AMS 553: Homework 2

1. (L&K 7.1) Consider the LCG $Z_i = (5Z_i + 3)(mod 16)$ and $Z_0 = 7$. Find Z_{500}, using only pencil and paper.

2. (L&K 7.2) For the following multiplicative LCGs, compute Z_i for enough values of $i \geq 1$ to cover an entire cycle:

   (a) $Z_i = (11Z_{i-1})(mod\ 16),\ Z_0 = 1$

   (b) $Z_i = (11Z_{i-1})(mod\ 16),\ Z_0 = 2$

   (c) $Z_i = (2Z_{i-1})(mod\ 13),\ Z_0 = 1$

   (d) $Z_i = (3Z_{i-1})(mod\ 13),\ Z_0 = 1$

   Note that (a) and (b) have m of the form 2^b; (c) is a PMMLCG, for which $a = 2$ is a primitive element modulo $m = 13$.

3. (L&K 7.3) Without actually computing any Z_i's, determine which of the following mixed LCGs have full period:

   (a) $Z_i = (13Z_{i-1} + 13)(mod\ 16)$

   (b) $Z_i = (12Z_{i-1} + 13)(mod\ 16)$

   (c) $Z_i = (13Z_{i-1} + 12)(mod\ 16)$

   (d) $Z_i = (Z_{i-1} + 12)(mod\ 13)$

   For part (d), also plot the pairs $(U_1, U_2), (U_2, U_3), \ldots$. Are there any obvious undesirable structure?

4. Without generating any Z_i's, determine which of the following multiplicative LCGs is an PMMLCG?

   (a) $Z_i = (2Z_{i-1})(mod\ 19)$

   (b) $Z_i = (4Z_{i-1})(mod\ 15)$

   (c) $Z_i = (97Z_{i-1})(mod\ 9021)$