Bayesian Data Analysis EC543

Instructor: M.A. Rahman

Submitted by: Mohit Shukla 11435

Date: April 23, 2015

Problem:

Go through the paper “Understanding the Metropolis-Hastings Algorithm’ by Siddhartha Chib and Edward Greenberg. Replicated the results of ARMA (p,q) model that is presented in Table1

Solution:

Code:

```
%%Mohit Shukla
%Q1

%Generate Y-Data series
y=zeros(100,1);
w=zeros(98,2);
y(1,1)=normrnd(0,1);
y(2,1)=normrnd(0,1);
y2=zeros(2,1);
phi1=1;
phi2=-0.5;
sum_w=zeros(2,2);
n=98;

%Generate lag y
for i=3:100
   eps=normrnd(0,1);
   y(i,1)=phi1*y(i-1,1) + phi2*y(i-2,1) + eps;
   w(i,1)=y(i-1,1);
   w(i,2)=y(i-2,1);
   sum_w=sum_w+w(i,:)'*w(i,:);
end
```

```
y2(1,1)=y(1,1);
y2(2,1)=y(2,1);

%Calculate G,Phi hat etc
G=sum_w;
phi_hat=zeros(1,2);
error=zeros(98,1);
sum_err_sq=0;
for i=3:100
    phi_hat=phi_hat+y(i,:)*w(i,:);
    error(i-2,1)=y(i,1)-w(i,1)*phi1-w(i,2)*phi2;
    sum_err_sq=sum_err_sq+(error(i-2,1)^2);
end

%Calculate Posterior parameters
    phi_hat=phi_hat*(G^-1);
    G_Inv=G^-1;
    V_Inv=[1-phi2^2,-phi1*(1+phi2);-phi1*(1+phi2),1-phi2^2];
    V=V_Inv^-1;
    V_post_Inv=(y2'*V_Inv*y2 + sum_err_sq);
    V_post=V_post_Inv^-1;

    s=50000;
    s0=1000;
    df=n/2;
    phi_prev=[1,-0.5];
    iteration=1;
    sigma_prev=1;
    sum=phi_hat;

%Gibbs Sampling
    for i=1:s
       sigma_sq_inv=wishrnd(V_post,df);
       sigma_sq=sigma_sq_inv^-1;
       phi_draw=mvnrnd(phi_hat,sigma_sq*G_Inv);
       sum=sum+phi_draw;
       %Acceptance region of phi1 and phi2
       if (phi_draw(1,2)+phi_draw(1,1)<1) &  (-
phi_draw(1,1)+phi_draw(1,2)<1) & (phi_draw(1,2)>-1)
        V2_inv=[1-phi_draw(1,2)^2,-phi_draw(1,1)*(1+phi_draw(1,2));-
phi_draw(1,1)*(1+phi_draw(1,2)),1-phi_draw(1,2)^2];
        V1_inv=[1-phi_prev(1,2)^2,-phi_prev(1,1)*(1+phi_prev(1,2));-
phi_prev(1,1)*(1+phi_prev(1,2)),1-phi_prev(1,2)^2];
        ratio=((det(V2_inv)^0.5)*exp(-
(y2'*V2_inv*y2)/(2*sigma_prev)))/((det(V1_inv)^0.5)*exp(-
(y2'*V1_inv*y2)/(2*sigma_prev)));
       if ratio>=1
            phi_prev=phi_draw;
       end
       u=rand;
       if ratio<1 & u<ratio
            phi_prev=phi_draw;
       end
       if i>s0   %Gibbs sampling after s0 burns
```

```
            gibbs_phi(iteration,1)=phi_prev(1,1);
            gibbs_phi(iteration,2)=phi_prev(1,2);
            gibbs_sigma(iteration,1)=sigma_sq;
            sigma_prev=sigma_sq;
            iteration=iteration+1;
        end
        end
    end

%Summary Phi
mean(gibbs_phi)
median(gibbs_phi)
min(gibbs_phi)
max(gibbs_phi)
std(gibbs_phi)

%Summary Sigma
mean(gibbs_sigma)
median(gibbs_sigma)
min(gibbs_sigma)
max(gibbs_sigma)
std(gibbs_sigma)
```

Analysis results showed that M-H algorithm has quickly and accurately produced a posterior distribution concentrated on the values that generated the data.