# ECN 506
# A Simple Model of Money

Consider an economy where individuals are endowed with $y$ units of the consumption good when young and zero units of the consumption good when old. Suppose the population is growing over time at a constant rate. In particular, assume that $N_{t+1} = nN_t$ for all $t \geq 0$ where $n > 1$. If it makes it easier, you can assume $n$ is equal to a number larger than 1 (e.g. set $n = 2$).

**Part I. Planner's problem**

1. **Find the planner's resource constraint in period *t* (do not assume a stationary equilibrium).**

   There are $N_t$ young individuals in period $t$ and each is endowed with $y$ units of the consumption good. There are $N_{t-1}$ old individuals in period $t$ and each is endowed with zero units of the consumption good. Therefore, the total endowment of the consumption good available to the planner is: $N_t y$. The planner chooses consumption of the young $c_{1,t}$ and consumption of the old $c_{2,t}$ in period $t$. Again, there are $N_t$ young in period $t$ and there are $N_{t-1}$ old in period $t$. The total consumption is therefore: $N_t c_{1,t} + N_{t-1} c_{2,t}$. We are told that the economy is growing at rate $n > 1$ and $N_{t+1} = nN_t$. This implies that $N_t = nN_{t-1}$, which can be rewritten as $N_{t-1} = \frac{1}{n} N_t$. The planner's resource constraint is therefore:

$$N_t c_{1,t} + N_{t-1} c_{2,t} \leq N_t y$$

$$N_t c_{1,t} + [\tfrac{1}{n}] N_t c_{2,t} \leq N_t y$$

$$c_{1,t} + [\tfrac{1}{n}]\, c_{2,t} \leq y$$

**2. What is the planner's resource constraint in a stationary equilibrium?**

In a stationary equilibrium we have $c_{1,t} = c_1$ for all $t \geq 1$ and $c_{2,t} = c_2$ for all $t \geq 1$. Substituting this into the planner's resource constraint from the previous part we get:

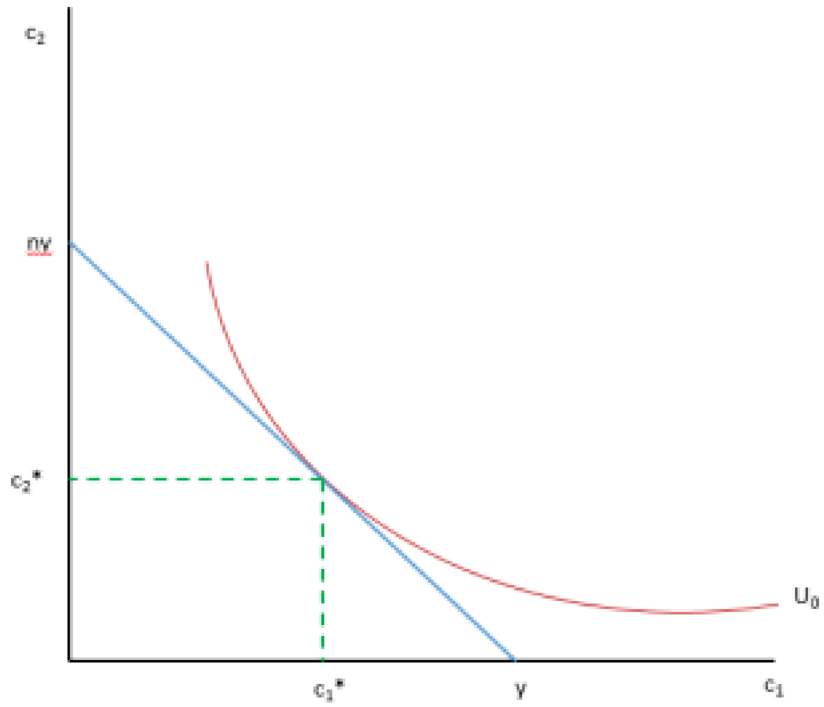
$$c_1 + \left[\frac{1}{n}\right] c_2 \leq y$$

