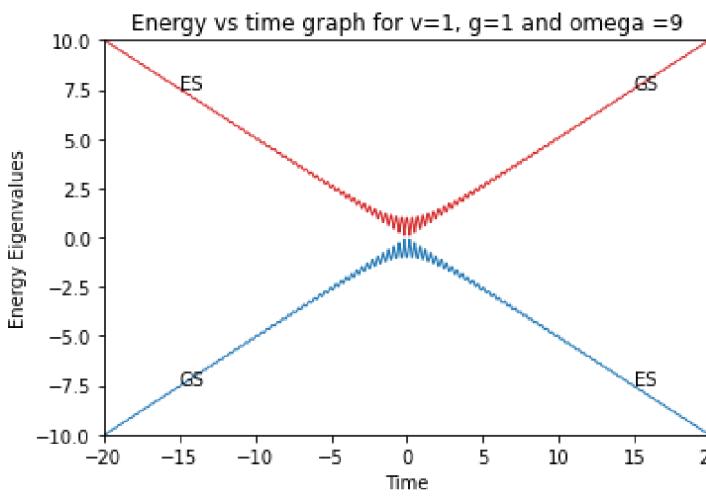
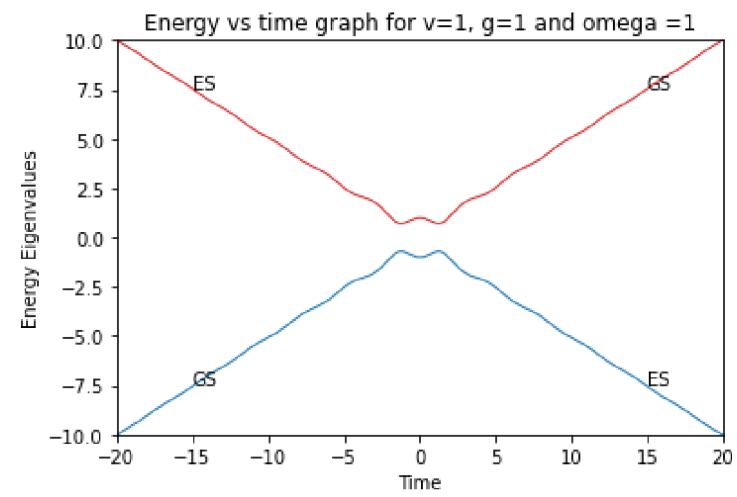
$\Omega=1$



