EXPIRING BUDGETS AND DYNAMIC COMPETITION IN YEAR-END PUBLIC AUCTIONS: THEORY AND EVIDENCE FROM CHILE

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Abstract. Data from Chile's public procurement system shows that auctions become less competitive toward the end of the fiscal year as the number of tenders increases because of use-it-or-lose-it budget provisions. A dynamic price-competition model with capacity constraints rationalizes this finding and has additional predictions that are confirmed by the data. In particular, firms infer a less competitive environment by the end of the year and frequently set their bids around the price cap. This result is driven by the possibility of becoming a single-bidder when competitors exhaust their capacity. Additionally, high-capacity firms usually skew their victories toward the end of the year, while smaller firms win more tenders in earlier periods.

"Evidence from various sources indicates that the quality of contract awards suffers during year-end surges and that the Government does not get full value for its money... the use or lose syndrome placed the Government negotiators in a poor bargaining position."
- United States General Accounting Office
(GAO, 1980)

1. Introduction

Public procurement is often criticized because use-it-or-lose-it provisions lead to an expenditure spike at the end of the year along with allegedly inefficient spending (McPherson, 2007). This has been thoroughly studied[1] and some demand-side explanations which could cause this behavior have been examined (Liebman and Mahoney, 2017). Nevertheless, there is no literature on the supply-side implications of this particular procurement scheme. In this paper we take this demand-side feature as given and we center our attention in the supply-side of this market.

We study the behavior of firms participating in year-end public auctions. As firms know about the demand scheme, they may optimally condition their intertemporal pricing strategies to face this higher expenditure prospect.[2] The implications of this are not obvious, as a static price-competition model would still yield a competitive outcome. Nevertheless, by modeling a two-period price-competition duopoly, we conclude that under the presence of capacity constraints, a mixed equilibrium exists in which firms profitably bid above the competitive benchmark and sort the timming of their sales depending on their capacity. This result is driven by the possibility of participating in a single-offer auction when a competitor exhausts its capacity. Thus, dynamic considerations regarding demand expansions may undermine competition.

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[1] For example, GAO (1980), NAO (2014), Pidgeon (2014), Engel et al. (2015), and CEP (2017).

[2] As a former public agency chief in Chile stated: *"The money you do not spend is lost. This is known by contractors, who knock your door at the end of the year because they know about the threat of underexecuting the budget."* (CEP, 2017).

There is a wide variety of papers about *static* capacity-constrained oligopolies.[3] Notwithstanding, the literature on *dynamic* capacity-constrained oligopolies is scarce.[4] We chose this setting because public auctions in Chile work in a similar fashion: large tenders are announced with a lot of anticipation, allowing firms to foresee demands months ahead, and these tenders require upfront security payments that directly constrain firms that opt in (besides from the productive constraints that may arise after providing the goods or services). Indeed, some firms may take advantage of this situation. For example, suppose an agency wishes to buy 1 unit of a good in November and 2 units of the same good in December. If a firm that may supply 3 units competes with another that may supply only 1 unit, the large firm may have incentives to lose the tender in November. In exchange, it would constitute a monopoly in December and may maximize its profits by charging the reserve value of the procuring agency. In effect, the central intuition of our model has to do with how large firms' outside option make it attractive for them to deviate from a static competitive equilibrium and price in a less aggressive fashion. Smaller firms should exert similar actions when internalizing what large firms do.

This theoretical model yields some key implications. A direct implication is that if more tenders are open in December (the last month of the Chilean fiscal year), then they must be less competitive, as smaller firms that exhausted their capacities would be out of the market. A less straightforward implication is that, despite not knowing how many competitors they are facing, some firms will infer their single-bidder condition and extract all the rents they can from the procuring agency. Additionally, this model implies that large firms are more likely to win tenders in December, while small firms concentrate their awards in the preceding months. Indeed, we confirm all these results by testing them on Chilean public procurement microdata.

Finally, there are some technical considerations that will rise from our model, such as a mixed strategy equilibrium with a relevant mass point in the pricing distribution of large firms. This mass point is located in the upper bound of the price support, and it is generated by the odds of being a single bidder in at least one auction. We find a significant fraction of bids in our data that effectively set prices as high as possible. This supports our theoretical claim and clearly evidence an environment that is far distant from the static competitive benchmark.

The remainder of this paper is organized as follows. Section 2 first explains a basic model and summarizes some implications. Our data is presented in Section 3 and empirical predictions are tested in Section 4. Section 5 concludes.

2. Basic Model

We develop a model that focuses on public tenders, and features capacity-constrained firms that compete in two periods to supply units of a single good. This model characterizes the optimal bidding strategies of firms and also sheds light on what a rational public agency could do to maximize its expected surplus. In this section we first explain a basic version of the model and generalize it in Appendix B. The setup of the model is characterized below.

2.1. **Setup.**

Consider a risk-neutral public agency that wants to buy 3 units of a non-divisible good. These 3 units have to be procured in two periods by opening three independent auctions of one unit each. The agency decides how many auctions to call in each period, as long as total purchases add up 3 units. Its discount factor is $\delta \in [0, 1]$.

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[3]A classic result due to Edgeworth (1925) indicates that *static* price-competing duopolies with capacity constraints usually don't achieve a pure strategy equilibrium. Nevertheless, Levitan and Shubik (1972) show that there exists a mixed-strategy equilibrium under certain conditions (for example, linear demand). Osborne and Pitchik (1986) further extend the equilibrium existence result to more general cases. Additionally, Griesmer and Shubik (1963) studied a static bidding setting with capacity constraints that is similar to ours, but without dynamics. Other well-known contributions that consider different capacity-constraining settings include Kreps and Scheinkman (1983), Peters (1984), Saloner (1987), Pal (1991), and Kovenock and Roy (1998).

[4]The price competition setting was thoroughly studied and popularized by Dudey (1992), who characterizes the cases in which pure and mixed strategy equilibria arise. Four more recent papers by Ghemawat and McGahan (1998), Garcia, Reitzes, and Stacchetti (2001), Van Den Berg et al. (2012), and Anton, Biglaiser, and Vettas (2014) complete, to the best of our knowledge, the list of papers that consider dynamic capacity-constrained competition models, where the latter is similar in spirit to ours.

There are two risk-neutral firms with different capacities that can supply this good through public auctions. The “low capacity” firm L can only supply one unit, either in the first or in the second period, whereas the “high capacity” firm H can supply a total of three units. It follows that the large firm faces no relevant constraints and may supply the whole market in any period.

The small firm can choose to participate in at most one auction and the chosen auction is private information, along with its bid. The large firm can privately choose to participate in up to three auctions and set prices that are also unknown to its competitor. Nevertheless, the past history of auctions is common knowledge.

Each purchased unit generates a social value of $V > 0$, which is totally internalized by the public agency. This is public information for both firms, which are also aware of their zero-cost symmetry, their capacity constraints, their common discount factor $\beta \in [0, 1)$ and the fact that the agency has to buy three units.

The winner of each auction is the firm with the lowest bid, and capacity is reduced only when a firm wins an auction. Ties are decided by lot and both firms and the agency wish to maximize their individual expected discounted surpluses.

We start by showing that, in each period, firms will bid in as many auctions as their maximum capacity allows them to and that if a firm optimally participates in more than one auction in a given period, then it will choose only one price strategy to characterize its bids. In every case, we assume that all bids are *serious*, as in standard auction theory (Krishna, 2003), which means that any price p submitted in a tender satisfies $p \in [0, V]$.

The fact that firms will bid in as many auctions as they can is a non-trivial feature of our model, particularly because not participating in an auction may be rewarded with additional future profits. Indeed, strategic capacity withholding may be relevant in other cases, such as electricity markets (Green, 2004; Bergler, Heim, and Hüschelrath, 2017). Nevertheless, we prove that this is not our case.

Intuitively, consider the small firm and suppose that there is one open auction in period 1. The small firm could eventually save its capacity and not participate in the present auction, so it can compete in period 2. Nevertheless, as prices are bounded by V, saving its unit will yield at the most a present discounted value of βV. Thus, the firm could simply submit a bid of $p = V$ in period 1. If it wins, it yields $V > \beta V$, whereas if it loses, it can still participate in period 2. Under any case, participating with a bid equal to V is at least as profitable as withholding capacity. The formal proof for any capacity constraint and any number of open auctions may be found on Lemma 1 in Appendix A.1.

The latter means that explicitly saving capacity for the future (Dechenaux and Kovenock, 2007) by not participating in a tender is weakly dominated by bidding in all possible tenders. Given the latter, we will not find any strategic capacity withholding.

Next we show that each firm will optimally choose a single price strategy to define all bids. We do not restrict this strategy to be a single price p^*, so we assume the existence of a pricing distribution s^* instead, which may either describe a mixed or a pure strategy (when s^* degenerates).

Indeed, as all auctions are symmetric and the competitor’s bids are private information, there is no reason to optimally define asymmetric pricing strategies. In fact, if any firm prices according to two different strategies s_1 and s_2, then it will be at least as profitable to choose one of them (the one that maximizes the individual expected profit derived from any single tender). A formal and generalized proof may be found on Lemma 2 in Appendix A.2.

Note that the last result implies that if a firm plays a pure strategy it will set the same price in all auctions. This is not true when firms play a mixed pricing strategy, as the proof would only imply that a given firm in a given period will choose the same pricing distribution on each auction, but not the same price. Nevertheless, as firms are risk neutral, they would enjoy the same expected benefits if they draw different prices for each tender from the same distribution or they simply take a single price from this distribution and bid this price in all tenders. In order to simplify further mathematical expressions, but without any impact on relevant results, we assume that when playing a mixed strategy, firms will bid the same price on all tenders they opt in a given period.[5] This assumption may be also justified by non-discriminatory pricing policies that are often required in public procurement. Indeed, under equivalent procurement conditions, firms are generally required to submit equivalent prices, and may be punished if they do not.

We now depart with the resolution of our model[6] by using backwards induction. We develop the competitive framework in period 2 and compute the expected utilities that each agent may achieve in this period.

2.2. **Period 2.** Recall that the agency can choose how many units to procure in each period. Depending on what happens in period 1, we may have the following scenarios in period 2:

(1) The agency purchased all three units of the good in period 1.
(2) The agency purchased two units of the good in period 1 and
  (a) the units were separately purchased from both firms; or
  (b) the units were purchased from a single (large) firm.
(3) The public agency purchased only one unit of the good in period 1 and
  (a) the unit was purchased from the small firm; or
  (b) the unit was purchased from the large firm.
(4) The public agency did not purchase any unit in period 1.

There are some cases with a trivial equilibrium in period 2. Indeed, in case 1, the game is over before period 2 starts; cases 2a and 3a constitute a monopoly in which the large firm prices V; and case 2b generates a Bertrand competition scheme in which both firms drive their profits to zero. The non-trivial cases, which we address formally below, are cases 3b and 4.

2.2.1. *Case 3b.* First note that the large firm will never price under $V/2$. If it did, it would profit less than V, whereas setting $p = V$ in both auctions yields a secure benefit of V, because the small firm cannot compete in both auctions. Knowing this, the small firm will also bid $V/2$ or more. This is formally addressed in Lemma 3 in Appendix A.3. The equilibrium of this case is presented below.

Proposition 1. *Given a firm with capacity 2 and another firm with capacity 1, both in period 2, there will exist a mixed-strategy Nash equilibrium in which the price distribution of the large firm stochastically dominates in first order that of the small firm. The small firm's distribution will be* $F(p) = 2 - \dfrac{V}{p}$ *and the large firm's distribution will be* $G(p) = 1 - \dfrac{V}{2p}$*, with a mass of* $\dfrac{1}{2}$ *when* $p = V$*.*

Proof. There are no Nash equilibria in pure strategies, as firms will always deviate by undercutting each other until they reach $\dfrac{V}{2}$ and then engage in an Edgeworth cycle (Maskin and Tirole, 1988).

Nevertheless, a mixed strategy equilibrium exists. Let p_L be the low-capacity firm's bid and p_H be the high-capacity firm's bid and let F and G be their respective right-continuous probability distributions with a possible finite amount of discontinuities,[7] and both sharing a common support equivalent to $\left[\dfrac{V}{2}, V\right]$.

[5] The result of imposing this assumption is that this model is equivalent to having two tenders, one in each period, in which units are sold to the lowest bidder, but excess demand is still covered by the other firm.

[6] The mathematical development of the following sub-section is equivalent to that of Griesmer and Shubik (1963) and Anton, Biglaiser, and Vettas (2014), even though the next one is different.

[7] Moreover, we may state that these functions are continuous in their whole domain, except, potentially, in when $p = V$. Indeed, if G had a positive mass in some $\bar{p} \in \left[\dfrac{V}{2}, V\right)$, then the large firm would be better off by reducing it and increasing the odds of bidding V. This would reduce the odds of winning an additional unit at a low price, but would increase the price

We know that the high-capacity firm has an expected profit of

$$\mathbb{E}\pi^H \equiv \int_{V/2}^{V} \left[p_H \int_{V/2}^{p_H} dF(p_L) + 2p_H \int_{p_H}^{V} dF(p_L) \right] dG(p_H),$$

while the small firm expects

$$\mathbb{E}\pi^L \equiv \int_{V/2}^{V} p_L \int_{p_L}^{V} dG(p_H) dF(p_L).$$

For F and G to conform a mixed-strategy equilibrium, these expected profits must remain unchanged whenever any of the firms decides to bid any price p in the support[8] defined by $p \in \left[\frac{V}{2}, V\right]$. Particularly, this must be true when any firm decides to bid $p = \frac{V}{2}$, so we will use this point to evaluate the expected profit and infer F and G. We use this point because it is the lowest price in the support of both distributions[9] and it is impossible for any of the distributions to have a positive mass accumulated in this point.

