Time : 3 Hours
Maximum Marks : 80

MATHEMATICS

CBSE
Sample Question Paper–2020-21
(Issued by Board on 9th October, 2020)

Sample Question Paper 1

Solved

General Instructions :

1. *This question paper contains two **parts A and B**. Each part is compulsory. Part A carries **24** marks and Part B carries **56** marks.*
2. ***Part-A** has Objective Type Questions and **Part-B** has Descriptive Type Questions.*
3. *Both Part A and Part B have choices.*

Part – A :

1. *It consists of two sections- **I and II.***
2. *Section **I** comprises of 16 very short answer type questions.*
3. *Section **II** contains **2** case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt **any 4 out of** 5 MCQs.*

Part – B :

1. *It consists of three sections-**III, IV and V.***
2. *Section **III** comprises of 10 questions of **2 marks** each.*
3. *Section **IV** comprises of 7 questions of **3 marks** each.*
4. *Section **V** comprises of 3 questions of **5 marks** each.*
5. *Internal choice is provided in **3** questions of Section –III, **2** questions of Section -IV and **3** questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART-A

Section-I

Question numbers 1 to 16 are very short answer type questions.

1. Check whether the function $f : \text{R} \rightarrow \text{R}$ defined as $f(x) = x^3$ is one-one or not.

OR

How many reflexive relations are possible in a set A whose $n(A) = 3$

2. A relation R in S = {1,2,3} is defined as R = {(1,1),(1,2),(2,2),(3,3)}. Which element(s) of relation R be removed to make R an equivalence relation?

3. A relation R in the set of real numbers R defined as R=$\{(a, b) : \sqrt{a} = b\}$ is a function or not. Justify

OR

An equivalence relation R in A divides it into equivalence classes A_1, A_2, A_3.

What is the value of $A_1 \cup A_2 \cup A_3$ and $A_1 \cap A_2 \cap A_3$.

4. If A and B are matrices of order $3 \times n$ and $m \times 5$ respectively, then find the order of matrix 5A – 3B, given that it is defined.

5. Find the value of A^2, where A is a 2×2 matrix whose elements are given by

$$a_{ij} = \begin{cases} 1 & \text{if} \quad i \neq j \\ 0 & \text{if} \quad i = j \end{cases}$$

OR

Given that A is a square matrix of order 3×3 and $|A| = -4$. Find $|\text{adj } A|$

6. Let A = $[a_{ij}]$ be a square matrix of order 3×3 and $|A| = -7$. Find the value of $a_{11} A_{21} + a_{12}A_{22} + a_{13}A_{23}$ where A_{ij} is the cofactor of element a_{ij}.

7. Find $\int e^x (1 - \cot x + \operatorname{cosec}^2 x) dx$

OR

Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x \, dx$

8. Find the area bounded by $y = x^2$, the x – axis and the lines $x = -1$ and $x = 1$.

9. How many arbitrary constants are there in the particular solution of the differential equation

$$\frac{dy}{dx} = -4xy^2 \, ; \, y(0) = 1$$

OR

For what value of n is the following a homogeneous differential equation: $\dfrac{dy}{dx} = \dfrac{x^3 - y^n}{x^2 y + xy^2}$

10. Find a unit vector in the direction opposite to $-\frac{3}{4}\hat{j}$

11. Find the area of the triangle whose two sides are represented by the vectors $2\hat{i}$ and $-3\hat{j}$.

12. Find the angle between the unit vectors $\hat{a}$ and $\hat{b}$, given that $|\hat{a} + \hat{b}| = 1$

13. Find the direction cosines of the normal to YZ plane?

14. Find the coordinates of the point where the line $\dfrac{x+3}{3} = \dfrac{y-1}{-1} = \dfrac{z-5}{-5}$ cuts the XY plane.

15. The probabilities of A and B solving a problem independently are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If both of them try to solve the problem independently, what is the probability that the problem is solved?

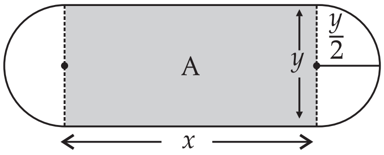
16. The probability that it will rain on any particular day is 50%. Find the probability that it rains only on first 4 days of the week.

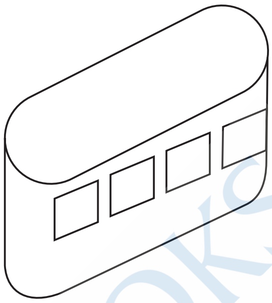
Section-II

Both the case study based questions are compulsory. Attempt any 4 sub parts from each question (17) and (18). Each question carries 1 mark.

17. An architect designs a building for a multi-national company. The floor consists of a rectangular region with semicircular ends having a perimeter of 200 m as shown below:

Design of Floor





Building

Based on the above information answer the following:

(i) If x and y represents the length and breadth of the rectangular region, then the relation between the variables is :

(a) $x + \pi y = 100$ **(b)** $2x + \pi y = 200$ **(c)** $\pi x + y = 50$ **(d)** $x + y = 100$

(ii) The area of the rectangular region A expressed as a function of x is :

(a) $\frac{2}{\pi}(100x - x^2)$ **(b)** $\frac{1}{\pi}(100x - x^2)$ **(c)** $\frac{x}{\pi}(100 - x)$ **(d)** $\pi y^2 + \frac{2}{\pi}(100x - x^2)$

(iii) The maximum value of area A is :

(a) $\frac{\pi}{3200}\text{m}^2$ **(b)** $\frac{3200}{\pi}\text{m}^2$ **(c)** $\frac{5000}{\pi}\text{m}^2$ **(d)** $\frac{1000}{\pi}\text{m}^2$