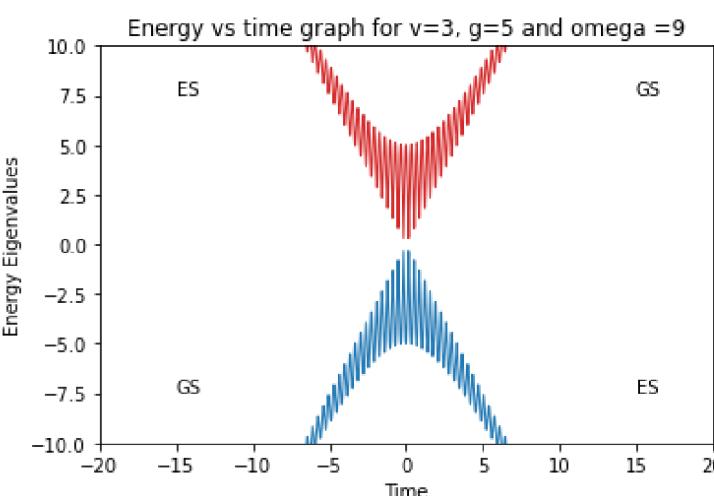
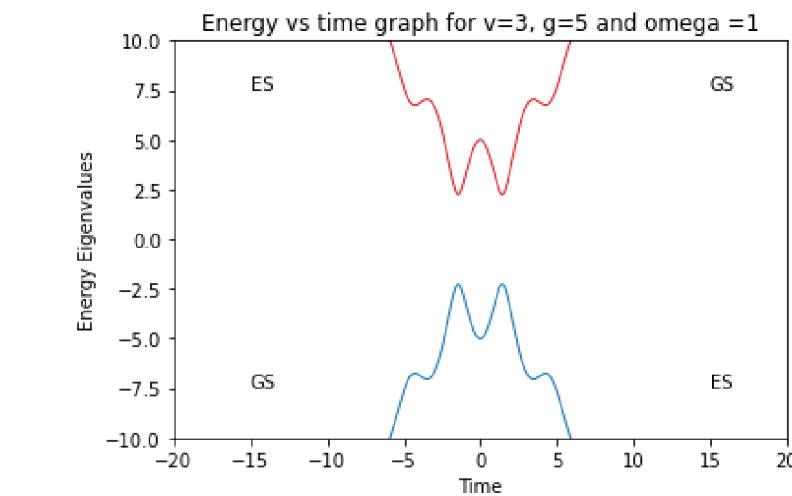
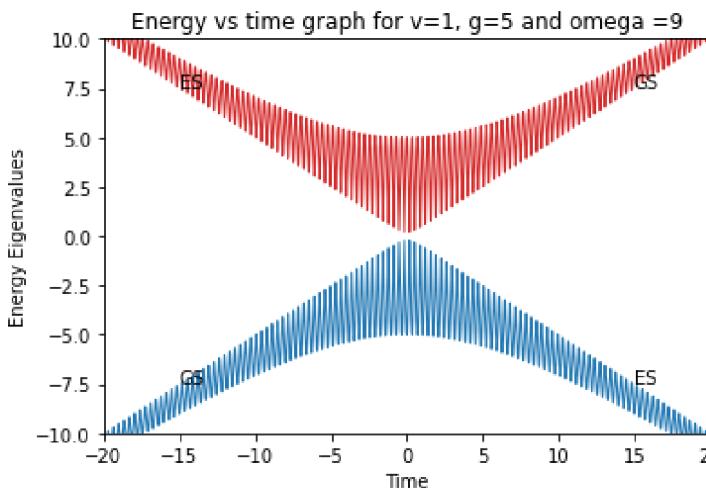
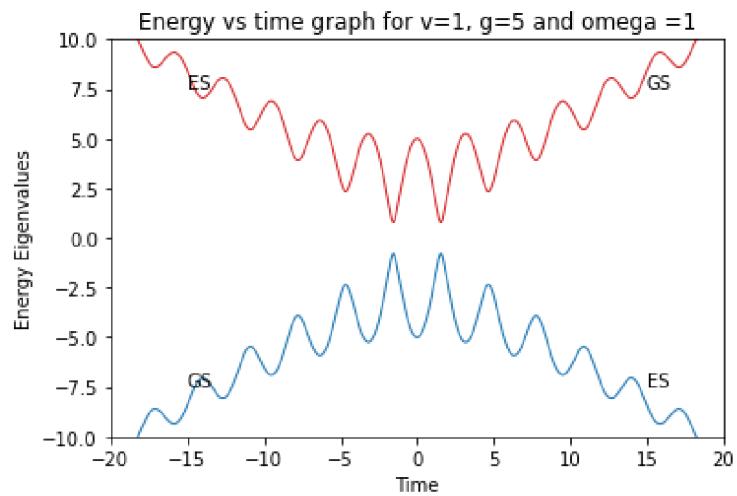
$\Omega=9$



$g\omega=1$



$g\omega=5$



# Transition Probability

Let us use our perturbation theory to calculate the probability  $P(t)$  of transition from  $|0\rangle$  to  $|t\rangle$ , at time  $t$ , under the effect of the perturbation  $B(t)$

The initial state at  $t=-\infty$  is given as

$$|\Psi(-\infty)\rangle = |0\rangle \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

At any time  $t$  our wave-function will be

$$|\Psi(t)\rangle = |t\rangle = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$

Our transition prob. amplitude will be  $\langle t|0\rangle$  and probability will  $|\langle t|0\rangle|^2$

$$i\hbar \frac{\partial}{\partial t} |\Psi\rangle = \hat{H}(t) |\Psi\rangle$$

Schrödinger's Eq.

$\Psi(n+1) = \Psi(n) + dt * \Delta$   
Difference Eq.

