

## ECN 506

### Central Banking and Money Supply

This problem set considers the model of liquidity, banking, reserves and lending presented in class. Individuals in the economy live for three periods and in each period  $N_t = nN_{t-1}$  young individuals are born. That is, the number of young born in each period grows at the rate  $n > 1$ . Denote the stock of fiat money in the economy in period  $t$  by  $M_t$  and suppose that the supply of fiat money grows at the rate  $z > 1$  such that  $M_t = zM_{t-1}$ . The young are endowed with  $y$  units of time and use all of this time to work. Each unit of labor produces one unit of the consumption good so that each young individual is endowed with  $y$  units of the consumption good when young. The individual is endowed with nothing when they are in middle-age and old. The consumption good cannot be stored across periods.

Individuals care about consuming in each period of their life and have two assets they can acquire. First, they can sell the consumption good to acquire fiat money at the price  $p_t = \frac{1}{v_t}$  in period  $t$ . Second, the individual can convert the consumption good into units of capital, where each unit of the consumption good can create one unit of capital. Capital is illiquid. That is, if an individual creates a unit of capital when young, it does not mature until the individual is old. Denote the two-period return on capital by  $X$  and the one period return on capital by  $x = X^{0.5}$ . Individuals cannot directly trade unmatured capital with each other.

1. First consider an economy without banks so that a young individual can consume, acquire money or invest in capital.

**(a) Suppose a young individual uses one consumption good to acquire money when young. How many consumption goods does the individual receive in middle-age from the acquired money?**

The individual receives the real rate of return on money:  $\frac{v_{t+1}}{v_t}$ . When young

the one unit of consumption good allows the individual to acquire  $p_t = \frac{1}{v_t}$  units of fiat money. In the second period the individual can purchase  $v_{t+1} * \frac{1}{v_t}$  consumption goods. The real rate of return on money is therefore:  $\frac{v_{t+1}}{v_t} = \frac{n}{z}$ .

**(b) How many consumption goods does the individual receive when old if they decide to hold the money for two periods until they are old?**

In each period the individual receives the real rate of return of money. After holding the money for one period the individual receives  $\frac{v_{t+1}}{v_t}$  consumption goods. If they use these consumption

goods to acquire money in middle-age, they receive  $\frac{v_{t+1}}{v_t} * p_{t+1} = \frac{v_{t+1}}{v_t} * \frac{1}{v_{t+1}} = \frac{1}{v_t}$  units of money. This makes sense.

This is the same amount of money they would have if they did not convert their money to goods at the start of the period. In the next period this money provides

$$\frac{1}{v_t} * v_{t+1} = \frac{nu_{t+2}}{v_{t+1}} * \frac{v_{t+1}}{v_t} = \left(\frac{n}{z}\right)^2 \text{ consumption goods. The two-period return on money is: } \left(\frac{n}{z}\right)^2.$$

**(c) Suppose a young individual uses one consumption good to create capital when young. How many consumption goods does this capital provide to the individual in middle-age?**

None. Capital takes two periods to mature and individuals are unable to trade unmatured capital.

**(d) How many consumption goods does the individual receive from the capital when they are old?**

If an individual uses one consumption good to create capital, this yields  $X$  units of capital when old.

**(e) Using the answers to the previous parts, explain why fiat money and capital can co-exist in this economy in equilibrium.**

Fiat money provides a lower two-period real return, but is more liquid. Capital has a higher real return, but individuals can only realize this return after two period. That is, capital is relatively illiquid.

**(f) Referring to the above parts, explain what is meant when we say that individuals have a liquidity mismatch problem in this economy.**

Individuals need to acquire fiat money to consume in middle-age, which has a relatively low real rate of return. Ideally, individuals could put all of their deposits in capital and somehow access this higher return in middle-age, however this is not possible as unmatured cannot be purchased or sold.

**2. Now we introduce banks that will act as liquidity providers in the economy. Suppose that banks are infinitely lived and are able to issue private IOU's, such that individual deposit assets with the bank and the bank can promise a return on the deposit. We start by assuming that there is no reserve requirement or lending by the Central Bank.**

**(a) Suppose a young individual wants to use one consumption good to acquire money. What return does the bank need to promise the individual to have them deposit the good with the bank instead?**

The bank needs to promise a one-period return on deposits at least as large as the real rate of return on money  $n/z$ .

**(b) Suppose that when the individual deposits a good with the bank, the bank uses this good to create capital. Further, suppose that when the bank offers a return on deposits that is equal to the real rate of return on money then the individual will choose to deposit with the bank instead of acquiring money. An young individual in period  $t$  deposits one good with the bank when young. Suppose no young individuals in period  $t + 1$  make deposits. How does the bank pay the young individual from  $t$  their promised return?**

The bank is able to pay returns from two sources, deposits and matured capital. Since no individuals are making deposits in  $t + 1$ , the bank would need to rely on capital that was created two periods ago (which we have not mentioned or discussed). If the bank is solely relying on deposits to repay this individual, then they would be unable to fulfill the obligation. However, there were likely individuals that made deposits in  $t-1$  with which the bank created

capital. The bank could use this matured capital to repay the individual on their deposit.

**(c) Suppose a young individual from period  $t$  deposits  $a_z$  goods. What does the bank do with the deposit? What return does the bank need to promise the individual on their deposit?**

The bank can use the deposit to repay the individual who deposited one good in period  $t$  as this is exactly the amount they owe that individual. On the deposit of  $n/z$  goods the bank must promise a return that is at least as large as the real rate of return on money  $v_{t+2}/v_{t+1} = n/z$ .