(iv) The CEO of the multi-national company is interested in maximizing the area of the whole floor including the semi-circular ends. For this to happen the valve of x should be

(a) 0 m **(b)** 30 m **(c)** 50 m **(d)** 80 m

(v) The extra area generated if the area of the whole floor is maximized is :

(a) $\frac{3000}{\pi}\text{m}^2$ **(b)** $\frac{5000}{\pi}\text{m}^2$

(c) $\frac{7000}{\pi}\text{m}^2$ **(d)** No change Both areas are equal

18. In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

Based on the above information answer the following:

(i) The conditional probability that an error is committed in processing given that Sonia processed the form is :

(a) 0.0210 **(b)** 0.04 **(c)** 0.47 **(d)** 0.06

(ii) The probability that Sonia processed the form and committed an error is :

(a) 0.005 **(b)** 0.006 **(c)** 0.008 **(d)** 0.68

(iii) The total probability of committing an error in processing the form is :

(a) 0 **(b)** 0.047 **(c)** 0.234 **(d)** 1

(iv) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Vinay is :

(a) 1 **(b)** 30/47 **(c)** 20/47 **(d)** 17/47

(v) Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form. The value of $\sum_{i=1}^{3} P(E_i | A) = 1$ is :

(a) 0 **(b)** 0.03 **(c)** 0.06 **(d)** 1

PART-B

Section-III

Question numbers 19 to 28 carry 2 marks each.

19. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), \frac{-3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

20. If A is a square matrix of order 3 such that $A^2=2A$, then find the value of $|A|$.

OR

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$.

Hence find A^{-1}.

21. Find the value(s) of k so that the following function is continuous at $x = 0$

$$f(x) = \begin{cases} \frac{1-\cos kx}{x \sin x} & \text{if } x \neq 0 \\ \frac{1}{2} & \text{if } x = 0 \end{cases}$$

.

22. Find the equation of the normal to the curve

$y = x + \frac{1}{x}, x > 0$ perpendicular to the line $3x - 4y = 7$.

23. Find $\int \frac{1}{\cos^2 x(1-\tan x)^2} dx$

OR

Evaluate $\int_0^1 x(1-x)^n dx$

24. Find the area of the region bounded by the parabola $y^2 = 8x$ and the line $x = 2$.

25. Solve the following differential equation: $\frac{dy}{dx} = x^3 \operatorname{cosec} y$, given that $y(0) = 0$.

26. Find the area of the parallelogram whose one side and a diagonal are represented by coinitial vectors $\hat{i} - \hat{j} + \hat{k}$ and $4\hat{i} + 5\hat{k}$ respectively

27. Find the vector equation of the plane that passes through the point (1,0,0) and contains the line $\vec{r} = \lambda \hat{j}$.

28. A refrigerator box contains 2 milk chocolates and 4 dark chocolates. Two chocolates are drawn at random. Find the probability distribution of the number of milk chocolates. What is the most likely outcome?

OR

Given that E and F are events such that P(E) = 0.8, P(F) = 0.7, P $(E \cap F) = 0.6$. Find P $(\bar{E} | \bar{F})$

Section-IV

Question numbers 29 to 35 carry 3 marks each.

29. Check whether the relation R in the set Z of integers defined as R = {(a, b) : $a + b$ is "divisible by 2"} is reflexive, symmetric or transitive. Write the equivalence class containing 0 *i.e.* [0].

30. If $y = e^{x\sin^2 x} + (\sin x)^x$, find $\frac{dy}{dx}$.

31. Prove that the greatest integer function defined by $f(x) = [x]$, $0 < x < 2$ is not differentiable at $x = 1$.

OR

If $x = a \sec\theta$, $y = b \tan\theta$. Find $\frac{d^2y}{dx^2}$ at $x = \frac{\pi}{6}$.

32. Find the intervals in which the function f given by $f(x) = \tan x - 4x$, $x \in \left(0, \frac{\pi}{2}\right)$ is

(a) strictly increasing **(b)** strictly decreasing

33. Find $\int \frac{x^2+1}{(x^2+2)(x^2+3)} dx$.

34. Find the area of the region bounded by the curves $x^2 + y^2 = 4$, $y = \sqrt{3}x$ and x – axis in the first quadrant

OR

Find the area of the ellipse $x^2 + 9y^2 = 36$ using integration

35. Find the general solution of the following differential equation: $x\,dy - (y + 2x^2)dx = 0$

Section-V

Question numbers 36 to 38 carry 5 marks each.

36. If A = $\begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}. Hence

Solve the system of equations;

$$x - 2y = 10$$
$$2x - y - z = 8$$
$$-2y + z = 7$$

OR

Evaluate the product AB, where

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

Hence solve the system of linear equations

$$x - y = 3$$
$$2x + 3y + 4z = 17$$
$$y + 2z = 7$$

37. Find the shortest distance between the lines

$$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$$

and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

If the lines intersect find their point of intersection

OR

Find the foot of the perpendicular drawn from the point (-1, 3, -6) to the plane $2x + y - 2z + 5 = 0$. Also find the equation and length of the perpendicular.

