

$$a_1 \equiv a_2 \pmod{m_2}$$

$$a_1 \equiv a_2 \pmod{m_3}$$

from Lemma

$$0 \equiv b \pmod{m}$$

$$0 \equiv b \pmod{n} \Rightarrow 0 \equiv b \pmod{m}$$

$$a_1 \equiv a_2 \pmod{m_k}$$

hence

$$a_1 \equiv a_2 \pmod{(m_1 \times m_2 \times \dots \times m_k)}$$

$$= a_2 \pmod{M}$$

\Rightarrow have only unique solution.

Elliptic curve

$$\text{equation: } y^2 = x^3 + ax + b$$

Ex. Find the points on EC $E_{13}(1,1)$. The eq. is $y^2 = x^3 + x + 1$ and the calculation is done modulo 13. points on the curve can be found.

$$y^2 = x^3 + x + 1 \pmod{13}$$

$$x=0, y^2 = 1 \pmod{13}$$

$$x=1, y^2 = 3$$

$$x=3, y^2 = 3^3 + 3 + 1 = 37 \pmod{13} = 11 \pmod{13}$$

$$x=4, y^2 = 64 + 4 + 1 = 69 \pmod{13} = 4$$

$$x=5, y^2 = 125 + 5 + 1 = 131 \pmod{13} = 1$$

$$x=6, y^2 = 216 + 6 + 1 = 223 \pmod{13} = 2$$

$$x=7, y^2 = 343 + 7 + 1 = 351 \pmod{13} = 0$$

$$x=8, y^2 = 512 + 8 + 1 = 521 \pmod{13} = 1$$

$$x=9, y^2 = 729 + 9 + 1 = 739 \pmod{13} = 10$$

$$x=10, y^2 = 1000 + 10 + 1 = 1011 \pmod{13} = 10$$

Let calculate

$$1^2 = 1, 2^2 = 4, 3^2 = 9$$

$$4^2 = 16, 5^2 = 25, 6^2 = 36$$

$$7^2 = 49, 8^2 = 64, 9^2 = 81$$

$$10^2 = 100, 11^2 = 121, 12^2 = 144$$

points: $(0,1), (0,12)$

$(1,2), (1,11)$

$(4,2), (4,11)$

$(5,1), (5,12)$

$(7,0), (7,0)$

$(8,1), (8,12)$

$(10,2), (10,11)$

$(12,5), (12,8)$



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$E(1,2)$ over $\mathbb{F}(11)$
 $y^2 = x^3 + ax + b$
 for $y^2 = x^3 + x + 3$

$1^2=1$
 $2^2=4$
 $3^2=9$
 $4^2=5$
 $5^2=3$
 $6^2=3$
 $7^2=5$
 $8^2=9$
 $9^2=4$
 $10^2=1$

$x=0 \quad y^2=3 \quad y=\pm\sqrt{3}$
 $x=1 \quad y^2=5$
 $x=2 \quad y^2=13 \pmod{11} = 2$
 $x=3 \quad y^2=27+3+3=33 \pmod{11} = 0$
 $x=4 \quad y^2=5$
 $x=5 \quad y^2=125+5+3=133 \pmod{11} = 5$
 $x=6 \quad y^2=216+6+3=225 \pmod{11} = 5$
 $x=7 \quad y^2=9$
 $x=8 \quad y^2=512+8+3=523 \pmod{11} = 6$
 $x=9 \quad y^2=729+9+3=741 \pmod{11} = 4$
 $x=10 \quad y^2=1000+10+3=1013 \pmod{11} = 1$

E

$(0,5) (0,6)$
 $(1,4) (1,7)$
 $(3,0)$
 $(4,4) (4,7)$
 $(5,1) (5,10)$
 $(6,4) (6,7)$
 $(7,3) (7,8)$
 $(9,2) (9,9)$
 $(10,1) (10,10)$

$\rightarrow \mathbb{F}_P, G, P$

$\alpha \in \mathbb{F}_P$
 $\alpha \in G$
 P, X, Y, Z
 P, X, Y, Z

$j=217 \quad w_2(217)=115 \quad s(217)=9$
 $i=2$
 $j = 217 + 503 + w_2(217) \pmod{255}$
 $= 217 + 115 + 115$



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Modular Exponentiation algorithm:-

basically the main agenda is to calculate,

$$x = a^b \text{ mod } n$$

For basic:

$x \leftarrow 1$
for $(i=0 \text{ to } b)$

$$x = (x \times a) \text{ mod } n$$

}

return x

convert b in binary

$$\Rightarrow b = \begin{array}{|c|c|c|c|c|c|} \hline b_{k-1} & b_{k-2} & \dots & b_1 & b_0 \\ \hline 2^{k-1} & 2^{k-2} & \dots & 2^1 & 2^0 \\ \hline \end{array}$$

$$\Rightarrow x = a^b \text{ mod } n$$

$$= (a^{b_{k-1}2^{k-1} + b_{k-2}2^{k-2} + \dots + b_12^1 + b_02^0}) \text{ mod } n$$

$$= (a^{b_{k-1}2^{k-1}} \cdot a^{b_{k-2}2^{k-2}} \cdot \dots \cdot a^{b_12^1} \cdot a^{b_02^0}) \text{ mod } n$$

$$= \left(\prod_{i=0}^{k-1} a^{b_i 2^i} \right) \text{ mod } n$$

for binary bit $b_i \begin{cases} \rightarrow 0 \\ \rightarrow 1 \end{cases}$

\Rightarrow when $b_i = 0$ then $a^{0 \cdot 2^i} = 1$
when $b_i = 1$ then $a^{1 \cdot 2^i} = (a^2)^i$

$$\Rightarrow x = \left(\prod_{i=0}^{k-1} (a^2)^{b_i} \right) \text{ mod } n$$

1. $x \leftarrow 1, y \leftarrow a$

2. for $(i=0 \text{ to } k-1)$

3. {

4. if $(b_i == 1)$

5. {

6. $x = (x \times y) \text{ mod } n$

7. }

8. }

9. return x

Ex: $9^8 \text{ mod } 11$

$$8 = 1000$$

$$\begin{array}{|c|c|c|c|} \hline 1 & 0 & 0 & 0 \\ \hline b_3 & b_2 & b_1 & b_0 \\ \hline \end{array}$$

$$x \leftarrow 1$$

$$y \leftarrow 9$$

$$i=0 \quad y \leftarrow (9 \times 9) \text{ mod } 11 = 4$$

$$i=1 \quad y \leftarrow (4 \times 4) \text{ mod } 11 = 5$$

$$i=2 \quad y \leftarrow (5 \times 5) \text{ mod } 11 = 3$$

$$i=3 \quad x \leftarrow (1 \times 3) \text{ mod } 11 = 3$$

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Euclid algorithm

→ This algorithm is used to find the GCD of two numbers

$$\text{GCD}(a, b) = ?$$

```

 $x_1 \leftarrow a; x_2 \leftarrow b$ 
while ( $x_2 \neq 0$ )
{
     $q = \lfloor x_1 / x_2 \rfloor$ 
     $r = x_1 - q \cdot x_2$ 
     $x_1 \leftarrow x_2$ 
     $x_2 \leftarrow r$ 
}
return  $x_1$ 

```

$$q = \frac{x_1}{x_2}$$

Extended euclid algorithm

→ This algorithm is extended version of computing GCD as well as multiplicative inverse of given two numbers

Ex: $a^{-1} \bmod b$

1. $x_1 \leftarrow a; x_2 \leftarrow b$

2. $t_1 \leftarrow 0; t_2 \leftarrow 1$

3. while ($x_2 > 0$)

4. {

5. $q = \lfloor x_1 / x_2 \rfloor$

6. $r = x_1 - q \cdot x_2$

7. $t = t_1 - q \cdot t_2$

8. $x_1 \leftarrow x_2$

9. $x_2 \leftarrow r$

10. $t_1 \leftarrow t_2$

11. $t_2 \leftarrow t$

12. }

13. if ($x_1 == 1$)

{
return t_1 ;
}

else
return NULL
}

Fermat's theorem

if p is prime number and $a \in \mathbb{Z}$ such that
 $p \nmid a$ (p does not divide a)

then $\boxed{a^{p-1} \equiv 1 \pmod{p}}$

Proof:-

Let $\mathbb{Z}_p^* = \{1, 2, 3, \dots, (p-1)\}$ since p is prime
multiply with a in each element of set

$$X = \{a, 2a, 3a, \dots, a(p-1)\} \pmod{p}$$

if apply \pmod{p} in each element of set

$$\Rightarrow X = \{a \pmod{p}, 2a \pmod{p}, \dots, a(p-1) \pmod{p}\}$$

Since a is coprime with p hence all the
elements of X belong to \mathbb{Z}_p^* only