**3. Draw a figure depicting the Golden Rule allocation. If you prefer, you can solve for the planner's optimal allocation using the utility function:**

$$u(c_{1,t}, c_{2,t+1}) = \ln c_{1,t} + \beta \ln c_{2,t+1}$$

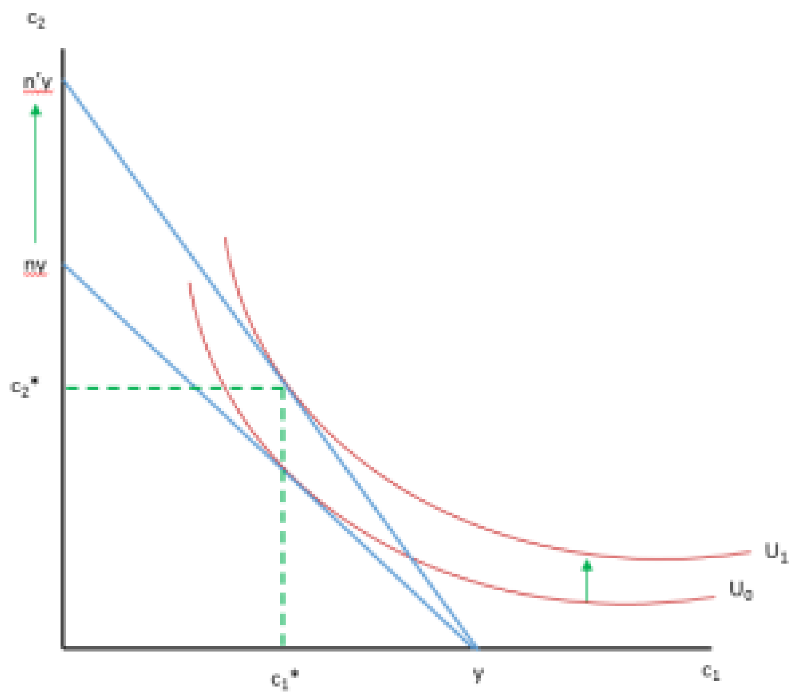
**where $\beta \in (0, 1)$.**

Referring to the planner's resource constraint from the previous part, the $x$ intercept on the planner's resource constraint is $c_1 = y$, which is obtained by setting $c_2 = 0$. The $y$ intercept on the planner's resource constraint is $c_2 = ny$, which is obtained by setting $c_1 = 0$. The figure below depicts the planner's resource constraint and a typical solution.



**4. How does an increase in *n* affect the Golden Rule allocation? That is, how does ($c^*_1$, $c^*_2$) change? Support your answer.**

As $n$ increases, the $c_2$ intercept in the planner's resource constraint increases. This reflects the fact that when $n$ increases the economy is growing at a faster rate and there is more of the endowment good available to be allocated. This increase in $n$ makes the future generations better off as there is more of the endowment good available to be consumed in each period. The figure below depicts the change when $n$ increases to $n$.



## Part II. Decentralized economy

1. **Suppose record-keeping is costless. Find an individual's lifetime budget constraint.**

   To find an individual's lifetime budget constraint, we start with the young budget constraint. The transfer made by a young individual in period $t$ is denoted by $\Phi_t$ and the transfer received when old is denoted by $\Phi^R_{t+1}$. Further, we define $\Phi^R_t = x_{t+1}\Phi_t$ where $x_{t+1}$ is the rate at which a transfer made when young is converted into a transfer received when old. Recall that $x_t$ is determined by setting the supply of transfers equal to the demand of transfers in each period.