Thus, we know that $F\left(\frac{V}{2}\right) = G\left(\frac{V}{2}\right) = 0$ and we may state that, for any $p \in \left[\frac{V}{2}, V\right)$,

$$\mathbb{E}\pi^H(p) = \mathbb{E}\pi^H\left(\frac{V}{2}\right) \iff pF(p) + 2p[1 - F(p)] = \frac{V}{2} \cdot 0 + V \cdot 1,$$

which means that the large firm's expected utility must be V and that the small firm must be pricing according to the distribution defined by

$$F(p) = 2 - \frac{V}{p}.$$

Analogously, for any $p \in \left[\frac{V}{2}, V\right)$, the small firm's expected utility must satisfy

$$\mathbb{E}\pi^L(p) = \mathbb{E}\pi^L\left(\frac{V}{2}\right) \iff p[1 - G(p)] = \frac{V}{2} \cdot 1,$$

which means that the small firm's expected utility must be $\frac{V}{2}$ and that the large firm must be pricing according to the distribution defined by

$$G(p) = \begin{cases} 1 - \dfrac{V}{2p} & p \in \left[\dfrac{V}{2}, V\right) \\ 1 & p = V \end{cases}.$$

The result is a tempered distribution, without a density defined at $p = V$ and with a mass of 0.5 accumulated at this point. Thus, $F > G$ for any $p \in \left(\frac{V}{2}, V\right)$, so distribution G first-order stochastically dominates distribution F, just as depicted in Figure 1.

Note that when playing according to these distributions, by construction, both firms will have the same expected utility whenever they deviate by bidding any price in the support. Additionally, we already explained why pricing outside this support is not optimal for any firm. Thus, there are no unilateral incentives to deviate and F and G conform a mixed strategy equilibrium. □

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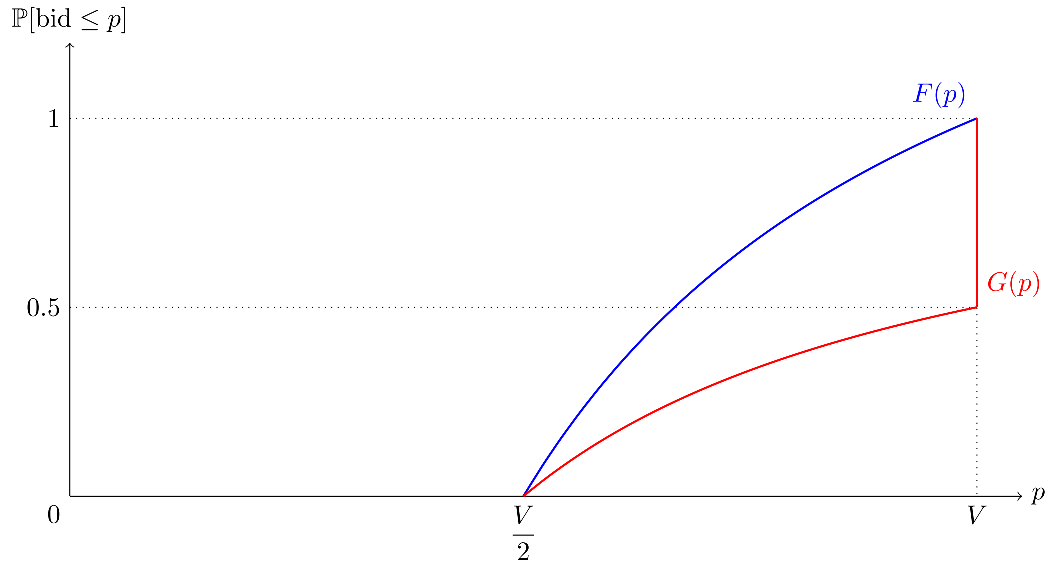
received in the secure unit. By construction, the second effect dominates the first one. After a first iteration, the small firm would also eliminate any positive mass for F in $\bar{p}$. Thus, if a discontinuity exists, it is located in $p = V$. A more formal proof may be found in Appendix A.4.

[8]By contradiction, if the expected utility of some firm increases for some $\bar{p} \in \left[\frac{V}{2}, V\right)$, then it would be better off by degenerating its distribution and playing a pure strategy given by pricing $\bar{p}$. Now suppose the expected utility decreases when bidding $\bar{p}$. Deleting $\bar{p}$ from the support would have no effect on the expected utility of the firm, as the set-measure of $\{\bar{p}\}$ is zero. But we already know the distributions are continuous in $\bar{p}$, and therefore the expected utilities too. Let $\mathbb{E}\pi^*$ denote the expected utility in equilibrium. If $\mathbb{E}\pi(\bar{p}) < \mathbb{E}\pi^*$, then there must exist some $\delta > 0$ such that $\forall p \in [\bar{p}, \bar{p} + \delta), \mathbb{E}\pi(p) < \mathbb{E}\pi^*$. But $\mathbb{P}(\bar{p} < p < \bar{p} + \delta) > 0$, so the firm would be better off by not bidding any price in $[\bar{p}, \bar{p} + \delta)$ and the support would be flawed.

[9]If it were not, any firm could increase its profits by bidding a shade above $\frac{V}{2}$.

Figure 1. Comparisson of optimal pricing distributions



Besides proving that the larger firm prices at higher levels, we also found a mass point on the pricing distribution. The intuition behind this result is that the large firm knows that it won't meet the smaller firm in both auction, as the latter will exhaust its capacity by participating in one out of two auctions.

Indeed, if the small firm could supply the whole market, this result would be lost and a Bertrand outcome would characterize the equilibrium. As this is not our case, both firms earn positive expected profits in equilibrium: the large firms earns V and the small one gets $V/2$ as outcome.

In case 4 both firms now have their full capacity and there are 3 auctions, where the small firm can only choose 1 to participate and the large firm may supply the whole market. Again, both firms know that the larger firm will only meet the smaller firm in one auction. Not surprisingly, we obtain a result similar to that of case 3b. We proceed analogously.

2.2.2. *Case 4.* Now both firms will never price under $3V/2$. Arguments are equivalent to those of the last case and this is formally addressed in Lemma 4 in Appendix A.5. The equilibrium of this case is as follows.

Proposition 2. *Given a firm with capacity 3 and another firm with capacity 1, there will exist a mixed-strategy Nash equilibrium in which the price distribution of the large firm stochastically dominates in first order that of the small firm. The small firm's distribution will be* $F^*(p) = 3 - \frac{2V}{p}$ *and the large firm's distribution will be* $G^*(p) = 1 - \frac{2V}{3p}$*, with a mass of* $\frac{2}{3}$ *when* $p = V$*.*

Proof. See Appendix A.6 □

Note that in this case they both price higher than in the two-auctions case and, furthermore, the large firm prices $p = V$ with higher chances. The reason is that there are even more chances of being a single-bidder and that the large firm has a secure sale of at least two units. Again, this is given because the small firm will always leave two auctions empty, so both firms will be able to attain positive profits in equilibrium. The small firm earns an expected profit of $2V/3$, while the large firm obtains $2V$.

TABLE 1. Expected payoffs in period 2

Scenario	Big Firm	Small Firm	Public Agency
1	0	0	0
2a	V	0	0
2b	0	0	V
3a	$2V$	0	0
3b	V	$\frac{V}{2}$	$\frac{V}{2}$
4	$2V$	$\frac{2}{3}V$	$\frac{1}{3}V$

In order to close this subsection, note that if firms play their equilibrium strategies, then the expected payoff in period two of both firms and the public agency may be summarized as in Table 1. Note how the expected utility for each firm is equivalent to its outside option in each scenario, that is, no firm yields extraordinary rents through this competitive process.

2.3. **Period 1.** We now consider period 1. To reduce notation, the L and H subscripts will be dropped, unless they are needed to clarify mathematical ambiguity. Additionally, we will avoid the strategy-space lemmas, as the idea is analogous in the forthcoming cases. Note that if the public agency does not call for tenders, then we are in case 4 and there is no action in period 1, so we start by analyzing the single-auction case in period 1.

Proposition 3. *If the public agency calls for a single auction in period 1, then there will be an equilibrium characterized by the large firm mixing prices $p \in [\beta V, V]$ according to a distribution $G(p) = 1 - \frac{\beta V}{2p - \beta V}$, with a mass of $\frac{\beta}{2-\beta}$ when $p = V$, and the small firm bidding βV. Additionally, infinite $\varepsilon-$equilibria exist in which, for any $M \in [0, 1)$, the large firm mixes prices $p \in [\beta V, V]$ according to a distribution $G^{**}(p) = 1 - \frac{\beta V M}{2p - \beta V}$, with a mass of $\frac{\beta M}{2-\beta}$ when $p = V$, and the small firm bids a shade under βV.*

Proof. See Appendix A.7 □

This equilibrium is intuitive when viewed as a Bertrand equilibrium in which firms undercut each other until they reach $p = \beta V$. At this point, the large firm will not find it profitable to undercut its rival, whereas the small firm would just bid a shade under this value. Just as in any static Bertrand-competition duopoly, the "efficient" firm with the lowest marginal cost will charge (a shade under) the marginal cost of the "inefficient" firm. The only difference in this case is that the cost is a result of the intertemporal feature of the model, which places a higher opportunity cost (or outside option differential) to the larger firm.

When there are two open auctions in period 1, one might *a priori* think of a similar equilibrium, as the same logic of a future-profit opportunity cost remains still. Indeed, if the small firm wins by bidding p, it earns p (case 2a) and if it loses, it earns a total expected discounted profit of 0 (case 2b). On the other hand, if the large firm bids p, it either earns $2p$ or $p + \beta V$. Same logic as in Proposition 3.

Nevertheless, there is a key difference: the small firm cannot supply the whole market in this period. Thus, we will not find a near-Nash equilibrium in which it plays a pure strategy, but rather a Nash equilibrium in mixed strategies for both firms.

Table 2. Agency's expected disbursements and surplus for each case

Tenders ($t=1, t=2$)	$t=1$ Expenditure	$t=2$ Expenditure	Discounted Surplus
Case 4: (0,3)	0	$\frac{8}{3}V$	$\frac{1}{3}\delta V$
Case 3: (1,2)	βV	$2V$	$(1-\beta)V$
Case 2: (2,1)	$\frac{(3-\beta)(1+\beta)V}{2}$	$\frac{(1+\beta)V}{2}$	$\frac{(1-\beta)(1-\beta+\delta)V}{2}$
Case 1: (3,0)	$\frac{8}{3}V$	0	$\frac{1}{3}V$

Proposition 4. *If there are two open auctions in period 1, there will exist a mixed-strategy Nash equilibrium in which the distribution of the less constrained firm stochastically dominates in first order that of the more constrained firm, with both firms bidding* $p \in \left[\frac{V+\beta V}{2}, V\right]$. *The small firm's distribution will be* $F^{***}(p) = 2 - \frac{V-\beta V}{p-\beta V}$ *and the large firm's distribution will be* $G^{***}(p) = 1 - \frac{V+\beta V}{2p}$, *with a mass of* $\frac{1+\beta}{2}$ *when* $p = V$.

Proof. See Appendix A.8 □

Finally, as a last case, the agency could open 3 tenders in the first period. If the purchasing scheme is credible, which means that there is no chance for the public agency to accrue additional units in period 2, then Proposition 2 characterizes the equilibrium.

2.4. **Agency's choice.** A fully rational public agency would choose to set a buying scheme that maximizes its expected discounted surplus accrued from the procured goods. If the agency has a discount factor of δ, then Table 2 summarizes its expected disbursements in each period and the total expected discounted surplus for each purchasing scheme.

Proposition 5. *If the public agency maximizes its expected discounted surplus, then, for any* $\beta > 2/3$, *it should purchase all three units in period 1.*

Proof. Straightforward after inspecting Table 2. □

Thus, assuming β is close enough to 1, as it would be in short time spans at the end of a fiscal year, the agency should optimally choose to purchase all three goods in a single period (ideally in period 1, unless $\delta = 1$). The intuition behind this result is that firms price more aggressively in absence of any future demand, as their outside option (or opportunity cost of winning) becomes zero. This replicates a one shot game in which firms can only profit because of the certainty of not overlapping all their bids.[10] Nevertheless, for case 1 to be optimal, a necessary condition is that the purchasing scheme is credible. This means that firms should be certain that no other purchase will be made in the final period, even if the agency has a positive surplus, which could be used to buy additional units in absence of commitment to the purchasing scheme.

This model[11] has various testable implications on what is to be expected when observing data derived from procurement systems that are close to our assumptions. First, firms price higher when the odds of being a single bidder are higher, even if they are not sure of their single-bidder condition. Secondly, if there are not enough firms to supply the whole market, then an expenditure spike at the end of a year should be accompanied by a reduction in competition. Finally, large firms are the ones that should be more likely to win tenders in the end of the year. In the following sections, we provide an empirical counterpart to these theoretical ideas.

[10]Indeed, if both firms had the capacity to individually attend the whole market, they would drive profits to zero.
[11]A more general case is presented in Appendix B.

3. The Data

3.1. **Database description.** In order to test model predictions, we use Chilean microdata on public procurement in 2014 and 2015. This dataset is provided in real-time by the Department of Public Procurement and Contracting (in Spanish: *Dirección de Compras y Contrataciones Públicas*, or DCCP) and contains high-frequency data on public purchases made in Chile. Additional details on our data may be found in Appendix C.1.