\Rightarrow we can say that

$$X = \mathbb{Z}_p^* \Rightarrow 1 \leq x_i \leq (p-1)$$

\Rightarrow Let assume $x_i \equiv 0 \pmod{p}$

$$\Rightarrow k \cdot a \equiv 0 \pmod{p} \quad \text{where } k \in \{1, \dots, (p-1)\}$$

from this either $k \pmod{p} = 0$ or $a \pmod{p} = 0$

\downarrow
not possible

Since k is from
 \mathbb{Z}_p^*

\downarrow
Contradict
the assumption

Let assume

$$* x_i \equiv x_j \pmod{p} \quad \text{where } i \neq j$$

$$* k_1 \cdot a \equiv k_2 \cdot a \pmod{p} \Rightarrow (k_1 - k_2)a \equiv 0 \pmod{p}$$

\Rightarrow either $(k_1 - k_2) \pmod{p} = 0$ or $a \pmod{p} = 0$

\downarrow
 $p \nmid a$ contradict the
assumption

* From above assumptions we concluded that

$$X \equiv Z_p^*$$

Lets multiply all element of $X \neq Z_p^*$ both side

~~\Rightarrow~~

$$X = \{a, 2a, 3a, \dots, a(p-1)\}$$

$$Z_p^* = \{1, 2, 3, \dots, (p-1)\}$$

~~\Rightarrow~~

$$\{a(1), a(2), a(3), \dots, a(p-1)\} \pmod p \equiv \{1, 2, 3, \dots, (p-1)\} \pmod p$$

$$\Rightarrow a^{p-1} (1 \cdot 2 \cdot 3 \cdot \dots \cdot (p-1)) \equiv \{1, 2, \dots, (p-1)\} \pmod p$$

eliminate common term from both side

$$\Rightarrow \boxed{a^{p-1} \equiv 1 \pmod p}$$

proved

$$\Rightarrow \frac{a^{p-1}}{a} \equiv \frac{1}{a} \pmod p \Rightarrow \boxed{a^{p-2} \equiv a^{-1} \pmod p}$$

Euler's totient function

if n is a prime number then $\phi(n) = (n-1)$

no. of elements with ' n '

Euler's theorem

1st version: if $\gcd(a, n) = 1$ then $\boxed{a^{\phi(n)} \equiv 1 \pmod n}$

2nd version: if $n = p \times q$ where p, q are coprime then $\boxed{a^{k\phi(n)+1} \equiv a \pmod n}$ where $k \in \mathbb{Z}$

Proof:

$$Z_n^* = \{x_1, x_2, \dots, x_{\phi(n)}\}$$

these only $\phi(n)$ values in Z_n^*

where $\phi(n) = (p-1)(q-1)$

multiply all the values of Z_n^* with a

$$X = \{ax_1, ax_2, \dots, ax_{\phi(n)}\}$$

apply mod n

$$X = \{ax_1 \bmod n, \dots, ax_{\phi(n)} \bmod n\}$$

all elements lies in Z_n^*

$$\Rightarrow X = Z_n^*$$

take product both side of all elements

$$\Rightarrow (ax_1 \cdot ax_2 \cdot \dots \cdot ax_{\phi(n)}) \bmod n \equiv (x_1 \cdot x_2 \cdot \dots \cdot x_{\phi(n)}) \bmod n$$

$$\Rightarrow \boxed{a^{\phi(n)} \equiv 1 \bmod n} \text{ proved.}$$

Corollary: Let assume a is coprime with n

$$\text{then } a^{\phi(n)} \equiv 1 \bmod n$$

apply exp(k) both side

$$\Rightarrow a^{k\phi(n)} \equiv 1^k \bmod n$$

\Rightarrow multiply both side with a

$$\Rightarrow \boxed{a^{k\phi(n)+1} \equiv a \bmod n} \text{ proved}$$

Since a is coprime
means a^{-1} exist under
mod n



Case 2: Let's assume a is not coprime with n .

