**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Sol:- given μ = 45 minutes, σ = 8 minutes

Let X be the time it takes to complete repair on customer’s car

P(X>50)=1 – P(X≤ 50)

Z=X-μ/σ

=(50-45)/8=0.625=73.4%(using norm table)

Prob that service manager will not meet his demand will be =100-73.4=26.6% or 0.2676

Option D is the answer

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Sol:-

|  |
| --- |
| Probabilty of employees greater than age of 44= P(X>44) |
|  |
| P(X > 44) = 1 - P(X ≤ 44)  Z=44-38/6=1=P(Z≤1)=84.1345%   |  | | --- | |  | |  |      |  | | --- | | So the probability of number of employees between 38-44 years of age = P(X<44)-0.5=84.1345-0.5= 34.1345% | | Therefore the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE. |  |  | | --- | |  | |  |  | |  |  | |

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Sol:- Probabilty of employees less than age of 30 = P(X<30)

P(X ≤ 30)= 30-38/6=P(Z≤-1.333)=9.12%

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|  |
|  | So the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48( or 36 employees). |
|  | Therefore the statement B of the question is also TRUE. |

1. If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid*normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Sol:- As we know that if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ) .

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

Therefore in the question

2X1~ N(2 u,4 σ^2) and

X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1-(X1+X2) = N( 4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Sol:- From the above details,we have to excluded area of .005 in each of the left and right tails. Hence, we want to find the 0.5th and the 99.5th percentiles Z score values

Using Python

Z value is given as stats.norm.ppf(pvalue)

Z value at 0.5th percentile is given as

                                         Z(0.5) = stats.norm.ppf(0.005)= -2.576

Z value at 99.5 percentile is given as

                         Z(99.5) = stats.norm.ppf(0.995) = 2.576

Z = (x - 100)/20 = > x = 20z+100

      a = -(20\*2.576) + 100= 48.5

      b = (20\*2.576)+100= 151.5

Two values symmetric about mean for the given standard normal distribution are[48.5,151.5]

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Sol:- 95% of the probability lies between 1.96 standard deviations of the mean.

Thus range is:

=(12-1.96\*5.12+1.96\*5)

=(Rs 99M,Rs 1026M)

1. Specify the 5th percentile of profit (in Rupees) for the company

Sol:- Fifth percentile is calculated as:

From p values of z score table, we get:

p-12/5=-1.644=12-8.22=3.78

Thus at $3.78M dollars, or Rs. 170.1M amount, 5th percentile of profit lies.

Or 5th percentile of profit is Rs. 170.1 Million.

C.Which of the two divisions has a larger probability of making a loss in a given year?

Sol:- Loss is when profit < 0

Thus: p < 0

The first division of company, thus have larger probability of making a loss in a given year.