Public tenders are categorized according to their "size", which is defined by the total value of the tender, the latter being estimated by the procuring agency. The value is estimated in "UTM", where 1 UTM is about 75 USD. Public tenders worth more than 1000 UTM are classified as "large" and are subject to additional regulations,[12] which we describe later.

Each tender may have multiple "lines" or items (a line does not necessarily have a single unit, but does identify a specific kind of good, for example, a dozen of tables), in which bids are separately submitted by firms.[13] Most tenders have a single line: 58.31% of all tenders and 66.64% of large tenders. This is relevant because public agencies are not required to estimate the value of the procured goods in each line, but only in the tender as a whole. Thus, single-line tenders allow us to infer the value of the item that is being procured, just as V in our model.

The definition of the estimated value of a tender is mandatory by law for all public agencies that wish to procure with this mechanism. This condition is imposed because different regulations are applied to tenders according to their size. Besides defining these regulations, the estimated value is also used as a pseudo-maximum payment threshold. Indeed, when a bid significantly surpasses this estimated value, it is generally discarded. In case all bids of a tender are significantly above this threshold, the tender is dismissed. In fact, if a public agency chooses to award the tender to a competitor with a bid that surpasses the estimated value, other competitors can file a complaint and shut the tender process down. Therefore, just as in the theoretical model, we will refer to this maximum-price proxy as V. It is important to note that, though the estimated value must be defined for each tender, it is not public in all of them. This will be further commented later.

Elaborating a bit more on the characteristics of our data, each observation is a bid that has information on:

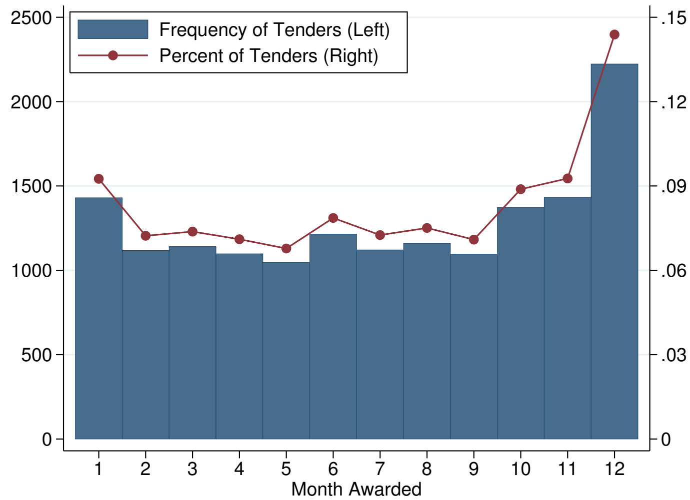
- A unique identifier of the tender and the line corresponding to the bid submitted.
- An 8-digit ISO code that identifies the good or service that is being procured in the specified line.
- A unique identifier of the procuring agency and the firm that submits the bid.
- The quantity and measurement unit of the item that is being demanded in the corresponding line. Firms cannot submit bids for less than the demanded quantity, so this is always equivalent to the supplied quantity as well.
- The unit price offered in the bid, all stated in Chilean Pesos (CLP).
- The quantity awarded, if any, to the firm that submitted the bid. It is important to note that agencies can split awards, that is, they can partially award the demanded quantity to different firms, even though firms cannot offer less than the total demand.[14]
- The estimated value of the tender (V) and an indicator of its public availability.
- The opening date of the tender, its bid-submission deadline and the award date. All these dates are publicly available ex-ante, which means that a firm that bids on a tender knows before hand the date in which the winner(s) will be announced.

[12]There is evidence that shows how these thresholds may induce manipulation in the valuation of the procured contracts. Just as Palguta and Pertold (2017), we also find an important bunching of estimated values just below the 1000 UTM threshold.

[13]Moreover, a firm can submit multiple bids in each line of a tender if it wishes to offer different options, say, qualities, of the same good or service. Nevertheless, this isn't a frequent practice and it is even unauthorized in some tenders, as firms should ideally chose their "best" bid to be submitted.

[14]Thus, firms must be always aware that the unit price they bid should not depend on the demanded quantity, like in a volume-basis discount setting.

FIGURE 2. Tenders awarded by month in our particular sample



3.2. **A particular sample.** Chilean public procurement regulations differ according to the size of a tender. Particularly, rules that govern the behavior of "large" tenders are more close to the assumptions in our theoretical model. Indeed, large tenders are announced with months of anticipation and require firms to pay upfront securities that constrain their financial capacity, besides from productive constraints derived from the provision of goods or services. Thus, in all forthcoming empirical exercises, we will only work with data that comes from tenders with $V > 1000$. Furthermore, we will only consider those tenders with a single line, a single winner and a single offer per firm. A more thorough discussion and justification of this decision may be found in Appendix C.2.

The particular sample that we use still preserves the procurement spike that is found when using the whole sample. As Figure 2 depicts, the amount of tenders in the last month of a fiscal year doubles the average amount of monthly tenders in the rest of the year.

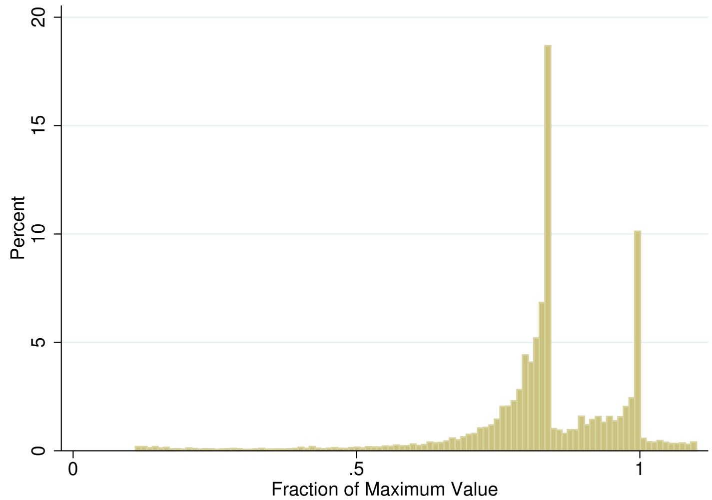
3.3. **Award mechanism.** Another important concern has to do with the award mechanism. In Chile, public tenders are awarded according to *scores* that are computed using different characteristics of an offer as inputs (for example, provider history, delivery times, quality of goods, etc.). This scoring function is publicly available in each public tender, but may vary from one case to another. As a mandatory condition, all scoring functions must consider the price as an input, but cannot solely rest on this variable as argument. Thus, in spite of being relevant to decide awards, prices do not uniquely define winners, so minimum-price bidders may lose a tender.

In Appendix D we assess the importance of being a minimum-price bidder, and conclude that it increases in over 20% the chances of winning a tender. Therefore, for sake of simplicity (and inability to construct the scoring functions), we will treat Chilean public tenders as first price auctions in which minimum-price bidders win.

3.4. **The VAT issue.** There are some tenders which require bids to be submitted without VAT. This makes the data on p/V distribute according to the bi-modal[15] density depicted in Figure 3. We do not have data to deterministically identify these two groups, so we perform a clustering exercise to infer for each bid if its corresponding tender did or did not consider VAT in its pricing scheme. This is done to avoid any scale

[15]Chile has a fixed VAT of 19%. Thus, if a firm wishes to bid the whole estimated value, but without VAT, it should submit a bid that accounts for $1/1.19 \approx 0.84$ of the total estimated value.

FIGURE 3. 100-bin histogram for p/V (11-110%)



problem due to the VAT when testing a prediction related to how firms set their prices.

This clustering exercise is different from popular classification algorithms, as we do not have data to characterize tenders in an informative way that relates to the VAT scheme. We first decompose the bidding distribution as a mixture of two distributions: one with VAT and another without VAT. We impose a penalizing factor that forces these two distributions to be similar, albeit a scale difference of 1.19 in their variable. We then assign individual tenders to one group or another in such a way that the observed relative frequencies, conditional on the average bid submitted in each tender, fit the two inferred distributions of the mixture. A thorough explanation of our method may be found in Appendix E.

The result of this classification exercise is a single-peaked density of p/V, with a great mass (around 25%) accumulated when $p/V = 1$. This approach allows us to create a comparable price proxy. We will indistinctly name this proxy as p/V hereafter and we will use it to test a key prediction of the model in the following section.

4. Testable Predictions

4.1. **Do firms price higher when they are alone?** In the last section we depicted how firms bid $p = V$ in a frequent way. Nevertheless, the model predicts that this behavior should be more common when firms are single-bidders, in spite of not having certainty of this condition. This is because they may infer an increase in their odds of being a single-bidder when a typical competitor exhausts its capacity.

Now that we have a homogeneous price index, we are able to test this prediction. In order to do so, we define a dummy variable D^s that equals 1 when a tender has a single bidder. We estimate

$$\frac{p_i}{V_i} = \alpha + \beta D_i^s + \mathbf{XB} + \varepsilon_i,$$

where $\mathbf{X}$ includes a series of controls: a dummy indicating if the tender publicly displays V, monthly dummies, segment dummies (3-digit ISO code) and agency dummies.

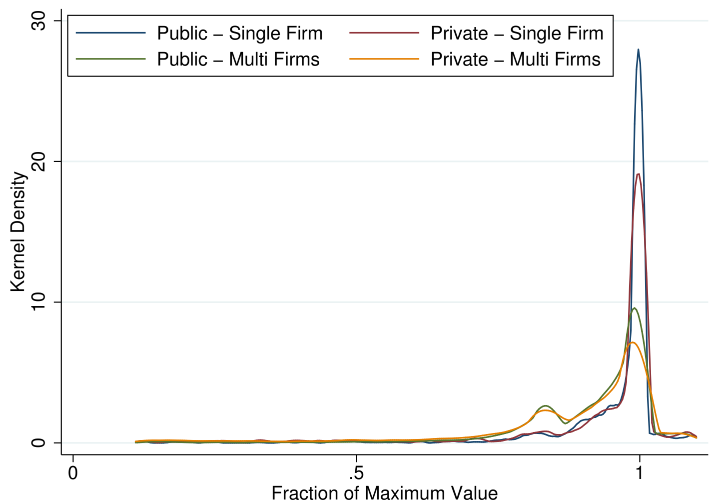
The results of these estimates using different controls are provided in Table 3. Note how single bidders set p/V around 4 percentage points higher than bids in multi-firm auctions. Recall that this result is given without any firm explicitly knowing about their single-bidder condition, they can only infer that there are no other competitors from past information.

TABLE 3. The effect of being a single bidder on prices

	(1)	(2)	(3)	(4)	(5)
Single Bidder	0.0374***	0.0386***	0.0393***	0.0411***	0.0419***
	(11.00)	(11.43)	(11.61)	(12.29)	(12.58)
V is public		0.0339***	0.0336***	0.0304***	0.0170***
		(19.41)	(19.20)	(17.52)	(7.70)
Constant	0.913***	0.893***	0.885***	0.955***	0.908***
	(1017.77)	(658.89)	(279.46)	(16.25)	(15.40)
Monthly Dummies	NO	NO	YES	YES	YES
Segment Dummies	NO	NO	NO	YES	YES
Agency Dummies	NO	NO	NO	NO	YES
N	34320	34320	34320	34320	34320

t statistics in parentheses; * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

FIGURE 4. Estimated densities for fraction awarded by type of tender



Moreover, an additional 2-3% increase in p/V is explained by the public display of V. The result is intuitive: when V is public, firms can anchor their prices without any uncertainty and tend to move p/V more closer to 1. In order to depict our findings, we plotted four superposed density plots defined by the combinations of our single-firm dummy and our public-V dummy. Figure 4 clearly shows how single-firm auctions with public V dramatically have their bids bunched around $p/V = 1$, with over 25% of bids taking this value (and around 19% for single-firm tenders with private V). As expected, competitive auctions present a lower concentration in this area (under 10%), where a private value of V reduces even more this bunching.

4.2. **Do Firms Compete Less at the End of the Year?** A key feature of the theoretical model is the fact that the year-end spike in tenders reduces competition, as other competitors may exhaust their capacity before the last period. Figure 5 plots the fraction of single-firm tenders and average amount of competitors per tender as the end of the year approaches.[16] It is apparent that average participation falls in November and December, while single-firm tenders increase.

[16]In this subsection, we constrain the sample to the second half of the year, as we find a lot of noise in the first half. Nevertheless, when considering the whole sample, results are similar but with lower magnitudes.

