CSCI 5561: Assignment #4
Convolutional Neural Network

1 Submission

- Assignment due: April 4, 2024 (11:59:59 pm)
- Individual assignment
- Up to 2 page summary write-up with resulting visualization (more than 2 page assignment will be automatically returned.).
- Submission through Canvas.
- Following skeletal functions are already included in the cnn.py file (see Canvas attachments):
  - `main_slp_linear`
  - `main_slp`
  - `main_mlp`
  - `main_cnn`
- List of function to submit:
  - `get_mini_batch`
  - `fc`
  - `fc_backward`
  - `loss_euclidean`
  - `train_slp_linear`
  - `loss_cross_entropy_softmax`
  - `train_slp`
  - `relu`
  - `relu_backward`
  - `train_mlp`
  - `conv`
  - `conv_backward`
  - `pool2x2`
  - `pool2x2_backward`
  - `flattening`
  - `flattening_backward`
  - `trainCNN`

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- A list of MAT files to submit that contain the following trained weights:
  - `slp_linear.mat`: `w`, `b`
  - `slp.mat`: `w`, `b`
  - `mlp.mat`: `w1`, `b1`, `w2`, `b2`
  - `cnn.mat`: `w_conv`, `b_conv`, `w_fc`, `b_fc`
- DO NOT SUBMIT THE PROVIDED IMAGE DATA
- Submissions that does not comply with its specification will not be graded.
- You are not allowed to use computer vision related package functions unless explicitly mentioned here. Please consult with TA if you are not sure about the list of allowed functions.

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2 Overview

Figure 1: You will implement (1) a multi-layer perceptron (neural network) and (2) convolutiona neural network to recognize hand-written digit using the MNIST dataset.

The goal of this assignment is to implement neural networks to recognize hand-written digits in the MNIST data.

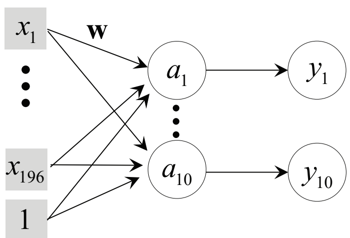
MNIST Data You will use the MNIST hand-written digit dataset to perform the first task (neural network). We have reduced the image size ($28 \times 28 \rightarrow 14 \times 14$) and subsampled the data. You can download the training and testing data from Canvas.

Description: The zip file includes two MAT files (`mnist_train.mat` and `mnist_test.mat`). Each file includes `im_*` and `label_*` variables:

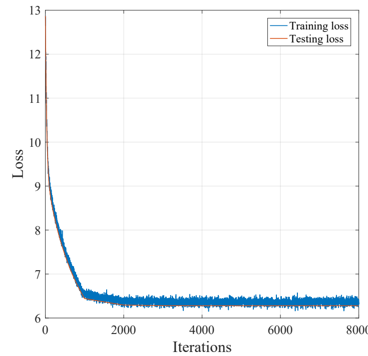
- `im_*` is a matrix ($196 \times n$) storing vectorized image data ($196 = 14 \times 14$)
- `label_*` is $1 \times n$ vector storing the label for each image data.

n is the number of images. You can visualize the i^{th} image, e.g.,
`plt.imshow(mnist_train['im_train'][:, 0].reshape((14, 14), order='F'), cmap='gray')`.

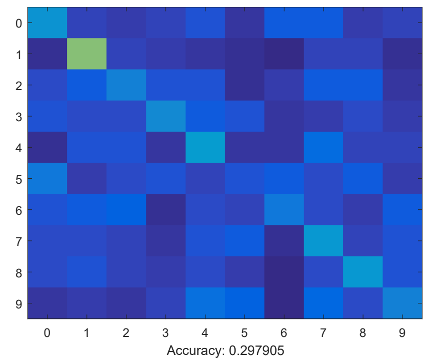
3 Single-layer Linear Perceptron



(a) Single linear perceptron



(b) Loss



(c) Confusion

Figure 2: You will implement a single linear perceptron that produces accuracy near 30%. Random chance is 10% on testing data.

