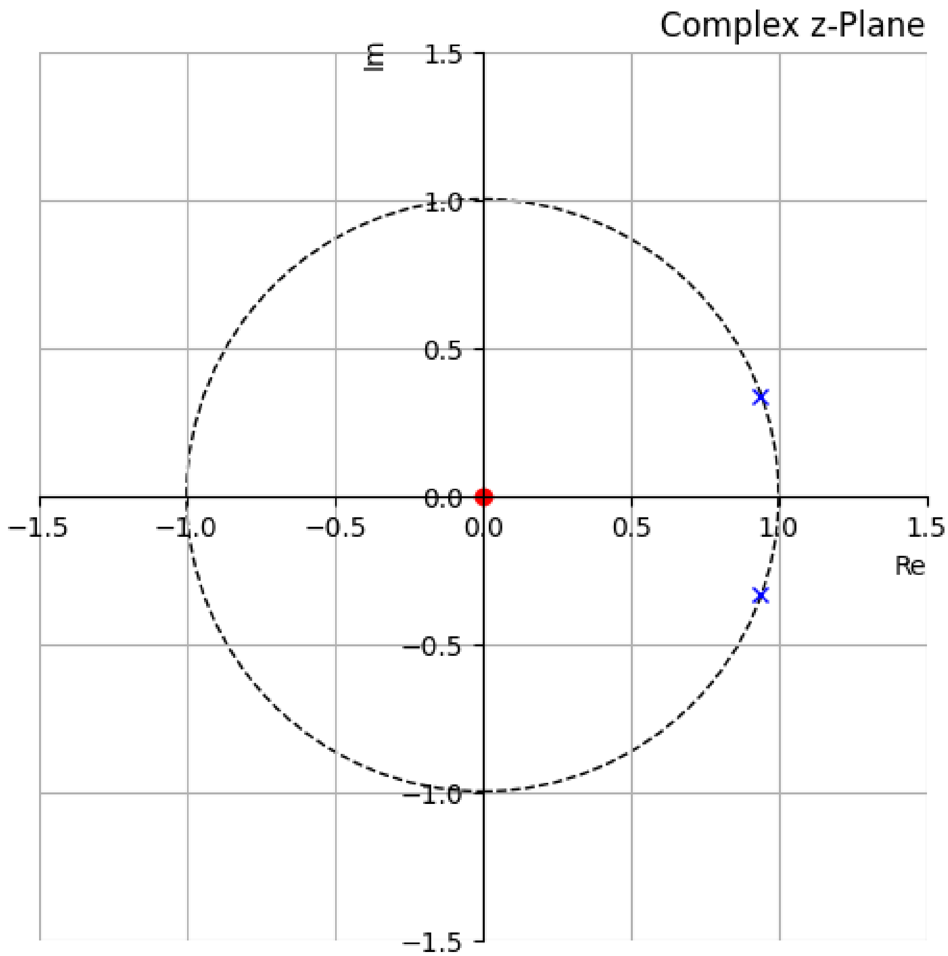
example

February 13, 2024

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt

Q=0.995
Omega= 2*np.pi*440/8000
p0 = Q*np.exp(1j*Omega)
p1 = Q*np.exp(-1j*Omega)
# Plot Poles and Zeros
from  matplotlib import patches
plt.figure(figsize=(8,6))
ax = plt.subplot(111)
r = 1.5; plt.axis('scaled'); plt.axis([-r, r, -r, r])
#ticks = [-1, 1]; plt.xticks(ticks); plt.yticks(ticks)
# Unit Circle
uc = patches.Circle((0,0), radius=1, fill=False, color='black', ls='dashed')
ax.add_patch(uc)
ax.spines['left'].set_position('center')
ax.spines['bottom'].set_position('center')
ax.spines['right'].set_visible(False)
ax.spines['top'].set_visible(False)
plt.xlabel('Re', horizontalalignment='right', x=1.0)
plt.ylabel('Im',  y=1.0)
plt.title('Complex z-Plane', loc='right')
plt.grid()
plt.plot(0,0,'ro')
plt.plot(np.real(p0),np.imag(p0),'bx')
plt.plot(np.real(p1),np.imag(p1),'bx')
```

[ ]:
```
[<matplotlib.lines.Line2D at 0x1c9546b0990>]
```



[ ]:
```
from scipy import signal as sig
Fs=8000
seq_len=Fs*2
omega=2*np.pi*440.0 /Fs
k1=2*Q*np.cos(omega)
k2=-pow(Q,2)
ximp=np.zeros(seq_len) #make impulse with a desired sequence length
ximp[0]=0.1 #impulse at the beginning
filtered=sig.lfilter([1], np.array([1, -k1,-k2]), ximp)
```

[ ]:
```
import IPython.display as ipd
ipd.Audio(filtered, rate=Fs)
```

[ ]: <IPython.lib.display.Audio object>