FIGURE 5. Fraction of single-firm tenders and average firms per tender (kernel-smoothed)

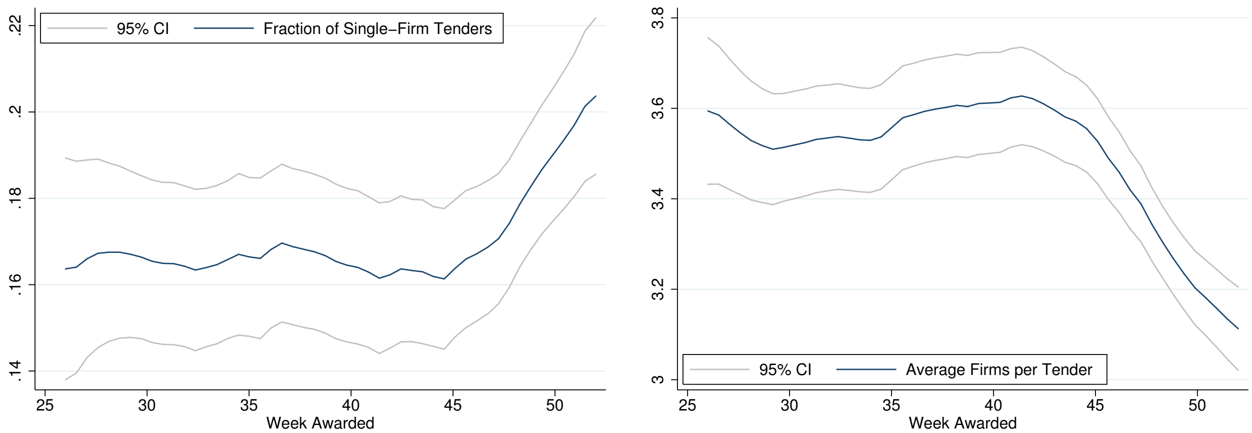


TABLE 4. Polynomial Fit of Single-Firm Tenders Fraction and Average Firms per Week

	(1)	(2)	(3)	(4)
	Single-Firm Fraction		Average Competitors	
Week	0.00135**	0.104*	-0.0172***	-0.782**
	(2.36)	(1.88)	(-5.16)	(-2.32)
Week^2		-0.00287**		0.0223**
		(-2.01)		(2.57)
Week^3		0.0000259**		-0.000207***
		(2.14)		(-2.85)
Constant	0.118***	-1.061	4.155***	12.51***
	(5.01)	(-1.52)	(29.09)	(2.93)
N	6991	6991	6991	6991

t statistics in parentheses; * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Indeed, when performing mean-difference tests comparing these outcomes in the last two months against the rest of the sample, we find a significant increase of 1.90 percentage points ($t = 2.08$) in the fraction of single bidders and a decrease of 0.27 competitors ($t = 4.96$). When repeating the exercise comparing December against the rest of the sample, results are stronger: single-bid tenders increase in 3.88 percentage points ($t = 3.80$) and average competitors decrease by 0.49 firms ($t = 7.98$).

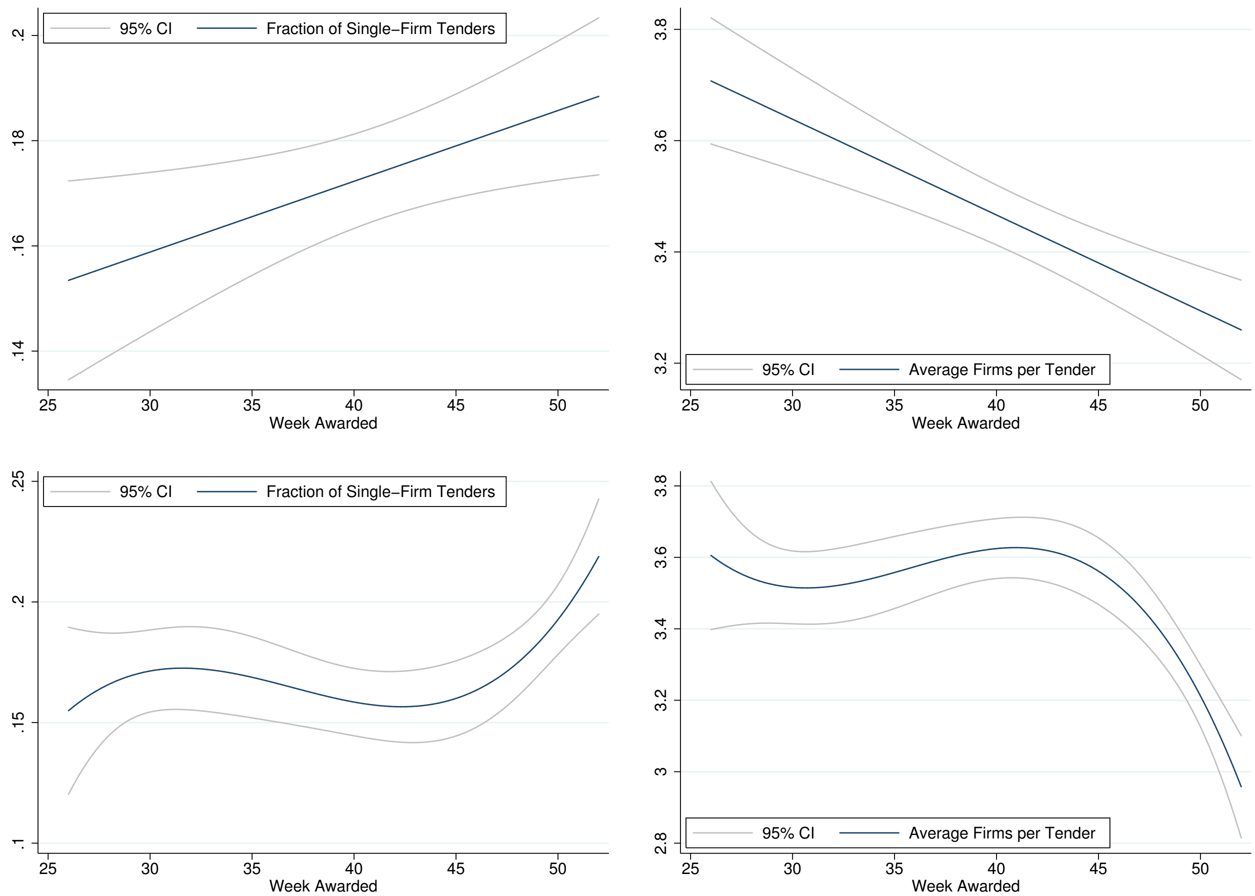
Additionally, we performed a polynomial fit of these variables by estimating

$$Y_t = \alpha + \sum_{k=1}^{n} \beta_k t^k + \varepsilon_t,$$

where Y_t can be the fraction of single-firm tenders or the average competing firms per tender in award-week t, and n is the degree of the estimated polynomial. Results for $n = 1$ and $n = 3$ are depicted in Figure 6 and tabulated in Table 4. These estimations confirm our previous comments.

4.3. **Do Large Firms Win More at the End of the Year?** The model does not only predict price surges and reduced competition when a competitor exhausts its capacity, I also provides an insight on what kind of firms usually win more tenders at the end of the year: large ones. Thus, we test the idea that large firms win a higher fraction tenders at the end of the year.

Figure 6. Fraction of single-firm tenders and average firms per tender (linear and cubic fit)



This poses some empirical challenges. First, explicit measures of capacity or firm size are not available in our data, so we must construct a suitable proxy. Secondly, creating this capacity (or size) indicator may introduce endogeneity issues in our estimations if we use the number of awarded tenders as independent variable.[17] Besides, as pointed out by Jofre-Bonet and Pesendorfer (2003), tenders won in 2014-2015 may be a proxy of capacity, but at the same time of exhausted capacity, so this will not allow us to correctly identify any relation. In order to overcome this limitation, we use *past* wins as a proxy of capacity. The idea is straightforward: firms that were able to win many tenders in the past are likely to be large firms, but their capacity should not be constrained in 2014-2015, as they had time to restore it. Furthermore, the relevant time gap may allow us to consider this capacity proxy as exogenous. Thus, we estimate

$$\frac{w_{i,Dec.}^{2014,2015}}{w_{i,Dec.}^{2014,2015}+w_{i,\sim Dec.}^{2014,2015}}=\alpha+\beta w_i^{2009-2013}+\varepsilon_i,$$

where the super index denotes the years from which the variables are retrieved: our dependent variable comes from pooling data from 2014 and 2015, while our regressor is computed by pooling data from 2009 through 2013.[18] The estimation results are tabulated in the first column of Table 5.

[17] For example, when estimating an equation like $\frac{w_{i,Dec.}}{w_{i,Dec.}+w_{i,\sim Dec.}}=\alpha+\beta w_{i,\sim Dec.}+\varepsilon_i$, where $w_{i,Dec.}$ (resp. $w_{i,\sim Dec.}$) are the tenders won (resp. not won) in December of 2014-2015, the estimate for β will definitely be inconsistent.

[18] At the cost of losing some power because of unmatched observations (firms that did not participate in some past year), we performed additional independent estimations in which we use w_i^t separately $\forall t \in \{2009,...,2013\}$ as right-hand variable. Results are similar, with a clear scale-effect difference. These results are presented in Appendix F.

TABLE 5. Estimates of the effect of firm size (2009-2013, pooled) on the fraction of tenders awarded in December of 2014-2015

	(1) Past Awards		(2) Percentile Dummies	
w	0.0000549***	(2.71)		
D_{50}			0.0375***	(2.78)
D_{75}			0.00599	(0.50)
D_{90}			0.00935	(0.81)
Constant	0.129***	(29.54)	0.0932***	(8.65)
$H_0 : D_{50} + D_{75} = 0$				
Coefficient (p-value)			0.0435***	[0.0017]
$H_0 : D_{50} + D_{75} + D_{90} = 0$				
Coefficient [p-value]			0.0528***	[0.0001]
N	4091		4091	

t statistics in parentheses; * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Our results suggest that larger firms win more on December, as a fraction of total yearly awards. A clearer way of testing our hypothesis and getting a more sensible interpretation of our parameter is by splitting the sample into small and large firms. This was performed by constructing binary variables defined as

$$D_{i,\tau}^{2009-2013} = \begin{cases} 1 & \text{if } w_i^{2009-2013} > percentile^t(\tau) \\ 0 & \sim \end{cases},$$

where τ may take different values to set different victory percentiles as a threshold.

The results of estimating the model described by

$$\frac{w_{i,Dec.}^{2014,2015}}{w_{i,Dec.}^{2014,2015} + w_{i,\sim Dec.}^{2014,2015}} = \alpha + \sum_{\tau} \beta_\tau D_{i,\tau}^{2009-2013} + \varepsilon_i$$

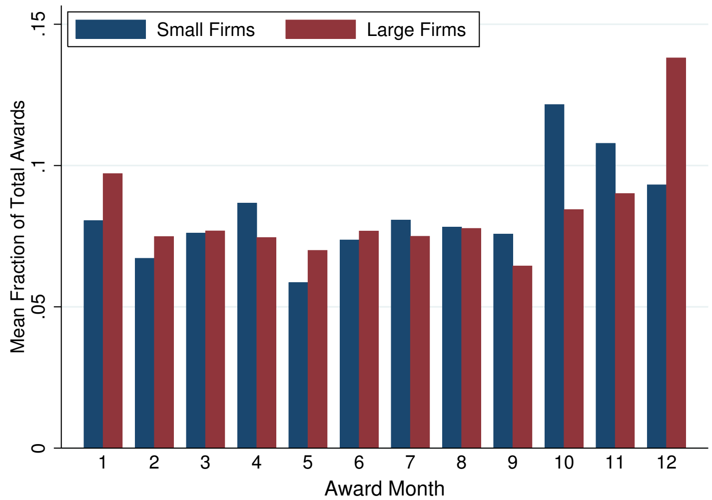
for $\tau = 50, 75, 90$ are summarized in the second column of Table 5.[19] Thus, large firms, specially those above the 90th percentile of past victories, are more prone to winning tenders in December. The effect of being a large firm may be quantified in around a 5% increase in the fraction of tenders won in December, with respect to small firms. This is depicted in Figure 7, where it is clearly shown that large firms significantly win a higher fraction of tenders in December, while small firms usually win more in the preceding months. This is exactly what our theoretical model predicts.[20]

Thus, in this section we have shown how three testeable predictions of the theoretical model actually find some empirical support in the Chilean case. Indeed, firms price higher when they are alone, compete less at the end of the year and large firms skew their victories toward the year-end.

[19]Again, we performed additional estimations in which we use $D_{i,\tau}^t$ separately $\forall t \in \{2009, ..., 2013\}$ as right-hand variables. Results are tabulated in Appendix F.

[20]As a robustness exercise, we repeated the preceding estimations, but changing $\frac{w_{i,Dec.}^{2014,2015}}{w_{i,Dec.}^{2014,2015} + w_{i,\sim Dec.}^{2014,2015}}$ to $\frac{w_{i,Month}^{2014,2015}}{w_{i,Month}^{2014,2015} + w_{i,\sim Month}^{2014,2015}}$, where $Month$ denotes any month different from December. The results are presented in Table 14 in Appendix F.

Figure 7. Tenders awarded per month, as a fraction of total awards, by firm size



5. Concluding Remarks

We developed a theoretical model in which risk-neutral firms with heterogeneous capacities compete in first price auctions to supply units of a procured good in different periods. The model predicts that firms will profitably bid above static competitive levels, as they have chances to meet again and compete in future auctions.

Additionally, high-capacity firms will usually price higher, as their back-up upon losing a tender is to increase their chances of being single bidders in a future tender and extracting monopolistic rents. Moreover, we conclude that these firms should bid the reserve price of an auction with a positive probability. This makes it more likely to observe large firms skewing their victories toward the end of the fiscal year.

The model's predictions fit well with Chilean microdata and, thus, it might provide some policy implications on how to induce additional competition in year-end auctions. One of these implications might be limiting the amount tenders awarded to large firms on short time spans. This would reduce the outside option of large firms, as they would not be able to fully take advantage of their capacity in a single (terminal) period. Obliging them to distribute their sales would force them to price more aggressively in non-terminal periods.

Another implication has to do with the timming of the procurement process. If the award dates of year-end tenders are bunched, then, in practice, public agencies would be recreating a one-shot game. Indeed, whenever the award date of a tender is scheduled after the bid-submission deadline of the next tender, firms are forced to choose the price to bid in the second auction before they know the result of the first one. Not knowing whether a competitor reduced its capacity may induce firms to price more aggressively, as they would not be able to infer a single-bidder condition.

