GOV1368 Section 7

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Recap

Last time (before the mock midterm) we covered:

- Endogeneity
- Instrumental Variables (IV)
- IV Identifying Assumptions

Today we will learn about another *flavor* of Instrumental Variables (IV) estimation. It is called the **Regression Discontinuity** (RD) design.

Agenda

Refresher: Instrumental Variables

Regression Discontinuity (RD)

Local Average Treatment Effect (LATE)

Application

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Refresher: Instrumental Variables

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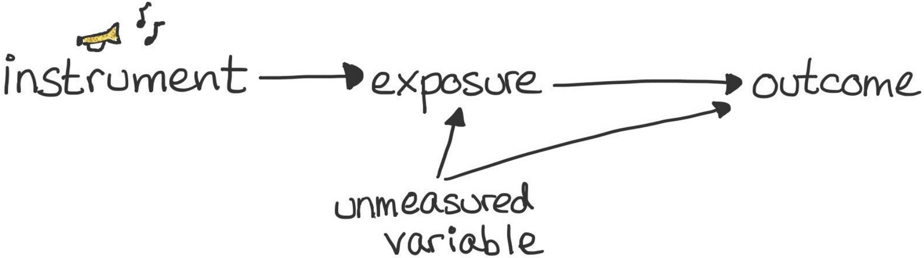

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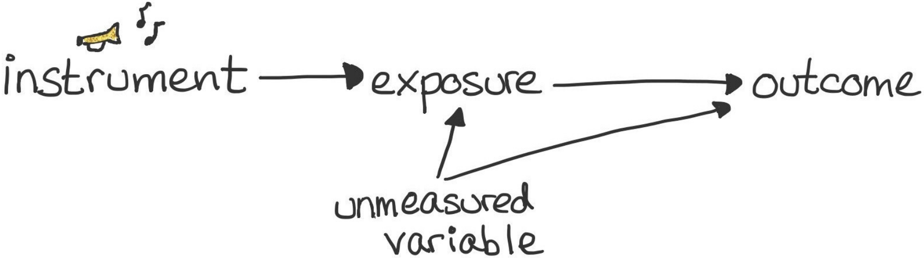

How can we use this to obtain a causal estimate?

Refresher: The IV-Wald Estimator

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$$\text{IV-Wald} \rightarrow \frac{\mathbb{E}[Y_i|Z_i = 1] - \mathbb{E}[Y_i|Z_i = 0]}{\mathbb{E}[D_i|Z_i = 1] - \mathbb{E}[D_i|Z_i = 0]} = \frac{\mathbb{E}[Score_i|Voucher] - \mathbb{E}[Score_i|NoVoucher]}{\mathbb{P}[PreK_i|Voucher] - \mathbb{P}[PreK_i|NoVoucher]}$$

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Another interpretation: it is the Intent To Treat (ITT) divided by the "first stage".

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A taxonomy of the population based on the potential outcomes:

- Compliers: $D_i = Z_i$, i.e. those who attend preschool only if they receive the voucher.
- Always-takers: $D_i = 1$, i.e. those who attend preschool regardless of the voucher.
- Never-takers: $D_i = 0$, i.e. those who don't attend preschool regardless of the voucher.
- Defiers: $D_i = 1 - Z_i$, i.e. those who attend preschool only if they don't receive the voucher. Assumption: there are <u>no defiers</u>.

Refresher: Identification Assumptions

Our identification strategy relies on the following assumptions:

1. **Relevance**: The instrument Z_i is *relevant* for the treatment D_i, i.e.

$$\mathbb{P}[D_i = 1 | Z_i = 1] \neq \mathbb{P}[D_i = 1 | Z_i = 0].$$

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2. **Exclusion**: The instrument Z_i affects the outcome Y_i *only* through the treatment D_i:

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3. **Monotonicity**: There are no defiers, i.e. $D_i \neq 1 - Z_i$.
4. **Independence**: The instrument Z_i is *independent* of the potential outcomes $Y_i(0)$ and $Y_i(1)$.