```
[ ]: import torch
import torch.nn as nn
# Device configuration
device = torch.device('cuda' if torch.cuda.is_available() else 'cpu')
print("device=", device)

infeatures= 1 # samples per input time step
hiddensize= 3 #number of hidden states, the memory elements
outputs=1 #1 samples per output time step
numlayers=1 #number of layers of the network
batch=1
```

```
device= cpu
```

```
[ ]: class RNNnet(nn.Module):
    def __init__(self, infeatures, hiddensize, outputs):
        super(RNNnet, self).__init__()

        # Define the model.
        self.rnn = nn.RNN(input_size=infeatures, hidden_size=hiddensize,␣
 ↪num_layers=numlayers, bias=False)
        #forward layer for output
        self.fo = nn.Linear(hiddensize, outputs, bias=False)

    def forward(self, x):
        h_0 = torch.zeros(numlayers, batch, hiddensize).to(device)
        out, hn = self.rnn(x, h_0)
        #Output is simply the hidden state of the last layer (if more than 1␣
 ↪layer)
        out = self.fo(out) #e.g. used to just keep first output
        return out
```

```
[ ]: def signal2pytorch(x):
    #Function to convert a signal vector x, like a mono audio signal, into a␣
 ↪3-d Tensor that Pytorch expects,
    #https://pytorch.org/docs/stable/nn.html
    #Argument x: a 1-d signal as numpy array
    #output: 3-d Pytorch Tensor.
    #for RNN Input: (siglen,batch,features)
    X = np.expand_dims(x, axis=-1) #add batch dimension (here only 1 )
    X = np.expand_dims(X, axis=-1) #add features dimension (here only 1 )
    X=torch.from_numpy(X)
    X=X.type(torch.Tensor)
    return X
```

```
[ ]: import numpy as np
Fs=8000
```

```
omega=2*np.pi*440.0 /Fs; #normalized frequency
Q=0.995 #determines speed of decay, the closer to 1 the longer
k1=2*Q*np.cos(omega)
k2=-pow(Q,2)
```

[ ]:
```
from scipy import signal as sig
import IPython.display as ipd
seq_len=Fs*2
ximp=np.zeros(seq_len) #make impulse with a desired sequence length
ximp[0]=0.1 #impulse at the beginning
filtered=sig.lfilter([1], np.array([1, -k1,-k2]), ximp)
print("The impulse response sound from the I.I.R. filter")
display(ipd.Audio(filtered, rate= Fs))
```

```
The impulse response sound from the I.I.R. filter

<IPython.lib.display.Audio object>
```

[ ]:
```
"""
RNN as IIR Filter: hidden states h as delay chain
Weights for updating h for shifting the delay line:
s=array([[0,0,0],[1,0,0],[0,1,0]])
In [4]: s
array([[0, 0, 0],
       [1, 0, 0],
       [0, 1, 0]])

Application of IIR coefficients a(1)=2, a(2)=3:
a=array([[1,2,3],[0,1,0],[0,0,1]])

