PROBLEM SHEET 1

Sets:

1. Mohit Jangir has a collection of 10 books, while Samridhi Jha has a collection of 12 books. If they both have 4 books in common, how many different books do they have in total?

2. Lokesh Sain has a collection of 6 CDs, while Jay Bhati has a collection of 8 CDs. If they both have 3 CDs in common, how many different CDs do they have in total?

3. Let A and B be sets such that $|A| = 5$ and $|B| = 6$. How many different functions are there from A to B?

4. Let A and B be sets. Prove that if A is a proper subset of B, then B has more elements than A.

5. Let A, B, and C be sets. Prove that if $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then $B = C$.

6. Let A and B be sets. Prove that if A is a subset of B, then $A \cap B = A$.

7. Let A, B, and C be sets. Prove that if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

8. Mohit Jangir has a collection of 15 pens, while Samridhi Jha has a collection of 12 pens. If they both have 6 pens in common, how many different pens do they have in total?

9. Lokesh Sain has a collection of 8 baseball cards, while Jay Bhati has a collection of 10 baseball cards. If they both have 3 baseball cards in common, how many different baseball cards do they have in total?

Fundamental operations of sets:

1. Ankit Kumawat has a collection of 15 DVDs, while Mohit Jangir has a collection of 20 DVDs. If they both have 8 DVDs in common, how many DVDs do they have in total?

2. Samridhi Jha has a collection of 30 stamps, while Lokesh Sain has a collection of 25 stamps. If they both have 15 stamps in common, how many stamps do they have in total?

3. Let A, B, and C be sets. Prove that $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$.

4. Let A, B, and C be sets. Prove that $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$.

5. Let A, B, and C be sets. Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

6. Let A, B, and C be sets. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

7. Let A and B be sets. Prove that $A \Delta B = (A \cup B) \setminus (A \cap B)$, where Δ denotes the symmetric difference.

8. Ankit Kumawat has a collection of 25 CDs, while Mohit Jangir has a collection of 18 CDs. If they both have 9 CDs in common, how many CDs do they have in total?
9. Samridhi Jha has a collection of 20 movies, while Lokesh Sain has a collection of 15 movies. If they both have 8 movies in common, how many movies do they have in total?

Principle of inclusion and exclusion:

1. In a group of 50 students, Mohit Jangir is friends with 35 students, Samridhi Jha is friends with 28 students, Lokesh Sain is friends with 20 students, Jay Bhati is friends with 18 students, and Ankit Kumawat is friends with 15 students. If any two of them have 10 friends in common, and any three of them have 5 friends in common, how many students are not friends with any of them?
2. Let A, B, and C be sets. Prove that $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.
3. In a class of 50 students, 28 like pizza, 25 like burgers, and 20 like both pizza and burgers. How many students in the class like neither pizza nor burgers?
4. Let A and B be sets. Prove that if $|A| = m$ and $|B| = n$, then the number of functions from A to B is n^m.
5. Let A and B be sets. Prove that if $|A \cap B| = k$, then the number of functions from A to B that map exactly k elements of A to B is (n choose k) * n^(m-k), where $n = |B|$ and $m = |A|$.
6. Let A, B, and C be sets. Prove that the number of elements in exactly one of the sets A, B, or C is $|A| + |B| + |C| - 2|A \cap B| - 2|A \cap C| - 2|B \cap C| + 3|A \cap B \cap C|$.

Principle of mathematical induction:

1. Prove that 1 + 2 + 3 + ... + n = n(n+1)/2 for all positive integers n.
2. Prove that 2^n > n^2 for all positive integers $n \geq 5$.
3. Prove that n! < n^n for all positive integers
4. Prove that for all positive integers n, 1 + 2 + 3 + … + n = (n(n+1))/2.
5. Prove that for all positive integers n, 2^n > n.
6. Prove that for all positive integers n, 3^n < n!.
7. Prove that for all positive integers n, 1 + 3 + 5 + … + (2n-1) = n^2.
8. Prove that for all positive integers n, 2 + 4 + 6 + … + 2n = n(n+1).