38. Solve the following linear programming problem (L.P.P) graphically.

Maximize $Z = x + 2y$

subject to constraints ;

$x + 2y \geq 100$

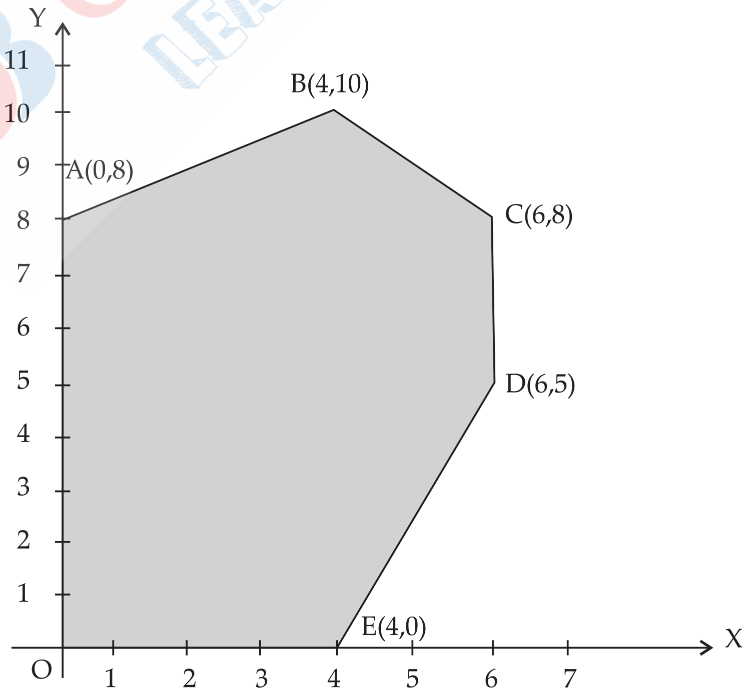
$2x - y \leq 0$

$2x + y \leq 200$

$x, y \geq 0$

OR

The corner points of the feasible region determined by the system of linear constraints are as shown below:



Answer each of the following :

(i) Let Z $= 3x - 4y$ be the objective function. Find the maximum and minimum value of Z and also the corresponding points at which the maximum and minimum value occurs.

(ii) Let Z $= px + qy$, where $p, q > 0$ be the objective function. Find the condition on p and q so that the maximum value of Z occurs at B(4, 10) and C(6,8). Also mention the number of optimal solutions in this case.

■■■

Time : 3 Hours
Maximum Marks : 80

MATHEMATICS
CBSE Solutions
With CBSE Marking Scheme-2020-21

1 Solution of Question Paper

PART-A

Section-I

1. Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R$ **1**

$\Rightarrow \quad (x_1)^3 = (x_2)^3$

$\Rightarrow \quad x_1 = x_2$

Hence $f(x)$ is one-one

OR

2^6 reflexive relations **1**

[CBSE Marking Scheme, 2020]

Detailed Answer :

Given, $n(A) = 3$

Total number of reflexive relations $= 2^{n(n-1)}$

$= 2^{3(3-1)} = 2^{3 \times 2} = 2^6$

2. (1, 2) **1**

3. Since $\sqrt{a}$ is not defined for $a \in (-\infty, 0)$ **1**

$\therefore \quad \sqrt{a} = b$ is not a function

OR

$A_1 \cup A_2 \cup A_3 = A$ and $A_1 \cap A_2 \cap A_3 = \phi$ **1**

4. 3×5 **1**

[CBSE Marking Scheme, 2020]

Detailed Answer :

$5[A]_{3 \times n} - 3[B]_{m \times 5} = [5A - 3B]_{3 \times 5}$

5. $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow \quad A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **1**

OR

$|\text{adj } A| = (-4)^{3-1} = 16$ **1**

[CBSE Marking Scheme, 2020]

Detailed Answer :

$|\text{adj } A| = |A|^{n-1}$

where n = order of matrix A

$|\text{adj } A| = (-4)^{3-1} = (-4)^2 = 16$

6. 0 **1**

7. $e^x(1 - \cot x) + C$ **1**

[CBSE Marking Scheme, 2020]

Detailed Answer :

$e^x(1 - \cot x + \text{cosec}^2 x)dx$

$= \int [e^x(1 - \cot x) + e^x(\text{cosec}^2 x)]dx$

$\left[\because \int e^x(f(x) + f'(x))dx = e^x f(x) + C\right]$

$= e^x(1 - \cot x) + C$

OR

$\because f(x)$ is an odd function

$\therefore \quad \int_{-\pi/2}^{\pi/2} x^2 \sin x \, dx = 0$ **1**

[CBSE Marking Scheme, 2020]

Detailed Answer :

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x dx = 0$

$\left[\because \int_{-a}^{a} f(x)dx = 0, \text{ if } f(x) \text{ is an odd function } \quad i.e., f(x) = -f(x)\right]$

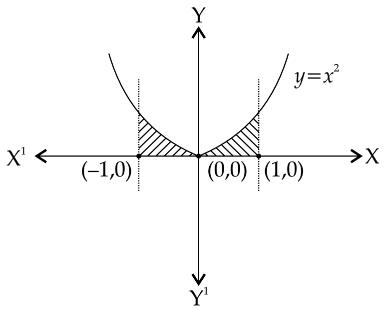
8. $A = 2\int_0^1 x^2 dx$

$$= \frac{2}{3}[x^3]_0^1$$ **1**

$$= \frac{2}{3} \text{ sq. unit}$$

[CBSE Marking Scheme, 2020]

Detailed Answer :



$$\text{Area of shaded region} = 2\int_0^1 x^2 dx$$

$$= \frac{2}{3}[x^3]_0^1 = \frac{2}{3} \text{ sq. unit}$$

9. 0 **1**

OR

3 **1**

[CBSE Marking Scheme, 2020]

Detailed Answer :

Homogeneous differential equation must have same degree in both numerator and denominator, it means

$$\frac{dy}{dx} = \frac{x^3 - y^n}{x^2y + xy^2}$$

so, $n = 3$

10. $\hat{j}$ **1**

11. $\frac{1}{2}|2\hat{i}\times(-3\hat{j})| = \frac{1}{2}|-6\hat{k}| = 3$ sq units **1**