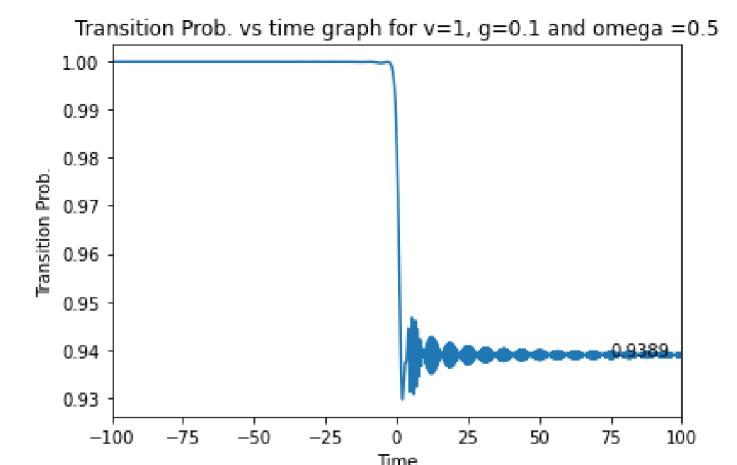
Solving it using RK4  
Numerical Method

Computing and  
plotting the transition  
probabilities wrt time

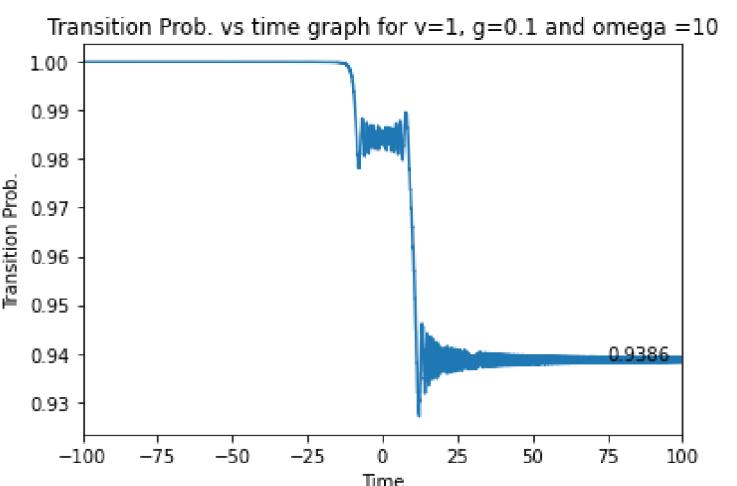
