ECN 506

Money Stock Fluctuations, Fully Backed Central Money

1. Consider an OLG economy with an infinite horizon where individuals live for two periods. Suppose the number of young born in a period is constant at $N_t = N$ (i.e. n = 1). There is a constant stock of fiat money $M_t = M$ (i.e. z = 1). In each young generation there are three types of individuals born: workers, bankers and entrepreneurs.
Workers and entrepreneurs both have y units of time available and use all of this time to work. Each unit of time spent working produces one unit of the consumption good. Here we assume that individuals only care about consuming when old. Each worker produces a different amount (some are more productive than others). Denote the amount produced by each young worker by s_i. Entrepreneurs have an additional ability to create capital, which has a one period return of $x > 1$ (and matures after one period).
Finally, bankers have no endowment and do not care about consumption. Bankers in this economy simply take deposits and promise a one-period return r^d, and make loans at the rate r. Bankers can perfectly locate entrepreneurs to make loans and can be easily located by depositors. Finally, when a depositor needs to make a withdrawal, there is a fixed cost φ that they must pay to do this that does not depend on the size of the withdrawal. You can think of this cost as the cost of going to the bank, periodically answering security questions, signing-in to your account etc.

(a) Young workers can either acquire money or deposit with a banker. How many goods do they receive in the next period if they acquire fiat money?
The worker receives the real rate of return on fiat money. For each good used to acquire money, the worker receive $\frac{vt+1}{vt} = \frac{n}{z}$ = 1 consumption good.

(b) How many goods do they acquire in the next period if they deposit?
The worker receives the return r^d on each unit but pays the transaction cost θ. Therefore, for each good deposited the worker receives $r^d - \theta$ goods when old.

(c) What is the rate r^d that must be offered by bankers? In equilibrium we have r^d. Why?
Bankers are risk neutral and do not care about consumption or profits and so the return promised is the maximum possible return x.

(d) What is the average rate of return received on money and deposits?
Individuals receive a rate of return equal to 1 on fiat money. The cost θ does not depend on the amount deposited. For an individual that deposits s_i goods they receive $xs_i - \theta$ goods when old. The average rate of return per deposited good is $\frac{xsi - \theta}{si}$ = $x - \frac{\theta}{si}$. This is increasing in the size of the deposit.

(e) Explain (using a figure and an intuitive argument) why some individuals will deposit and others will acquire money.
The figure is the one from the lecture and the textbook. Intuitively, the average rate of return on money is constant and equal to one. Conversely, the average rate of return on deposits is strictly increasing in the size of the deposit. For very small deposits, the return on money is higher, while for very high deposits, the return on deposits is higher. There is a point in between s^* where the return is equal. This is intuitive. If you have very little to deposit, it is not worth depositing as the cost is too high. If you have a lot to deposit, then this cost is relatively small compared to the return you will get on your deposits.

(f) What does the banker do with deposits?
Bankers use all deposits to find entrepreneurs and make loans. With these loans the entrepreneur will create capital.

(g) At what rate r^l does the banker lend to entrepreneurs?
Bankers lend at rate x. If they choose a higher amount then no entrepreneurs would take the loan. If the rate was smaller, then any other entrepreneur would have an incentive to pay a slightly higher rate and the bank would lend to them instead. This drives the rate to x.

(h) What do entrepreneurs do with the goods they produce?
They consume some of the goods and use the rest to create capital. Capital matures in one period so there is no need to acquire money for entrepreneurs.

(i) Assume bankers loan at rate r = x and pay rate rd = x on deposits and do not consume. Why do we include them in our model?
The bankers are included for two reasons. First, this allows workers to access capital creation through deposits even if they cannot identify entrepreneurs directly. Second, and most importantly, the inclusion of banks allows the cost θ to be introduced. This cost is really what drives the result that individuals acquire both money and make deposits in equilibrium.

(j) Suppose the total real deposits in the economy are H_t and total real currency is Q_t. What is the price of the good?
The money stock is M and real currency is Q_t. Nominal currency is p_tQ_t. In equilibrium all fiat money must be used as nominal currency so that $M = p_tQ_t$ so that $p_t = \frac{M}{Qt}$.

(k) Find a relationship between the monetary base M_t and the money stock $(M1)_t$.
Monetary base is total nominal currency $p_tQ_t = M$ plus total nominal deposits p_tH_t. We can rewrite total nominal deposits as $p_tH_t = \frac{Ht}{Qt}M$. This gives us:

$(M1)_t = p_tQ_t + p_tH_t$
$= M + \frac{Ht}{Qt}M$
$= (1 + \frac{Ht}{Qt})M.$

(l) What is the money multiplier? How does it better help us match evidence in the data compared to our previous money multiplier $\frac{1}{\gamma}$?
The money multiplier is $(1 + H_t/Q_t)$. The term $\frac{Ht}{Qt}$ is the relative demand for deposits to currency. This is something that can fluctuate over time (as demand for deposits and currency fluctuate), which better matches evidence in the data than the multiplier $1/\gamma$, which is very static in reality.

(m) In the data, we observe the following relationships: (i) positive correlation between innovations in the nominal money stock and real output; (ii) innovations in the nominal money stock precede innovations in real output; (iii) positive relationship between changes in interest rates and the nominal money stock. Consider an increase in the productivity of capital from *x* to *x*. Explain how the model is able to generate these observations.
An increase in productivity from x to x' > x decreases the minimum deposit size. In the real sector the increase in the return to capital increases capital investment, which increases the real output of the economy in the next period. In the nominal sector, this increases nominal deposits and reduces nominal currency held which increases the money multiplier and increases the money stock $(M1)_t$ in the period. We see that the unexpected increase in the return to capital results in an increase in the money stock in the same period and an increase in real output tomorrow. These are the documented facts.