12. $|\hat{a}+\hat{b}|^2 = 1$ **1**

$$\Rightarrow \hat{a}^2 + \hat{b}^2 + 2\hat{a}.\hat{b} = 1$$

$$\Rightarrow 2\hat{a}.\hat{b} = 1-1-1$$

$$\Rightarrow \hat{a}\cdot\hat{b} = -\frac{1}{2}$$

$$\Rightarrow |\hat{a}||\hat{b}|\cos\theta = -\frac{1}{2}$$

$$\Rightarrow \theta = \pi - \frac{\pi}{3}$$

$$\Rightarrow \theta = \frac{2\pi}{3}$$

13. 1, 0, 0 **1**

[CBSE Marking Scheme, 2020]

Detailed Answer :

The coordinate of x-axis must be 1 and the coordinate of y-axis and z-axis must be 0 because we have to find the direction cosines of the normal to yz-plane. So, Direction cosines are (1, 0, 0).

14. (0, 0, 0) **1**

15. $1-\frac{2}{3}\times\frac{3}{4} = \frac{1}{2}$ **1**

16. $\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^7$ **1**

[CBSE Marking Scheme, 2020]

Detailed Answer :

P(rain on any particular day) = 50%

$$= \frac{50}{100} = \frac{1}{2}$$

$$\text{P(rain on first four days of week)} = \left(\frac{1}{2}\right)^4\left(1-\frac{1}{2}\right)^3$$

$$= \left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^7 = \frac{1}{128}$$

Section-II

17. (i) (b) **1**

[CBSE Marking Scheme, 2020]

Detailed Answer :

$$\text{Perimeter} = x + x + \frac{\pi y}{2} + \frac{\pi y}{2}$$

$$200 = 2x + \frac{2\pi y}{2}$$

$$200 = 2x + \pi y \qquad ...(A)$$

(ii) (a) **1**

[CBSE Marking Scheme, 2020]

Detailed Answer :

$$\text{Area (A)} = x \times y$$

$$= x\times\left(\frac{200-2x}{\pi}\right) \quad \text{[from (A)]}$$

$$= \frac{2}{\pi}[100x - x^2] \qquad ...(B)$$

(iii) (c) **1**

[CBSE Marking Scheme, 2020]

Detailed Answer :

$$\frac{dA}{dx} = \frac{2}{\pi}[100-2x]$$

$$\frac{dA}{dx} = \frac{4}{\pi}[500 - x]$$

For maxima,

$$\frac{dA}{dx} = 0$$

$$x = 50 \quad ...(C)$$

$$A = \frac{2}{\pi}[100 \times 50 - 50 \times 50]$$

[from (B)]

$$= \frac{2}{\pi}[5000 - 2500]$$

$$= \frac{2}{\pi} \times 2500 = \frac{5000}{\pi}\ m^2$$

(iv) (a) **1**

(v) (d) **1**

18. (i) (b) **1**

(ii) (c) **1**

[CBSE Marking Scheme, 2020]

Detailed Answer :

P(sonia processed the form and committed an error) = 20% × 0.4

$$= \frac{20}{100} \times 0.04 = \frac{1}{5} \times 0.04$$

$$= 0.008$$

(iii) (b) **1**

(iv) (d) **1**

(v) (d) **1**

[CBSE Marking Scheme, 2020]

Detailed Answer :

$$\sum_{j=1}^{3} P\left(\frac{E_i}{A}\right) = P\left(\frac{E_1}{A}\right) + P\left(\frac{E_2}{A}\right) + P\left(\frac{E_3}{A}\right) = 1$$

[∴ sum of all occurence of an event is equal to 1]

PART-B

Section-III

19. $$\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right) = \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2} - x\right)}{1 - \cos\left(\frac{\pi}{2} - x\right)}\right] \quad \text{½}$$

$$= \tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4} - \frac{x}{2}\right)\cos\left(\frac{\pi}{4} - \frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}\right]$$

$$\tan^{-1}\left[\cot\left(\frac{\pi}{4} - \frac{x}{2}\right)\right] = \tan^{-1}\left[\tan\frac{\pi}{2} - \left(\frac{\pi}{4} - \frac{x}{2}\right)\right] \quad \mathbf{1}$$

$$\tan^{-1}\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right] = \frac{\pi}{4} + \frac{x}{2} \quad \text{½}$$

20. $$A^2 = 2A$$

$$\Rightarrow |AA| = |2A|$$

$$\Rightarrow |A||A| = 8|A| \quad \text{½}$$

$$(\because |AB| = |A||B| \text{ and } |2A| = 2^3|A|)$$

$$\Rightarrow |A|(|A| - 8) = 0 \quad \mathbf{1}$$

$$\Rightarrow |A| = 0 \text{ or } 8 \quad \text{½}$$

OR

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$$

$$7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\Rightarrow A^2 - 5A + 7I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \quad \mathbf{1}$$

$$\Rightarrow A^{-1}(A^2 - 5A + 7I) = A^{-1}0$$

$$\Rightarrow A - 5I + 7A^{-1} = 0$$

$$\Rightarrow 7A^{-1} = 5I - A$$

$$\Rightarrow A^{-1} = \left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right)$$

$$\Rightarrow A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \quad \mathbf{1}$$

