KU LEUVEN

Learning Constraints for Personnel Rostering Problems Using Tensors

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Overview

- Problem Introduction
- Algorithm – COUNT-OR
- Experiments
- Results
- Submissions
- Future Work

Problem Introduction

Objective

	Mon			Tue			-		
Nurse	S1	S2	S3	S1	S2	S3	-	-	-
Sophie	1	0	1	0	0	0	-	-	-
Hanna	0	1	0	0	0	0	-	-	-
Ruben	1	0	1	0	0	0	-	-	-
Sam	0	0	0	1	0	0	-	-	-

Constraints

- Max/Min number of working days
- Max/Min number of employees each day/shift
- Max/Min number of consecutive working days
- Max/Min number of shifts per day

Why is it useful?

"We had a scheduling conflict, so tomorrow's meeting has been moved to yesterday."

- Removes the dependency on domain experts
- Saves a lot of time and money
- Reduces the chances of making any error

Algorithm – COUNT-OR

Data Representation

	Mon			Tue			-		
Nurse	S1	S2	S3	S1	S2	S3	-	-	-
Sophie	1	0	1	0	0	0	-	-	-
Hanna	0	1	0	0	0	0	-	-	-
Ruben	1	0	1	0	0	0	-	-	-
Sam	0	0	0	1	0	0	-	-	-

X

Rank $\quad Dimensions$
$D = \{Nurses, Days, Shifts\}$

Shape $\quad Distinct\ Values\ for\ each\ Dimensions$
$\{4,7,3\}$

Tensor Slices

$$D' = \{Nurses, Days\}$$

	Mon			Tue			-		
Nurse	S1	S2	S3	S1	S2	S3	-	-	-
Sophie	1	0	1	0	0	0	-	-	-
Hanna	0	1	0	0	0	0	-	-	-
Ruben	1	0	1	0	0	0	-	-	-
Sam	0	0	0	1	0	0	-	-	-

Tensor Slices

$$D' = \{Shifts, Days\}$$

	Mon			Tue			-		
Nurse	S1	S2	S3	S1	S2	S3	-	-	-
Sophie	1	0	1	0	0	0	-	-	-
Hanna	0	1	0	0	0	0	-	-	-
Ruben	1	0	1	0	0	0	-	-	-
Sam	0	0	0	1	0	0	-	-	-

Tensor Slices

$$D' = \{Days\}$$

	Mon			Tue			-		
Nurse	S1	S2	S3	S1	S2	S3	-	-	-
Sophie	1	0	1	0	0	0	-	-	-
Hanna	0	1	0	0	0	0	-	-	-
Ruben	1	0	1	0	0	0	-	-	-
Sam	0	0	0	1	0	0	-	-	-

Function Definitions – NonZero

$$D' = \{Nurses, Days\}$$

	Mon			Tue			-		
Nurse	S1	S2	S3	S1	S2	S3	-	-	-
Sophie	1	0	1	0	0	0	-	-	-
Hanna	0	1	0	0	0	0	-	-	-
Ruben	1	0	1	0	0	0	-	-	-
Sam	0	0	0	1	0	0	-	-	-

$$Y[e] = I(\boldsymbol{X}[e] \neq \mathbf{0}) \; for \; each \; e \in \bigotimes(D')$$

Function Definitions – NonZero

$$D' = \{Nurses, Days\}$$

	Mon			Tue			-		
Nurse	S1	S2	S3	S1	S2	S3	-	-	-
Sophie	1			0			-	-	-
Hanna	1			0			-	-	-
Ruben	1			0			-	-	-
Sam	0			1			-	-	-

$$Y[e] = I(\boldsymbol{X}[e] \neq \mathbf{0}) \; for \; each \; e \in \otimes(D')$$