In [2]: a
Out[2]:
array([[1, 2, 3],
       [0, 1, 0],
       [0, 0, 1]])
First apply shift operator s and then coeffcents, h multiplied from right:
dot(a,s)
array([[2, 3, 0],
       [1, 0, 0],
       [0, 1, 0]])
this is w_hh.
""";
```

[ ]:
```
rnn = RNNnet(infeatures, hiddensize, outputs)
#Shifted diagonal matrix and IIR Filter coefficients:
rnn.state_dict()['rnn.weight_hh_l0'].data.copy_(torch.diag(torch.
 ↪ones(hiddensize-1),diagonal=-1))
```

```
rnn.state_dict()['rnn.weight_hh_l0'][0,:].data.copy_(torch.tensor([ k1, k2,0.
 ↪0]))
#Vector with a "1" for input x:
rnn.state_dict()['rnn.weight_ih_l0'].data.copy_(torch.zeros((hiddensize,1)))
rnn.state_dict()['rnn.weight_ih_l0'][0,0].data.copy_(torch.tensor(1.0))
#Weight for linear output layer to select h[0]:
rnn.state_dict()['fo.weight'][0,:].data.copy_(torch.tensor([1.0,0.0 ,0.0]))
ww = rnn.state_dict() #read obtained weights
print("weights =", ww)
```

```
weights = OrderedDict([('rnn.weight_ih_l0', tensor([[1.],
        [0.],
        [0.]])), ('rnn.weight_hh_l0', tensor([[ 1.8724, -0.9900,  0.0000],
        [ 1.0000,  0.0000,  0.0000],
        [ 0.0000,  1.0000,  0.0000]])), ('fo.weight', tensor([[1., 0., 0.]]))])
```

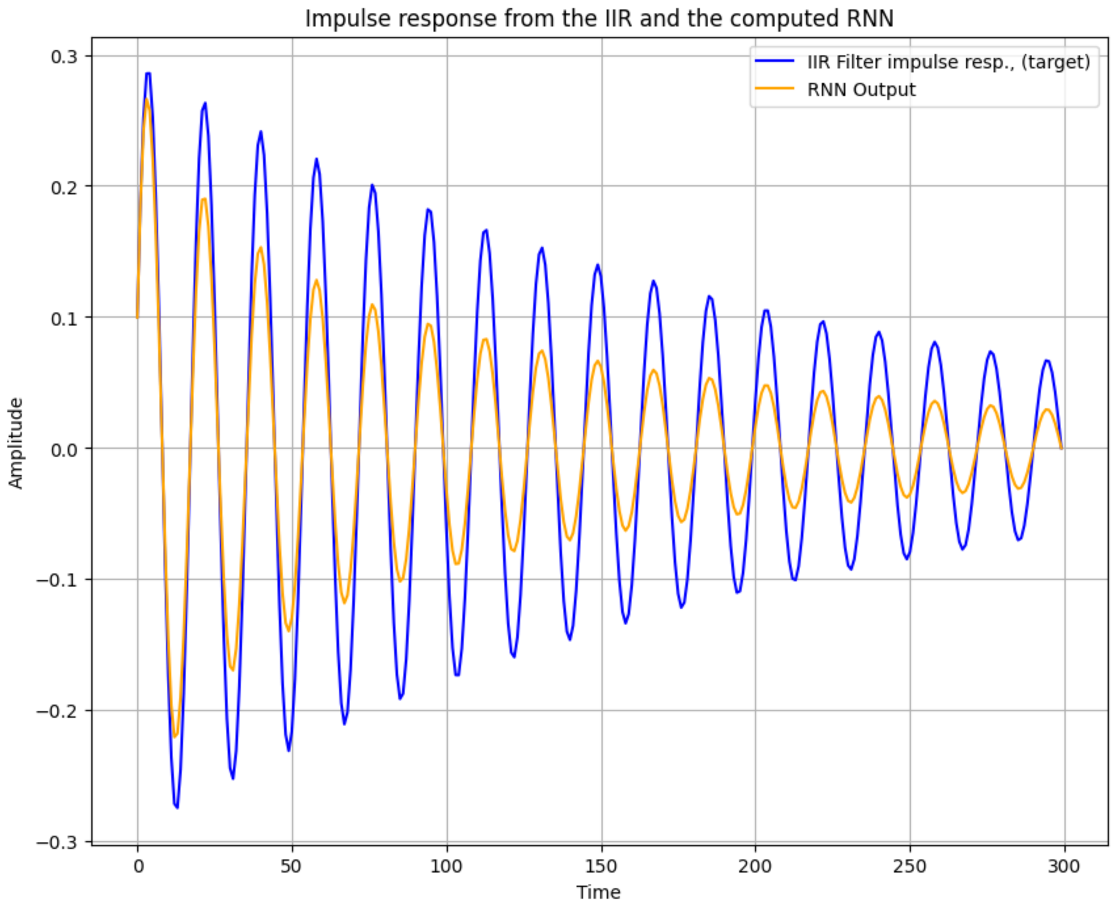
[ ]:
```
#the input, converted from the Numpy array:
inputsig=signal2pytorch(ximp)
#Run Recurrent Neural Network:
outsig= rnn(inputsig)
```

[ ]:
```
outsig=outsig.detach()
outsig=np.array(outsig) #turn into numpy array
outsig=outsig[:,0,0]
print("The sound from the recurrent neural network")
display(ipd.Audio(outsig, rate=Fs))
```