You will implement a single-layer *linear* perceptron (Figure 2(a)) with stochastic gradient descent method. We provide `main_slp_linear` where you will implement `get_mini_batch` and `train_slp_linear`.

```
def get_mini_batch(im_train, label_train, batch_size)
  ...
  return mini_batch_x, mini_batch_y
```

Input: `im_train` and `label_train` are a set of images and labels, and `batch_size` is the size of the mini-batch for stochastic gradient descent.

Output: `mini_batch_x` and `mini_batch_y` are cells that contain a set of batches (images and labels, respectively). Each batch of images is a matrix with size 196×`batch_size`, and each batch of labels is a matrix with size 10×`batch_size` (one-hot encoding). Note that the number of images in the last batch may be smaller than `batch_size`.

Description: You should randomly permute the the order of images when building the batch, and whole sets of `mini_batch_*` must span all training data.

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```
def fc(x, w, b)
  ...
  return y
```

Input: x$\in \mathbb{R}^{m\times 1}$ is the input to the fully connected layer, and w$\in \mathbb{R}^{n\times m}$ and b$\in \mathbb{R}^{n\times 1}$ are the weights and bias.
Output: y$\in \mathbb{R}^{n\times 1}$ is the output of the linear transform (fully connected layer).
Description: `FC` is a linear transform of $\mathbf{x}$, i.e., $\mathbf{y} = \mathbf{w}\mathbf{x} + \mathbf{b}$.

```
def fc_backward(dl_dy, x, w, b, y)
  ...
  return dl_dx, dl_dw, dl_db
```

Input: `dl_dy` $\in \mathbb{R}^{1\times n}$ is the loss derivative with respect to the output $\mathbf{y}$.
Output: `dl_dx` $\in \mathbb{R}^{1\times m}$ is the loss derivative with respect the input $\mathbf{x}$, `dl_dw` $\in \mathbb{R}^{1\times(n\times m)}$ is the loss derivative with respect to the weights, and `dl_db` $\in \mathbb{R}^{1\times n}$ is the loss derivative with respect to the bias.
Description: The partial derivatives w.r.t. input, weights, and bias will be computed. `dl_dx` will be back-propagated, and `dl_dw` and `dl_db` will be used to update the weights and bias.

```
def loss_euclidean(y_tilde, y)
  ...
  return l, dl_dy
```

Input: `y_tilde` $\in \mathbb{R}^m$ is the prediction, and y$\in \{0,1\}^m$ is the ground truth label.
Output: l$\in \mathbb{R}$ is the loss, and `dl_dy` is the loss derivative with respect to the prediction.
Description: `loss_euclidean` measure Euclidean distance $L = \|\mathbf{y} - \widetilde{\mathbf{y}}\|^2$.