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Appendix A. Lemmas and Proofs

A.1. **No Strategic Capacity Withholding.**

Lemma 1. *There is no strategic capacity withholding. This means that if a firm with capacity $k \in \mathbb{N}$ faces $a \in \mathbb{N}$ open auctions in a single period, then the firm optimally participates in $\min\{k, a\}$ auctions, that is, in as many as it can participate in.*

Proof. Given the setup of the model, a firm with capacity k_t that participates in n auctions in period t, by bidding $\{p_{it}\}_{i=1}^{n}$, earns a total discounted utility of

$$\pi_t(p_t|n, p'_t, k'_t) := \sum_{i=1}^{n} p_{it} \cdot \mathbf{1}[p_{it} < p'_{it}] + \beta\pi_{t+1}(p_{t+1}|k_{t+1}, p'_{t+1}, k'_{t+1}) \quad \forall t \in \{1, 2\},$$

where p' and k' indicate the competitor's pricing scheme and available capacity and where π_{t+1} denotes the future profit, which does not depend on the present prices p_{it}, but rather on the future capacity of both firms and their future prices. In period $t = 2$, $\pi_3 = 0$, and in case the other firm does not participate in auction i in period t, then $\mathbf{1}[p_{it} < p'_{it}] = 1$.

Recall that capacities are public, but prices and participations are private information. Therefore, firm's objective is to maximize its expected discounted profit in any given period, given some unknown strategy profile p'_t of its competitor. This objective is defined by

$$\mathbb{E}\pi_t(p_t|n, p'_t, k'_t) := \sum_{i=1}^{n} p_{it} \cdot \mathbb{P}[p_{it} < p'_{it}] + \beta\mathbb{E}\pi^*_{t+1}(p_{t+1}|k_{t+1}, p'_{t+1}, k'_{t+1}) \quad \forall t \in \{1, 2\}.$$

Without loss of generality, let $n \leq \min\{k_t, a_t\}$, as it can neither be greater than the firm's capacity k_t, nor the number of open auctions a_t. We will now prove that if if it chooses to participate in $n < \min\{k_t, a_t\}$, then this strategy is always weakly dominated by participating in $n + 1$ tenders.

Because winning a tender constrains the firm's capacity, k_{t+1} is non-increasing in the tenders won today and also in n. We also know that the upside of withholding capacity is that the firm may profit from an additional sale in the future. Thus, because $\mathbb{E}\pi_{t+1}$ is non-decreasing in k_{t+1}, it must be non-increasing in n. Nevertheless, as prices are bounded by V, we know that $\mathbb{E}\pi_{t+1}(\cdot|k_{t+1} + 1) - \mathbb{E}\pi_{t+1}(\cdot|k_{t+1}) \leq V$.

Now notice that participating in an additional tender by pricing V either yields a contemporaneous profit increment of V (when winning) or simply preserves k_{t+1} (when losing). Thus, submitting a bid in an additional tender with a price of V will either increase π_t in V -against a possible loss bounded by βV- or it will simply maintain π_t constant. Thus, we conclude that $\mathbb{E}\pi_t$ is non-decreasing in n for any $n < \min\{k_t, a_t\}$. □

A.2. **Single Pricing Strategy.**

Lemma 2. *Suppose that a firm optimally participates in n open auctions in a single period. Then, there exists a single pricing strategy $s^*_t : [0, V] \to [0, 1]$ such that bidding according to $s^*_t(p)$ in each of them is at least as profitable as any other pricing strategy.*

Proof. The proof is straightforward for period 2. First, note that if auctions are independent, then the firm's problem is equivalent to solving

$$\max_s \mathbb{E}_s p_{i2} \cdot \mathbb{P}[p_{i,2} < p'_{i2}] \qquad \forall i \in \{1, ..., n\}.$$

Thus, if a solution exists, there must be a function s^* that characterizes optimal bids for all tenders. This would yield an expected utility equivalent to

$$\mathbb{E}\pi^*_2 := \sum_{i=1}^{n} \mathbb{E}_{s^*} p_{i2|n} \cdot \mathbb{P}[p_{i,2} < p'_{i2}].$$

Analogously, because tenders are independent, in period 1 the firm will choose the same pricing strategy for all bids. The only difference is given by the effect that winning an auction in period 1 may have on future profits. Nevertheless, as commented above, profits in period 2 do not depend on the particular result of an auction in period 1, but on the final remaining capacity.

Thus, for each period there will exist only one pricing strategy that may characterize optimal bids in every tender of the period.

□

A.3. **Optimal Pricing Space for Case 3b.**

Lemma 3. *Given a firm with capacity 2 and another firm with capacity 1, both in period 2, the effective strategy space for each firm is the set defined by* $p \in \left[\frac{V}{2}, V\right]$, *that is, bidding any price above* V *or below* $\frac{V}{2}$ *is strictly dominated by other price for each firm.*

Proof. We know that bidding above V yields zero payoff, so any price above V is not a serious bid and is weakly dominated by any price in $[0, V]$.

Now consider the less constrained firm. If it chooses a price p below $\frac{V}{2}$, then it can profit up till a total of $2p < V$, whereas bidding $p = V$ will definitely yield at least V, because the more constrained firm may win at most one auction. Thus, any price below $\frac{V}{2}$ is strictly dominated by a price equal to V for the less constrained firm.

Having this in mind, the more constrained firm will also find it profitable to bid at least $\frac{V}{2}$, as any price below this threshold is dominated by it.

Additionally, note that the minimum price of the support effectively must be $\frac{V}{2}$, as if it were not, the large firm could profitably deviate by bidding a shade over $\frac{V}{2}$. Similar arguments apply to assure that the maximum price of the support is V.

□

A.4. **Optimal Function-Space Definition.** We know that every distribution function G can be broken down into a unique weighted average of two distributions: a pure continuous one and a step function (Loève, 1977). Thus, if the index c denotes a continuous function and s denotes a step function, then for some weight $w \in [0, 1]$ and any distribution G we may state that

$$G(x) = wG_c(x) + (1 - w)G_s(x), \qquad \forall x \in \mathrm{Dom}G.$$

By contradiction, suppose that an optimal pricing distribution G with support $[\underline{p}, \bar{p}]$ in our model has n discontinuities located in $\{p_i\}_{i=1}^n$, with respective masses of $\{m_i\}_{i=1}^n \in (0, 1)$, such that $0 < \sum_{i=1}^n m_i < 1$. The latter implies that $G(p_i) - \lim_{\delta \to 0^+} G(p_i - \delta) = m_i$ for any $i \in \{1, ..., n\}$.

Now set a continuous distribution $\hat{G}$ defined by successive vertical translations of G, such that

$$\hat{G}(x) := \frac{G(x) - \sum_{i=1}^n m_i \mathbf{1}[x \geq p_i]}{1 - \sum_{i=1}^n m_i}.$$

Additionally, define distribution $\bar{G}$ as a step function equivalent to

$$\bar{G}(x) := \frac{\sum_{i=1}^n m_i \mathbf{1}[x \geq p_i]}{\sum_{i=1}^n m_i}.$$

Note that both functions are right-continuous distribution functions and that

$$G(x) = \left(1 - \sum_{i=1}^n m_i\right)\hat{G}(x) + \left(\sum_{i=1}^n m_i\right)\bar{G}(x),$$

so if $w := 1 - \sum_{i=1}^n m_i$, $G_c := \hat{G}$ and $G_s := \bar{G}$, we arrive at the first decomposition stated in this proof.

We also know[21] that for any Borel function f, the Lebesgue-Stieltjes integral defined by $\int_{\underline{p}}^{\bar{p}} f(x)dG(x)$ is equivalent to

$$w \int_{\underline{p}}^{\bar{p}} f(x)dG_c(x) + (1-w) \int_{\underline{p}}^{\bar{p}} f(x)dG_s(x).$$

Note that all our expected utilities may be arranged to fit the last expression, so in order to prove that an optimal distribution G cannot have any discontinuity located in $p_i \in [\underline{p}, \bar{p})$, we only need to prove that there exists other distribution G^* that generates a higher expected utility.

In effect, given any G decomposed as above in G_c and G_s, define

$$G^*(x) := wG_c(x) + (1-w)\mathbf{1}[x = \bar{p}].$$

Note that this new distribution has the exact same continuous part as G, but instead of having n multiple steps, it only has a single step with a mass of $\sum_{i=1}^n m_i$ in the upper bound of the support.

Thus, any computed expected utility in our model would share the first term in the right-hand-side of the last equation, but there will be a key difference in the second. Note that in our case, for the large firm, f always takes the following form

$$f(x) = k + (ax - b)\mathbb{P}[p < p'] + (cx - d)(1 - \mathbb{P}[p < p']),$$

where $a, b, c, d, k \in \mathbb{R}_0^+$ and x' is the unknown bid of the other firm. Note that for the large firm f is non-decreasing in x and recall that the definition of the Lebesgue-Stieltjes integral for a pure step function in our case is

$$\int_{\underline{p}}^{\bar{p}} f(x)dG_s(x) := \sum_{i=1}^n f(p_i)m_i.$$

Finally, to complete the proof, notice that this last expression multiplied by $(1-w)$ is a convex combination of $f(p_i)\forall i \in \{1, ..., n\}$. But f is non-decreasing, so this convex combination cannot be greater than $f(\bar{p})$. So if G had multiple discontinuities, the expected utility derived from it would always be less or equal to that derived from distribution G^*. This is a contradiction with the fact that G is optimal.

Thus, every distribution G decomposed in G_c and G_s according to w is weakly dominated by a distribution G^* decomposed in G_c and $\mathbf{1}[x = \bar{p}]$ according to w.

A.5. **Optimal Pricing Space for Case 4.**

Lemma 4. *Given three open auctions and both firms with full capacity in period 2, the effective strategy space for each firm is the set defined by* $p \in \left[\frac{2}{3}V, V\right]$*, that is, bidding any price above* V *or below* $\frac{2}{3}V$ *is strictly dominated by other price for each firm.*

Proof. We know that bidding above V yields zero payoff, so any price above V is not a serious bid and is weakly dominated by any price in $[0, V]$.

Notice that if the large firm chooses a price p below $\frac{2}{3}V$, then it can profit up till a total of $3p < 2V$, whereas bidding $p = V$ will yield a deterministic profit of $2V$, because it will always be a single-bidder in two auctions. Thus, any price below $\frac{2}{3}V$ is strictly dominated by a price equal to V for the large firm.

Knowing this, the small firm will replicate the same space of strategies.

Additionally, note that the minimum price of the support effectively must be $\frac{2}{3}V$, as if it were not, the large firm could profitably deviate by bidding a shade over $\frac{2}{3}V$. Similar arguments apply to assure that the

[21] We now intensively rely in the definitions found in the first appendix of Bühlmann (1970).

maximum price of the support is V.

□

A.6. **Proof of Proposition 2.** Analogous to Proposition 1 with the new support constraint of $p \in \left[\frac{2}{3}V, V\right]$ and the following expected utilities:

$$\mathbb{E}\pi^H \equiv \int_{2V/3}^{V} \left[2p_H \int_{2V/3}^{p_H} dF^*(p_L) + 3p_H \int_{p_H}^{V} dF^*(p_L) \right] dG^*(p_H),$$

$$\mathbb{E}\pi^L \equiv \int_{2V/3}^{V} p_L \int_{p_L}^{V} dG^*(p_H) dF^*(p_L).$$

The final result is that the smaller firm will bid according to $F^*(p) = 3 - \frac{2V}{p}$ and the large firm will price following $G^*(p) = 1 - \frac{2V}{3p}$, with a mass of $\frac{2}{3}$ at $p = V$.

A.7. **Proof of Proposition 3.** If the small firm wins the tender in period 1 by submitting a bid of p, then we are in case 3a and its total discounted profit is p. On the other hand, if the small firm loses the tender in period 1, then we are in case 3b, and it earns an expected discounted profit of $\frac{\beta V}{2}$. Thus, the expected discounted profit for the small firm is

$$p\mathbb{P}[p < p_H] + \frac{\beta V}{2}(1 - \mathbb{P}[p < p_H]),$$

which is equivalent to

$$\frac{\beta V}{2} + \left(p - \frac{\beta V}{2}\right) \mathbb{P}[p < p_H].$$

Note how this may be interpreted as the expected markup with respect to the expected discounted profit that the small firm will achieve if it forfeits the present sale: the one obtained in period 2 with a single-unit capacity facing a competitor with two units of capacity. In other words, the objective of the firm is to maximize its expected discounted earnings above its outside option.

Thus, it is straightforward to note that the small firm would never submit a bid under $\frac{\beta V}{2}$, as it would be better off by forfeiting today's sale and having the chance to compete in period 2.

Similarly, if the large firm bids p, it will earn an expected discounted profit of $p + \beta V$ if it wins (case 3b) and a deterministic discounted profit of $2\beta V$ if it loses (case 3a). Thus, its expected discounted profit is

$$(p + \beta V)\mathbb{P}[p < p_L] + 2\beta V(1 - \mathbb{P}[p < p_L]),$$

which is equivalent to

$$2\beta V + (p - \beta V)\mathbb{P}[p < p_L].$$

Again, the high-capacity firm wishes to maximize the expected markup over its outside option: if it wins today, it will lose a discounted value of βV tomorrow (competing and earning an expected profit of V vis a vis being a monopolist and earning $2V$). Thus, it will never submit a price under βV.

