UNIT=5

Part = II

Test of significance

Method 2: Difference of proportions

Part-7

Method ⇒ 2 **Difference of Proportion**

Working Rule :-

① Given :- n_1 ⇒ First sample size

n_2 ⇒ Second sample size.

X_1 ⇒ Number of successes in the first sample ⇒ Attribute I

X_2 ⇒ Number of successes in the second sample ⇒ Attribute II

② To find ~~population~~ sample proportion

$\hat{P}_1 = \frac{X_1}{n_1}$ | $\hat{P}_2 = \frac{X_2}{n_2}$

⑤ Set Hypothesis

$H_0 : \mu = \mu_0$ } two tailed test.

$H_1 : \mu \neq \mu_0$

③ To find :- Difference = $\hat{p}_1 - \hat{p}_2$

④ To find standard Error :-

$S.E = \sqrt{\quad} = \sqrt{\hat{p}(1-\hat{p}) \cdot \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$

where $\hat{p} = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2}$

$= \frac{x_1 + x_2}{n_1 + n_2}$

⑥ To find :- $|Z| = \frac{\text{Diff}}{S.E}$ } Test Statistic

$\frac{\hat{p}_1 - \hat{p}_2}{S.E}$

or

$|Z_{cal}| \Rightarrow ?$

⑦ L.O.S :- Critical value of z at 5% L.O.S i.e

$Z_\alpha = 1.96$ or $Z_{Tab} = 1.96$

⑧ Decision :- ① If $Z_{cal} > Z_{Tab}$ ⇒ H_0 reject.

② If $Z_{cal} < Z_{Tab}$ ⇒ H_0 Accept.

Q4 In a random sample of 2000 peoples from colony A, 900 were found to be consumers of rice. In a sample of 1500 people from colony B, 800 were found to be consumers of rice. Do the sample support the assumption that there is no significant difference between two colonies A & B, is so far as the proportion of rice consumers is concerned.

Sol

① Given :-

$n_1 \Rightarrow$ First sample size $\Rightarrow 2000$

$n_2 \Rightarrow$ Second sample size $\Rightarrow 1500$

$X_1 \Rightarrow$ No. of succ. in the first sample $\Rightarrow 900$

$X_2 \Rightarrow$ No. of succ. in the second sample $\Rightarrow 800$

② To find sample proportion :-

$$\hat{P}_1 = \frac{X_1}{n_1} \qquad \hat{P}_2 = \frac{X_2}{n_2}$$

$$\hat{P}_1 = \frac{900}{2000} = 0.45 \qquad \hat{P}_2 = \frac{800}{1500} = 0.53$$

③ To find the Difference $= |\hat{P}_1 - \hat{P}_2| = |0.45 - 0.53|$

Difference $= |-0.08| = -0.08$

④ **S.E** $= \sqrt{\hat{P}(1-\hat{P})(1/n_1 + 1/n_2)}$ where $\hat{P} = \frac{n_1\hat{P}_1 + n_2\hat{P}_2}{n_1+n_2}$

$= 0.0171$

⑤ Set Hypothesis :-

$H_0: \hat{p}_1 = \hat{p}_2$ i.e There is no difference between the proportions of rice consumers of two colonies

$H_1: \hat{p}_1 \neq \hat{p}_2$ i.e There is difference between the

⑥ Test statistic :- $|Z| = \frac{Diff}{S.E}$

or

$$|Z_{cal}| = \frac{Diff}{S.E} = \frac{0.08}{0.0171} = |-4.68|$$

$$\boxed{Z_{cal} = 4.68}$$

⑦ L.O.S :- $Z_\alpha = 1.96$ at 5% L.O.S

$$\boxed{Z_{Tab} = 1.96}$$

⑧ Decision :- $Z_{cal} > Z_{Tab}$

$4.68 > 1.96$

⇒ H_0 reject

⇒ H_1 accept

⇒ we say that colony A & colony B differ significantly in respect of rice consumption.

Q2 In a simple of 600 men from a certain large city, 400 are found to be smokers. In another sample of 900 from the other city, 450 are smokers. Do the data indicate that the cities are significantly different with respect to prevalence of smoking among men?