⇒ then

Case 1: $a \in \{1 \times p, 2 \times p, \dots, p \times (p-1)\}$ since $n = p \times q$

Case 2: $a \in \{1 \times q, 2 \times q, \dots, q \times (p-1)\}$

Let take case 1:

⇒ $a = k_1 p$ where $k_1 \in \{1, 2, \dots, (p-1)\}$

from Fermat's Law $2 \times a$

$$\Rightarrow a^{p-1} \equiv 1 \pmod{p}$$

here $p-1 = \phi(p)$ here

$$\Rightarrow a^{\phi(p)} \equiv 1 \pmod{p}$$

apply $\phi(p)$ in power both side

$$a^{\phi(p) \cdot \phi(p)} \equiv 1 \pmod{p}$$

$$\Rightarrow a^{\phi(n)} \equiv 1 \pmod{p}$$

$$\Rightarrow a^{k\phi(n)} \equiv 1 \pmod{p}$$

$$\Rightarrow 2) a^{k\phi(n)} \equiv 1 \pmod{q}$$

$$\Rightarrow a^{k\phi(n)} = (2k_2 + 1) \cdot$$

multiply with a both side

$$\Rightarrow a^{k\phi(n)+1} = a^{2k_2+1}$$

$$a^{k\phi(n)+1} = k_1 \cdot k_2 \cdot p \cdot 2 + a$$

$$a^{k\phi(n)+1} = k_3 \cdot n + a$$

apply mod n both side

$$a^{k\phi(n)+1} \equiv k_3 n \pmod{n} + a \pmod{n}$$

$$\boxed{a^{k\phi(n)+1} \equiv a \pmod{n}}$$

we can write

$$1 = a^{k\phi(n)} - 2k_2$$

$$\Rightarrow a^{k\phi(n)} = 1 + 2k_2$$



→ Hash function

→ Learning hash function

Miller Rabin Algorithm for primality testing

Input: n where $n \geq 2$

Output: Composite (Yes) / No (Prime)

Steps:

calculate k and m from n such that $(n-1) = 2^k \cdot m$ [note: m is odd]

1. $a \leftarrow \mathbb{R} \{2, \dots, n-1\}$
2. $b \leftarrow a^m \bmod n$
3. if $(b \equiv 1 \bmod n)$ { return No }
4. ~~else~~
4. for $(i=0 \text{ to } (k-1))$
 - 5. if $(b \equiv 1 \bmod n)$ then return No;
 - 6. else
 - 7. $b \leftarrow b^2 \bmod n$
 - 8. }
8. return Yes;

Ex $n=561$

$$n-1 = 560 \Rightarrow 2^4 \cdot 35$$

$$k=4, m=35$$

1. take $a=2$ $\{2, \dots, 560\}$
2. $b \leftarrow 2^{35} \bmod 561 \equiv 263$

Proof: Yes biased Monte-carlo algorithm

Assume: n is prime

from algo $a^m \not\equiv 1 \bmod n$

$$\text{for } i=0 \quad a^m \not\equiv -1 \bmod n$$

$$i=1 \quad a^{2m} \not\equiv -1 \bmod n$$

$$\begin{array}{r} 2 \cdot 560 \\ 7 \cdot 280 \\ 2 \cdot 140 \\ 2 \cdot 70 \\ 35 \\ \hline 2^{35} \cdot 560 \\ 512 \cdot 560 \\ 48 \\ \hline 2^{10} \cdot 560 \\ 2^{560} \equiv 1 \bmod 561 \\ 2^{5 \cdot 35} \end{array}$$



ce-image problem Resistant:

→ Hash function

c. c. determining both functions

Muller Rabin Algorithm for primality testing

Input: n where $n \geq 2$

output: Composite (Yes) / No (Prime)

stop!

calculate k and m from n such that $(n-1) = 2^k \cdot m$ [note: m is odd]

- ```

1. $a \leftarrow \mathbb{R} \setminus \{2, \dots, n-1\}$
2. $b \leftarrow a^m \bmod n$
3. if $(b \equiv 1 \bmod n)$ { return No }
4. else
5. for $(i = 0 \text{ to } (k-1))$
6. { if $(b \equiv 1 \bmod n)$ then return No;
7. else
8. { $b \leftarrow b^2 \bmod n$
9. }
10. }
11. return Yes;

```

Ex  $n = 561$

$$n-1 = 566 \Rightarrow 2^4 \cdot 35$$

$$K=4, m=35$$

1. take  $b = 2$  and ... 561
2.  $b \leftarrow 2^{25} \bmod 561 \equiv 283$

Proof: xor based Montecarlo algorithm

Assumo:  $n$  è primo

from algo  $a^m \not\equiv 1 \pmod n$

$$f(x) = 0 \quad a_m \not\equiv -1 \pmod{n}$$
$$i=1 \quad a^{2m} \equiv -1 \pmod{m}$$

not on OnePlus

$$\begin{array}{r} 2560 \\ 7280 \\ \hline 2170 \\ \hline 270 \\ \hline 35 \end{array}$$