# Plots for Transition Probability

**g=0.1**

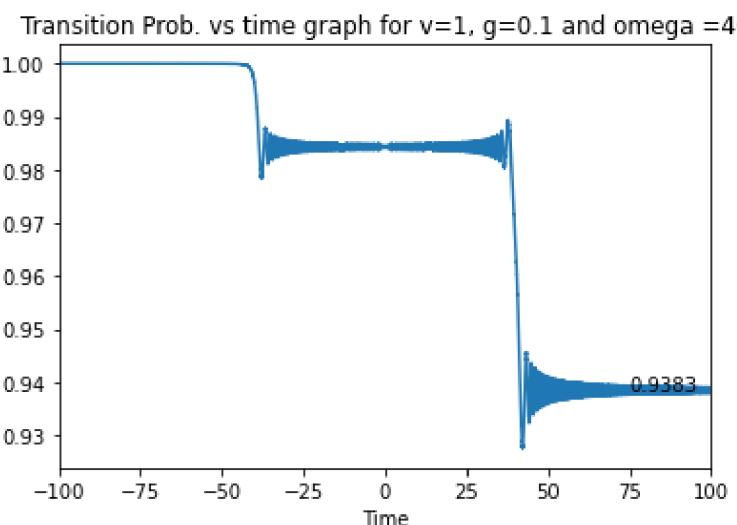
**$\Omega=0.5$**



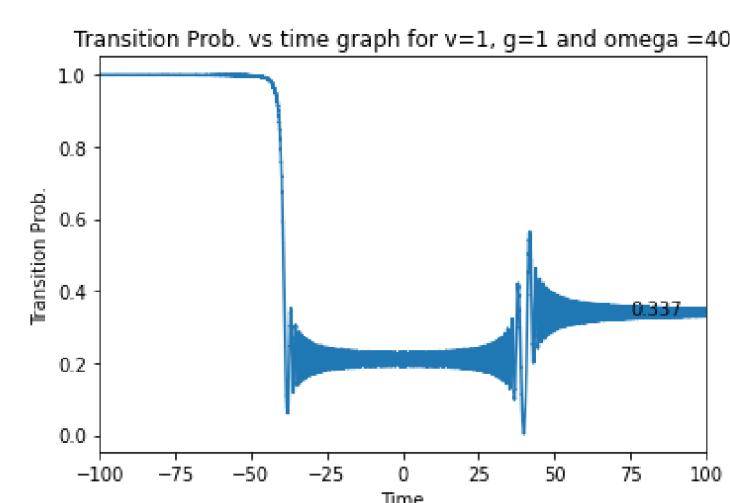
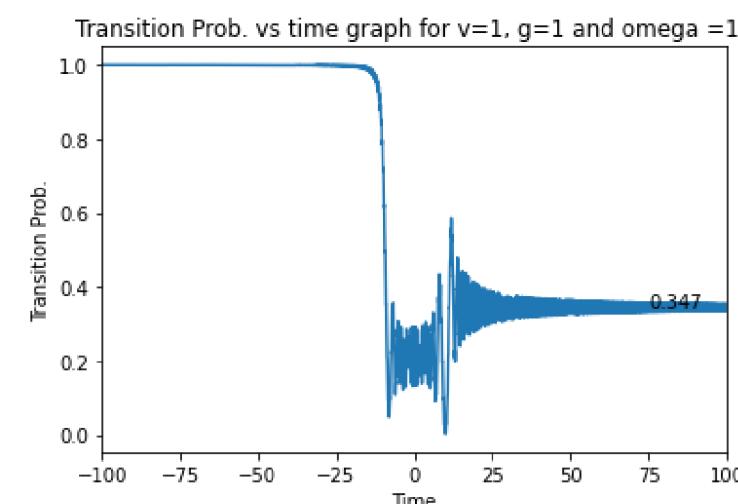
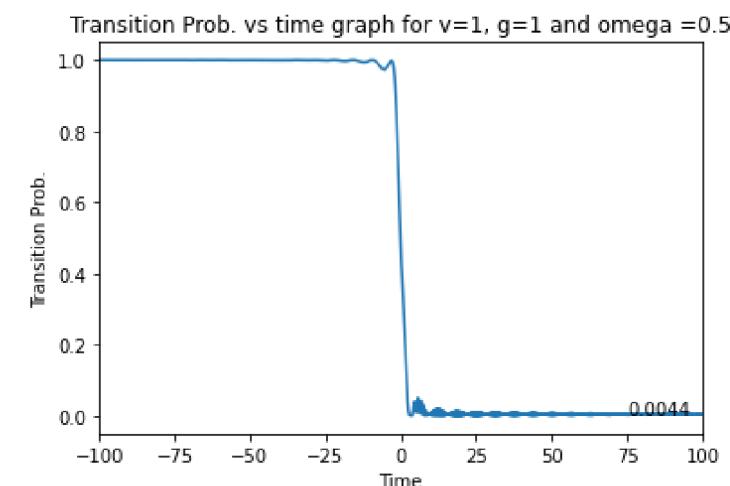
**$\Omega=10$**