```
The sound from the recurrent neural network
```

```
<IPython.lib.display.Audio object>
```

[ ]:
```
import matplotlib.pyplot as plt
plt.figure(figsize=(10,8))
plt.plot(filtered[:300], 'blue')
plt.plot(outsig[:300],'orange')
plt.legend(('IIR Filter impulse resp., (target)','RNN Output'))
plt.xlabel('Time')
plt.ylabel('Amplitude')
plt.title("Impulse response from the IIR and the computed RNN")
plt.grid()
```



[ ]:
```
#input is the pulse, with the 0.1 in the beginning, target is the output of the
 ↪IIR filter:
target= signal2pytorch(filtered)
print("target.type()=", target.type())
print("target=", target)
```

```
target.type()= torch.FloatTensor
target= tensor([[[ 1.0000e-01]],

        [[ 1.8724e-01]],

        [[ 2.5157e-01]],

        …,

        [[-2.8209e-36]],

        [[-1.4916e-36]],
```

```
        [[ 0.0000e+00]]])
```

[ ]:
```
#random starting point as initialization:
rnn.state_dict()['rnn.weight_hh_l0'].data.copy_(torch.
 ↪randn(hiddensize,hiddensize))
#rnn.state_dict()['rnn.weight_hh_l0'].data.copy_(torch.
 ↪zeros(hiddensize,hiddensize))
rnn.state_dict()['rnn.weight_ih_l0'].data.copy_(torch.randn((hiddensize,1)))
#rnn.state_dict()['rnn.weight_ih_l0'].data.copy_(torch.zeros((hiddensize,1)))
#rnn.state_dict()['fo.weight'][0,:].data.copy_(torch.tensor([ 1.0, 0.0 ,0.0]))
#rnn.state_dict()['fo.weight'][0,:].data.copy_(torch.zeros(hiddensize))
rnn.state_dict()['fo.weight'][0,:].data.copy_(torch.randn(hiddensize))
```

[ ]:
```
tensor([-0.8782, -1.3228, -0.0902])
```

[ ]:
```
rnn = RNNnet(infeatures, hiddensize, outputs).to(device)
ww = rnn.state_dict() #read current weights
print("weights =", ww)
print('Total number of parameters: %i' % (sum(p.numel() for p
in rnn.parameters() if p.requires_grad)))
loss_fn = nn.MSELoss(reduction='sum')
#learning_rate = 1e-3
optimizer = torch.optim.Adam(rnn.parameters())
for epoch in range(500):
    Ypred=rnn(inputsig)
    loss=loss_fn(Ypred, target)
    if epoch%10==0:
        print(epoch, loss.item())
    optimizer.zero_grad()
    loss.backward()
    optimizer.step()
```

