ECN 506

Inflation, Barter and Commodity Money

7 Exercises

4.1. Let $N_t = nN_{t-1}$ and $M_t = zM_{t-1}$ for every period t, where z and n are both greater than 1. The money created each period is used to finance a lump-sum subsidy of a_t^* goods to each *young* person.

a. Find the equation for the budget set of an individual in the monetary equilibrium. Graph it. Show an arbitrary indifference curve tangent to the set and indicate the levels of c_1 and c_2 that would be chosen by an individual in this equilibrium.

b. On the graph you drew in part a, draw the feasible set. Take advantage of the fact that the feasible set line goes through the monetary equilibrium (c_1^*, c_2^*). Label your graph carefully, distinguishing between the budget and feasible sets.

c. Prove that the monetary equilibrium does not maximize the utility of the future generations. Support your assertion with references to the graph you drew of the budget and feasible sets.

4.2. Consider an economy with a shrinking stock of fiat money. Let $N_t = N$, a constant, and $M_t = zM_{t-1}$ for every period t, where z is positive and less than 1. The government taxes each old person τ goods in each period, payable in fiat money. It destroys the money it collects.

a. Find and expl...

a). We know that z and n are > 1. Therefore there is a growth n in the population.

Z ~~is~~ shows that the inflation rates or that the price of goods in the future will be more giving me ~~less~~ incentive to work today.

young budget constraint

$$C_{1_t} + \Phi_t \le y + a_t^*$$

y transfers to old as Φ^R_{t+1} or $V_{t+1} M_t$

or $C_{1_t} + V_t M_t \le y + a_t^*$

Old budget const.

$C_{2t+1} \le V_{t+1} M_t$ → transferred to old

$$\frac{C_{2t+1}}{V_{t+1}} \le M_t$$

$$C_{2t+1} \cdot \frac{V_t}{V_{t+1}} \le V_t M_t$$

Therefore the budget is

$$C_{1t} + C_{2t+1}\left(\frac{v_t}{v_{t+1}}\right) \leq y + a^*_t$$

Since $\frac{v_{t+1}}{v_t} = \frac{n}{z}$; we get new eqn

$$C_{1t} + C_{2t+1} \times \frac{z}{n} \leq y + a^*_t$$

$$C_{1t} + C_{2t+1}\frac{z}{n} - y \leq a^*_t$$

Demand for real money

$$M_t - M_{t-1} = M_t - \frac{M_t}{z} = M_t\left[1-\frac{1}{z}\right] = \text{new money}$$

Demand $= M_t V_t \left[1-\frac{1}{z}\right]$ | Supply $= N_t a_t^*$

~~Deman~~ Market clearing condition
occurs at demand = Supply

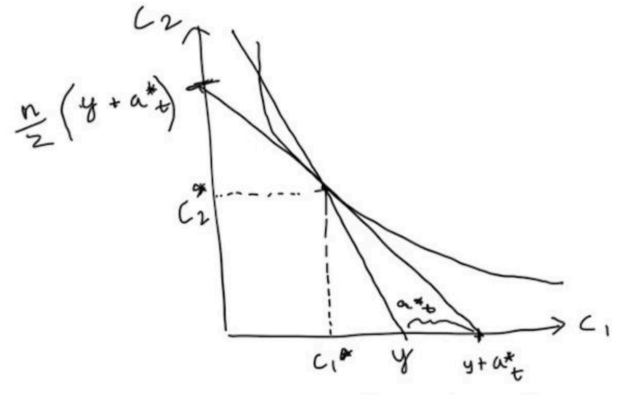
$$M_t V_t\left[1-\frac{1}{z}\right] = N_t a^*_t$$

$$a_t = \frac{M_t V_t\left[1-\frac{1}{z}\right]}{N_t}$$

Cal. return of money $= \frac{v_{t+1}}{v_t}$

Tot. dmd. for mony $= N_t V_t M_t = N_t(y + a_t^* - c_1)$

Indifference Curve



When ; Lifetime budget curve $= c_1 + c_2\left(\frac{v_t}{v_{t+1}}\right) \leq y + a_t^*$