Note how the large firm's outside option differential is halved for the small firm's case. Indeed, if the small firm loses today, it will only increment its future expected profits by $\frac{\beta V}{2}$, while the larger increases its expected profits by βV if it forfeits the present sale. Thus, as both firms may supply the whole market in this period, this setting is similar to a Bertrand situation with different opportunity costs. This will be relevant to understand the equilibrium in this case.

Having this intuition clear, the formal proof is straightforward. As we know, the large firm always bids $p \geq \beta V$. Knowing this, after one iteration deleting strictly dominated strategies, the smaller firm would never bid under βV.

Now suppose the existence of two price distributions, F^{**} for the small firm and G^{**} for the large firm, with a common support of $[\beta V, V]$, that conform an equilibrium. Then, for any $p \in [\beta V, V]$, these distributions must satisfy

$$\frac{\beta V}{2} + \left(p - \frac{\beta V}{2}\right)[1 - G^{**}(p)] = \frac{\beta V}{2} + \left(\beta V - \frac{\beta V}{2}\right)[1 - G^{**}(\beta V)]$$

and

$$2\beta V + (p - \beta V)[1 - F^{**}(p)] = 2\beta V + (\beta V - \beta V)[1 - F^{**}(\beta V)].$$

The second condition directly implies that $F^{**}(p) = 1$ for any $p \neq \beta V$, which means that F^{**} degenerates into a pure strategy: bidding βV. However, the first one implies that $G^{**} = 1 - \dfrac{\beta V}{2p - \beta V}[1 - G^{**}(\beta V)]$, with a mass of $\dfrac{\beta}{2 - \beta}[1 - G^{**}(\beta V)]$ when $p = V$.

Note how the large firm can have an infinite set of distributions depending on the mass it assigns to βV. While any of these distributions yield for the large firm a constant expected discounted profit of $2\beta V$, the small firm is not indifferent among them, as its expected utility would be $\dfrac{\beta V}{2}[2 - G^{**}(\beta V)]$.

Thus, the only distribution that generates a Nash equilibrium is that in which $G^{**}(\beta V) = 0$, that is, when the large firm does not price βV. In any other case, the small firm would be better off by bidding a shade under βV and generating a near-Nash equilibrium.

A.8. **Proof of Proposition 4.** Note that the large firm will not bid under $\dfrac{V + \beta V}{2}$, as it would yield less than $V + \beta V$, which is the amount earned by bidding V (its outside option). The small firm will thus delete all strategies under $\dfrac{V + \beta V}{2}$.

Now suppose the existence of two distributions, F^{***} and G^{***} respectively, that conform an equilibrium for the small and large firm. Then these distributions must satisfy

$$p[1 - G^{***}(p)] = \frac{V + \beta V}{2}\left[1 - G^{***}\left(\frac{V + \beta V}{2}\right)\right]$$

and

$$2p - (p - \beta V)F^{***}(p) = V + \beta V - \frac{V - \beta V}{2}F^{***}\left(\frac{V + \beta V}{2}\right).$$

Just as in Proposition 1, both distributions must have no mass in $\dfrac{V + \beta V}{2}$, as the large firm would benefit from moving any positive mass to increasing the odds of bidding V. Thus, as $F^{***}\left(\dfrac{V + \beta V}{2}\right) = G^{***}\left(\dfrac{V + \beta V}{2}\right) = 0$, we may infer that

$$G^{***} = 1 - \frac{V + \beta V}{2p}, \text{ with a mass of } \frac{1 + \beta}{2} \text{ when } p = V$$

and

$$F^{***}(p) = 2 - \frac{V - \beta V}{p - \beta V}.$$

Therefore, just as shown in Figure 1, the large firm will bid higher prices than the small one, as G^{***} stochastically dominates F^{***} in first order.

To ensure that this conforms a mixed-strategy equilibrium, note that when pricing according to these optimal strategies, the expected discounted profit for the small firm is $\dfrac{V + \beta V}{2}$ for any price in the support, with no chances of profiting in period 2. On the other hand, the large firm obtains an expected discounted profit of $V + \beta V$ for any price in the support, which includes the possibility of earning V in period 2 with a probability of $\dfrac{1 + \beta}{2}$. Again, firms will not have any profitable deviation from this equilibrium.

Appendix B. A More General Case

After revising the basic model, we explore a natural generalization of the setting. In this generalization we will not impose a particular capacity on firms nor a given amount of units to be purchased. Nevertheless, we will impose some constrains on these values in order to maintain a similar structure to that of the basic model. These constrains will allow us to find simple analytic solutions at the cost of less flexibility. This appendix intends to be shorter than Section 2, as the key ideas remain unchanged.

B.1. **Setup.** Consider that the same agency must buy a total of $A+B$ units, where $B \in \mathbb{N} \cup \{0\}$ are procured in period 1 and $A \in \mathbb{N} \cup \{0\}$ in period 2. Suppose that the small firm now has a capacity of $\mathbb{N} \ni K_1 \leq \min\{A, B\}$, whereas the large firm has a capacity of $\mathbb{N} \ni K_2 \geq A+B$, that is, the latter may still supply the whole market and the former cannot fully supply more than one period's demand. Consider that the constrain on K_1 applies only when $\min\{A, B\} > 0$ and replace it by $\mathbb{N} \ni K_1 \leq \max\{A, B\}$ if $\min\{A, B\} = 0$. All the other assumptions are maintained.

B.2. **Period 2.** Suppose that after period 1, the firms are still left with $K_1' \in \mathbb{N} \cup \{0\}$ and $K_2' \in \mathbb{N}$ units of capacity, where $K_1' < A$ and $K_2' \geq A$. Using the same mechanics of the toy model, we can infer that the strategy space is bounded by $\left[\frac{A-K_1'}{A}V, V\right]$. We also know that the small firm has an expected utility equal to

$$K_1' p[1-G(p)],$$

while the large firm's utility is

$$Ap[1-F(p)] + (A-K_1')pF(p),$$

where the notation is the usual one (the one defined in the basic model).

After imposing the equilibrium conditions and solving for F and G, we may conclude that the optimal distributions that characterize the mixed-strategy equilibrium are

$$F(p) = \frac{A}{K_1'} - \frac{(A-K_1')V}{K_1' p}$$

and

$$G(p) = 1 - \frac{(A-K_1')V}{Ap},$$

with a mass of $\frac{A-K_1'}{A}$ when $p = V$.

Note how the plus point in G depends positively on A and negatively on K_1'. The intuition is simple: when there are more *uncompetitive* tenders (tenders in which the small firm will not participate), the large firm has additional incentives to price as high as possible.

The expected profit for firm 1 (the small firm) in any case is $\frac{A-K_1'}{A}VK_1'$, while the expected profit for firm 2 (the large firm) is $(A-K_1')V$. It is interesting to note that: 1) they both end up earning their outside option and 2) the ratio of profits is always equal to $\frac{\mathbb{E}\pi_1}{\mathbb{E}\pi_2} = \frac{K_1}{A}$, which means that profit inequality is driven by the gap between the small firm's capacity and the total demand.

B.3. **Period 1.** Just as in the basic model, we now continue with period 1, in which the agency purchases B units and where the evolution of capacity is ruled by

$$K_i' = K_i - \min\{B, K_i\} \cdot \mathbf{1}[\text{firm } i \text{ wins}], \quad \forall i \in \{1, 2\}.$$

Recall that, given a capacity of K_1' for the small firm, the large firm obtains benefits of $(A-K_1')V$ in period 2. Having this in mind, the large firm will never bid under $p = V\left(1 - \frac{(1-\beta)K_1}{B}\right)$, as if it does so and wins the tenders in period 1, it will contemporaneously earn less than $Bp = BV - (1-\beta)K_1$ and it will obtain a discounted future profit of $\beta(A-K_1)V$. This latter outcome is dominated by bidding V, as it would obtain a present sale of $V(B-K_1)$, plus a discounted benefit of βAV.

Noting that $Bp+\beta(A-K_1)V < V(B-K_1+\beta A)$ is enough to consider that $\underline{p} := V\left(1-\frac{(1-\beta)K_1}{B}\right)$ is a lower bound for prices in period 1, both for the large firm and for the small firm that would replicate the strategy space defined by $p \in [\underline{p}, V]$.

Thus, if G^* is the large firm's price distribution, the small firm's expected profit is

$$K_1 p[1-G^*(p)] + \beta\frac{A-K_1}{A}VK_1G^*(p),$$

or, equivalently, this expression may be rearranged as

$$\beta\frac{A-K_1}{A}VK_1 + K_1\left(p-\beta\frac{A-K_1}{A}V\right)[1-G^*(p)],$$

that is, the firm's objective is the expected markup against its outside option upon losing in period 1.

On the other hand, if F^* is the small firm's distribution, then the large firm's expected utility is

$$[Bp+\beta(A-K_1)V][1-F^*(p)] + [(B-K_1)p+\beta AV]F^*(p)$$

or, as a markup objective,

$$\beta(A-K_1)V + (B-K_1)p + K_1(p-\beta V)[1-F^*(p)].$$

Note that if $K_1 = B$, that is, if the small firm can supply the whole market in period 1, then there is no secure payoff for the large firm in period 1. As in Proposition 3, this would force an equilibrium in which the small firm will bid βV as a pure strategy.

Indeed, imposing the equilibrium conditions we finally conclude that

$$F^*(p) = \frac{\frac{B}{K_1}(p-V) + V(1-\beta)}{p-\beta V}$$

as long as $K_1 < B$, and it degenerates into bidding $p = \beta V$ when $K_1 = B$.

Additionally,

$$G^*(p) = \begin{cases} \dfrac{p-\underline{p}}{p-\beta V\dfrac{A-K_1}{A}} & p \in [\underline{p}, V) \\ 1 & p = V \end{cases},$$

which forces a mass of $1-\frac{A}{B}\frac{(1-\beta)K_1}{A(1-\beta)+\beta K_1}$ when $p = V$.

Thus, when playing these strategies, the expected discounted profits for both firms are $\mathbb{E}\pi_L = K_1\underline{p}$ and $\mathbb{E}\pi_H = (B-K_1+\beta A)V$, so

$$\frac{\mathbb{E}\pi_1}{\mathbb{E}\pi_2} = \frac{K_1}{B}\frac{B-K_1+\beta K_1}{B-K_1+\beta A}.$$

Again, just as in period 2, profit inequality is driven because of the inability of the small firm to fully supply the market (in at least one period[22]).

[22] Note that if $A = B = K_1$, then this quotient is 1, as the small firm will be able to supply half of the "total" dynamic market. Thus, this enforces an equilibrium in which both firms split the market and earn the same amount of discounted profits.

B.4. **Agency's Choice.** Just as in the basic model, given that firms will price according to their equilibrium distributions, and assuming β is close enough to 1, the public agency would maximize its expected surplus by purchasing all units in a single period.

To illustrate this,[23] assume that $\min\{A, B\} > 0$, $\delta = \beta = 1$ and fix some $K_1 \in \mathbb{N}$. The expected discounted payment would then be

$$\mathbb{E}\pi_L + \mathbb{E}\pi_H = (A + B)V,$$

i.e. the agency completely pays for the social value of the procured goods.

Now, alternatively, assume $\min\{A, B\} = 0$. In this case, the resulting payment would be

$$\mathbb{E}\pi_L + \mathbb{E}\pi_H = \frac{K_1}{A + B}(A + B - K_1)V < (A + B)V.$$

Thus, under this model's setting, a rational agency that aims to maximize the expected surplus on the purchase of goods could benefit from a buying scheme that skews all procurement at the end of the year. This conclusion, opposed to typical assessments on year-end public expenditure spikes, actually supports the idea of highly concentrated tenders at the end of the year, as under certain conditions it may work as a mechanism to induce firms to price more aggressively and enhance competition.

[23]The formal math for any β and δ close to 1 yields the same conclusion. Notwithstanding, this illustrating example captures the same idea behind the formal case and considerably reduces the necessary math to compute the solution.

Table 6. Observations by public tender size

Size (UTM)	Frequency (Bids)	Percent	Cumulative
(0,100]	17,266,611	54.30	54.30
(100,1000]	10,599,837	33.33	87.63
(1000,∞)	3,932,158	12.37	100.00
Total	31,798,606		

Appendix C. Additional Details on our Data

C.1. **Characteristics of our Database.** We focus on data coming from tenders in 2014 and 2015.[24] For this particular dataset, we have a grand total of 33,106,087 observations, which describe every single bid that was made on a tender with an award date in 2014-2015. Of these observations, 31,810,669 correspond to bids made on tenders that were actually awarded (about 96%). Over 99.9% of them were bids made on public tenders (31,798,606, the rest were made on, for example, private tenders). These 31,798,606 bids were submitted in 411,269 public tenders.

Public tenders are categorized according to their "size", which is defined by the total value of the tender, the latter being estimated by the procuring agency. The measurement unit is the Chilean Monthly Taxing Unit (in Spanish, *Unidad Tributaria Mensual* or UTM), which is equivalent to about 75 USD. Public tenders with an estimated value above 1000 UTM are classified as "large" and are subject to additional regulations. There are 26,925 large tenders in the data, and about 12.37% of our bids are submitted in them.