```
def train_slp_linear(mini_batch_x, mini_batch_y)
  ...
  return w, b
```

Input: `mini_batch_x` and `mini_batch_y` are cells where each cell is a batch of images and labels.

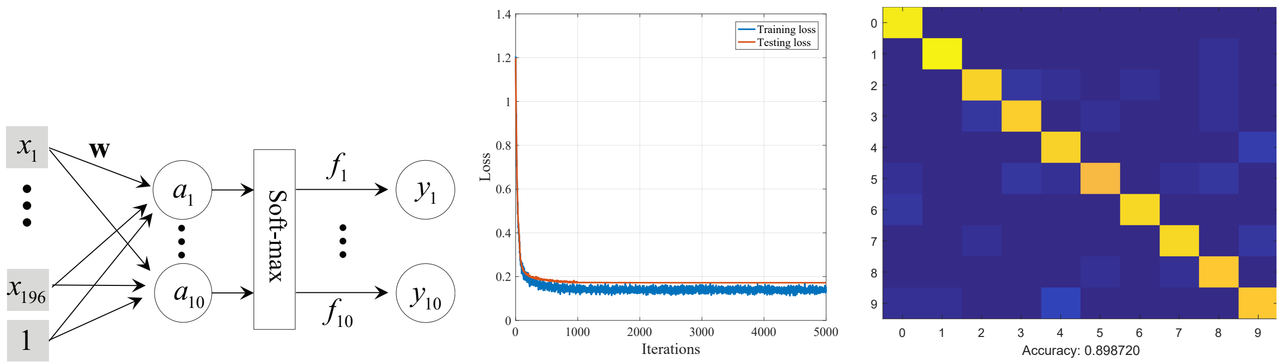
Output: `w` $\in \mathbb{R}^{10\times 196}$ and `b` $\in \mathbb{R}^{10\times 1}$ are the trained weights and bias of a single-layer perceptron.

Description: You will use `fc`, `fc_backward`, and `loss_euclidean` to train a single-layer perceptron using a stochastic gradient descent method where a pseudo-code can be found below. Through training, you are expected to see reduction of loss as shown in Figure 2(b). As a result of training, the network should produce more than 25% of accuracy on the testing data (Figure 2(c)).

Algorithm 1 Stochastic Gradient Descent based Training

Set the learning rate γ
Set the decay rate $\lambda \in (0, 1]$
Initialize the weights with a Gaussian noise $\mathbf{w} \in \mathcal{N}(0, 1)$
$k = 1$
for iIter = 1 : nIters **do**
  At every 1000^{th} iteration, $\gamma \leftarrow \lambda\gamma$
  $\frac{\partial L}{\partial \mathbf{w}} \leftarrow 0$ and $\frac{\partial L}{\partial \mathbf{b}} \leftarrow 0$
  for Each image $\mathbf{x}_i$ in k^{th} mini-batch **do**
    Label prediction of $\mathbf{x}_i$
    Loss computation l
    Gradient back-propagation of $\mathbf{x}_i$, $\frac{\partial l}{\partial \mathbf{w}}$ using back-propagation.
    $\frac{\partial L}{\partial \mathbf{w}} = \frac{\partial L}{\partial \mathbf{w}} + \frac{\partial l}{\partial \mathbf{w}}$ and $\frac{\partial L}{\partial \mathbf{b}} = \frac{\partial L}{\partial \mathbf{b}} + \frac{\partial l}{\partial \mathbf{b}}$
  end for
  k++ (Set $k = 1$ if k is greater than the number of mini-batches.)
  Update the weights, $\mathbf{w} \leftarrow \mathbf{w} - \frac{\gamma}{R}\frac{\partial L}{\partial \mathbf{w}}$, and bias $\mathbf{b} \leftarrow \mathbf{b} - \frac{\gamma}{R}\frac{\partial L}{\partial \mathbf{b}}$
end for

4 Single-layer Perceptron



(a) Single-layer perceptron (b) Loss (c) Confusion

Figure 3: You will implement a single perceptron that produces accuracy near 90% on testing data.

You will implement a single-layer perceptron with *soft-max cross-entropy* using stochastic gradient descent method. We provide `main_slp` where you will implement `train_slp`. Unlike the single-layer linear perceptron, it has a soft-max layer that approximates a max function by clamping the output to $[0, 1]$ range as shown in Figure 3(a).

```
def loss_cross_entropy_softmax(x, y)
  ...
  return l, dl_dy
```

Input: `x` $\in \mathbb{R}^{m\times 1}$ is the input to the soft-max, and `y`$\in \{0,1\}^m$ is the ground truth label.
Output: `L`$\in \mathbb{R}$ is the loss, and `dl_dy` is the loss derivative with respect to $\mathbf{x}$.
Description: `Loss_cross_entropy_softmax` measure cross-entropy between two distributions $L = \sum_i^m \mathbf{y}_i \log \widetilde{\mathbf{y}}_i$ where $\widetilde{\mathbf{y}}_i$ is the soft-max output that approximates the max operation by clamping $\mathbf{x}$ to $[0, 1]$ range:

$$\widetilde{\mathbf{y}}_i = \frac{e^{\mathbf{x}_i}}{\sum_i e^{\mathbf{x}_i}},$$

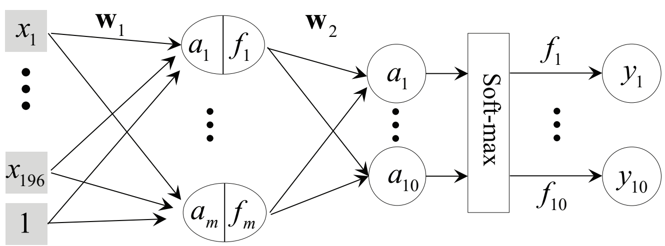
where $\mathbf{x}_i$ is the i^{th} element of $\mathbf{x}$.

