GOV1368 Section 9

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Recap

Last time we talked about:

- RD and IV to estimate the impact of class size on academic achievement
- Issues arising from manipulation around the threshold
- Research ethics

Today we will revisit Difference-in-Differences analysis in the context of estimating the causal effect of charter expansions on district-level outcomes.

Agenda

Difference-in-Differences

The Market-Level Effects of Charter Schools on Student Outcomes

Application

Contents

Review: DiD Basics

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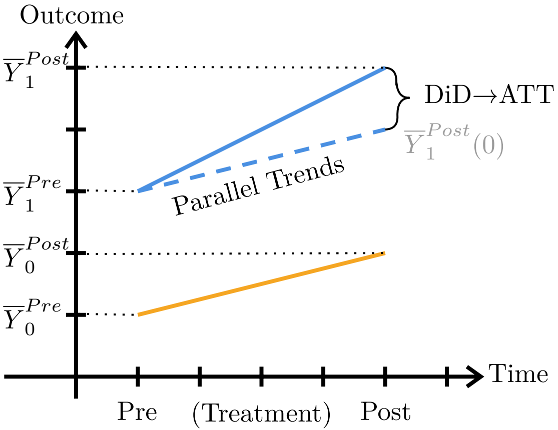
With these four ingredients, provided that the parallel trends (PT) assumption holds, we can estimate the ATT by computing a difference in differences:

$$(\bar{Y}_1^{Post} - \bar{Y}_1^{Pre}) - (\bar{Y}_0^{Post} - \bar{Y}_0^{Pre})$$

DiD Visualized



DiD as Linear Regression

Suppose we have a dataset with a sample of 3 random variables:

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$$Y_{it} = \alpha + \beta Treat_i + \gamma Post_t + \delta Treat_i \times Post_t + \varepsilon_{it}$$

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Turns out the linear regression estimator of δ is equivalent to a DiD estimator!

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Let's compute 4 conditional means:

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Now taking a *difference in differences* of these we get

$$\begin{aligned}
DiD \rightarrow & (\mathbb{E}[Y_{it}|Treat_i = 1, Post_t = 1] - \mathbb{E}[Y_{it}|Treat_i = 1, Post_t = 0]) \\
& - (\mathbb{E}[Y_{it}|Treat_i = 0, Post_t = 1] - \mathbb{E}[Y_{it}|Treat_i = 0, Post_t = 0]) \\
= & [(\cancel{\alpha} + \cancel{\beta} + \gamma + \delta) - (\cancel{\alpha} + \cancel{\beta})] - [(\cancel{\alpha} + \gamma) - \cancel{\alpha}] \\
= & \cancel{\gamma} + \delta - \cancel{\gamma} \\
= & \delta
\end{aligned}$$

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The Market-Level Effects of Charter Schools on Student Outcomes

One-line summary: Chen and Harris (2023) analyze the nationwide effects of charter school market share on student outcomes, finding modest improvements in test scores and graduation rates, particularly in urban districts, through both participant and competitive mechanisms.

They do this by estimating a *generalized difference-in-differences* (GDD) regression model.

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GDD in a nutshell:

From: $Y_{it} = \alpha + \beta Treat_i + \gamma Post_t + \delta \mathbf{Treat}_i \times Post_t + \varepsilon_{it}$

To: $Y_{it} = \alpha + \beta Treat_i + \gamma Post_t + \delta \mathbf{Share}_i \times Post_t + \varepsilon_{it}$

Generalized Difference-in-Differences (GDD)

From the paper:

The dependent variables in our GDD analysis are test scores ($Test_{isgt}$) and high school graduation (GR_{ist}) in district i state s and grade g during year t (test scores are grade-specific; graduation is not). We include geographic unit fixed effects (usually geographic school districts) as well as state-grade-year fixed effects. This implies the following models:

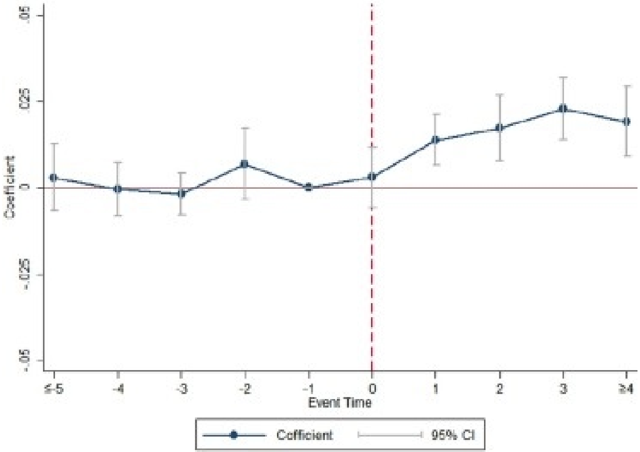
$$GR_{ist} = \alpha + \beta_1 \; ChartShare_{\;ist} + X_{ist}\gamma + \mu_i + \lambda_{st} + \varepsilon_{ist}$$

$$Test_{isgt} = \alpha + \beta_2 \; ChartShare_{\;isgt} + X_{ist}\gamma + \mu_i + \lambda_{sgt} + \varepsilon_{isgt}$$

Treatment is the continuous, time-varying charter market share, $ChartShare_{ist}$. The coefficients of interest are represented by β.

Results

(a) Graduation rate



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Application: The Market-Level Effects of Charter Schools

`Stata`