```
weights = OrderedDict([('rnn.weight_ih_l0', tensor([[ 0.4776],
        [-0.4512],
        [-0.3389]])), ('rnn.weight_hh_l0', tensor([[ 0.2738, -0.1824,  0.1227],
        [-0.3203, -0.1178, -0.1392],
        [ 0.0519,  0.4503, -0.4261]])), ('fo.weight', tensor([[-0.2246, -0.3554,
0.2195]]))])
Total number of parameters: 15
0 4.391043663024902
10 4.390157222747803
20 4.389317989349365
30 4.388519287109375
40 4.387748718261719
50 4.386981964111328
60 4.386186122894287
70 4.385321140289307
80 4.384339332580566
```

```
90 4.383176803588867
100 4.381751537322998
110 4.379958629608154
120 4.377667427062988
130 4.374718189239502
140 4.370918273925781
150 4.36605167388916
160 4.359899997711182
170 4.352312088012695
180 4.343331336975098
```

```
---------------------------------------------------------------------------
KeyboardInterrupt                         Traceback (most recent call last)
Cell In[46], line 10
      8 optimizer = torch.optim.Adam(rnn.parameters())
      9 for epoch in range(500):
---> 10     Ypred=rnn(inputsig)
     11     loss=loss_fn(Ypred, target)
     12     if epoch%10==0:

File c:
 ↪\Users\mohit\AppData\Local\Programs\Python\Python311\Lib\site-packages\torch\nn\modules\mod
 ↪py:1511, in Module._wrapped_call_impl(self, *args, **kwargs)
   1509     return self._compiled_call_impl(*args, **kwargs)  # type:
 ↪ignore[misc]
   1510 else:
-> 1511     return self._call_impl(*args, **kwargs)

File c:
 ↪\Users\mohit\AppData\Local\Programs\Python\Python311\Lib\site-packages\torch\nn\modules\mod
 ↪py:1520, in Module._call_impl(self, *args, **kwargs)
   1515 # If we don't have any hooks, we want to skip the rest of the logic in
   1516 # this function, and just call forward.
   1517 if not (self._backward_hooks or self._backward_pre_hooks or self.
 ↪_forward_hooks or self._forward_pre_hooks
   1518         or _global_backward_pre_hooks or _global_backward_hooks
   1519         or _global_forward_hooks or _global_forward_pre_hooks):
-> 1520     return forward_call(*args, **kwargs)
   1522 try:
   1523     result = None

Cell In[35], line 11, in RNNnet.forward(self, x)
      9 def forward(self, x):
     10     h_0 = torch.zeros(numlayers, batch, hiddensize).to(device)
---> 11     out, hn = self.rnn(x, h_0)
     12     #Output is simply the hidden state of the last layer (if more than
 ↪layer)
     13     out = self.fo(out) #e.g. used to just keep first output
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   1518         or _global_backward_pre_hooks or _global_backward_hooks
   1519         or _global_forward_hooks or _global_forward_pre_hooks):
-> 1520     return forward_call(*args, **kwargs)
   1522 try:
   1523     result = None

File c:
↪\Users\mohit\AppData\Local\Programs\Python\Python311\Lib\site-packages\torch\nn\modules\rnn
↪py:554, in RNN.forward(self, input, hx)
    552 if batch_sizes is None:
    553     if self.mode == 'RNN_TANH':
--> 554         result =␣
↪_VF.rnn_tanh(input, hx, self._flat_weights, self.bias, self.num_layers,
    555␣
↪                              self.dropout, self.training, self.bidirectional
    556                               self.batch_first)
    557     else:
    558         result = _VF.rnn_relu(input, hx, self._flat_weights, self.bias,
↪self.num_layers,
    559                               self.dropout, self.training, self.
↪bidirectional,
    560                               self.batch_first)

KeyboardInterrupt:
```

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