21. $$\lim_{x \to 0} \frac{1 - \cos kx}{x \sin x} = \lim_{x \to 0} \frac{2\sin^2\left(\frac{kx}{2}\right)}{x \sin x}$$

$$= \lim_{x \to 0} \frac{\frac{2\sin^2\left(\frac{kx}{2}\right)}{x^2}}{\frac{x \sin x}{x^2}}$$

$$= \frac{\lim_{x \to 0} \frac{2\sin^2\left(\frac{kx}{2}\right)}{\left(\frac{kx}{2}\right)^2} \times \left(\frac{k}{2}\right)^2}{\lim_{x \to 0} \frac{\sin x}{x}}$$

$$= \frac{2 \times 1 \times \frac{k^2}{4}}{1} \quad \text{1½}$$

$\because f(x)$ is continuous at $x = 0$

$$\therefore \quad \underset{x \to 0}{\text{Lt}} f(x) = f(0)$$

$$\Rightarrow \quad \frac{k^2}{2} = \frac{1}{2}$$

$$\Rightarrow \quad k^2 = 1$$

$$\Rightarrow \quad k = \pm 1 \qquad \tfrac{1}{2}$$

22. $$y = x + \frac{1}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = 1 - \frac{1}{x^2}$$

$\because$ normal is perpendicular to $3x - 4y = 7$

$\therefore$ tangent is parallel to it

$$1 - \frac{1}{x^2} = \frac{3}{4}$$

$$\Rightarrow \quad x^2 = 4$$

$$\Rightarrow \quad x = 2 \qquad (\because x > 0) \ \mathbf{1}$$

when $\quad x = 2$

$$y = 2 + \frac{1}{2} = \frac{5}{2}$$

$\therefore$ Equation of Normal :

$$y - \frac{5}{2} = -\frac{4}{3}(x - 2)$$

$$\Rightarrow \quad 8x + 6y = 31 \qquad \mathbf{1}$$

23. $$I = \int \frac{1}{\cos^2 x(1 - \tan x)^2} dx$$

Put, $\quad 1 - \tan x = y$

So that, $\quad -\sec^2 x \, dx = dy \qquad \mathbf{1}$

$$= \int \frac{-1 dy}{y^2}$$

$$= -\int y^{-2} dy$$

$$= +\frac{1}{y} + c$$

$$= \frac{1}{1 - \tan x} + c \qquad \mathbf{1}$$

OR

$$I = \int_0^1 x(1-x)^n dx$$

$$I = \int_0^1 (1-x)[1-(1-x)]^n dx \qquad \tfrac{1}{2}$$

$$I = \int_0^1 (1-x)x^n dx$$

$$= \int_0^1 (x^n - x^{n+1}) dx$$

$$I = \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1 \qquad \mathbf{1}$$

$$I = \left[\left(\frac{1}{n+1} - \frac{1}{n+2}\right) - 0\right]$$

$$= \frac{1}{(n+1)(n+2)} \qquad \tfrac{1}{2}$$

24. $$\text{Area} = 2\int_0^2 \sqrt{8x} dx$$

$$= 2 \times 2\sqrt{2} \int_0^2 x^{\frac{1}{2}} dx \qquad \mathbf{1}$$

$$= 4\sqrt{2}\left[\frac{2}{3} x^{\frac{3}{2}}\right]_0^2 \qquad \tfrac{1}{2}$$

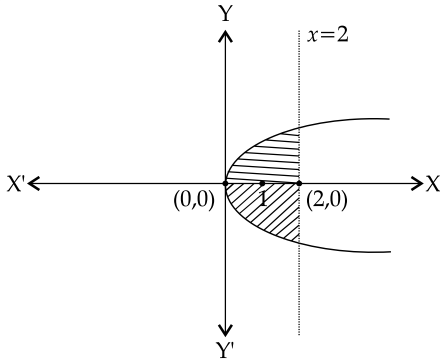
$$= \frac{8}{3}\sqrt{2}[2^{\frac{3}{2}} - 0]$$

$$= \frac{8\sqrt{2}}{3} \times 2\sqrt{2}$$

$$= \frac{32}{3} \text{ sq units} \qquad \tfrac{1}{2}$$

[CBSE Marking Scheme, 2020]

Detailed Answer :



$$\text{Area of shaded region} = 2\int_0^2 \sqrt{8x} dx$$

$$= 2 \times 2\sqrt{2} \int_0^2 x^{1/2} dx$$

$$= 4\sqrt{2}\left[\frac{2}{3} x^{3/2}\right]_0^2$$

$$= \frac{8\sqrt{2}}{3}\left[2^{3/2} - 0\right]$$

$$= \frac{8\sqrt{2}}{3} \times \left[2^{1+1/2}\right]$$

$$= \frac{8\sqrt{2}}{3}\left[2^1.2^{1/2}\right]$$

$$= \frac{8\sqrt{2}}{3}.2\times\sqrt{2} = \frac{8\times2\times2}{3}$$

$$= \frac{32}{3} \text{ sq. units}$$

25. $\frac{dy}{dx} = x^3 \text{cosec } y \; ; \; y(0) = 0$

$\int \frac{dy}{\text{cosec } y} = \int x^3 dx$ ½

$\int \sin y \; dy = \int x^3 dx$

$-\cos y = \frac{x^4}{4} + c$ 1

$-1 = c \quad (\because y = 0, \text{ when } x = 0)$

$\cos y = 1 - \frac{x^4}{4}$ ½

26. Let $\vec{a} = \hat{i} - \hat{j} + \hat{k}$

$\vec{d} = 4\hat{i} + 5\hat{k}$

$\because \quad \vec{a} + \vec{b} = \vec{d}$

$\therefore \quad \vec{b} = \vec{d} - \vec{a}$

$= 3\hat{i} + \hat{j} + 4\hat{k}$ ½

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & -\hat{j} & +\hat{k} \\ 1 & -1 & 1 \\ 3 & 1 & 4 \end{vmatrix}$$