```
def train_slp(mini_batch_x, mini_batch_y)
  ...
  return w, b
```

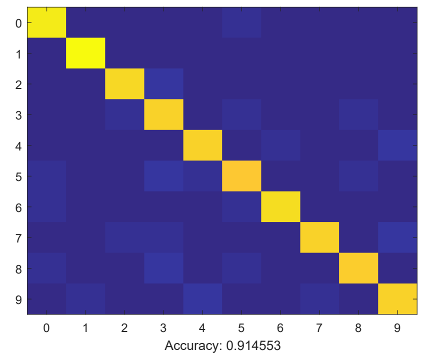
Output: `w` $\in \mathbb{R}^{10\times 196}$ and `b` $\in \mathbb{R}^{10\times 1}$ are the trained weights and bias of a single-layer perceptron.
Description: You will use the following functions to train a single-layer perceptron using a stochastic gradient descent method: `fc`, `fc_backward`, `loss_cross_entropy_softmax`

Through training, you are expected to see reduction of loss as shown in Figure 3(b). As a result of training, the network should produce more than 85% of accuracy on the testing data (Figure 3(c)).

5 Multi-layer Perceptron



(a) Multi-layer perceptron



(b) Confusion

Figure 4: You will implement a multi-layer perceptron that produces accuracy more than 90% on testing data.

You will implement a multi-layer perceptron with a single hidden layer using a stochastic gradient descent method. We provide `main_mlp`. The hidden layer is composed of 30 units as shown in Figure 4(a).

```
def relu(x)
  ...
  return y
```

Input: `x` is a general tensor, matrix, and vector.
Output: `y` is the output of the Rectified Linear Unit (ReLu) with the same input size.
Description: `ReLu` is an activation unit ($\mathbf{y}_i = \max(0, \mathbf{x}_i)$). In some case, it is possible to use a Leaky ReLu ($\mathbf{y}_i = \max(\epsilon \mathbf{x}_i, \mathbf{x}_i)$ where $\epsilon = 0.01$).

```
def relu_backward(dl_dy, x, y)
  ...
  return dl_dx
```

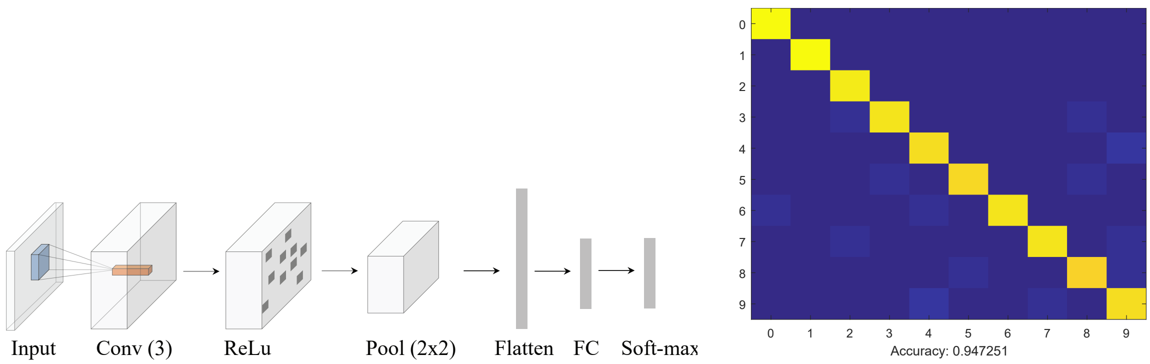
Input: `dl_dy` $\in \mathbb{R}^{1\times z}$ is the loss derivative with respect to the output $\mathbf{y} \in \mathbb{R}^{z}$ where z is the size of input (it can be tensor, matrix, and vector).
Output: `dl_dx` $\in \mathbb{R}^{1\times z}$ is the loss derivative with respect to the input $\mathbf{x}$.