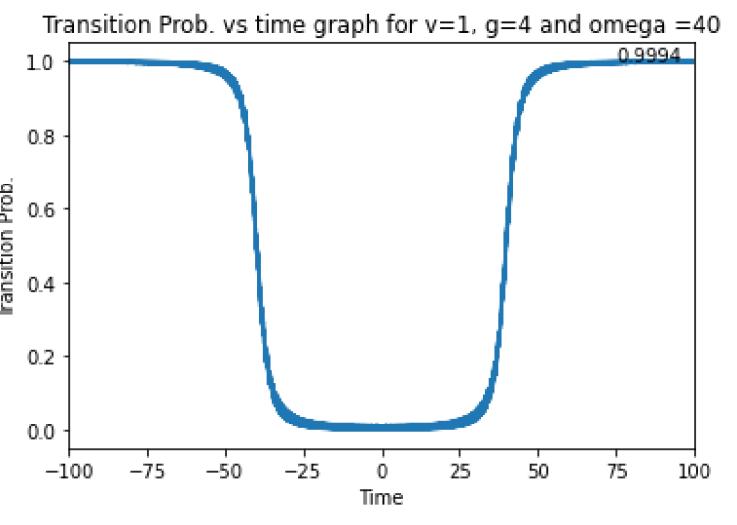
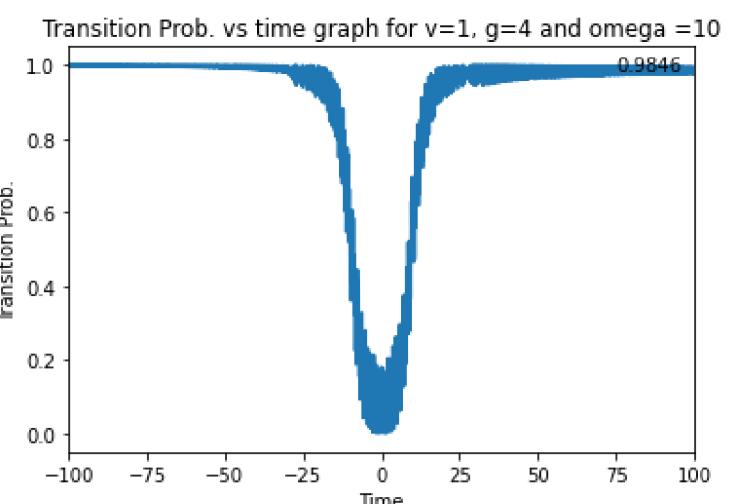
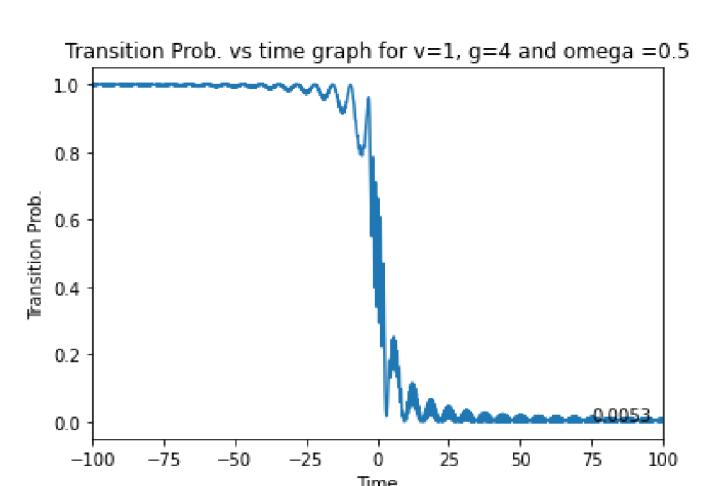
**$\Omega=40$**