$= -5\hat{i} - 1\hat{j} + 4\hat{k}$ 1

Area of parallelogram $= |\vec{a}\times\vec{b}|$

$= \sqrt{25+1+16}$

$= \sqrt{42}$ sq units ½

27. Let the normal vector to the plane be $\vec{n}$

Equation of the plane passing through (1, 0, 0), *i.e.*, $\hat{i}$ is

$(\vec{r} - \hat{i}).\vec{n} = 0$ ...(i) 1

$\because$ plane (i) contains the line $\vec{r} = \vec{0} + \lambda\hat{j}$

$\therefore \quad \hat{i}.\vec{n} = 0$ and $\hat{j}.\vec{n} = 0$

$\Rightarrow \quad \vec{n} = \hat{k}$

Hence equation of the plane is $(\vec{r} - \hat{i}).\hat{k} = 0$

i.e., $\vec{r}.\hat{k} = 0$ 1

28. Let x denote the number of milk chocolates drawn

X	P(x)
0	$\frac{4}{6}\times\frac{3}{5} = \frac{12}{30}$
1	$\left(\frac{2}{6}\times\frac{4}{5}\right)\times2 = \frac{16}{30}$
2	$\frac{2}{6}\times\frac{1}{5} = \frac{2}{30}$

1½

Most likely outcome is getting one chocolate of each type

OR

$$P(\bar{E}|\bar{F}) \; \frac{P(\bar{E}\cap\bar{F})}{P(\bar{F})} = \frac{P(\overline{E\cup F})}{P(\bar{F})}$$

$$= \frac{1 - P(E\cup F)}{1 - P(F)}$$ ...(i) 1

Now $P(E\cup F) = P(E) + P(F) - P(E\cap F)$

$= 0.8 + 0.7 - 0.6 = 0.9$ ½

Substituting value of $P(E\cup F)$ in (1)

$P(\bar{E}|\bar{F}) = \frac{1-0.9}{1-0.7} = \frac{0.1}{0.3} = \frac{1}{3}$ ½

Section-IV

29. (i) Reflexive :

Since, $a + a = 2a$ which is even

$\therefore (a, a) \in R \; \forall \; a \in Z$

Hence R is reflexive. ½

(ii) Symmetric :

If $(a, b) \in R$, then $a + b = 2\lambda$

$\Rightarrow \quad b + a = 2\lambda$

$\Rightarrow (b, a) \in R$, Hence R is symmetric. 1

(iii) Transitive :

If $(a, b) \in R$ and $(b, c) \in R$

then $a + b = 2\lambda$ ...(i)

and $b + c = 2\mu$ ...(ii)

Adding (i) and (ii) we get

$a + 2b + c = 2(\lambda + \mu)$

$\Rightarrow \quad a + c = 2(\lambda + \mu - b)$

$\Rightarrow \quad a + c = 2k$

where $\lambda + \mu - b = k$

$\Rightarrow (a, c) \in R$

Hence R is transitive

$[0] = \{...-4, -2, 0, 2, 4...\}$ 1½

30. Let $u = e^{x\sin^2 x}$ and $v = (\sin x)^x$ ½

so that $y = u + v$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{...(i)}$$

Now, $u = e^{x\sin^2 x}$

Differentiating both sides w.r.t. x, we get

$$\Rightarrow \quad \frac{du}{dx} = e^{x\sin^2 x}[x(\sin 2x) + \sin^2 x]$$

...(ii) **1**

Also, $v = (\sin x)x$

$\Rightarrow \quad \log v = x \log(\sin x)$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{v}\frac{dv}{dx} = x\cot x + \log(\sin x) \quad \mathbf{1}$$

$$\frac{dv}{dx} = (\sin x)^x[x\cot x + \log(\sin x)]$$

...(iii)

Substituting from – (ii), – (iii) in – (i) we get

$$\frac{dy}{dx} = e^{x\sin^2 x}[x\sin 2x + \sin^2 x] + (\sin x)^x[x\cot x + \log(\sin x)]$$

½

31. $$\text{RHD} = \underset{h\to 0}{\text{Lt}} \frac{f(1+h) - f(1)}{h}$$

$$= \underset{h\to 0}{\text{Lt}} \frac{[1+h]-[1]}{h}$$

$$= \underset{h\to 0}{\text{Lt}} \frac{(1-1)}{h} = 0 \quad \mathbf{1}$$

$$\text{LHD} = \underset{h\to 0}{\text{Lt}} \frac{f(1-h) - f(1)}{-h}$$

$$= \underset{h\to 0}{\text{Lt}} \frac{[1-h]-[1]}{-h} = \underset{h\to 0}{\text{Lt}} \frac{0-1}{-h}$$

$$= \underset{h\to 0}{\text{Lt}} \frac{1}{h} = \infty \quad \mathbf{1}$$

Since, RHD $\neq$ LHD

Therefore $f(x)$ is not differentiable at $x = 1$ **1**

OR

$$y = b\tan\theta$$

$$\Rightarrow \quad \frac{dx}{d\theta} = b\sec^2\theta \quad \text{...(i)}$$

$$x = a\sec\theta$$

$$\Rightarrow \quad \frac{dx}{d\theta} = a\sec\theta\tan\theta \quad \text{...(ii)}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta}$$

$$= \frac{b}{a}\text{cosec}\theta \quad \mathbf{1½}$$

Differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{-b}{a}\text{cosec}\theta\cot\theta \times \frac{d\theta}{dx}$$

$$= \frac{-b}{a}\text{cosec}\theta\cot\theta \times \frac{1}{a\sec\theta\tan\theta}$$

