One Approach for Training of Recurrent Neural Network Model of IIR Digital Filter

Stefanova S.A.

Department of Electronic Engineering and Technologies, Technical University of Sofia,
8 Kliment Ohridski Str.,
Sofia, 1797, Bulgaria

Abstract - **One approach for training of recurrent neural network model of 1-D IIR digital filter is proposed. The sensitivity coefficients method has been applied in the training process of the neural network. The set of time domain data is generated and used as a target function in the training procedure. The modeling results have been obtained for two different cases - for 4-th order bandpass IIR digital filter and for partial response IIR digital filter. The frequency domain behavior of the neural network model and the target IIR filter has been investigated. The analysis of the frequency responses shows good approximation results.**

I. INTRODUCTION

In the last years various approaches for modeling and simulation of the analog and discrete dynamic systems based on different type of neural networks have been developed. Basic results related to the discrete dynamical systems approximation using neural networks are discussed in [1]. Many approaches based on neural network have been successfully applied to the problems of design, modeling and simulation of analog filters [2], linear and non-linear [3], [4] digital filters,1-D and 2-D non-recursive (FIR) digital filters [5], [6], [7], [8], 1-D and 2-D recursive (IIR) digital filters [9], [10], [11], [12] and adaptive digital filters [4].

One approach for the 1-D FIR digital filter design based on the weighted mean square method and neural network to state the approximation problem is proposed in [5]. The optimal design approaches of high order FIR digital filters based on the parallel algorithm of neural networks, which its activation matrix is produced by cosine and sine basis functions are given in [6],[7] . The main idea is to minimize the sum of the square errors between the amplitude response of the desired FIR filter and that of the designed by training the weight vector of neural networks, then obtaining the impulse response of FIR digital filter. The reference [8] provides an approach for the design of 2-D FIR linear-phase digital filters based on a parallel back-propagation neural networks algorithm.

One method for time domain design of 1-D and 2-D recursive digital filter, based on recurrent neural network is proposed in [9]. An approach for the training algorithm of a fully connected recurrent neural network where each neuron is modeled by an IIR filter is proposed in [10]. The weights of each layer in the network are updated by optimizing IIR filter coefficients and the optimization is based on the recursive least squares method. A Hopfield-type neural network for the design of IIR digital filters with given amplitude and phase responses is discussed in [11]. A method for 2-D recursive digital filters design is investigated in [12], where the design problem is reduced to a constrained minimization problem the solution of which is achieved by the convergence of an appropriate neural network.

Some methods for the non-linear digital filters design using neural networks are considered in [3]. A design method for nonlinear adaptive digital filters using parallel neural networks is given in [4].

A various methods have been developed for the purposes of the training process of the recurrent neural networks [13], [14], [15]. The back-propagation through time method is the basic approach for recurrent neural network training, discussed in [13]. Some training algorithms based on the sensitivity theory are considered in [14], [15]. The application of the Lagrange multipliers method as a training procedure of the recurrent neural network model of IIR digital filter is discussed in [14], [16]. Reference [9] describes the learning algorithm for the neural network structure based on the state space representation of 1-D and 2-D IIR digital filters.

In this paper one approach for training of IIR digital filter model based on two layer recurrent neural network is proposed. The structure and mathematical description of the recurrent neural network is given in section II. The training process of the neural network using sensitivity coefficients method is discussed in Section III. The neural network model has been trained in such a way that with given predetermined input signal, the output variable approximates the target function in mean square sense. The neural network model of the IIR digital filter is considered in section IV. The effectiveness of the proposed neural network model of the IIR digital filter is demonstrated in Section V. The modeling results have been obtained for two different cases - for 4-th order bandpass IIR digital filter and for partial response IIR digital filter which is a special case of the Nyquist recursive digital filter [17]. The frequency domain behavior of the neural network model and the target IIR filter has been investigated and the analysis of the frequency responses shows good approximation results. Some simulations are realized using harmonic signals with different frequencies in the filter's passband and stopband. The proposed approach can be extended and successfully applied to the case of modeling of nonlinear IIR digital filters.

K. Elleithy et al. (eds.), *Technological Developments in Networking*, *Education and Automation*,
DOI 10.1007/978-90-481-9151-2_38,

II. Structure and Description of the Neural Network

The two layer recurrent neural network is considered. The structure of neural network is shown in Fig. 1.

The recurrent neural network is described with the recurrent system of equations as follows:

$$\begin{bmatrix} s_1(k) \\ s_2(k) \\ \cdots\cdots \\ s_n(k) \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} \cdots & w_{1n} \vdots \cdots w_{1n_z} \\ w_{21} & w_{22} \cdots & w_{2n} \vdots \cdots w_{2n_z} \\ \cdots\cdots & \cdots\cdots & \vdots \cdots\cdots \\ w_{n1} & w_{n2} \cdots & w_{nn} \vdots \cdots w_{nn_z} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \cdots\cdots \\ x_n(k) \\ u_1(k) \\ \cdots\cdots \\ u_{n_u}(k) \end{bmatrix}, \tag{1}$$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \cdots\cdots \\ x_n(k+1) \end{bmatrix} = \begin{bmatrix} f(s_1(k)) \\ f(s_2(k)) \\ \cdots\cdots \\ f(s_n(k)) \end{bmatrix} \quad k = k_0, k_0+1, \ldots, k_f - 1,$$

where f(.) is the activation function for the first layer of the neural network

Let be introduced the vector function **f(s)** as follows:

$$\mathbf{f}(\mathbf{s}) = [f(s_1), f(s_2), \ldots, f(s_n)]^T,$$

then the neural network description in the matrix form is:

$$\mathbf{x}\,(k+1) = \mathbf{f}(\mathbf{W}\mathbf{z}(k)), \quad \mathbf{x}(k_0) = \mathbf{x}_0, \tag{2}$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k)\ , \tag{3}$$