$$\begin{array}{r} 2560 \\ 512 \\ \hline 43 \end{array}$$

$$\begin{array}{r} 1024 \\ 512 \\ \hline 215 \end{array}$$

$$2^{560} \equiv 1 \pmod{101}$$

$$2^{35}$$



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According to Fermat's law

$$a^{n-1} \equiv 1 \pmod{n} \text{ where } n \nmid a$$

$$(n-1 = 2^k \cdot m)$$

Given  $a^{2^{k-1}m} \not\equiv -1 \pmod{n}$

~~$$a^{2^{k-1}m} \equiv -1 \pmod{n}$$~~

$$a^{2^k m} \equiv 1 \pmod{n}$$

apply root both side

$$a^{2^{k-1}m} \equiv \pm 1 \pmod{n}$$

and we know  $a^{2^{k-1}m} \not\equiv -1 \pmod{n}$

hence  $a^{2^{k-1}m} \equiv 1 \pmod{n}$

apply  $\sqrt{\phantom{x}}$  on both side

$$a^{2^{k-2}m} \equiv \pm 1 \pmod{n}$$

$$\rightarrow a^{2^{k-2}m} \equiv 1 \pmod{n}$$

it contradict our  
assumption

∴



log problem Resistant:  
→ Hash function

designing hash function

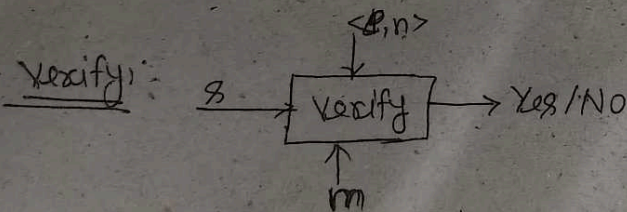
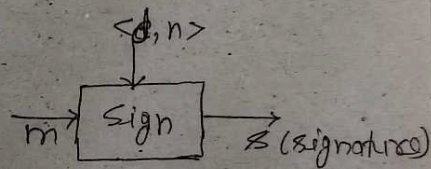
## RSA Digital Signature

keyGen:

1.  $p \leftarrow \text{large prime}, q \leftarrow \text{large prime}$
2. Compute  $n \leftarrow p \times q$
3. Compute  $\phi(n) \leftarrow \phi(p) \times \phi(q) = (p-1)(q-1)$
4.  $d \leftarrow_R \mathbb{Z}_{\phi(n)}^*$
5.  $e \leftarrow d^{-1} \bmod \phi(n)$

Output: privatekey  $\langle d, n \rangle$   
publickey  $\langle e, n \rangle$

Sign: Given:  $m, \langle d, n \rangle$   
 $m \in \mathbb{Z}_{\phi(n)}^*$   
1. calculate the hash of  $m$   
2.  $s \leftarrow \{H(m)\}^d \bmod n$



$\Rightarrow$  calculate  $(s)^e \bmod n = H(m) \bmod n$   
if  $s^e \bmod n = H(m) \bmod n$  then return Yes  
otherwise No.

Correctness Proof:

$$\begin{aligned} \text{take LHS: } s^e \bmod n &= (H(m)^d)^e \bmod n \\ &= H(m)^{ed} \bmod n \\ &= H(m) \bmod n \\ &= \text{RHS proved} \end{aligned}$$





### Soundness Proof:

RSA digital signature sounds as long as RSA problem is hard and underlying hash function is collision resistant.

### 1st) define Attacker problem

#### Attacker prob:

Given:  $\langle e, n \rangle$ ,  $q_1$  (no. of hash queries)  $\in \text{poly}$ ,  $q_2$  (no. of sign queries)  $\in \text{poly}$