   We start by writing down the young budget constraint. The young budget constraint is:

   $$c_{1,t} + \Phi_t \leq y.$$

   The old budget constraint is:

   $$\begin{aligned} c_{2,t+1} &\leq \Phi^R_{t+1} \\ &= x_{t+1}\Phi_t \end{aligned}$$

   We can combine the young and old budget constraints to get:

   $$c_{1,t} + \frac{c_{2,t+1}}{x_{t+1}} \leq y$$

   As mentioned, $x_{t+1}$ is determined by setting the supply of transfers equal to the demand for transfers. There are $N_t$ young individuals in period $t$ and there are $N_{t-1}$ old individuals. Furthermore, we have that $N_t = nN_{t-1}$, which can be rewritten as $N_{t-1} = \frac{1}{n}N_t$. The supply of transfers is $N_t\Phi_t$, but from the young budget constraint, $\Phi_t = y - c_{1,t}$, so the supply of transfers is $N_t(y - c_{1,t})$. The demand for transfers is

   $$\begin{aligned} N_{t-1}\Phi^R_t &= [\tfrac{1}{n}]N_t\Phi^R_t \\ &= [\tfrac{1}{n}]N_t x_t \Phi_{t-1} \\ &= [\tfrac{1}{n}]N_t x_t (y - c_{1,t-1}) \end{aligned}$$

Setting supply of transfers equal to demand for transfers we get:

$$N_t(y - c_{1,t}) = [\tfrac{1}{n}]N_t x_t (y - c_{1,t-1})$$

In a stationary equilibrium $c_{1,t} = c_1$ for all $t \geq 1$ so we get:

$$N_t(y - c_1) = [\tfrac{1}{n}]N_t x_t(y - c_1)$$
$$\Rightarrow x_t = n$$

Substituting into the lifetime budget constraint we get:

$$c_1 + [\tfrac{1}{n}]c_2 \leq y$$

The individual's lifetime budget constraint is the same as the planner's resource constraint when the economy is growing at rate $n$ and there is perfect and costless record-keeping.

**2. Now suppose that record-keeping is prohibitively costly. What is ($c^*_{1,t}$, $c^*_{2,t+1}$) for an individual born in period *t*?**

When record-keeping is prohibitively costly there is no way to sustain trade between individuals and the equilibrium is autarky. That is, each individual consumes their endowment $c_1 = y$ when young and consumes $c_2 = 0$ when old. This results from the fact that a young individual does not have a trading partner that has the good that they want when they are old. Other young individuals also don't have the good when old and an old individual is no longer in the economy in the next period.

**3. Suppose a fixed supply of money is introduced into the economy. Write down an individual's lifetime budget constraint and solve for the rate of return of money.**

This is very similar to the previous part and we follow the same steps. We start by writing down the young budget constraint. A young individual in period *t* chooses their consumption $c_{1,t}$ and the amount of real money to hold $v_t m_t$, where $m_t$ is the units of money the individual acquires and $v_t$ is the value of a unit of money in terms of the consumption good in period *t*. The young budget constraint is:

$$c_{1,t} + v_t m_t \leq y.$$

When old, the individual's $m_t$ units of money is now worth $v_{t+1}m_t$ units of the consumption good. The old budget constraint is:

$$c_{2,t+1} \leq v_{t+1}m_t.$$

Rearranging the old budget constraint and combining it with the the young budget constraint we get:

$$C_{1,t} + \frac{v_t}{v_{t+1}} C_{2,t+1} \leq y.$$

This is our usual budget constraint with money. From here, we need to find the real rate of return of fiat money. To do this we set the demand for money equal to the supply of money in period $t$ to get an expression for $v_t$. Note: we will get the same expression we derived in class, but it is good practice to derive it each time you are asked.