Additionally, "small" tenders are those worth less than 100 UTM, while "medium" tenders have an estimated value between 100 and 1000 UTM. These disjoint categories and the observations they account for are described in Table 6.[25]

As commented in the text, tenders may have multiple lines and firms submit bids in particular lines, instead of participating in the tender as a whole. Additionally, firms may submit multiple bids in each line. For example, consider a tender with three lines and four competing firms: A, B, C and D. For instance, this tender may only have a single bid from firm A in the first line, while, at the same time, it may receive two bids from firm A and one bid from firm B in the second line. Finally, it can perfectly have a bid from firm A, firm B, firm C and firm D in its third line. Descriptive statistics on tender lines by size are provided in Table 7.

C.2. **Regulations Concerning Large Tenders.** There are some regulatory and legal considerations that must be taken into account to understand how public tenders work in Chile. As explained before, tenders have different rules according to their size. Particularly, "large" tenders have the following key features:

- Public tenders over 1000 UTM must have a special three-person committee that evaluates all the bids. This committee must be constituted by qualified public officials that are internal or external to the procuring agency and its mission is to guarantee a competitive process. Thus, when analyzing data coming from tenders with $V > 1000$, we are less likely to be facing issues derived from corruption or other anti-competitive concerns.
- Public tenders over 1000 UTM must allow firms to have at least 20 days to prepare their bids once the terms and conditions of the tender are released. This constraint may be relaxed to 10 days in case of an urgency and whenever the nature of the procured goods or services allows competing firms to swiftly prepare correctly designed bids. This constraint is scaled to 30 days when the estimated

[24]An effective "consumption-smoothing policy" was applied in 2016, which dramatically reduced the year-end spike in expenditure. We pool data from 2014 and 2015 to have a larger sample size in some forthcoming exercises that importantly constrain the data.

[25]Since 2016, larger tenders are required to be further classified into two new UTM ranges: $(2000, 5000]$ and $(5000, \infty)$. This is because of special rules applied to these even larger tenders. Nevertheless, less than 2% of tenders in our data specified these classifications in 2014-2015, the vast majority was simply bunched in the $(1000, \infty)$ range. Thus, we just aggregate these tenders that were identified in $(2000, 5000]$ and $(5000, \infty)$ and considered them as in the $(1000, \infty)$ range.

TABLE 7. Descriptive statistics of lines by public tender size

Size (UTM)	(0,100]	(100,1000]	(1000,∞)	Total
Mean	4.75	6.99	7.67	5.38
Std. Dev.	9.13	19.57	34.14	14.65
Observations	302,984	81,360	26,925	411,269
Percentile 1	1	1	1	1
Percentile 5	1	1	1	1
Percentile 10	1	1	1	1
Percentile 25	1	1	1	1
Percentile 50	1	1	1	1
Percentile 75	4	3	3	4
Percentile 90	12	17	14	13
Percentile 95	20	35	30	23
Percentile 99	44	94	126	59

value is over 5000 UTM, without exceptions. Therefore, when analyzing data coming from tenders with $V > 1000$, we can assume firms had enough time to strategically prepare their bids.

- Public tenders over 2000 UTM must require an upfront payment from bidders in order to guarantee that the offer is "serious". This payment is rapidly refunded (less than 10 days) if a firm is not awarded any quantity, but it is retained for firms that win a tender until the goods or services are provided, plus *at least* 60 days after the contract is over. This upfront guarantee can be anywhere between 5% and 30% of V. It is also important to note that the threshold was only recently updated to 2000 UTM: the guarantee used to be mandatory for all public tenders above 1000 UTM, and this is still *de facto* being applied in this way.[26] Therefore, when analyzing data coming from tenders with $V > 1000$, we can assume firms directly constrain their financial capacity when winning a tender.

Given the foregoing, and considering that the theoretical model's predictions require firms to really exhaust their capacity in a relevant way, strategically define their optimal bids, and truly compete against each other, we will only work with data that comes from tenders with $V > 1000$. Furthermore, we will only consider those tenders with a single line, a single winner and a single offer per firm. All these conditions help us ensure that we can correctly identify V, that there will not be any split auction and that firms directly compete with a single optimally-chosen bid.

[26] In fact, tenders with an estimated value below 1000 UTM can also require this upfront payment.

TABLE 8. Marginal effect of being the low-price bidder on the probability of winning a tender

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Minimum Bid	0.210***	0.210***	0.210***	0.201***	0.201***	0.209***	0.191***
	(46.90)	(46.61)	(46.57)	(44.17)	(44.25)	(46.38)	(41.80)
p/V		3.71e-13	-8.68e-14	1.06e-12	3.22e-13	-9.58e-15	1.92e-13
		(0.03)	(-0.01)	(0.08)	(0.02)	(-0.00)	(0.01)
December Dummy			0.019***	0.017***	0.017***	0.019***	0.013**
			(3.12)	(2.63)	(2.72)	(3.01)	(1.99)
Segment Dummy:							
3-digit ISO	NO	NO	NO	YES	NO	NO	NO
2-digit ISO	NO	NO	NO	NO	YES	NO	NO
1-digit ISO	NO	NO	NO	NO	NO	YES	YES
Agency Dummy:	NO	NO	NO	NO	NO	NO	YES
N	49126	48532	48532	48532	48532	48532	48526

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

APPENDIX D. MINIMUM BIDS AND CHANCES OF WINNING A TENDER

To assess the importance of being a minimum-price bidder on the odds of winning a tender, we estimate a simple probit model of the form

$$\mathbf{1}[bid_i\ wins] = \alpha + \beta\mathbf{1}[bid_i\ is\ the\ lowest] + \mathbf{XB} + \varepsilon_i,$$

where $\mathbf{X}$ includes a series of controls: p_i/V_i, a dummy indicating if the tender was awarded on December, segment dummies[27] and agency dummies.

The marginal effects derived from the estimated parameters are printed in Table 8. Thus, minimum-price bidders significantly increase their odds of winning in around 20%, even when controlling by the fraction of the bid p with respect to V. The results support a somewhat controversial premise: lowering prices does not increase the chances of winning, unless one crosses a relative threshold, such being the low bidder. Indeed, this is justified because many tenders have discrete scoring functions that reward the low-price bidder much more than in other continuous-scoring functions. An example is a scoring function that assigns 100 points to the minimum price, 50 points to the runner-up and 0 points thereafter.[28]

Taking into account that the unconditional chance of winning a tender in our sample is around 30% (Table 9), we may still state that offering the lowest price considerably increases the odds of winning.

TABLE 9. Winners and low-price bidders

Minimum Bid	Frequency	Wins	$\mathbb{P}[win]$
No	32,373	7,864	0.2429
Yes	16,753	7,588	0.4529
Total	49,126	15,452	0.3145

[27]The 8-digit ISO code that defines the procured good may be shortened with its first digits to form a "segment" that groups similar goods. We use three, two and one digit groups to define different segments.

[28]As a robustness check, and considering the fact that some discontinuous scoring functions also assign a benefit to second-price bidders, we estimated the foregoing model including different price-rank dummies. The low-bidder still has significant advantage against others. With the sole exception of the second-price bidder, the minimum bidder has over 20% of additional chances of winning an average tender vis a vis any other competitor. When compared to the second-price bidder, this difference shrinks to 6∼10%.

Appendix E. Classification Method: VAT vs. no-VAT Tenders

Firstly, we constructed a 100-bin histogram that describes the density (percentage) associated to each 1% bin from 11 to 110%. This is just to exclude large "non-serious" bids (which are rarely observed, but make our data noisier) and small accumulations of data on the lower part of the support, as we have evidence on data manipulation practices that overweight small fractions.[29] In effect, we discarded all observations that do not satisfy $p/V \in [0.11, 1.1]$. Thus, we created a function $g : \{0.1 + 0.01k\}_{k=1}^{100} \to [0, 1]$ with a discrete domain that describes the density of p/V. The result of this 100-bin histogram is depicted in Figure 3.

The second step in our procedure is to decompose this density $g(x)$ into two parts: a density for tenders with VAT and another for tenders without VAT. In essence, we state that

$$g(x) \approx \alpha f(1.19x) + (1 - \alpha) f(x),$$

where $\alpha \in (0, 1)$ is the share of tenders that asked for bids without VAT (or, equivalently, expressed V with VAT included) and $f : \{0.1 + 0.01k\}_{k=1}^{100} \to [0, 1]$ *should* be a common density, as we have no reasons to believe that this scale adjustment modifies the optimal bidding behavior.

Thus, we have to identify both, $f(x)$ and α. In order to identify them, we used a numerical method approach that solved the following problem

$$\underset{\alpha, \{f_{nv}(x_i)\}_{i=1}^{100}, \{f_v(x_i)\}_{i=1}^{100}}{\arg\min} \sum_{i=1}^{100} [g(x_i) - \alpha f_{nv}(x_i) - (1 - \alpha) f_v(x_i)]^2 + \omega \sum_{i=1}^{100} [f_{nv}(1.19x_i) - f_v(x_i)]^2,$$

where f_{nv} and f_v are functions denoting the density for bids in tenders without VAT and with VAT, respectively, and $\omega > 0$ is an exogenous relative-penalizing factor.

The intuition behind this unconstrained[30] objective function in is as follows: the idea is to minimize the sum of the squared deviations of this density mixture against the original density g, but also considering the sum of the squared deviations of both functions f_{nv} and f_v, as they should be nearly equal. The trade-off between these two objectives is summarized in ω: as $\omega \to \infty$, $f_{nv}(x_i) - f_v(x_i) \to 0$, while when $\omega = 0$, $g(x_i) - \alpha f_{nv}(x_i) - (1 - \alpha) f_v(x_i) = 0$, for all x_i.

Note that the solution of this problem depends on the value assigned to ω. Nevertheless, as shown in the following tables, results are quite robust to different "reasonable" values for ω. Indeed, as seen in Table 10and Table 11, the functions do not differ importantly and α is always around 67%. Thus, we infer that about two thirds of the sample are bids that do not contain VAT.

The last step of this process is to assign each tender into either a VAT or no VAT group, such that these classifications yield distributions for p/V that are close enough to those we inferred from the calibration exercise. At this point we face a fundamental complication: we do not have relevant observable characteristics that may be used to map each observation to a cluster. Thus, as it is impossible to perform any typical clustering task, we opted for a more straightforward but noisy approach.

We computed the mean of p/V for each tender and constructed 100 bins to group these means. Then we simply flipped a weighted coin for each tender, conditional on its bin. The weighted coin assigned a probability of being in the VAT group equivalent to $\dfrac{f_v(x)}{f_{nv}(x) + f_v(x)}$, where x is evaluated at the center of each corresponding bin. Thus, tenders are classified according to the odds of finding their (average) bids in one group or another.

[29]In some occasions, public agencies nominally reduce quantities in tenders, in spite of purchasing higher quantities in the effective purchase order. For example, an agency that wants to buy 20 computers might input a quantity of 1 and ask for a "unit price" in the tender. Thus, if the estimated value is 1000 per computer and a firm bids 900, then the fraction in the data will only be 4.5% $((1 \times 900)/(20 \times 1000))$, as opposed to the real 90% $((20 \times 900)/(20 \times 1000))$ that should be expected.

[30]In the following results, f_{nv} is constrained to be non-increasing in the last 10 percentage points of its domain in order to avoid non-reasonable spikes that arise to forcefully fit the upper side of g.