Function Definitions – Sum

$$D' = \{Nurses, Days\}$$

	Mon			Tue			-		
Nurse	S1	S2	S3	S1	S2	S3	-	-	-
Sophie	1	0	1	0	0	0	-	-	-
Hanna	0	1	0	0	0	0	-	-	-
Ruben	1	0	1	0	0	0	-	-	-
Sam	0	0	0	1	0	0	-	-	-

$$Y[e] = Sum\ of\ values(\boldsymbol{X}[e])\ for\ each\ e \in \bigotimes(D')$$

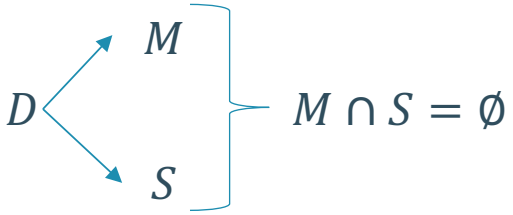
Function Definitions – Sum

$$D' = \{Nurses, Days\}$$

	Mon			Tue			-		
Nurse	S1	S2	S3	S1	S2	S3	-	-	-
Sophie	2			0			-	-	-
Hanna	1			0			-	-	-
Ruben	2			0			-	-	-
Sam	1			1			-	-	-

$$Y[e] = Sum\ of\ values(\boldsymbol{X}[e])\ for\ each\ e \in \bigotimes(D')$$

Function Definitions – Count



$M = \{Nurses\}$ $S = \{Days\}$

	Mon			Tue			-		
Nurse	S1	S2	S3	S1	S2	S3	-	-	-
Sophie	1	0	1	0	0	0	-	-	-
Hanna	0	1	0	0	0	0	-	-	-
Ruben	1	0	1	0	0	0	-	-	-
Sam	0	0	0	1	0	0	-	-	-

$$COUNT(X, M, S) = Sum(NonZero(X, M \cup S), S)$$

Function Definitions – Count

$$M = \{Nurses\} \quad S = \{Days\}$$

	Mon			Tue			-		
Nurse	S1	S2	S3	S1	S2	S3	-	-	-
Sophie	1			0			-	-	-
Hanna	1			0			-	-	-
Ruben	1			0			-	-	-
Sam	0			1			-	-	-

$$COUNT(X, M, S) = Sum(\mathbf{NonZero}(X, M \cup S), S)$$

Function Definitions – Count

$$M = \{Nurses\} \quad S = \{Days\}$$

	Mon			Tue			-		
Nurse	S1	S2	S3	S1	S2	S3	-	-	-
Sophie	1			0			-	-	-
Hanna	1			0			-	-	-
Ruben	1			0			-	-	-
Sam	0			1			-	-	-

$$COUNT(X, M, S) = \boldsymbol{Sum}(NonZero(X, M \cup S), S)$$

Function Definitions – Count

$$M = \{Nurses\} \quad S = \{Days\}$$

	Mon			Tue			-		
Nurse	S1	S2	S3	S1	S2	S3	-	-	-
Count	3			1			-	-	-

$$\boldsymbol{COUNT}(X, M, S) = Sum(NonZero(X, M \cup S), S)$$

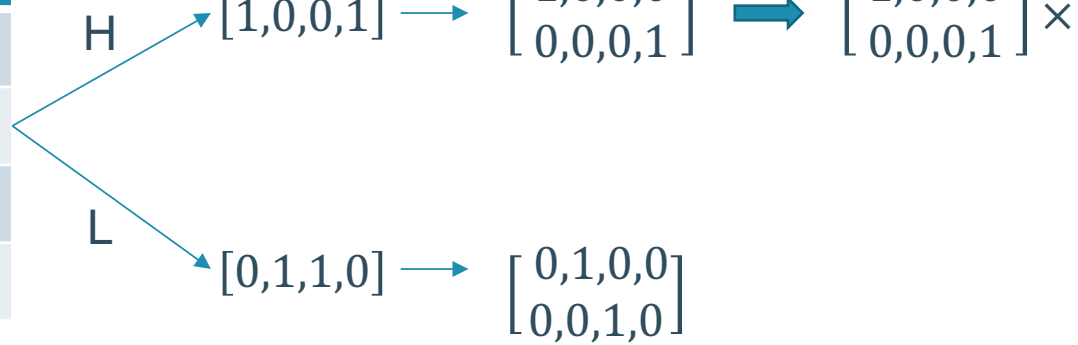
Constraints List

M	S	Count
{Days}	{Nurses}	# of working days/Nurse
{Days, Shifts}	{Nurses}	# of working shifts/Nurse
{Days}	{Nurses}	# of employees / day
{Shifts}	{Days}	# of shifts for each day with at least one nurse working
{Slots}	{Nurses, Days}	# of working shifts per day / nurse
{Days}	{Nurses, Slots}	# of working days in the same shift / nurse
{Nurses}	{Days, Shifts}	# of nurses pre shift per day