Find:  $(m', s')$

s.t.:  $s^e \equiv H(m') \pmod{n}$

where  $(m', s') \neq (m_i, s_i) \forall i$

### 2nd) RSA problem:

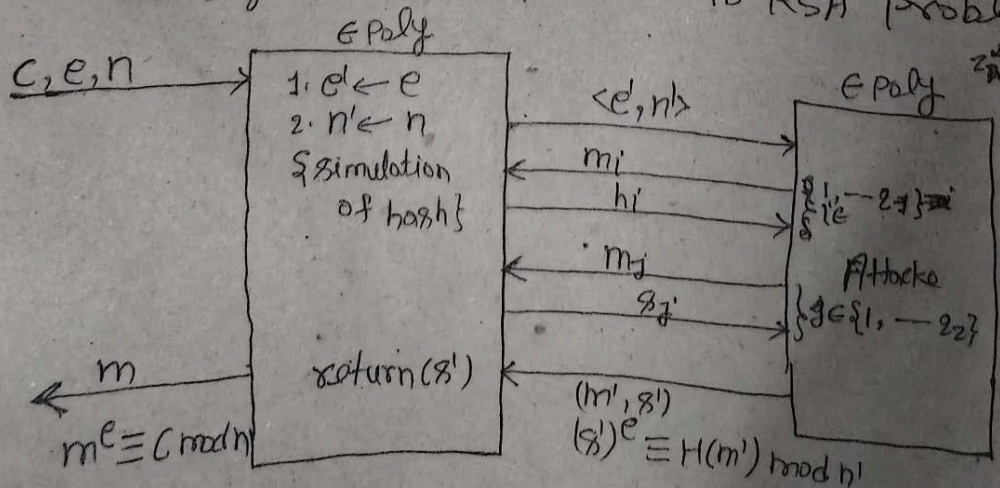
Given:  $c, \langle e, n \rangle$

Find:  $m$

s.t.:  $c \equiv m^e \pmod{n}$  is hard

### Start of proof:

Let assume RSA problem is easy, then reduce the ~~Digital~~ attacker problem to RSA problem





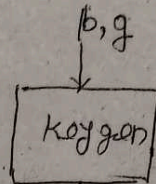
→ Hash function

for designing hash function

## ElGamal Encryption Scheme

### 1. KeyGen:

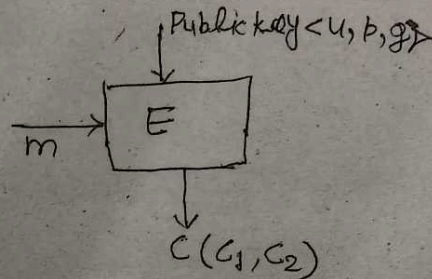
1.  $p \leftarrow \text{large prime}$
2.  $g \leftarrow \text{generator of } \mathbb{Z}_p^*$
3.  $s \leftarrow_R \mathbb{Z}_p^*$
4.  $u \leftarrow g^s \bmod p$



PrivateKey  $\langle s, p, g \rangle$   
PublicKey  $\langle u, p, g \rangle$

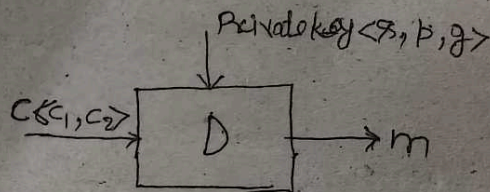
### 2. Encryption:

1.  $r \leftarrow_R \mathbb{Z}_p^*$
2.  $C_1 \leftarrow g^r \bmod p$
3.  $C_2 \leftarrow (m \cdot u^r) \bmod p$



### 3. Decryption:

1. Calculate  $C_1^s \bmod p$
2. Calculate  $(C_2 \cdot C_1^{-s}) \bmod p$
3.  $m = (C_2 \cdot C_1^{-s}) \bmod p$



### Correctness Proof:

$$\begin{aligned} \text{R.H.S. } & (C_2 \cdot C_1^{-s}) \bmod p \\ & [(m \cdot u^r) \cdot (g^{-rs})] \bmod p \\ & = [m \cdot u^r \cdot g^{-rs}] \bmod p \\ & = [m \cdot (g^s)^r \cdot g^{-rs}] \bmod p \quad \text{put } u = g^s \\ & = [m \cdot g^{rs} \cdot g^{-rs}] \bmod p = [m \cdot g^0] \bmod p \\ & = m = \text{L.H.S. proved} \end{aligned}$$





## Soundness proof of ElGamal Encryption:

~~At~~ ElGamal Encryption sounds as long as Diffie-Hellman problem is hard.