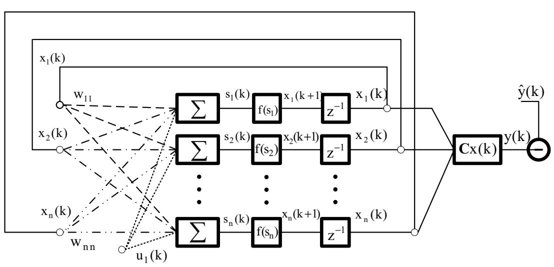

Fig. 1. Recurrent neural network structure for digital filter modeling

where

$\mathbf{x} \in \mathbf{R}^n$ is a state space vector,

$\mathbf{u} \in \mathbf{R}^{n_u}$ is a vector of neural network input excitations,

$\mathbf{y} \in \mathbf{R}^m$ is a vector of neural network output,

$\mathbf{W} \in \mathbf{R}^{n \times n_z}$ and $\mathbf{C} \in \mathbf{R}^{m \times n}$, $m \le n$, $\mathbf{C} = \lfloor c_{ij} \rfloor$,

$c_{ij} = \begin{cases} c_i & i = j \\ 0 & i \ne j \end{cases}$ are matrixes of system coefficients,

n - is a number of neurons at the first layer,

n_u - is a number of input excitations,

$n_z = n + n_u$ - is a number of neural network inputs.

The equations (2), (3) can be used to describe the recurrent neural network shown in Fig. 1.

The following additional vectors have been defined:

- a vector of the first layer inputs of neural network

$$\mathbf{z}(k) = [x_1(k), x_2(k), \ldots x_n(k), u_1(k), u_2(k), \ldots, u_{n_u}(k)]^T,$$
$$\mathbf{z} \in \mathbf{R}^{n_z} \tag{4}$$

- a vector of the neural network outputs

$$\mathbf{y}(k) = [y_1(k), y_2(k) \ldots y_m\ (k)]^T \tag{5}$$

- a vector of the neural network weighting coefficients

$$\mathbf{p} = \lfloor w_{11}\ w_{12} \ldots w_{1n_z}, w_{21}\ w_{22} \ldots w_{2n_z} \ldots . w_{nn_z}, c_1, c_2 \ldots c_m \rfloor,$$
$$\mathbf{p} \in \mathbf{R}^{n_p} \tag{6}$$

where $n_p = n.n_z + m$ is the number of neural network coefficients and the elements of matrix $\mathbf{W} \in \mathbf{R}^{n \times n_z}$ are introduced row by row.

Mean square error objective function is defined in following form:

$$J(\mathbf{p}) = \frac{1}{2} \sum_{k=k_0}^{k_f - 1} \sum_{i=1}^{m} [y_i(k) - \hat{y}_i(k)]^2, \tag{7}$$

where $\{\hat{y}_i(k)\}$ is a set of samples of the target .

III. Algorithm of Sensitivity Coefficients

The main problem in the neural network training process is the gradient calculation of the mean square objective function (7) with respect to weights.

For that reason, it is necessary to determine the gradient components of this objective function.

$$\frac{\partial J(p)}{\partial p_l} = \sum_{k=k_0}^{k_f - 1} \sum_{i=1}^{m} [y_i(k) - \hat{y}_i(k)] \frac{\partial y_i}{\partial p_l}, \quad l = 1, \ldots, n_p \tag{8}$$

For the first layer of the neural network the derivatives $\frac{\partial y_i}{\partial p_l}$ have been determined from (9) as follows:

$$\frac{\partial y_i(k)}{\partial p_l} = c_i \frac{\partial x_i(k)}{\partial p_l}, \ i = 1, \cdots, m,\ l = 1, \cdots n_w,\ n_w = n.n_z \tag{9}$$

The derivatives $\frac{\partial x_i(k)}{\partial p_l}$ from (9) can be obtained from the recurrent system:

$$\frac{\partial x_i(k+1)}{\partial p_l} = \frac{\partial x_i(k+1)^f}{\partial p_l} + \sum_{j=1}^{n} \frac{\partial x_i(k+1)^f}{\partial x_j(k)} \frac{\partial x_j(k)}{\partial p_l};$$
$$;\ i = 1, \ldots, n;\ l = 1, \ldots, n_w; \tag{10}$$

$$\frac{\partial x_i(k_0)}{\partial p_l} = 0 \quad ; \qquad k = k_0, k_0 + 1, \ldots, k_f - 1$$

where

$$\frac{\partial x_i(k+1)^f}{\partial w_{qj}} = \begin{cases} z_j . f'(s_i)\ ; & i = q = 1, \cdots, n\ ;\ j = 1, \cdots, n_z \\ 0 \qquad ; & i \ne q\ ; \qquad j = 1, \cdots, n_z \end{cases} \tag{11}$$

$$\frac{\partial x_i(k+1)^f}{\partial x_j(k)} = w_{ij} f'(s_i)\ ; \quad i = 1, \cdots, n\ ; \quad j = 1, \cdots, n \tag{12}$$