$$= c_1 + c_2\frac{z}{n} \leq y + a_t^*$$

2. Textbook 4.2

(a) The real rate of return of money is given by $\frac{v_{t+1}}{v_t}$ So we need to find an expression for v_t, which is the value of a unit of money in period t in terms of units of the consumption goods. This is a "price", which we find by equating the supply of real money and demand for real money in period t. We start by writing down the young budget constraint. In period t, a young individual chooses consumption $c_{1,t}$ and real money to acquire given by $v_t m_t$, where m_t is the units of money the individual acquires. The young budget constraint is:

$$c_{1,t} + v_t m_t \leq y,$$

which will hold with equality (since the young individual will not waste goods). The term $v_t m_t$ is the real money demanded by a single individual and rearranging the young budget constraint can be written as:

$$v_t m_t = y - c_{1,t}$$

There are N_t young in period t, so the total money demanded in period t by the young is:

$$N_t v_t m_t = N_t(y - c_{1,t}).$$

The real supply of money in period t is $M_t v_t$, so setting supply of real money equal to demand of real money we get:

$$M_t v_t = N_t(y - c_{1,t})$$

$$\Rightarrow v_t = \frac{N_t(y - c_{1,t})}{M_t}$$

This is the same formula we always find for the value of money in a period. The value of a single unit of money in terms of the consumption good is equal to the total demand for real money divided by the total units of money in period t.

The real rate of return of money is therefore:

$$\frac{v_{t+1}}{v_t} = \left(\frac{N_{t+1}(y - c_{1,t+1})}{M_{t+1}} / \frac{N_t(y - c_{1,t})}{M_t}\right)$$

In a stationary equilibrium we have $c_{1,t} = c_1$ for all t, so this becomes:

$$\begin{aligned}
\frac{v_{t+1}}{v_t} &= \left(\frac{N_{t+1}(y - c_{1,t+1})}{M_{t+1}} / \frac{N_t(y - c_{1,t})}{M_t}\right) \\
&= \left(\frac{N_{t+1}}{M_{t+1}} / \frac{N_t}{M_t}\right) \\
&= \left(\frac{N_{t+1}}{N_t} / \frac{M_{t+1}}{M_t}\right) \\
&= \frac{1}{z}
\end{aligned}$$

This is greater than one since the money supply is shrinking ($z < 1$). Note that this is the same expression for the real rate of return of money that we have found many times. I showed all the steps here, but you may skip some of these steps if you are comfortable doing so. The result is also intuitive. Since the population is staying constant and the supply of money is decreasing then the real demand for money will be constant over time and at the same time there are less units of money that can be used to acquire goods. This means that the value of money should be increasing over time, which is what $\frac{v_{t+1}}{v_t} > 1$ means.

(b) We know the planner's solution maximizes the utility of the future generations. Since the planner and individuals maximize the same utility function then if we want to show that the monetary equilibrium does not maximize the utility of the future generations then we simply need to show that an individual's lifetime budget constraint is not the same as the planner's resource constraint. We have already found the young budget constraint and the real rate of return of money. The next step is to write down the old budget constraint. An old individual that was born in period t chooses consumption $c_{2,t+1}$. When they were young they acquired m_t units of money and these are each worth v_{t+1} of the consumption good. Finally, in the question it says that the old are taxed τ units of the good. The way this works in our economy is the following. In period $t + 1$, the government taxes each old individual τ units of the consumption good. The government then takes this consumption good and sells it in the economy for money, which it then destroys. This is how the government decreases the money supply. Going back to the old budget constraint we get:

$$C_{2,t+1} \leq v_{t+1} m_t - \tau$$

$$\Rightarrow \frac{C_{2,t+1} + \tau}{v_{t+1}} \leq m_t$$

Combining this with the young budget constraint we get:

$$C_{1,t} + \frac{v_{t+1}}{v_t} C_{2,t+1} \leq y - \frac{v_{t+1}}{v_t} \tau$$

We already solved for the real rate of return of money in the previous part, which is $1/z$ so the lifetime budget constraint in a stationary equilibrium is:

$$c_1 + zc_2 \leq y - z\tau.$$

Now let's write down the planner's constraint. In period t there are N_t young and N_{t-1} old. Each young consumes $c_{1,t}$ and each old consumes $c_{2,t}$. Furthermore, each young is endowed with y units of the consumption good and each old is endowed with nothing. Consider a stationary equilibrium again. The planner's resource constraint is:

$$N_t c_{1,t} + N_{t-1} c_{2,t+1} \leq N_t y$$

$$\Rightarrow c_1 + c_2 \leq y$$

This is not the same as the individual's budget constraint so clearly the monetary equilibrium does not maximize the utility of the future generations (i.e. it is inefficient). You can draw these constraints to help interpret the result. Since the value of money is increasing individuals will want to acquire more money when young relative to the planner's solution. Therefore, in the monetary equilibrium the old consume more than they do in the planner's solution. As a result the initial old prefer this monetary equilibrium to the planner's solution (part c).

2. Textbook 4.3

(a) We are told there is a constant population with N = 1000. Furthermore, we are told that the money supply doubles in each period and is initially M_0 = 10000. This means that $M_1 = 2M_0 = 2(10000) = 20000$. The acquired goods are given to the old in each period and I denote the amount given to each old in period t by a_t. Each young is endowed with y = 20 units of the consumption good and nothing when old. Preferences are such that individuals always want to save 10 units of the consumption good at the equilibrium value of money. Since the endowment y doesn't change then we have c_1 = 10 for every period.

To find the real rate of return of money, $\frac{v_{t+1}}{v_t}$ we need to find an expression for v_t using supply and demand of real money. To do this, we can follow exactly what we did in part (a) of the previous question and we get the real rate of return of money:

$$\begin{aligned}\frac{v_{t+1}}{v_t} &= \left(\frac{N_{t+1}}{M_{t+1}} / \frac{N_t}{M_t}\right)\\ &= \left(\frac{N_{t+1}}{N_t} / \frac{M_{t+1}}{M_t}\right)\\ &= \frac{1}{z}\\ &= \frac{1}{2}\end{aligned}$$

(b) To find how many goods each old person receives, we need to write down the government's budget constraint. Since the money supply doubles in each period, we have that $M_t = 2M_{t-1}$ or that $M_{t-1} = \frac{1}{2}M_t$. The total units of money created by the government is:

$$\begin{aligned}M_t - M_{t-1} &= M_t - \frac{1}{2}M_t\\ &= (1 - \frac{1}{2})M_t\\ &= \frac{1}{2}M\end{aligned}$$

The real value of goods acquired is therefore the amount of money created times the value of money: $\frac{1}{2}M_t v_t$. We need to find out what the value of money is v_t, but we solved for this in part (a) of the previous question so we have:

$$\begin{aligned}v_t &= \frac{N_{t+1}(y - c_{1,t})}{M_t}\\ &= \frac{1000(20-10)}{M_t}\\ &= \frac{10000}{M_t}\end{aligned}$$

The supply of money, M_t is different in each period so here we can pick a period and plug in the amount of money for that period. Instead I will leave it in terms of M_t and will show that it drops out of my expression for a_t. Therefore the total goods acquired by the government by expanding the money supply is:

$$
\begin{aligned}
(1 - \tfrac{1}{z})M_t v_t &= (1 - \tfrac{1}{2})M_t v_t \\
&= \tfrac{1}{2}M_t \cdot \tfrac{10000}{M_t} \\
&= \tfrac{1}{2}(10000) \\
&= 5000
\end{aligned}
$$

The government transfers the acquired goods to the old. In period t there are $N_{t-1} = 1000$ old and the total amount of the consumption good transferred to the old by the government is $1000a_t$, where each receives a_t units. We find a_t by writing down the government's budget constraint, which says that the total goods acquired by expanding money is equal to the total transfer to the old. This means that:

$$
\begin{aligned}
N_{t-1}a_t = 1000a_t &= (1 - \tfrac{1}{z})M_t v_t \\
&= 5000,
\end{aligned}
$$

which means that:

$$a_t = \frac{5000}{1000} = 5.$$

You can also arrive at this answer by plugging in all the given numbers from the beginning, or deriving everything in terms of the variables and then inputting the given values at the very end. Note, the way I found this, I considered any period t, which means that this is the amount of goods transferred to the old regardless of the period. This makes sense. As the money supply grows the value of money decreases over time, which means that more money must be introduced by the government to acquire the same amount of goods. However, given the goods acquired by the government, the amount of the consumption goods given to each old person is constant over time.

