Practical 1 : 8 pts vs 5+1pts

The goal of this practical is to compare the classical linear 8 points algorithm and the linear 5 points knowing the vertical direction of the camera.
Let's consider 50 points randomly distributed in a cube of size [-300*300]*[-300*300]*[-300*300] in the world frame (O_w, X_w, Y_w, Z_w).
Let's note respectively, $(O_{c_1}, X_{c_1}, Y_{c_1}, Z_{c_1})$ and $(O_{c_2}, X_{c_2}, Y_{c_2}, Z_{c_2})$ the camera positions. We suppose a calibrated camera posed at a rotation R_i and T_i of the world coordinate $(X_w = R_i X_{c_i} + T_i)$.
The image points are noted P_i.

Preliminary questions

1. Let us note (R_t, T_t) the transformation between two camera positions, give the relation between R_t, T_t and R_1, R_2, T_1, T_2.
2. Compute de essential matrix E_t thanks to R_t and T_t.
3. Verify the epipolar geometry of the first point.
4. Compute the essential matrix E.

8 points algorithm

1. Estimate E_t using the 8 points algorithm. (use function `fundmatrix`)
2. Estimate E_t when some outliers are included in the data. (use function `ransacfitfundmatrix`)
3. Decompose E_t in R_t and T_t. (use function `PoseEMat`)

5+1 points algorithm

1. Suppose roll and pitch angles are known. Let's pose $R_{iderot} = R_{pi} R_{ri}$ $(i = 1, 2)$, build virtual cameras where their optical axis are vertical and let us note PV_i the image points.
2. Let us note R and $T = (T_x, T_y, T_z)^T$ the transformation between two virtual positions, give the relation between R, T and R_{y1}, R_{y2}, T_1, T_2.
3. Show that the essential matrix can be defined as :

$$E = \begin{bmatrix} -T_z \sin(\alpha) & -T_z \cos(\alpha) & T_y \\ T_z \cos(\alpha) & -T_z \sin(\alpha) & -T_x \\ -T_y \cos(\alpha) + T_x \sin(\alpha) & T_x \cos(\alpha) + T_y \sin(\alpha) & 0 \end{bmatrix} \tag{1}$$

$$\text{where} \quad \alpha = tan^{-1}\left(-\frac{R(1,2)}{R(1,1)}\right) \tag{2}$$

4. Propose a method to estimate E in a function noted `Essmatrix5pt_IMU` and a robust method noted `RansacEssmatrix5pt_IMU`.

Comparison

1. Test 1 : example with different datas, propose a test with different positions of the second camera $(R_1 = I, T_1 = 0)$ with angles of rotation between 0° and 45° and translation of 0 to 100

2. Test 2 : example with noise, propose a test with different camera positions ($R_1 = I, T_1 = 0$) with angles of rotation between 0° and 45° and translation of 0 to 100 AND white noise in image points of camera 2 between 0 to 1 pixel std (use RANSAC functions).
3. Test 3 : example with noise on IMU informations, propose a test with different camera positions ($R_1 = I, T_1 = 0$) with angles of rotation between 0° and 45° and translation of 0 to 100, white noise in image points of camera 2 between 0 to 1 pixel std AND white noise in IMU between 0° to 2° (use RANSAC functions).
4. Conclude...