**g=1**



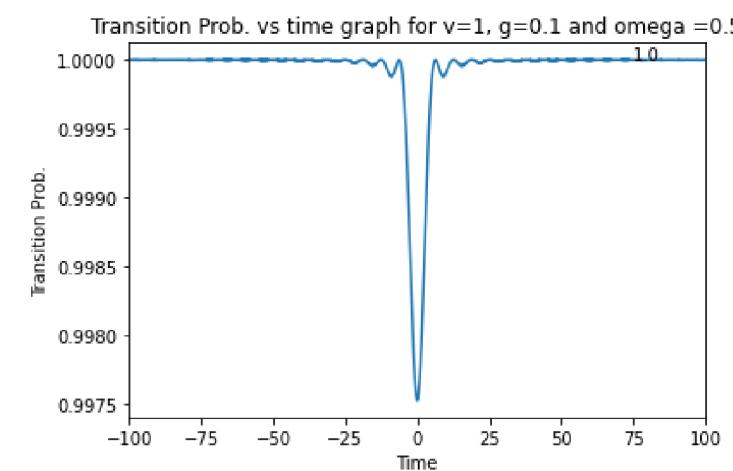
**g=4**



# Probability Plots when $\phi$ is an odd multiple of $\pi/2$

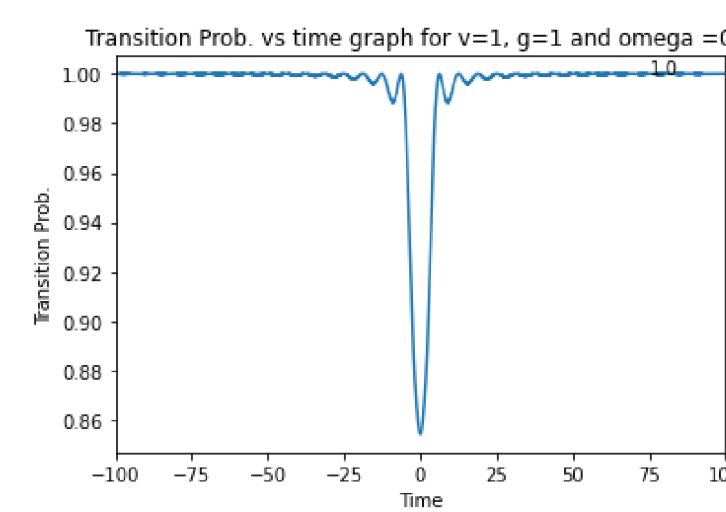
$\Omega=0.5$

$g=0.1$



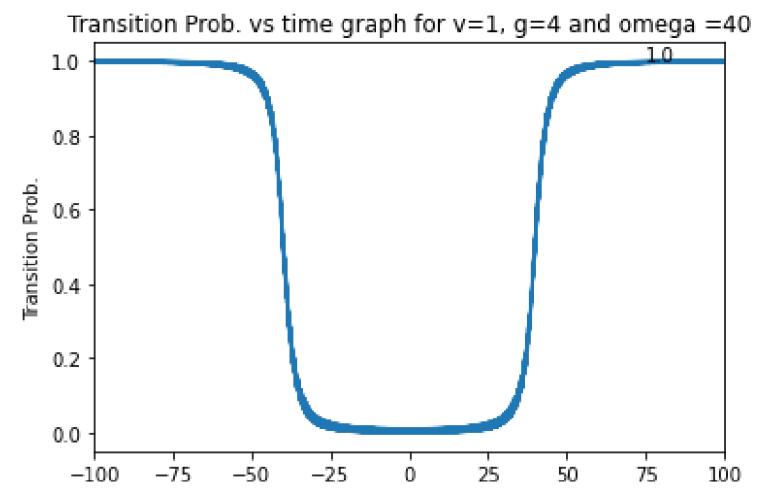