$$COUNT(X, M, S) = Sum(NonZero(X, M \cup S), S)$$

Background Knowledge

Nurses	Skill Level
Sophie	High
Hanna	Low
Ruben	Low
Sam	High

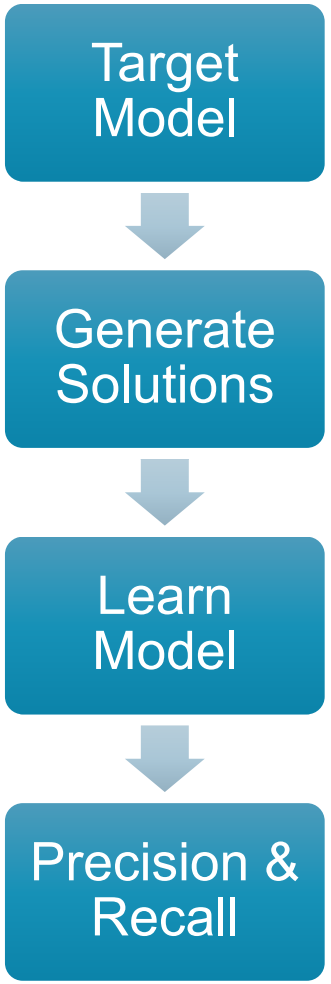


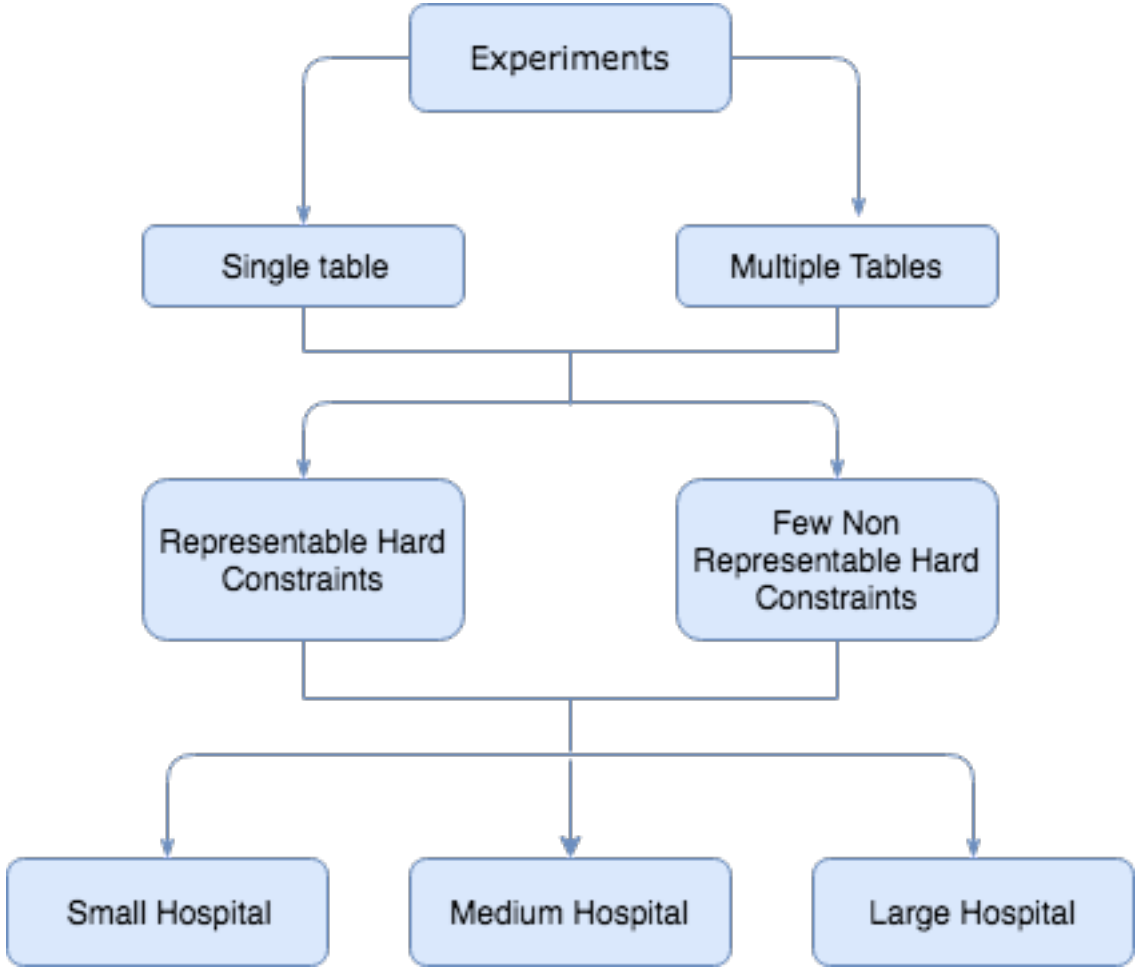
	Mon			Tue		
Nurse	S1	S2	S3	S1	S2	S3
Sophie	1	0	1	0	1	0
Hanna	0	1	0	0	1	0
Ruben	0	0	1	0	0	1
Sam	0	0	0	0	0	0