Table 10. Mixture callibration for different values of $\omega \geq 1$

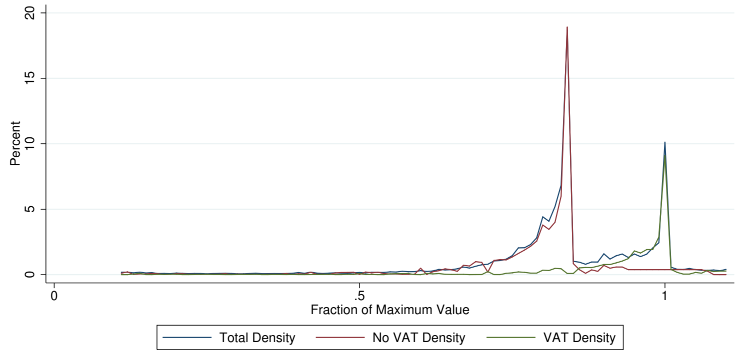
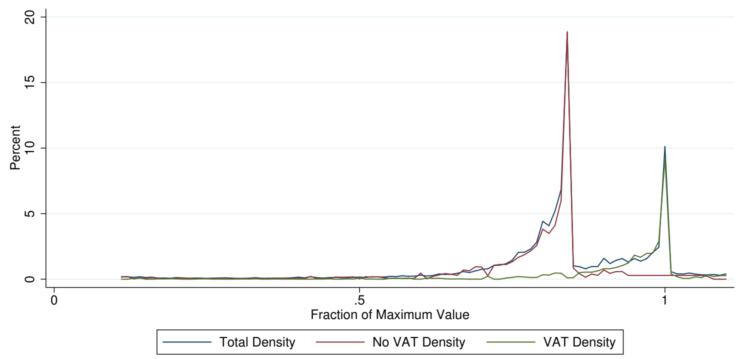
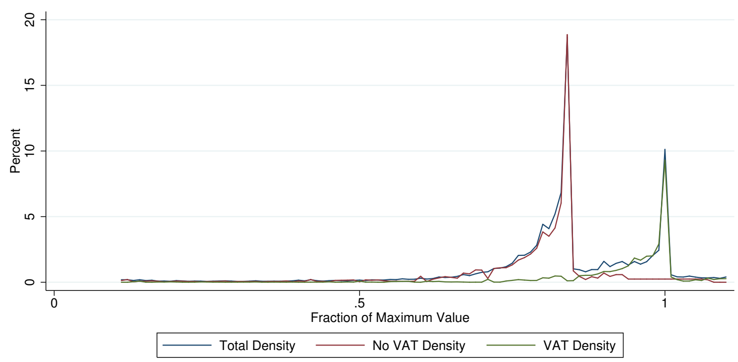
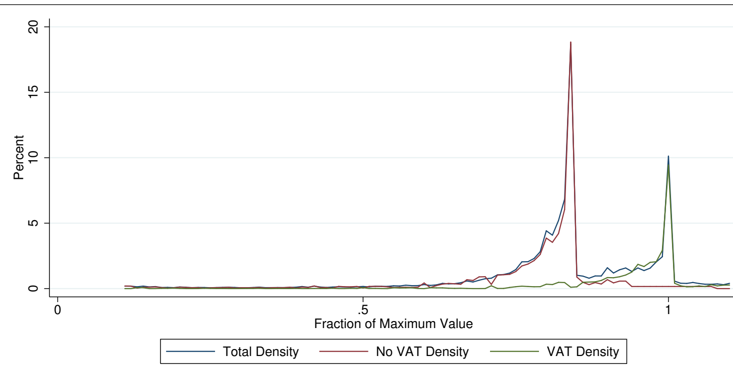
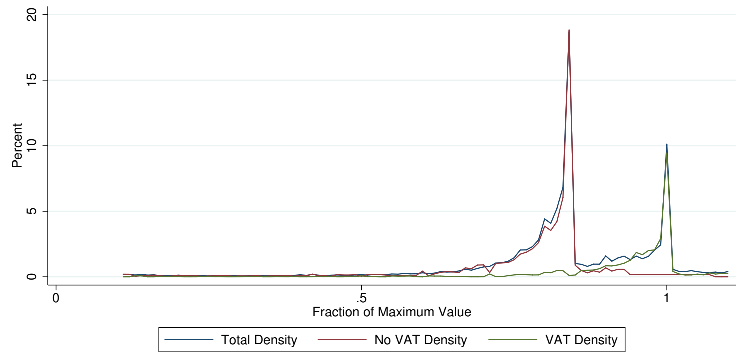
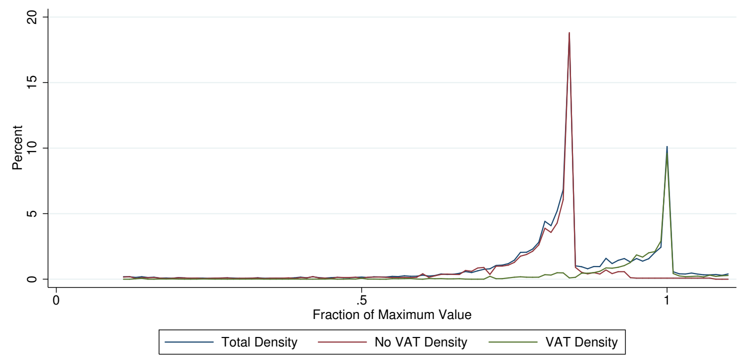
$\omega = 10 \implies$ $\alpha = 0.675$ $\min = 4.859$ RMSE=0.212	
$\omega = 3 \implies$ $\alpha = 0.672$ $\min = 4.227$ RMSE=0.188	
$\omega = 2 \implies$ $\alpha = 0.670$ $\min = 3.950$ RMSE=0.178	
$\omega = 1 \implies$ $\alpha = 0.667$ $\min = 3.347$ RMSE=0.157	

Table 11. Mixture callibration for different values of $\omega \leq 1$

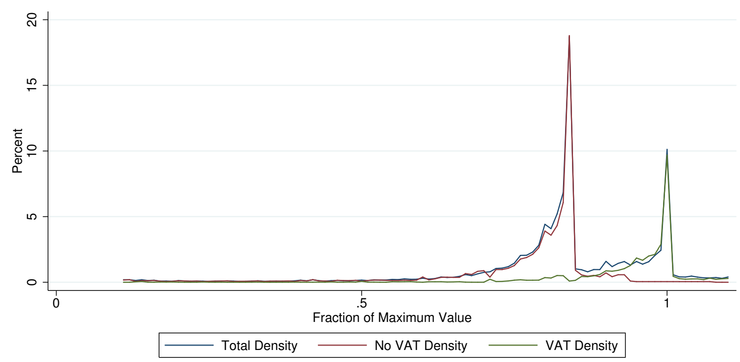
$\omega = 1 \implies$
$\alpha = 0.667$
min $= 3.347$
RMSE$=0.157$



$\omega = 1/2 \implies$
$\alpha = 0.664$
min $= 2.729$
RMSE$=0.134$



$\omega = 1/3 \implies$
$\alpha = 0.662$
min $= 2.354$
RMSE$=0.120$



$\omega = 1/10 \implies$
$\alpha = 0.659$
min $= 1.318$
RMSE$=0.074$

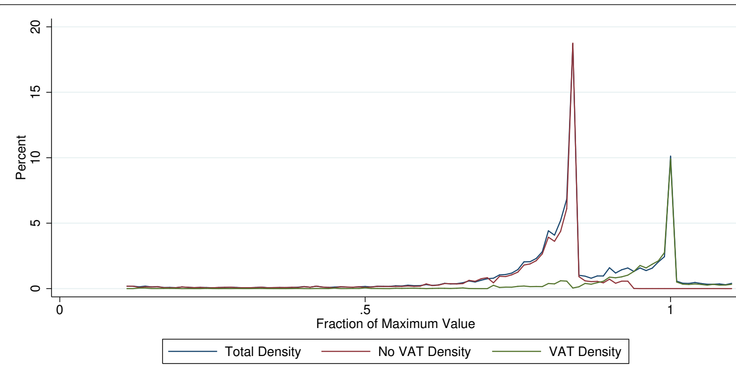
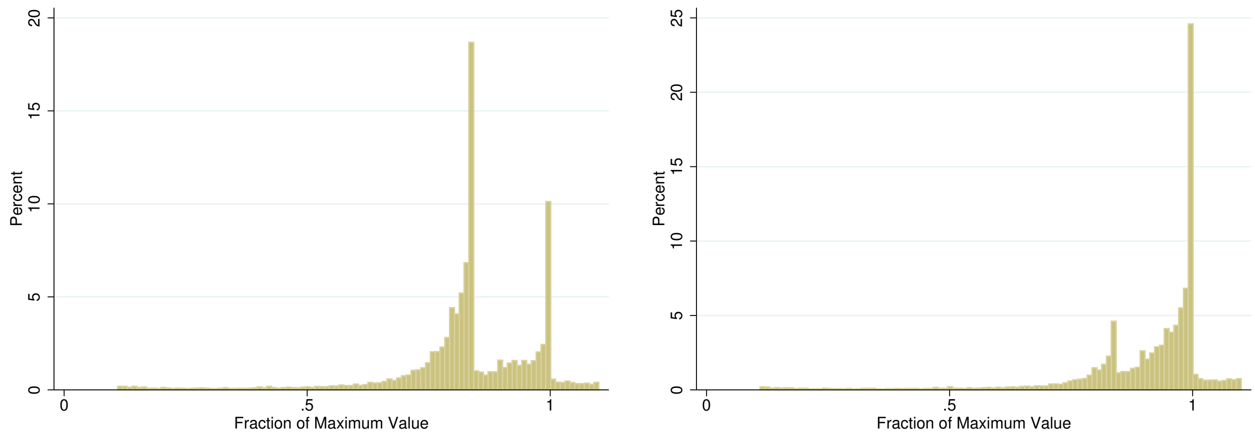


Figure 8. 100-bin histogram for p/V before and after VAT adjustment (11-110%)



After the latter, individual bids are associated to their tender's group and those that were assigned to the no-VAT group are simply multiplied by 1.19. Thus, the result of this exercise is a single-peaked density of p/V, with a great mass (around 25%) accumulated when $p/V = 1$. The resulting histogram is in Figure 8.

Appendix F. Additional Estimations

Table 12. Estimates of the effect of past wins (2009 through 2013, separately and 2009-2013, pooled) on the fraction of tenders awarded in December of 2014-2015

	(1) 2009	(2) 2010	(3) 2011	(4) 2012	(5) 2013	(6) 2009-2013
w	0.000191	0.000397***	0.000272***	0.000131	0.000154**	0.0000549***
	(1.34)	(3.28)	(2.84)	(1.60)	(2.44)	(2.71)
Constant	0.137***	0.140***	0.134***	0.135***	0.132***	0.129***
	(21.61)	(22.35)	(23.88)	(25.99)	(27.50)	(29.54)
N	1929	2020	2441	2847	3286	4091

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 13. Estimates of the effect firm size (2009 through 2013, separately and 2009-2013, pooled) on the fraction of tenders awarded in December of 2014-2015

	(1) 2009	(2) 2010	(3) 2011	(4) 2012	(5) 2013	(6) 2009-2013
D_{50}	-0.0135	-0.00753	0.0269*	-0.0152	0.0133	0.0375***
	(-0.79)	(-0.47)	(1.79)	(-1.14)	(1.03)	(2.78)
D_{75}	0.0215	0.0141	-0.0118	-0.00364	-0.00339	0.00599
	(1.19)	(0.79)	(-0.77)	(-0.25)	(-0.27)	(0.50)
D_{90}	0.0139	0.0390**	0.0422***	0.0338*	0.0367***	0.00935
	(0.76)	(2.03)	(2.59)	(1.96)	(2.61)	(0.81)
Constant	0.136***	0.136***	0.114***	0.144***	0.118***	0.0932***
	(11.96)	(12.51)	(10.84)	(13.37)	(12.59)	(8.65)
$H_0 : D_{50} + D_{75} = 0$						
Coefficient	0.0080	0.0066	0.0150	-0.0188	0.0099	0.0435***
p-value	0.6394	0.7016	0.3260	0.2538	0.4534	0.0017
$H_0 : D_{50} + D_{75} + D_{90} = 0$						
Coefficient	0.0219	0.0456***	0.0573***	0.0150	0.0465***	0.0528***
p-value	0.2057	0.0093	0.0003	0.3511	0.0010	0.0001
N	1929	2020	2441	2847	3286	4091

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

TABLE 14. Robustness Check: Effect of past wins and size on the fraction of tenders awarded in non-December months

	(1) Jan	(2) Feb	(3) Mar	(4) Apr	(5) May	(6) Jun	(7) Jul	(8) Aug	(9) Sep	(10) Oct	(11) Nov	(12) Dec
First Model: Total Past Victories												
w	5.89e-6	1.67e-6	-2.27e-6	-9.34e-6	-9.51e-6	-3.24e-6	-1.83e-6	-4.52e-6	-4.36e-6	-1.12e-5	2.53e-7	5.49e-5***
	(0.34)	(0.11)	(-0.14)	(-0.58)	(-0.63)	(-0.21)	(-1.15)	(-0.28)	(-0.30)	(-0.65)	(0.01)	(2.71)
Constant	0.0943***	0.0736***	0.0768***	0.0768***	0.0685***	0.0764***	0.0765***	0.0780***	0.0664***	0.0907***	0.0929***	0.129***
	(25.11)	(22.12)	(22.45)	(21.97)	(21.18)	(23.12)	(22.37)	(22.56)	(21.00)	(24.49)	(25.09)	(29.54)
Second Model: Relative Size Dummies												
D_{50}	0.0100	0.0105	0.00451	-0.0101	0.0152	-0.000273	-0.00253	0.000420	-0.00687	-0.0398***	-0.0185	0.0375***
	(0.86)	(1.02)	(0.43)	(-0.94)	(1.52)	(-0.03)	(-0.24)	(0.04)	(-0.70)	(-3.48)	(-1.62)	(2.78)
D_{75}	0.0258**	0.00146	-0.00465	-0.00485	-0.00329	0.00430	-0.0160*	0.00510	-0.0115	0.000351	-0.00275	0.00599
	(2.53)	(0.16)	(-0.50)	(-0.51)	(-0.37)	(0.48)	(-1.72)	(0.54)	(-1.34)	(0.03)	(-0.27)	(0.50)
D_{90}	-0.0285***	-0.0102	-0.00172	0.00326	-0.00432	0.00153	0.0200**	-0.0115	0.00873	0.00632	0.00698	0.00935
	(-2.86)	(-1.16)	(-0.19)	(0.35)	(-0.50)	(0.17)	(2.21)	(-1.25)	(1.04)	(0.64)	(0.71)	(0.81)
Constant	0.0805***	0.0672***	0.0761***	0.0867***	0.0586***	0.0737***	0.0807***	0.0782***	0.0758***	0.122***	0.108***	0.0932***
	(8.70)	(8.18)	(9.01)	(10.04)	(7.34)	(9.03)	(9.56)	(9.16)	(9.71)	(13.32)	(11.80)	(8.65)
... + $D_{75} = 0$												
Coefficient	0.0358***	0.0120	-0.0001	-0.0150	0.0119	0.0040	-0.0185**	0.0055	-0.0184**	-0.0395***	-0.0212**	0.0435***
p-value	0.0025	0.2560	0.9902	0.1763	0.2448	0.7006	0.0873	0.6141	0.0664	0.0008	0.0699	0.0017
... + $D_{90} = 0$												
Coefficient	0.0073	0.0018	-0.0019	-0.0117	0.0076	0.0056	0.0015	-0.0059	-0.0096	-0.0331***	-0.0143	0.0528***
p-value	0.5197	0.8624	0.8585	0.2705	0.4400	0.5801	0.8836	0.5710	0.3155	0.0032	0.2048	0.0001
N	4091	4091	4091	4091	4091	4091	4091	4091	4091	4091	4091	4091

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$