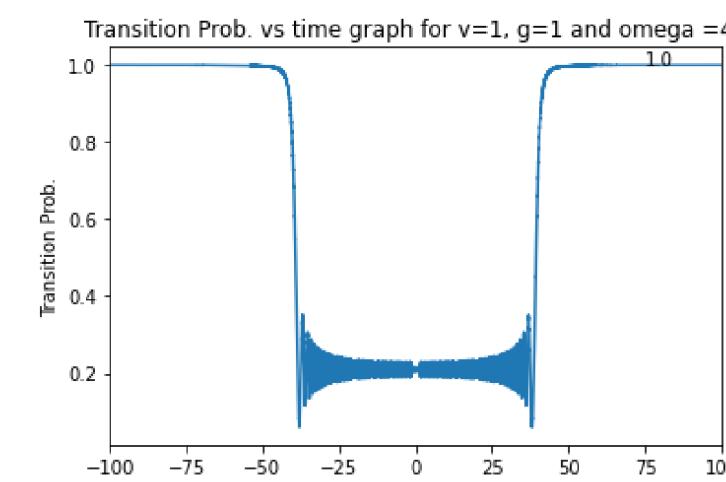
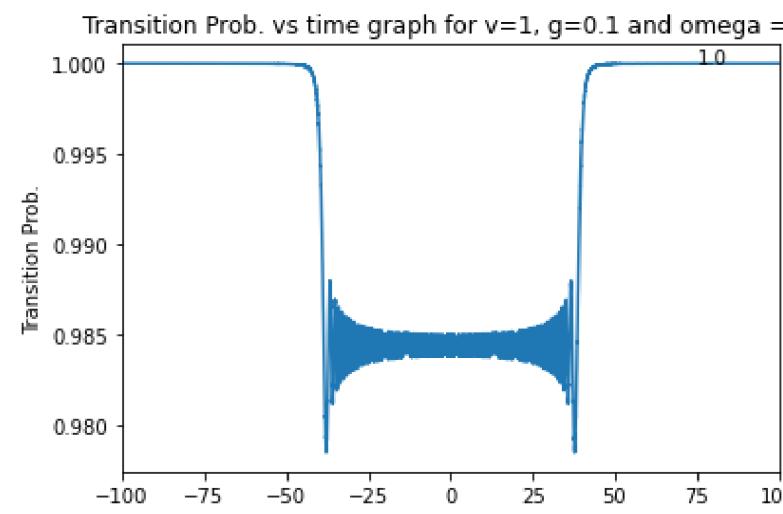
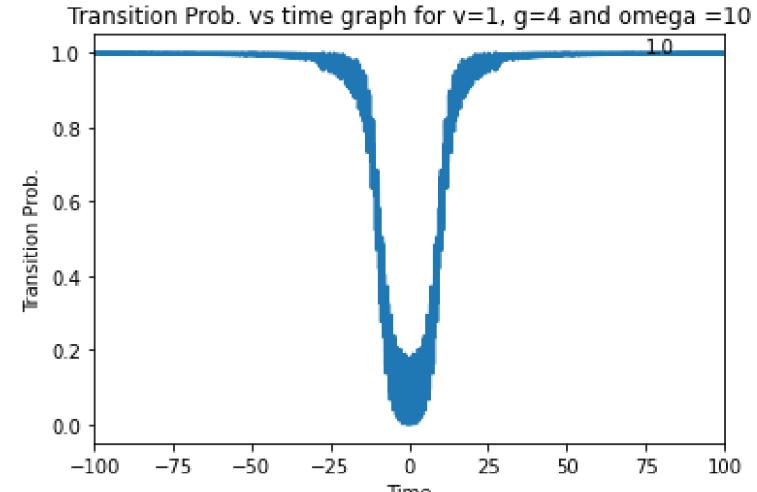
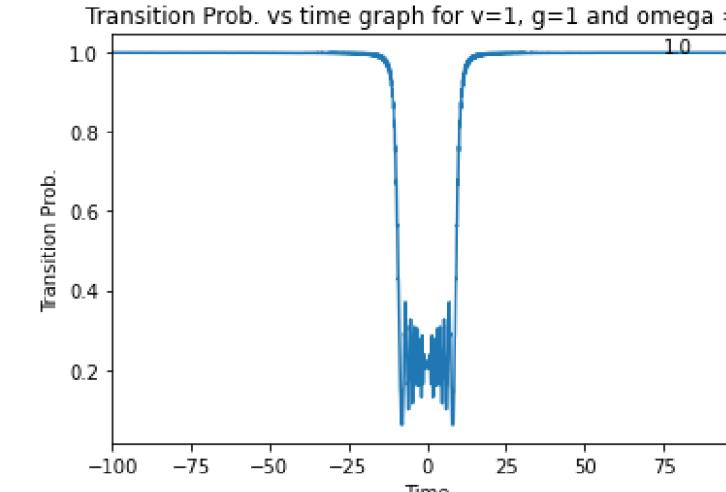
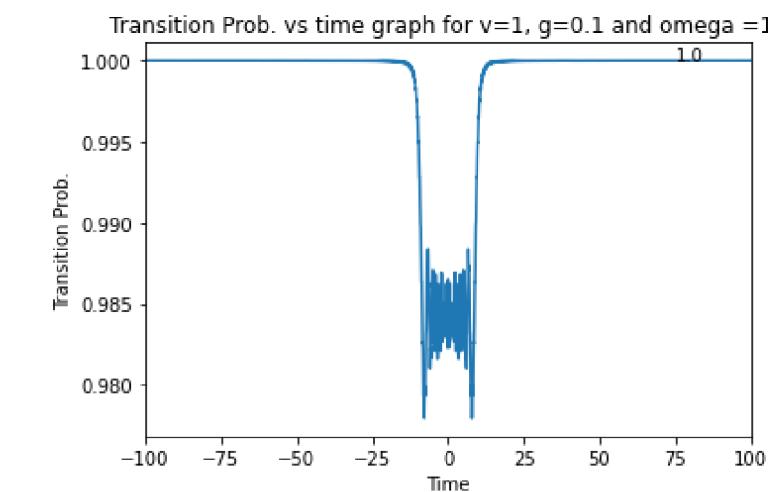
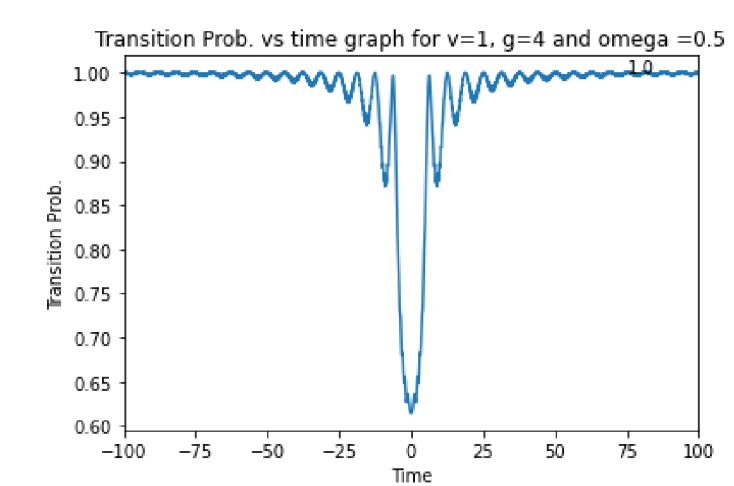
$\Omega=10$