2. What is fully-backed central bank money? Why is it important to consider a model with fully-backed central bank money?
Full backed central bank money is money that is backed by a real asset. This is important as it allows the monetary authority (central bank) to pay a return on fiat money. In the economy, this is the case with reserves. The Federal Reserve pays interest on reserves.

3. What are open-market operations?
Open market operations are the method central banks typically execute their monetary policy, which involves the buying and selling of assets in the market. To remove fiat money from the economy, the monetary authority can sell debt (issue bonds?) in the market. Conversely, to expand the money supply or supply liquidity to the market they can sell assets in the market or enter into repurchase agreements to temporarily increase liquidity. This is a measure the Fed has been using to stabilize rates in the repo market.

4. Lemons problem. Here we consider the market for mortgage backed securities before and during the crisis to understand how information played a role in propagating the recession and how quantitative easing (QE) helped to partially solve this problem.

(a) First we consider the pre-crisis period. There is a market for mortgage-backed securities. There are a large number of assets that are each worth \$100. There are a large number of buyers and sellers in the market and here we consider one pair of them. Buyers approach sellers and offer a price *p*. Sellers choose to accept or reject price offers. If the seller accepts, then they receive the price *p* and the buyer receives the good with value *v* = 100. Assume that if the seller is indifferent between selling and not then they will just sell the goods (it is a buyer's market).

i. For any price *p*, what will the seller do?

Given a price, the seller will only sell their asset if the price p is greater than or equal to the value of the asset $p > v_i$. Since the value of all goods is \$100 then the buyer will offer a price 100 and the seller will accept.

ii. Given the optimal strategy of sellers in the previous part, what will a buyer offer as a price?

Buyers offer a price $p^* = 100$.

iii. Does either the seller or buyer have an incentive to change their action given what the other is doing? This equilibrium concept is called a Nash Equilibrium, which some of you may be familiar with.

Neither the buyer or sell has an incentive to deviate. If the buyer offers a higher price then the seller accepts and the payoff to the buyer is less. If they offer a lower price then the offer is accepted and they receive nothing. They are indifferent so they don't have an incentive to change their action. Similarly, the sell has no incentive to deviate. If they choose a different strategy then they can only be worse off (i.e. decline an offer that they should have accepted or accept an offer they shouldn't have).

iv. If there are 1000 pairs of buyers and sellers, how many goods are traded? What is the total dollar amount of all trades?

1000 goods are traded. The total value is $1000 * 100 = 1,000,000$.

(b) Now suppose there is a shock in the housing market and some individuals in the economy stop paying their mortgage (not because they want to, but because they cannot afford to). This affects the quality of MBS in the economy. Now suppose the value of an MBS can take on one of 100 possible values $v_i \in \{1, 2, ..., 99, 100\}$. It can take on each value with the same probability 1/100 = 0.01 = 1%. Sellers know the particular value of their asset because they are receiving payments from it each period so they know the particular value v_i. Suppose that buyers cannot identify the

quality of a particular asset but know that MBS take on each of the 100 values with the same probability. Here we consider a possible trade between a buyer and seller. (Note: the expected value calculations are easier if you include a good with value 0 and assume each occurs with probability 1/101.).

i. From the buyer's perspective, what is the expected value of MBS in the market?
The expected value of the good is \$50.50 = 1 ∗ 0.01 + 2 ∗ 0.01 + ... + 99 ∗ 0.01 + 100 ∗ 0.01.

ii. Suppose you are a seller with a particular asset with value v_i (say v_i = 50 for example). Suppose the buyer offers price *p*. When will you accept the price? When will you not accept the price? Hint: write down the seller's payoff from accepting the price *p* and rejecting the price *p*.
Given a price, the seller will only sell their asset if the price p is greater than or equal to the value of the asset $p > v_i$. This is the same strategy as before.

iii. Suppose the buyer offers a price equal to the expected value of the asset. What is the expected value of the asset they will get *if trade occurs*. Is this a good idea for the buyer? Why or why not?
Only sellers with good with value $v_i \leq 50.50$ will accept. The average value of the good they will get is 25.50 and they pay a price $p = 50.50$. This is not a good trade for the buyer. They should decrease their price.

iv. Use the argument from the previous part (and iterate...called backward induction) to find the optimal price that buyers will offer.
If the buyer offers 25.50 instead then only sellers with goods with value $v_i \leq 25.50$ will accept so the expected value of the good is \$13. This continues and drives the price down to zero. Buyers offer a price $p = 1$ and only sellers with the worst good sell.

v. What value assets are traded? If there are 1000 pairs of buyers and sellers and 1% of sellers have each type of asset, how many trades take place? What is the total dollar amount of all trades?
Only 10 goods are traded and the total nominal value of the trades is \$10.

vi. What does the answer in the previous part say about liquidity in the market for MBS in the financial crisis?
With asymmetric information prices fall and liquidity dries up. This is what happened in the market for MBS.

vii. Quantitative easing refers to the purchase of distressed assets in the market by the central bank through open market operations. If you are the central bank, how would you solve this liquidity problem through the use of open market operations?

The goal here is to change the beliefs of buyers about the distribution of asset quality in the market. This can be done by buying low quality assets in the market through open-market operations. If this is done then buyers have more certainty about the types of assets in the market, prices will increase and liquidity will improve. More trades will take place. The key is that the central bank needs to change the belief about the distribution so that individuals believe they are very likely to get a good asset.