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CS-301/2401

B.Tech. (Semester-III) Exam.–2015

Discrete Structures

Time : Three Hours

Maximum Marks : **100**

Note : Attempt questions from all sections.

SECTION - A

(Short-answer Type Questions)

Note : Attempt **any ten** questions. Each question carries 4 marks. 10×4=40

1. What can you say about the sets A and B if we know that

   (i) A- B = A ?

   (ii) A - B = B - A ?

2. How many relations are there on the set {a,b,c,d}?

   $2^4 = 16$

[P. T. O.

3. Let the functions f and g be defined by $f(x) = 2x+1$ and $g(x) = x^2-2$. Find the formula defining the composition function gof.

4. Let G be a group of order P. Where P is a prime. Find all subgroups of G.

5. In an integral domain D, show that if $ab = ac$ with $a \neq 0$ then $b = c$.

6. Suppose $f : G \rightarrow G^1$ is a group homomorphism, prove that

   (i) $f(e) = e^1$

   (ii) $f(a^{-1}) = f(a)^{-1}$

7. Which of these are posets?

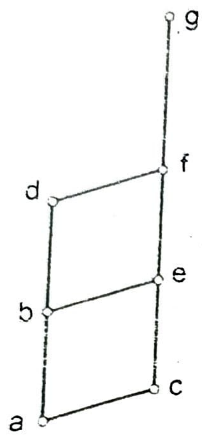
   (i) $(Z, =)$

   (ii) $(Z, \geq)$

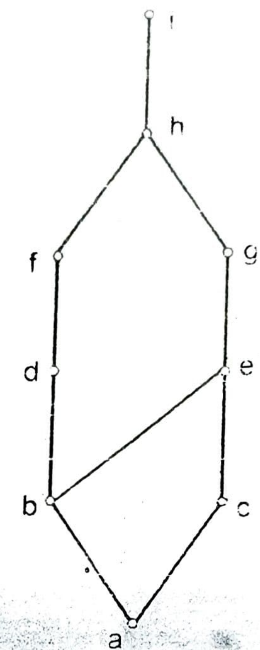
   Where Z is the set of integers.

8. Determine whether the posets with these Hasse diagrams are lattices.

(i)



(ii)



9. Change the Boolean function

$$f(x,y,z) = x'yz + xyz + x'yz' + xyz'$$

into disjunctive normal form of two variables.

10. Prove that $P \wedge (\sim P)$ is a contradiction.

11. Negate each of the following statements

(i) $\forall x\ (x+1>x)$ (ii) $\exists x\ (x^2<0)$ $\exists x \sim (x+1>x)$

$\neg$ $\sim \forall x(x+1>x) \equiv$ $x+1 \leq x$

$x \not> x+1$

12. Show that $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$ are logically equivalent.

[P. T. O.

13. Find the generating function $A(z)$ for the sequence $a_r = 3^r, r \geq 0$

14. Explain the following

(i) Isomorphic graphs

(ii) Regular graphs

15. Show that any tree is a bipartite graph.

SECTION - B

(Long Answer type questions)

Note : Attempt **any three** questions. Each question carries 20 marks. 20x3=60

1. Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric and / or transitive, where $(x,y) \in R$ if and only if

(i) $x \pm y$

(ii) $xy \geq 1$

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for the

(iii) $x = y + 1$

(iv) x is a multiple of y

(v) $x = y^2$

2. Let * be the operation on the set R of real numbers defined by $a*b = a + b + 2ab$

(i) Find $2 * 3$, $3 * (-5)$

(ii) Is (R, *) a semi group? Is it commutative?

(iii) Find the identity element

(iv) Which elements have inverses and what are they?

3. (a) Reduce the following Boolean products to either O or a fundamental product.

(i) $xyz\cancel{y}x$ (b) $xyz\cancel{y}x'z'$ $xyz' + yx'z'$

(b) Use a Karnaugh map to find a minimal sum of products form for E = $xyz + xyz' + x'yz' + x'y'z' + x'y'z$

4. Translate each of these statements into logical expression using predicates, quantifier and logical connectives.

   (i) No one is perfect.

   (ii) Not everyone is perfect

   (iii) All your friends are perfect

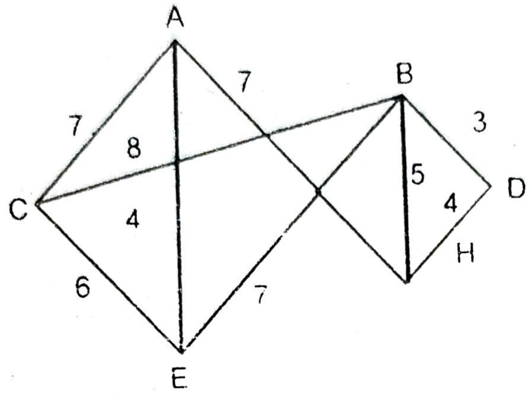
   (iv) One of your friends is perfect

   (v) Everyone is your friend and is perfect.

   (vi) Not everybody is your friend or someone is not perfect.

5. (a) What is the chromatic number of the complete bipartite graph K_{min} where m and n are positive integers.

(b) Find a minimal spanning tree of the weighted graph shown below :



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6. Let G be a finite graph with n ≥ 1 Vertices. Then prove that the following are equivalent :

(i) G is a tree

(ii) G is a cycle-free and has n-1 edges.

(iii) G is connected and has n-1 edges.