$g=1$



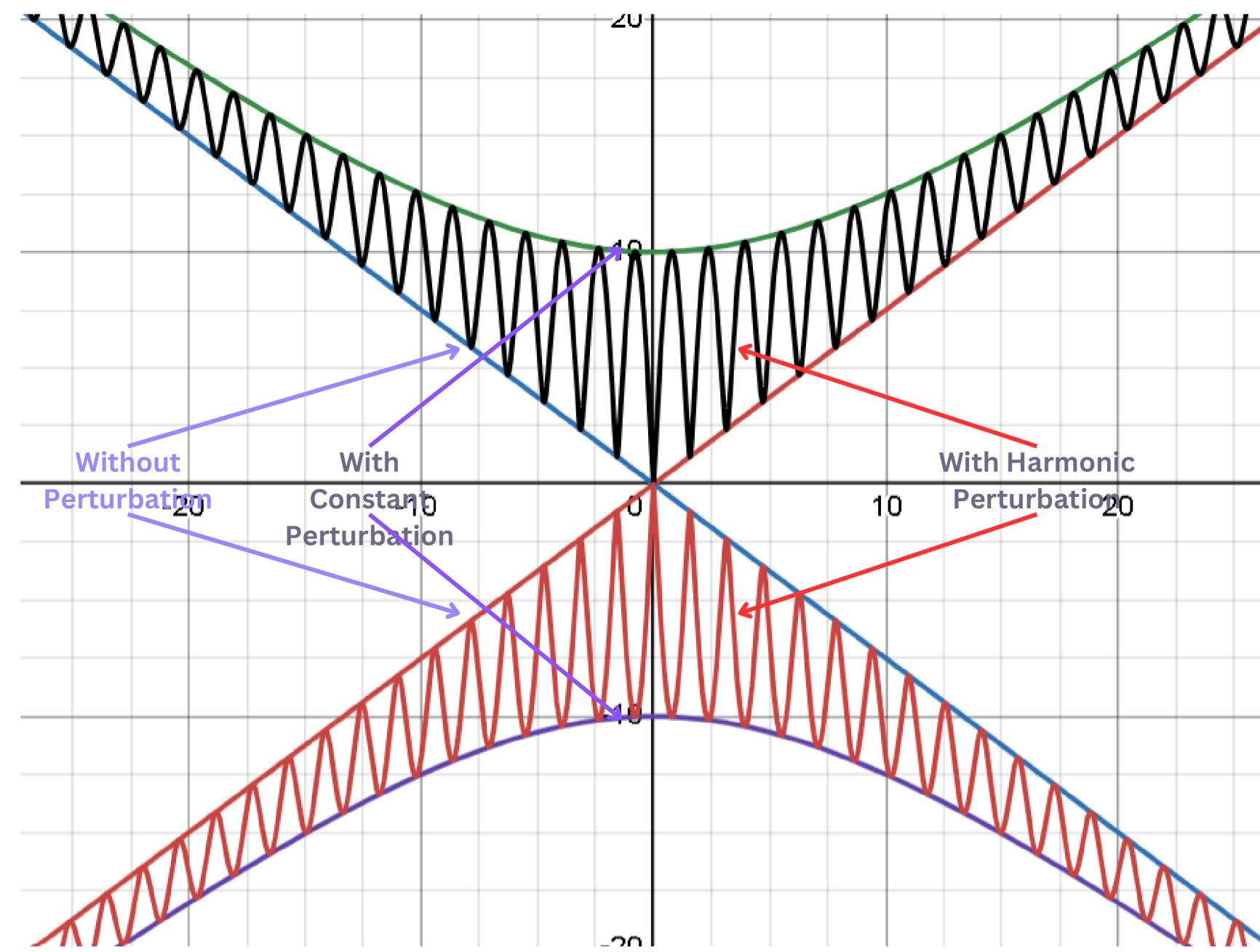
$\Omega=40$

$g=4$



# Comparison with Standard Landau-Zener

## Comparison of Eigenvalues



## Comparison of Trans. Prob.