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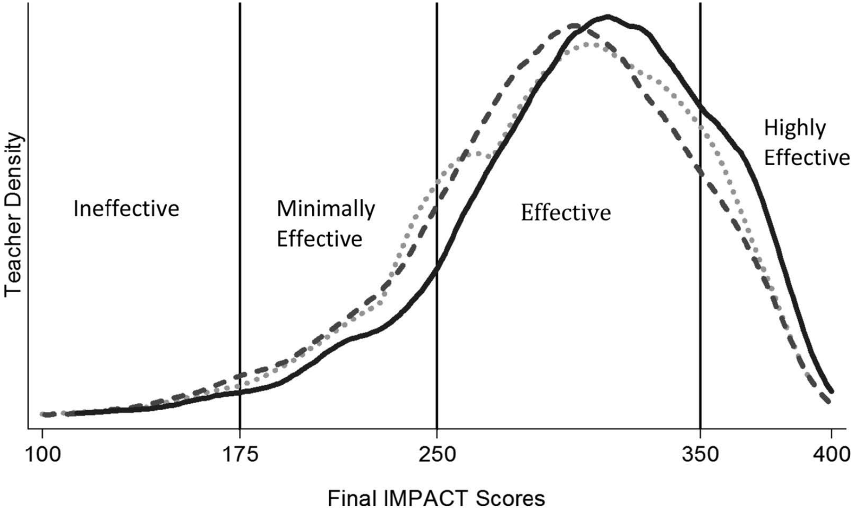
Threshold-Crossing as IV

Let's think of a special scenario where there is a cutoff/threshold c such that a variable S_i crossing c determines the take up of a treatment D_i. For example, in the IMPACT study, teachers with scores S_i over $c = 350$ points could receive substantial bonuses D_i.



Regression Discontinuity

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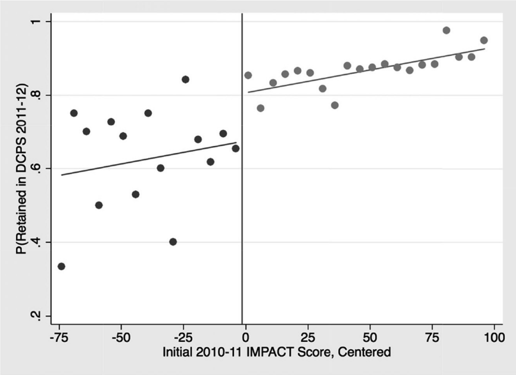
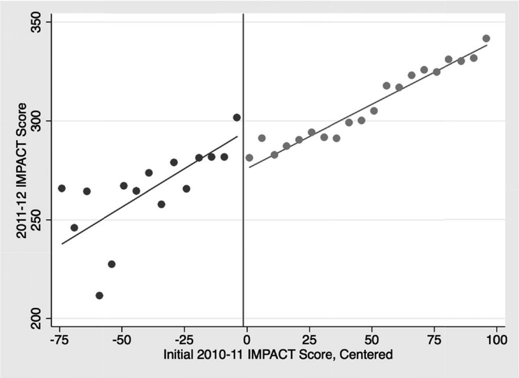
You basically fit "two regressions" and measure the *discontinuity* around the cutoff c:





A simple Wald-like RD formula

Before:

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Now:

$$\text{“RD-Wald”} \rightarrow \frac{\mathbb{E}[Y_i|S_i \rightarrow c^+] - \mathbb{E}[Y_i|S_i \rightarrow c^-]}{\mathbb{E}[D_i|S_i \rightarrow c^+] - \mathbb{E}[D_i|S_i \rightarrow c^-]}$$

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We want to compare the average outcome for units *just above* the cutoff vs units *just below* the cutoff. It is *as if* we had a *local randomization* of units into a treatment group or a control group. We can't compare units too far away from the cutoff, because we would confound the effect of the treatment D_i with the effect of having a very different score S_i.

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If there is "imperfect compliance", i.e. crossing the threshold does not determine the treatment D_i in a one-to-one fashion, we need to adjust this difference by the change in take-up of the treatment (just as with our basic IV estimator). When this is the case, we talk about a *fuzzy* RD. Otherwise, if crossing c implies $D_i = 1$ for all units, and not crossing it implies $D_i = 0$ for all units, we call this a *sharp* RD (the denominator is 1).

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RD estimates a LATE

From the paper (Dee and Wyckoff, 2015, page 269):

> *We are also careful to emphasize the stylized nature of the causal estimands that result from these RD designs. In particular, it should be noted that the "localness" of these RD estimates implies that they do not necessarily identify the average treatment effect (ATE) associated with the introduction of IMPACT. However, these results do provide credible evidence on the effects of the types of novel performance incentives IMPACT introduced.*

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Indeed, the estimand estimated by this estimator is a LATE, just like in IV estimation. The difference is that the *compliers* are now defined as units arbitrarily close to c that would change their treatment adoption if and only if they cross c.

Local Average Treatment Effect

Our target estimand is still the Local Average Treatment Effect (LATE):

$$LATE := \mathbb{E}[Y_i(1) - Y_i(0) | D_i(S_i > c) > D_i(S_i < c), S_i \approx c]$$

One key *assumption* for an RD estimate to be valid is that there is **no manipulation** of the running variable S_i around the cutoff c. We also need other covariates to evolve *smoothly* around the cutoff (exclusion restriction).

Contents

Application: School Spending and Educational, Economic Outcomes

`Stata`