[using (ii)]

$$= \frac{-b}{a.a}\cot^3\theta \quad \mathbf{1}$$

$$\left.\frac{d^2y}{dx^2}\right]_{\theta=\frac{\pi}{6}} = \frac{-b}{a}\left[\cot\frac{\pi}{6}\right]^3$$

$$= \frac{-b}{a}(\sqrt{3})^3 = -\frac{3\sqrt{3}b}{a.a} \quad ½$$

32. $$f(x) = \tan x - 4x$$

$$f'(x) = \sec^2 x - 4 \quad ½$$

(a) For $f(x)$ to be strictly increasing

$$f'(x) > 0$$

$$\Rightarrow \quad \sec^2 x - 4 > 0$$

$$\Rightarrow \quad \sec^2 x > 4$$

$$\Rightarrow \quad \cos^2 x < \frac{1}{4}$$

$$\Rightarrow \quad \cos^2 x < \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \quad -\frac{1}{2} < \cos x < \frac{1}{2}$$

$$\Rightarrow \quad \frac{\pi}{3} < x < \frac{\pi}{2} \quad ½$$

(b) For $f(x)$ to be strictly decreasing

$$f'(x) < 0$$

$$\Rightarrow \quad \sec^2 x - 4 < 0$$

$$\Rightarrow \quad \sec^2 x > 4$$

$$\Rightarrow \quad \cos^2 x > \frac{1}{4}$$

$$\Rightarrow \quad \cos^2 x > \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \quad \cos x > \frac{1}{2} \quad \left[\because x \in \left(0, \frac{\pi}{2}\right)\right]$$

$$\Rightarrow \quad 0 < x < \frac{\pi}{3} \quad \mathbf{1}$$

33. Put $x^2 = y$ to make partial fractions ½

$$\frac{x^2+1}{(x^2+2)(x^2+3)} = \frac{y+1}{(y+2)(y+3)}$$

$$= \frac{A}{y+2} + \frac{B}{y+3}$$

$\Rightarrow \quad y + 1 = A(y + 3) + B(y + 2)$ ...(i) ½

Comparing coefficients of y and constant terms on both sides of (i) we get

$$A + B = 1 \text{ and } 3A + 2B = 1$$

Solving, we get $A = -1, B = 2$ **1**

$$\int \frac{x^2+1}{(x^2+2)(x^2+3)}dx = \int \frac{-1}{x^2+2}dx + 2\int \frac{1}{x^2+3}dx$$

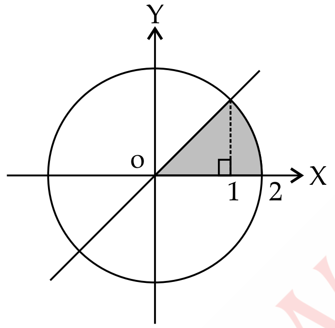
$$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C \quad \mathbf{1}$$

34. Solving $y = \sqrt{3}x$ and $x^2 + y^2 = 4$

We get $x^2 + 3x^2 = 4$

$\Rightarrow \quad x^2 = 1$

$\Rightarrow \quad x = 1$ ½



½

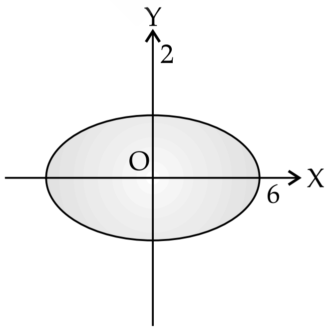
$$\text{Required Area} = \sqrt{3}\int_0^1 x\,dx + \int_1^2 \sqrt{2^2 - x^2}\,dx \quad ½$$

$$= \frac{\sqrt{3}}{2}[x^2]_0^1 + \left[\frac{x}{2}\sqrt{2^2-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]_1^2 \quad \mathbf{1}$$

$$= \frac{\sqrt{3}}{2} + \left[2\times\frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2\times\frac{\pi}{6}\right] = \frac{2\pi}{3} \text{ sq units} \quad ½$$

OR

$$\text{Required Area} = \frac{4}{3}\int_0^6 \sqrt{6^2 - x^2}\,dx \quad ½$$



$$= \frac{4}{3}\left[\frac{x}{2}\sqrt{6^2-x^2} + 18\sin^{-1}\left(\frac{x}{6}\right)\right]_0^6 \quad \mathbf{1}$$

$$= \frac{4}{3}\left[18\times\frac{\pi}{2} - 0\right] = 12\pi \text{ sq units} \quad \mathbf{1}$$

35. The given differential equation can be written as

$$\frac{dy}{dx} = \frac{y+2x^2}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{1}{x}y = 2x$$

Here $\quad P = -\frac{1}{x}$

$Q = 2x$ ½

$$\text{I.F.} = e^{\int P dx}$$

$$= e^{-\int \frac{1}{x}dx} = e^{-\log x} = \frac{1}{x} \quad \mathbf{1}$$

The solutions is :

$$y \times \frac{1}{x} = \int\left(2x \times \frac{1}{x}\right)dx \quad 1$$

$$\Rightarrow \quad \frac{y}{x} = 2x + c$$

$$\Rightarrow \quad y = 2x^2 + cx \quad ½$$

Section-V

36. $|A| = 1(-1-2) - 2(-2-0)$

$= -3 + 4 = 1$ ½

A is nonsingular, therefore A^{-1} exists

$$\text{Adj } A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow \quad A^{-1} = \frac{1}{|A|}(\text{Adj } A)$$

$$= \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \quad 1½$$

The given equations can be written as :

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} \quad ½$$

Which is of the form A'X = B

$$\Rightarrow \quad X = (A')^{-1}B = (A^{-1})'B \quad \mathbf{1}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$\Rightarrow \quad x = 0$