**(d) In period  $t + 2$  the bank owes the young person from the previous period (the one who deposited  $a_z$  goods) a return on their deposit. How do they pay this return?**

Again the bank has two sources of consumption goods to repay the deposit. First, there may be individuals depositing goods in  $t + 2$ . The bank can use these deposited goods to fulfill their obligation. Alternatively, the bank has capital that it created in period  $t$  that is maturing and producing  $X$  goods per unit of capital. The bank can also use these goods to fulfill its obligation.

**(e) In period  $t + 2$ , what is the “profit” of the bank? Suppose individual in the economy own the bank and profits in each period are paid as dividends. Are individuals better or worse off with the bank present?**

If the bank is offering a return on deposits equal to the real rate of return on fiat money then from the one consumption good deposit in period  $t$ , it owes  $(n/z)^2$  goods in period  $t + 2$ . To pay this the bank has capital that matures and produces  $X$  consumption goods. Therefore the net gain is  $X - (n/z)^2 > 0$  by assumption.

**(f) Suppose now that the individual in period  $t + 1$  deposits  $a_z + 1$  goods instead of just  $a_z$  goods. What does the bank do with this deposit? How many consumption goods does the bank owe this individual on their deposit in period  $t + 2$ ?**

The bank owes the individual who deposited in period  $t$   $(n/z)$  goods. The bank can use  $n/z$  goods from the deposit to fulfill its obligation to the individual from period  $t$  and create one unit of capitals with the additional unit of consumption good that the individual deposited. Effectively, on this additional unit of deposit this puts us back into the same situation except with an individual depositing an additional good in period  $t + 1$ . The bank owes the individual

$$[(n/z) + 1] \frac{v_{t+1}}{v_t} = [(n/z) + 1] * (n/z) \text{ consumption goods in period } t + 2.$$

**(g) How does the bank pay the individual the return on the  $a_z + 1$  deposit in period  $t + 2$ ?**

The bank pays the return in period  $t + 2$  through a combination of matured capital (created in period  $t$ ) and new deposits (in  $t + 2$ ).

**(h) Is there demand for fiat money in this economy?**

There is no demand for fiat money in this economy. As a result, it is hard to say what the “real rate of return on fiat money” is since it is not demanded.

**3. Now to ensure there is demand for fiat money, the central bank introduces a reserve requirement  $\gamma$ , which says that for every unit of deposit, the bank must keep a proportion  $\gamma$  of the deposit as fiat money. Suppose  $\gamma = 0.10$ .**

**(a) Repeat the parts of the previous question, now considering the reserve requirements.**

**(b) What is the total deposit in period  $t$ ? What is held as reserves? Is the reserve requirement satisfied?**

In period  $t$  there is one good deposited. A fraction  $\gamma$  of these goods are held as reserves. Yes the reserve requirement is satisfied.

**(c) What is the total deposit in period  $t + 1$ ? What is held as reserves? Is the reserve requirement satisfied?**

In period  $t+1$  there are  $n/z$  goods deposited. In period  $t$  the bank held  $\gamma$  goods as reserves, which it converts into  $\gamma p_t = \gamma \frac{1}{v_t}$  units of money. In period  $t$  it can use this  $\gamma \frac{1}{v_t}$  units of fiat money

to obtain  $v_{t+1} v_{t+1} * \gamma (\frac{1}{v_t}) = \gamma \frac{v_{t+1}}{v_t} = \gamma (\frac{n}{z})$  units of the

consumption good, which it now holds as reserves. The reserve requirement is satisfied as reserves are a fraction  $\gamma$  of total deposits.

**(d) What is the maximum return the bank can offer individuals on their deposits?**

The bank earns a one-period return equal to  $r = (1 - \gamma)x + \gamma(n/z)$  so the bank can pay this return.

**(e) Is output of the economy greater with or without the reserve requirement? If the reserve requirement reduces investment and output, why would a central bank choose to implement one?**

Without investment in capital and capital output decreases as the reserve requirement increases.

**(f) Now suppose the central bank allows banks to borrow up to  $\delta = 0.5$  of their required reserves from the central bank and charges interest on the loan that is less than the one-period return on capital  $\psi < x = X^{0.5}$ . Does the bank choose to borrow from the central bank?**

The bank will choose to borrow from the central bank. Now for each consumption good deposited, the bank keeps proportion  $\gamma$  in fiat money and receives the rate of return on fiat money  $n/z$  and directly invests proportion  $1 - \gamma$  in capital and receives one period return  $x = X^{0.5}$ . Additionally the bank borrows  $\delta\gamma$  from the central bank, invests it in capital and receives the one-period return  $x$  and pays the rate  $\Psi$  for borrowing. The bank has:

$$\gamma(n/z) + (1 - \gamma)x + \delta\gamma(x - \Psi)$$

Clearly as long as the one-period return on capital,  $x$ , is greater than the cost of borrowing,  $\Psi$ , then the bank should borrow from the central bank as the average return increases.

**(g) By how much does investment increase with central bank lending?**

As per above, investment in capital increases by  $\delta\gamma$ .

**(h) What is the maximum return the bank is able to offer individuals on their deposits?**

The maximum return is given above.