```
def train_mlp(mini_batch_x, mini_batch_y)
  ...
  return w1, b1, w2, b2
```

Output: `w1` $\in \mathbb{R}^{30\times196}$, `b1` $\in \mathbb{R}^{30\times1}$, `w2` $\in \mathbb{R}^{10\times30}$, `b2` $\in \mathbb{R}^{10\times1}$ are the trained weights and biases of a multi-layer perceptron.

Description: You will use the following functions to train a multi-layer perceptron using a stochastic gradient descent method: `fc`, `fc_backward`, `relu`, `relu_backward`, `loss_cross_entropy_softmax`. As a result of training, the network should produce more than 90% of accuracy on the testing data (Figure 4(b)).

6 Convolutional Neural Network



(a) CNN (b) Confusion

Figure 5: You will implement a convolutional neural network that produces accuracy more than 92% on testing data.

You will implement a convolutional neural network (CNN) using a stochastic gradient descent method. We provide `main_cnn`. As shown in Figure 4(a), the network is composed of: a single channel input ($14 \times 14 \times 1$) $\rightarrow$ Conv layer (3×3 convolution with 3 channel output and stride 1) $\rightarrow$ ReLu layer $\rightarrow$ Max-pooling layer (2×2 with stride 2) $\rightarrow$ Flattening layer (147 units) $\rightarrow$ FC layer (10 units) $\rightarrow$ Soft-max.

```
def conv(x, w_conv, b_conv)
  ...
  return y
```

Input: `x` $\in \mathbb{R}^{H \times W \times C_1}$ is an input to the convolutional operation, `w_conv` $\in \mathbb{R}^{h \times w \times C_1 \times C_2}$ and `b_conv` $\in \mathbb{R}^{C_2 \times 1}$ are weights and bias of the convolutional operation.
Output: `y` $\in \mathbb{R}^{H \times W \times C_2}$ is the output of the convolutional operation. Note that to get the same size with the input, you may pad zero at the boundary of the input image.
Description: You can use np.pad for padding 0s at boundary. Optionally, you may use `im2col`[1] to simplify convolutional operation.

[1] `https://leonardoaraujosantos.gitbook.io/artificial-inteligence/machine_learning/deep_learning/convolution_layer/making_faster`

```
def conv_backward(dl_dy, x, w_conv, b_conv, y)
  ...
  return dl_dw, dl_db
```

Input: `dl_dy` is the loss derivative with respect to $\mathbf{y}$.
Output: `dl_dw` and `dl_db` are the loss derivatives with respect to convolutional weights and bias $\mathbf{w}$ and $\mathbf{b}$, respectively.
Description: Note that for the single convolutional layer, $\frac{\partial L}{\partial \mathbf{x}}$ is not needed. Optionally, you may use `im2col` to simplify convolutional operation.

```
def pool2x2(x)
  ...
  return y
```

Input: $\mathtt{x} \in \mathbb{R}^{H\times W\times C}$ is a general tensor and matrix.
Output: $\mathtt{y} \in \mathbb{R}^{\frac{H}{2}\times\frac{W}{2}\times C}$ is the output of the 2×2 max-pooling operation with stride 2.

```
def pool2x2_backward(dl_dy, x, y)
  ...
  return dl_dx
```

Input: `dl_dy` is the loss derivative with respect to the output $\mathbf{y}$.
Output: `dl_dx` is the loss derivative with respect to the input $\mathbf{x}$.

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```
def flattening(x)
  ...
  return y
```

Input: `x` $\in \mathbb{R}^{H\times W\times C}$ is a tensor.
Output: `y` $\in \mathbb{R}^{HWC}$ is the vectorized tensor (column major).

```
def flattening_backward(dl_dy, x, y)
  ...
  return dl_dx
```

Input: `dl_dy` is the loss derivative with respect to the output $\mathbf{y}$.
Output: `dl_dx` is the loss derivative with respect to the input $\mathbf{x}$.

```
function train_cnn(mini_batch_x, mini_batch_y)
  ...
  return w_conv, b_conv, w_fc, b_fc
```

Output: `w_conv` $\in \mathbb{R}^{3\times3\times1\times3}$, `b_conv` $\in \mathbb{R}^{3}$, `w_fc` $\in \mathbb{R}^{10\times147}$, `b_fc` $\in \mathbb{R}^{10\times1}$ are the trained weights and biases of the CNN.
Description: You will use the following functions to train a convolutional neural network using a stochastic gradient descent method: `conv`, `conv_backward`, `pool2x2`, `pool2x2_backward`, `Flattening`, `flattening_backward`, `fc`, `fc_backward`, `relu`, `relu_backward`, `loss_cross_entropy_softmax`. As a result of training, the network should produce more than 92% of accuracy on the testing data (Figure 5(b)).