### Attacker problem:

Given:  $g, p, C, u$

Find:  $m$

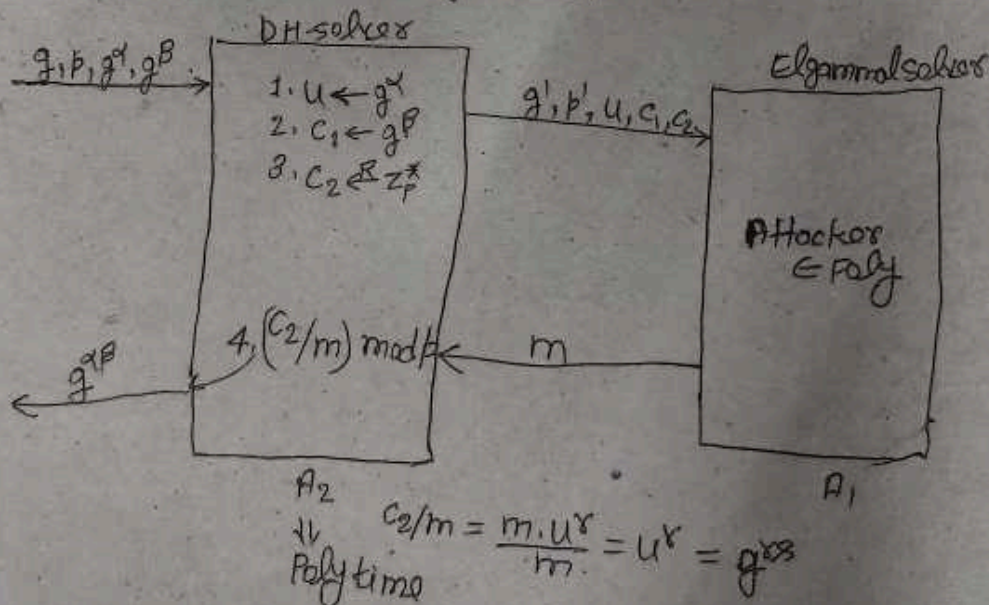
Sit.: decryption of  $C$  under the priv key corresponding to  $u$  is  $m$

Assumption: DH problem is hard

### DH-Problem:

Given:  $g, p, g^A, g^B$

Find:  $g^{AB} \bmod p$



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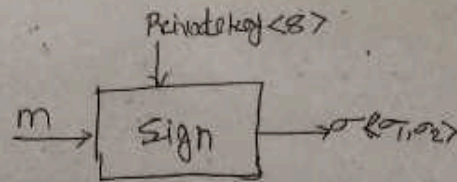
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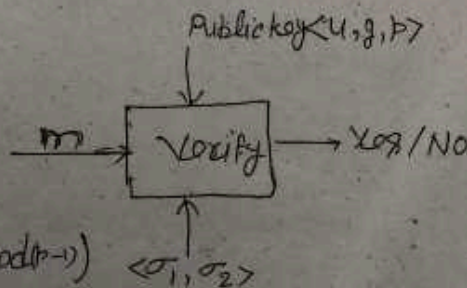
ed for designing hash function  
random oracle model then

### Sign:

1.  $r \leftarrow \mathbb{Z}_{(p-1)}^*$
2.  $\sigma_1 \leftarrow g^r \bmod (p-1)$
3.  $\sigma_2 \leftarrow ((m - g \cdot \sigma_1) \cdot r^{-1}) \bmod (p-1)$



### Verify:



if  $(u^{\sigma_1} \cdot g^{\sigma_2}) \bmod (p-1) = g^m \bmod (p-1)$   
then return Yes otherwise  
false.

### Correctness Proof:

$$\begin{aligned}
 & \text{L.H.S } u^{\sigma_1} \cdot g^{\sigma_2} \bmod (p-1) \\
 &= [u^{\sigma_1} \cdot g^{\sigma_2}] \bmod (p-1) \\
 &= [u^{\sigma_1} \cdot g^{(m - g \cdot \sigma_1) \cdot r^{-1}}] \bmod (p-1) \quad \because r \cdot r^{-1} = 1 \\
 &= [u^{\sigma_1} \cdot g^{m - g \cdot \sigma_1}] \bmod (p-1) \\
 &= [g^{\sigma_1} \cdot g^{m - g \cdot \sigma_1}] \bmod (p-1) \\
 &= [g^{\sigma_1} \cdot g^{m - g \cdot \sigma_1}] \bmod (p-1) \\
 &= g^m \bmod (p-1)
 \end{aligned}$$

put  $u = g^g \bmod p$



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### Soundness Proof:

Given:  $m', \sigma'$ ,  $u, g, p$   
 $\langle \sigma'_1, \sigma'_2 \rangle$

$$u^{m' \cdot \sigma'_1 \sigma'_2} = g^{m'} \pmod{p}$$

$$g^{\sigma'_1} \cdot g^{\sigma'_2} = g^{m'} \pmod{p}$$

Apply log

$$\Rightarrow \sigma'_1 + \sigma'_2 = m' \pmod{p-1}$$

$$z = \{(m' - \sigma'_2) \sigma'_1^{-1}\} \pmod{p-1}$$

$z$  is not known to attacker



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21/10/2024

## Hash Function

SHA-1, 256

MD5

$$H: \{0,1\}^* \rightarrow \{0,1\}^l \quad l: \text{digest size}$$

In practical implementation:

$$H: \{0,1\}^* \rightarrow \{0,1\}^l \quad \text{where } n \gg l$$

$$\left. \begin{array}{l} \text{Given: } m \in \{0,1\}^n \\ \textcircled{1} \text{ Find: } m' \in \{0,1\}^n \\ \text{s.t. : } H(m) = H(m') \end{array} \right\} \begin{array}{l} \rightarrow \text{This should be Hard} \\ \text{2nd pre-image problem} \end{array}$$

$$\left. \begin{array}{l} \text{pre-image} \\ \text{problem} \end{array} \right\} \begin{array}{l} \text{Given: } h \in \{0,1\}^l \\ \text{Find: } m \in \{0,1\}^n \\ \text{s.t. : } H(m) = h \end{array}$$

↓  
should be Hard

$$\left. \begin{array}{l} \text{collision problem} \\ \text{Given:} \\ \text{Find: } (m, m') \in \{0,1\}^n \times \{0,1\}^n \\ \text{s.t. : } H(m) = H(m') \end{array} \right\}$$

↓  
Should be hard  
must be



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Pre-image problem Resistant:

→ Hash function

→ Random Oracle model: used for designing hash function  
if ~~the~~ hash func<sup>n</sup> follows random oracle model then  
it must be collision resistant.

How we prove?

→ using statistical test we can verify that <sup>hash</sup> function is  
collision resistant.

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Random Oracle Model:



- ① Repeatability (same i/p → same o/p)
2. Independence: if two i/p highly correlated but o/p are non-correlated
- ③ Non-correlability.

$(x_1, y_1) (x_2, y_2) \dots (x_n, y_n)$   
→  $f(x_{n+1}) = ?$

\* ROM designed for hash function.

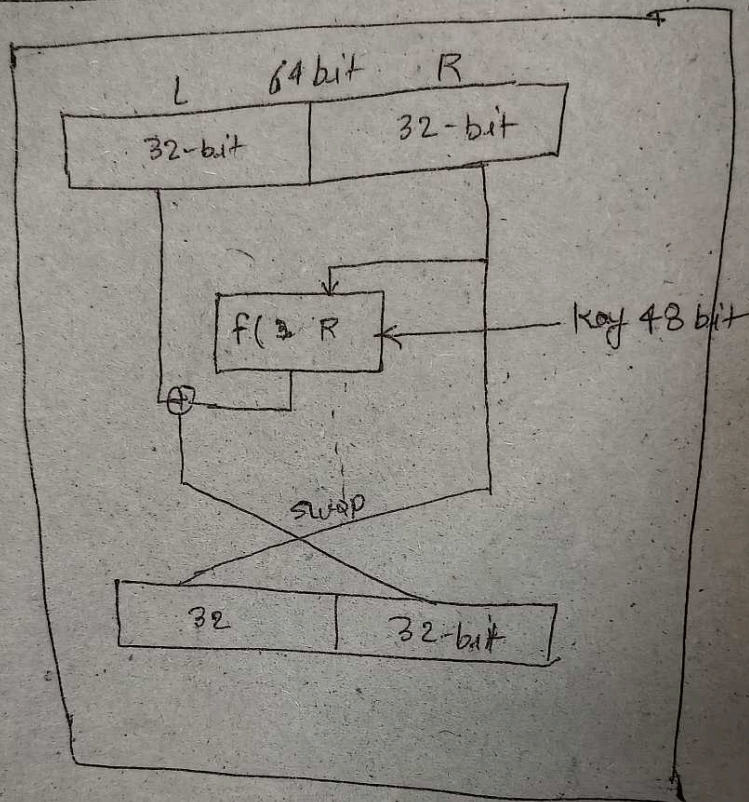




$$s = \{(m' - \sigma_1^{-1} \sigma_2^{-1}) \sigma_1^{-1}\} \bmod (p-1)$$

s is not known to attacker

DES



- 9) Extended Euclid algo ✓
- 10) Fermat's Theorem ✓
- 11) Euler's Theorem (both version) ✓
- 12) Structure of DES & AES ✓
- 13) Digital Signature ✓
- 14) Elliptic curve cryptography ✓