(c) The price of the consumption goods in period 1 is the inverse of the real value of money in period 1, v_1. From part (b), the real value of money in period t is given by:

$$v_t = \frac{10000}{M_t}$$

Initially $M_0 = 10000$ and in each period the money supply doubles, so in period $t = 1$ we have $M_1 = 20000$. Therefore we get:

$$v_1 = 10000/20000 = 1/2$$

The price of the good in period 1 is the inverse of the value of money, $p_t = \frac{1}{v_t}$ and therefore we have:

$$
\begin{aligned}
p_t &= \frac{1}{v_t} \\
p_t &= \frac{1}{v_t} = 1/0.5 = 2.
\end{aligned}
$$

3. Explain what is meant by the term "inflation tax". Use budget constraints and figures to support your answer. If it helps, you can assume we are in the simple economy where money grows at the rate $z > 1$, the population is constant, the young are endowed with y units of the consumption good, and the old are endowed with zero units of the consumption good.

When the government or central bank introduces new money this is a form of revenue generation known as "seigniorage". However, the government is not able to generate this revenue from nothing, and therefore the revenue generated by the expansion of money (in real terms) must come from somewhere. The real revenue generated by the expansion of money comes from the existing holders of money, which is devalued when new money is introduced. This is intuitive. Suppose you work when you are young and are paid 10000 units of money. If the government then doubles the amount of money in circulation in the next period, then your 10000 units of money are now going to be worth half as much. The government's gain is your loss. When the stock of money is increased at the rate $z > 1$, the value of money decreases at the rate ^{1}z and at the same time, prices of goods increase at the rate $z > 1$. Since prices are increasing, there is inflation, which is the inverse of the rate of return of money. An inflation tax refers to the devaluation of existing money when there is an increase in the money supply/inflation.

4. Describe and explain the relationship between the real rate of return of fiat money and inflation. How does a growing money supply affect inflation? You can assume the same economy as the previous question. The real rate of return of money $\frac{v_{t+1}}{v_t}$ and inflation $\frac{p_{t+1}}{p_t}$ are inverses of each other. The value of money in period t is v_t, which is a measure of the value of a single unit of money in terms of units of the consumption goods. Conversely, the price $p_t = \frac{1}{v_t}$ is the units of money required to buy a single unit of the consumption good. We have:

$$\frac{v_{t+1}}{v_t} = \frac{1}{\left(\frac{p_{t+1}}{p_t}\right)}$$

Consider the case where the money supply grows at rate $z > 1$ and the population is constant. The real rate of return of money is $\frac{v_{t+1}}{v_t} = \frac{1}{z}$, while inflation is equal to $\frac{p_{t+1}}{p_t} = z$. As the stock of money increases, the value of each unit of money decreases so we have a decreasing value of money. Conversely, as the stock of money increases, the price of a unit of consumption good in terms of units of money increases, so there is inflation.

5. Explain why inflation is not an efficient tax. Support your answer with the use of budget constraints and figures. You can assume the same economy as question 2.
This solution follows the lecture notes at the end of lecture 3 or the beginning of lecture 4 and uses the same figures. Using the labor/time interpretation of the model, an inflation tax is not efficient because it decreases the return to work for young individuals and causes them to choose inefficiently high leisure when they are young (c_1 in the monetary equilibrium is inefficiently high). This is intuitive. If you work, you produce a good that you sell in the market for units of money. In the next period, this money can buy fewer goods because there is inflation (increasing prices). This decreasing return to work provides an incentive for each young to supply less labor and take more leisure while young compared to the efficient solution (the planner's solution).