The real supply of money in period $t$ is $v_tM_t$. The real demand for money in period $t$ is $N_t(y - c_{1,t})$. That is, there are $N_t$ young individuals in period $t$ demanding money and each demands $y - c_{1,t}$ real money. Setting these equal to each other and rearranging we get:

$$\frac{v_{t+1}}{v_t} = \left(\frac{N_{t+1}(y - c_{1,t+1})}{M_{t+1}} / \frac{N_t(y - c_{1,t})}{M_t}\right)$$

In a stationary equilibrium $c_{1,t} = c_1$ for all $t \geq 1$ so we get the expression for the real rate of return of fiat money as:

$$\begin{aligned}\frac{v_{t+1}}{v_t} &= \left(\frac{N_{t+1}(y - c_1)}{M_{t+1}} / \frac{N_t(y - c_1)}{M_t}\right)\\ &= \left(\frac{N_{t+1}}{N_t} / \frac{M_{t+1}}{M_t}\right)\\ &= n\end{aligned}$$

The individual's lifetime budget constraint therefore becomes:

$$c_1 + [\tfrac{1}{n}]c_2 \leq y$$

**4. Suppose the money supply grows at the same rate as the population and the revenue from the issuance of new money is used to finance a government purchase that does not benefit the individuals in our economy. Write down the individual's lifetime budget constraint and solve for the rate of return of money.**

In this case, individuals don't benefit from the expansion of the money supply in that they don't receive a transfer from the government when they are old. As a result the lifetime budget constraint remains:

$$C_{1,t} + [\frac{v_t}{v_{t+1}}]C_{2,t+1} \leq y.$$

However, the real rate of return of fiat money will change. Denote the growth rate of money by $z$ so that $M_{t+1} = zM_t$. If the growth of money is equal to population growth then $z = n$. In a stationary equilibrium the expression for the real rate of return of money is unchanged as is given by

$$\begin{aligned}\frac{v_{t+1}}{v_t} &= \left(\frac{N_{t+1}(y-c_1)}{M_{t+1}} / \frac{N_t(y-c_1)}{M_t}\right)\\ &= \left(\frac{N_{t+1}}{N_t} / \frac{M_{t+1}}{M_t}\right)\\ &= \frac{n}{z} = 1\end{aligned}$$

Therefore the lifetime budget constraint becomes:

$$c_1 + c_2 \leq y.$$

Note, that this is not the same as the planner's resource constraint in a growing economy.

**5. Suppose there is no money in the economy, but the initial old are endowed with a stock of gold that doesn't change over time. Find the value of gold and the individual's lifetime budget constraint.**

We can follow the exact same process as we did in lecture. If we do so, we get the value of gold as:

$$v_t^g = \frac{N_t(y - c_1)}{M^g}$$

The individual's lifetime budget constraint is given by:

$$C_{1,t} + [\frac{v_t^g}{v_{t+1}^g}]C_{2,t+1} \leq y$$

Given the expression for $v^g_t$ above, we get the real rate of return of gold as:

$$\frac{v_{t+1}^g}{v_t^g} = \left(\frac{N_{t+1}(y - c_1)}{M^g} / \frac{N_t(y - c_1)}{M^g}\right)$$
$$= \left(\frac{N_{t+1}}{N_t}\right)$$
$$= n = 1$$

Therefore the individual's lifetime budget constraint becomes:

$$C_1 + [\frac{1}{n}]C_2 \leq y$$

**6. Explain what is meant by the intrinsic value of gold *v*˜.**

The intrinsic value of gold is the amount of the consumption good that must be consumed to give an individual the same utility as consuming one unit of gold. This value tells us whether an individual should trade or consume their gold.

**7. Suppose the value of gold is greater than *v*˜. Will the equilibrium involve the consumption of gold? Why or why not?**

The equilibrium will not involve the consumption of gold. For any unit of gold, an individual can trade the gold for the consumption good at the rate $v^g$ and consume this consumption good, which will give them more utility than consuming their gold by the definition of ˜*v*.

**8. Suppose you are the government that oversees the economy and you have a choice between the monetary economy in part (3) or the commodity money**

**equilibrium in part (7), what would you choose? Explain and support your answer.**

The monetary equilibrium is more efficient and is preferred. The reason is that in the equilibrium where gold is traded, there is a consumption good that is valued that is not consumed as it is being used as a unit of trade. By introducing money into the same economy, we can get the same equilibrium plus the initial old are able to consume their gold, which makes them strictly better off.