	Mon			Tue		
Nurse	S1	S2	S3	S1	S2	S3
Sophie	1	0	1	0	1	0
Sam	0	0	0	0	0	0

Experiments

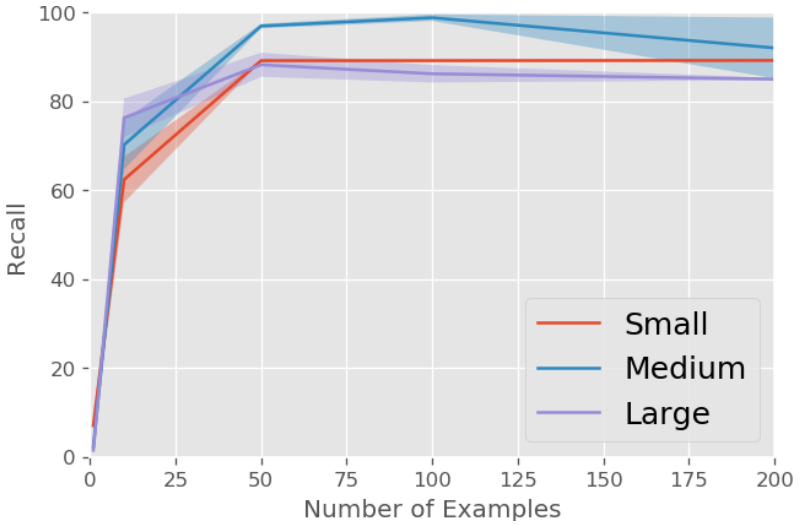
Experiments

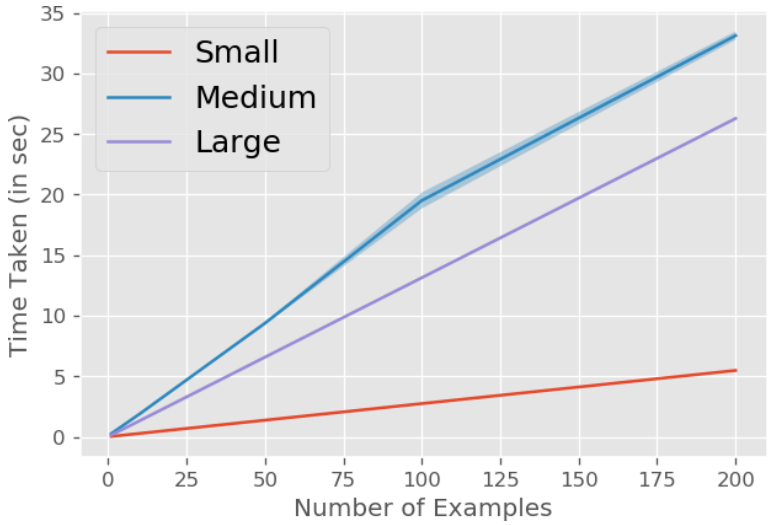




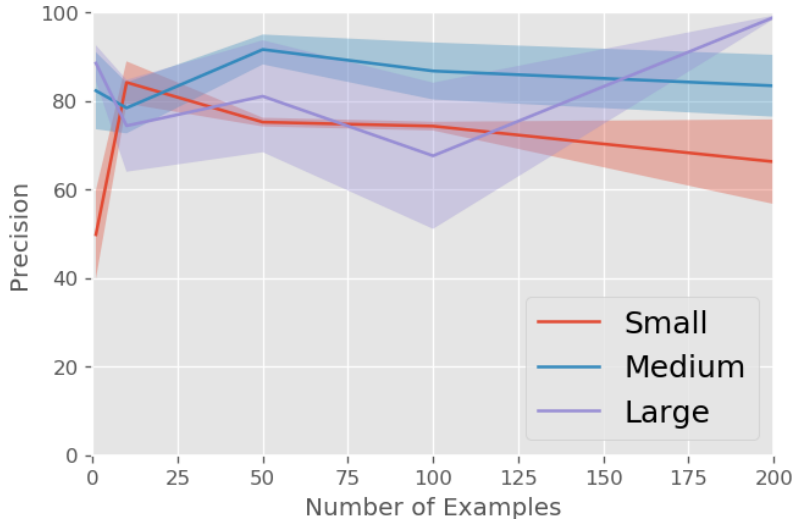
Results

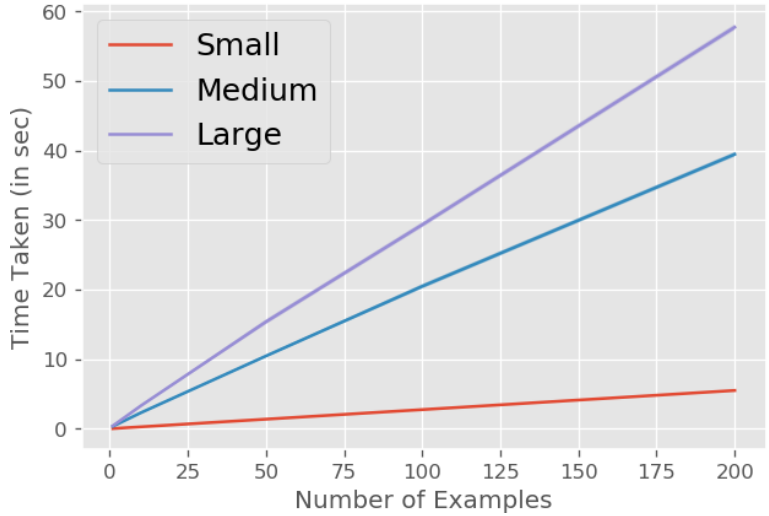
Results – Representable Hard Constraint

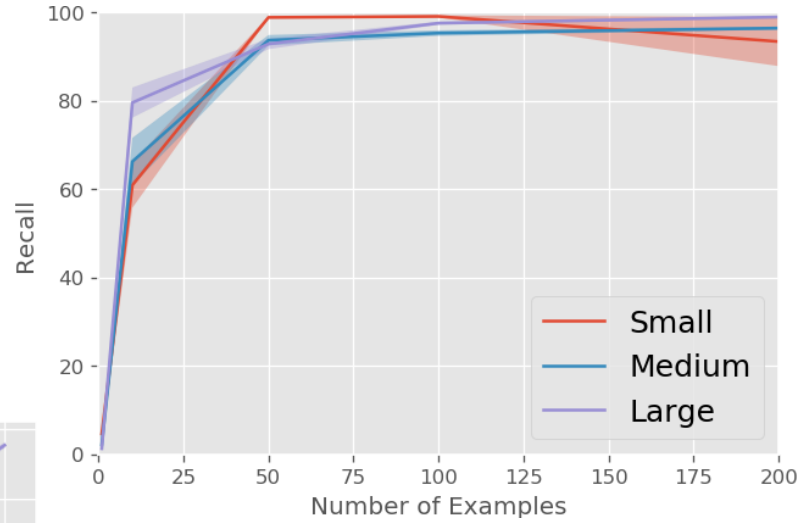




Results – Non-Representable Hard Constraint







Submissions & Future Work

Submissions

- Abstract accepted in EURO 2018 – 29th European Conference on Operational Research
- Workshop paper accepted in IJCAI 2018 – Data Science meets Optimization

Future Work

Extend the algorithm to support other problems

Make the algorithm interactive

Learning Soft Constraints

Careful omission of constraints

THANK YOU

QUESTIONS ?