Sol

Soln ① Given:- $n_1 \Rightarrow 600$

$n_2 \Rightarrow 900$

$X_1 \Rightarrow 400$

$X_2 \Rightarrow 450$

② To find sample proportion :-

$\hat{p}_1 = \frac{X_1}{n_1} = \frac{2}{3} = 0.667$ | $\hat{p}_2 = \frac{X_2}{n_2} = \frac{1}{2} = 0.5$

$\boxed{\cancel{q_1 = 1 - p_1 = 0.333}}$ | $\boxed{\cancel{q_2 = 1 - p_2 = 0.5}}$

③ To find Difference :- $\hat{p}_1 - \hat{p}_2 = 0.167$

④ To find S.E :- $S.E = \sqrt{\cancel{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}} = 0.0257$

⑤ $H_0 : \hat{p}_1 = \hat{p}_2 \Rightarrow$ There is no diff b/w the cities w.r.t prevalence of smoking among the men

$H_1 : \hat{p}_1 \neq \hat{p}_2$

⑥ $|Z_{cal}| = \frac{Diff}{S.E} = 6.59$

⑦ $\overset{1.05}{} Z_{Tab} = 1.96$

⑧ Decision :-

$Z_{cal} > Z_{Tab}$

$6.57 > 1.96$

$\Rightarrow$ So H_0 Reject

$\Rightarrow H_1$ accept

$\Rightarrow$ There is difference b/w the cities w.r.t prevalence of smoking among men

to be hidden in sampling.

Q3 Before an increase in excise duty on tea, 800 people out of a sample of 1000 persons were found to be tea drinkers. ~~in a sample of 12~~ After an increase in the duty, 800 persons were known to be tea drinkers in a sample of 1200 people. Do you think that there has been a significant decrease in the consumption of tea after the increase in the excise duty?

Sol ① Given:-

$n_1 \Rightarrow 1000$

$n_2 \Rightarrow 1200$

$X_1 \Rightarrow 800$

$X_2 \Rightarrow 800$

② Sample proportion :-

$\hat{p}_1 = \frac{X_1}{n_1} = \frac{800}{1000} = \frac{4}{5}$

$\boxed{P_1 = 0.8}$

$\therefore \cancel{q_1 = 1 - p_1 = 0.2}$

$\boxed{\cancel{q_1 = 0.2}}$

$\hat{P}_2 = \frac{X_2}{n_2} = \frac{800}{1200} = \frac{2}{3}$

$\boxed{P_2 = 0.6}$

$\cancel{q_2 = 1 - p_2 = 0.4}$

$\boxed{\cancel{q_2 = 0.4}}$

③ Difference $= \hat{p}_1 - \hat{p}_2 = 0.2$

to be hidden in sampling

(4) $S.E = \sqrt{\cancel{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}} \Rightarrow ?$

(5) Set Hypothesis :-

$H_0 : \hat{p}_1 = \hat{p}_2 \rightarrow$ There is no significant difference in the consumption of tea before & after increse of exercise duty

$H_1 : \underbrace{\hat{p}_1 \neq \hat{p}_2}_{}$

Two tailed test.

(6) $|Z_{cal}| = \frac{Diff}{S.D} = 9.5$

$|Z_{cal}| = |9.5| = 9.5$

$\boxed{Z_{cal} \Rightarrow 9.5}$

(7) L.O.S :- $Z_{Tab} = 1.96$

(8) Decision :- $Z_{cal} > Z_{Tab}$

$9.5 > 1.96$

$\Rightarrow$ Ho reject

$\Rightarrow$ H_1 Accept.

$\Rightarrow$ There is a significant difference in the consumption of tea before & after increse of exercise duty

to be hidden un ...

Q4 In two large populations there are 30% and 25% respectively of fair haired people. Is this difference likely to be hidden in samples of 1200 & 900 respectively from the two populations.

Sol^n ① Given :— $n_1 = 1200$

$n_2 = 900$

$\hat{P}_1$ = Proportion of fair haired people in the first population = 30% = 0.3

$\hat{P}_2$ = Proportion of fair haired people in the second population = 25% = 0.25

② Difference $\hat{P}_1 - \hat{P}_2 = 0.3 - 0.25 = 0.05$

③ To find standared Error :-

$$S.E = \sqrt{\hat{P}(1-\hat{P}) \cdot \left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \cancel{\quad}$$

$$\text{where } \hat{P} = \frac{n_1 \hat{P}_1 + n_2 \hat{P}_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2} = 0.2785$$

④ Set Hypothesis :- (two-tailed test)

$H_0 : P_1 = P_2$ i.e sample proportions are equal i.e the difference in population proportion is likely to be hidden in sampling.