$y = -5$

$z = -3$ 1½

OR

$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \qquad 1\frac{1}{2}$$

$$\Rightarrow \quad AB = 6I$$

$$\Rightarrow \quad A\left(\frac{1}{6}B\right) = I$$

$$\Rightarrow \quad A^{-1} = \frac{1}{6}(B) \qquad 1$$

The given equations can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$AX = D$$

where $D = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$$\Rightarrow \quad X = A^{-1}D$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\Rightarrow \quad = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} \qquad 1$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$x = 2$

$y = -1$

$z = 4$ $\qquad 1\frac{1}{2}$

37. We have $\quad a_1 = 3\hat{i}+2\hat{j}-4\hat{k}$

$b_1 = \hat{i}+2\hat{j}+2\hat{k}$

$a_2 = 5i-2j$

$b_2 = 3\hat{i}+2\hat{j}+6\hat{k}$

$$\vec{a_2}-\vec{a_1} = 2\hat{i}-4\hat{j}+4\hat{k} \qquad 1$$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix}$$

$$= \hat{i}(12-4)-\hat{j}(6-6)+\hat{k}(2-6)$$

1

$$\vec{b_1}\times\vec{b_2} = 8\hat{i}+0\hat{j}-4\hat{k} = 8\hat{i}-4\hat{k}$$

$$\because (\vec{b_1}\times\vec{b_2}).(\vec{a_2}-\vec{a_1}) = 16-16 = 0 \qquad 1$$

$\therefore$ The lines are intersecting and the shortest distance between the lines is 0.

Now for point of intersection

$$3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})$$

$$= 5\hat{i}-2\hat{j}+\mu(3\hat{i}+2\hat{j}+6\hat{k})$$

$\Rightarrow \quad 3+\lambda = 5+3\mu$ ...(i)

$2+2\lambda = -2+2\mu$ ...(ii)

$-4+2\lambda = 6\mu$ ...(iii) **1**

Solving (i) and (ii) we get $\mu = -2$ and $\lambda = -4$

Substituting in equation of line we get

$$\vec{r} = 5i-2j+(-2)(3\hat{i}+2\hat{j}-6\hat{k})$$

$$= -\hat{i}-6\hat{j}-12\hat{k}$$

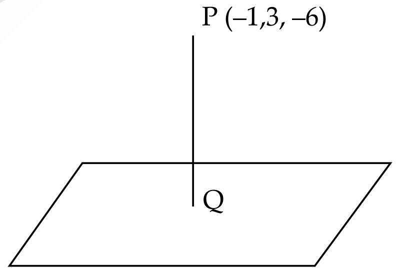
Point of intersection is $(-1, -6, -12)$ $\qquad$ **1**

OR

Let P be the given point and Q be the foot of the perpendicular.

Equation of PQ $\dfrac{x+1}{2} = \dfrac{y-3}{1}$

$$= \frac{z+6}{-2} = \lambda \qquad 1\frac{1}{2}$$



Let coordinates of Q be $(2\lambda-1, \lambda+3, -2\lambda-6)$

Since Q lies in the plane $2x+y-2z+5 = 0$

$\therefore \ 2(2\lambda-1)+(\lambda+3)-2(-2\lambda-6)+5 = 0$

$\Rightarrow 4\lambda-2+\lambda+3+4\lambda+12+5 = 0$ $\qquad \frac{1}{2}$

$\Rightarrow \quad 9\lambda+18 = 0$

$\Rightarrow \quad \lambda = -2$

$\therefore$ coordinates of Q are $(-5, 1, -2)$ $\qquad$ **1**

Length of the perpendicular

$$= \sqrt{(-5+1)^2+(1-3)^2+(-2+6)^2} = 6 \text{ units} \qquad 2$$

38. Max $\quad Z = 3x+y$

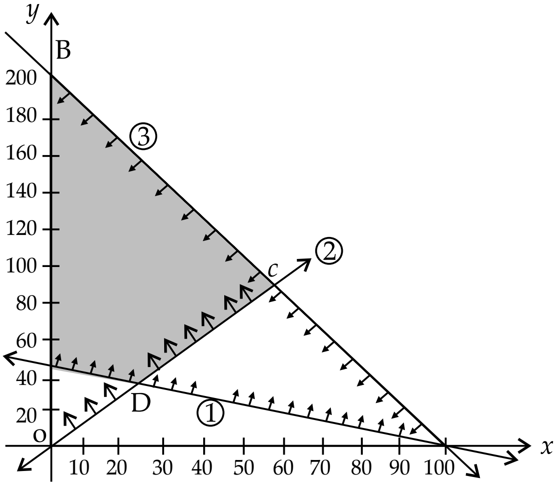
Subject to $\quad x+2y \geq 100$ ...(i)

$2x-y \leq 0$ ...(ii)

$2x+y \leq 200$ ...(iii)

$x \geq 0$

$y \geq 0,$



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Corner Points	$Z = 3x + y$
A(0, 50)	50
B(0, 200)	200
C(50, 100)	250
D(20, 40)	100

1

Max $\qquad z = 250$

at $\qquad x = 50$

$y = 100$ **1**

OR

(i)

Corner Points	$Z = 3x - 4y$
O(0, 0)	0
A(0, 8)	–32
B(4, 10)	–28
C(6, 8)	–14
D(6, 5)	–2
E(4, 0)	12

1½

Max $\qquad Z = 12$ at E(4, 0)

Min $\qquad Z = -32$ at A(0, 8) **1**

(ii) Since maximum value of Z occurs at B(4, 10) and C(6, 8)

$\therefore \qquad 4p + 10q = 6p + 8q$

$\Rightarrow \qquad 2q = 2p$ **2**

$\Rightarrow \qquad p = q$

Number of optimal solution are infinite. **½**

●●●