$H_1: P_1 \neq P_2$

(5) Test static:-

$$Z = \frac{Diff}{S.E}$$

or

$$Z_{cal} = \frac{Diff}{S.E} = 2.538$$

$\therefore |Z_{cal}| = |2.538| = 2.538$

$$\boxed{Z_{cal} = 2.538}$$

(6) L.O.S :- $\boxed{Z_{Tab} = 1.96}$

(7) Decision:- $Z_{cal} > Z_{tab}$

$2.538 > 1.96$

$\Rightarrow$ Ho reject

$\Rightarrow$ H_1 Accepted

$\Rightarrow$ we say these samples will reveal the difference in the population proportions

(4) **Q5** 500 articles from a factory are examined and found to be 2% defective. 800 similar articles from a second factory are found to have only 1.5% defective. Can it reasonably be concluded that the product of the first factory are inferior to those of second?

Sol^n ① Given :- $n_1 = 500$

$n_2 = 800$

$\hat{P}_1$ = proportion of defective from first factory = 2%

$\hat{P}_1 = 0.02$

$\hat{P}_2$ = proportion of defective from second factory = 1.5%

$\hat{P}_2 = 0.015$

② Difference $= \hat{P}_1 - \hat{P}_2 = 0.005$

③ To find S.E :- $S.E = \sqrt{\hat{p}(1-\hat{p}) \cdot (1/n_1 + 1/n_2)}$

where $\hat{p} = \dfrac{n_1 \hat{P}_1 + n_2 \hat{P}_2}{n_1 + n_2}$

$\boxed{\hat{P} = 0.01692}$

$\boxed{S.E = 0.0071}$

(4) Set Hypothesis :-

$H_0 : P_1 = P_2$ ⇒ There is no significant differena between the two products or

The products do not differ in quality

$H_1 : P_1 \neq P_2$ (two-tailed test)

(5) Test statistic :-

$$Z_{cal} = \frac{\text{Diff}}{\text{S.D}} = 0.68$$

$$|Z_{cal}| = |0.68| = 0.68$$

$$\therefore \boxed{Z_{cal} = 0.68}$$

(6) <u>L.O.S</u> :- $Z_{Tab} = 1.96$

(7) <u>Decision</u> :- $\underset{0.68}{Z_{cal}} < \underset{1.96}{Z_{Tab}}$

⇒ Ho Accepted.

⇒ The products do not differe in quality

Q6 In a year there are 956 births in a town A of which 52.7% were males while in town A and B combined, this proportion in a total of 1406 births was 0.496. Is there any significant difference in the proportion of male births in the two towns?

Sol^n ① Given :- $n_1 = 956$

$\hat{P}_1 = 52.7\% = 0.525$

To find :- $n_2 \Rightarrow ?$

$\hat{P}_2 \Rightarrow ?$

According to condition I^st

$n_1 + n_2 = 1406$

$956 + n_2 = 1406$

$n_2 = 1406 - 956$

$\boxed{n_2 = 450}$

~~Now, at this stage~~

~~$n_1 = 956$, $n_2 = 450$~~

~~$P_1 = 0.525$~~

Since we know that

$$\hat{P} = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} \quad \text{--- (A)}$$

Given $\hat{P} = 0.496$

By eq ①

$$0.496 = \frac{956 \times 0.525 + 450 P_2}{1406}$$

$$\therefore \boxed{\hat{P}_2 = 0.4343}$$

<u>Now</u> $n_1 = 956$

$n_2 = 450$

$\hat{P}_1 = 0.525$

$\hat{P}_2 = 0.4343$

(2) Diff $= \hat{P}_1 - P_2$

(3) S.E $= \sqrt{\hat{P}(1-\hat{P}) \cdot \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$

where $\hat{P} = \dfrac{n_1 \hat{P}_1 + n_2 \hat{P}_2}{n_1 + n_2} =$

(4) Set Hypothesis :-

$H_0 : P_1 = P_2 \Rightarrow$ There is no difference b/w the proportion of male births in the two towns

$H_0 : P_1 \neq P_2$ } Two-tailed Test.

(5) Test Statics :-

$$Z_{cal} = \frac{Diff}{S.E} = 3.32$$

(6) L.O.S $\quad Z_{Tab} = 1.96$

(7) Decision $\quad Z_{cal} > Z_{Tab}$

$$3.32 > 1.96$$

$\Rightarrow$ Ho is rejected.

$\Rightarrow$ H_1 is accepted

$\Rightarrow$ There is significant difference in the proportion of male birth in the two towns.