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Direction and Range Assignment for Directional Antennae in Wireless Networks

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Abstract In recent years, significant interest has been attracted towards using directional antennae in wireless and sensor networks due to its decreased energy consumption, increased security and decreased radio wave overlapping. The problem is not polynomially solvable in general and effort have been made in recent years to present approximation algorithms to solve this problem. The use of approximation algorithms have become possible by using unit disk graphs, Euclidean minimum spanning trees, grouping of relatively close nodes together, traveling salesman problem and Hamiltonian cycle. In this paper, we present a solution to 1D symmetric connectivity problem using dynamic programming with $O(n)$ runtime complexity.

1 Introduction

In Antennas Connectivity problem, given a set of points, the goal is to assign directions and minimum coverage radius to the antennas positioned on each point, in a way that the corresponding connectivity graph becomes symmetrically connected.

Definition 1 Given the set V of points in the Euclidean plane, $\forall v : V.\phi_v$ and $\forall v : V.r_v$, we define $G = (V, E)$ as directed connectivity graph of V, when edge $u \to v$ exists in E if and only if v is covered by u with r_u radius and ϕ_u direction.

We say that a directed graph $G = (V, E)$ is symmetrically connected if it's strongly connected and for every uv in E, vu is in E too.

The decision problem for Range Assignment problem in two dimensions is proved to be NP-complete by reduction to Hamiltonian Tour problem, when $\alpha < \frac{2\pi}{3}$ [5]. Also it is proved that optimization version of this problem is NP-complete for $\alpha = \frac{pi}{2}$ [3]. The Range Assignment for Directed Antennas problem is probably NP-hard in general therefore in most of the works in this area, special cases of the problem were studied or approximation algorithms were presented.
We can group the solutions for this problem into three categories:

- Papers that restrict the positioning and gradient of the antennas [1].
- Papers that use Unit Disk Graph to design an approximation algorithm [4].

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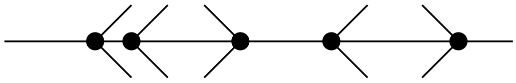

Fig. 1 Direction of the antennas

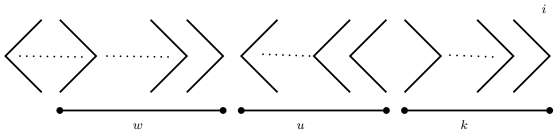

Fig. 2 An arbitrary optimal direction assignment

– Papers that use grids to design an approximation algorithm [2].

In this paper, we obtain a dynamic programming solution to the Symmetric Connectivity Problem in one dimension.

2 Dynamic Algorithm

In this paper, we present a solution to one dimensional symmetric connectivity problem using dynamic programming. First we provide some observations to prove an $O(n)$ runtime complexity for this algorithm.

Problem 1 Given a set V of points on a line, find the direction and minimum radius for the antennas positioned on each of these points, in a way that the resulting connectivity graph of this network would be symmetrically connected.

In the one dimensional setting, each antenna can cover one direction so we use $\prec$ and $\succ$ symbols in order to show the direction of each antenna. Also as shown in Figure 1, we use the shapes $<$ and $>$ to show right and left directions respectively.

Observation 1 *In any correct direction assignment for antennas in one dimension, the leftmost and the rightmost antennas are always directed to right and left respectively.*

Proof Otherwise there is no way to connect the leftmost and rightmost antennas to others.

Consider an optimal direction assignment A with size i. Without loss of generality, we suppose that $k \geqslant 1$ of rightmost antennas have $\succ$ direction which we call k-set. By Observation 1 we know that at least one antenna to the left of k-set has $\prec$ direction in A. Again, without loss of generality we assume that $u \geqslant 1$ consecutive antennas to the left of the k-set have $\prec$ direction (u-set) and w consecutive antennas to the left of u-set have the $\succ$ direction (w-set) as shown in Figure 2.

We use d_{ik} to show the minimum radius for an optimal assignment, in order to have a symmetric connectivity graph for the set $V = x_1...x_i$ of points, in which k of the rightmost antennas have $\succ$ direction. It is obvious that in order to find the solution we have to solve $d_n = min_{k=1...n} d_{nk}$. By looking at Figure 2, we see that by setting $d_{ik} \geqslant max\{x_i - x_{i-k}, x_{i-k+1} - x_{i-k-u+1}\}$ which are actually the distance between the rightmost and leftmost antennas in both k-set and u-set respectively, all the points in these two sets would be strongly connected in the connectivity graph. Also by the assumption that $w \geqslant 1$, in order to have symmetric connectivity on the set $V = x_1...x_i$ of points, the leftmost antenna in the k-set should be able to connect to the rightmost antenna with $\prec$ direction to the left of w-set. We assume that the points $x_1...x_{i-k-u}$ are strongly

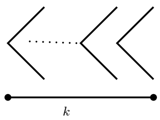

Fig. 3 k antennas in same direction

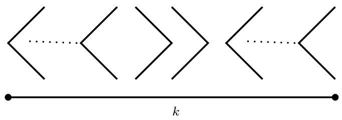

Fig. 4 Except for the first and last antennas, each antenna can have an arbitrary direction.

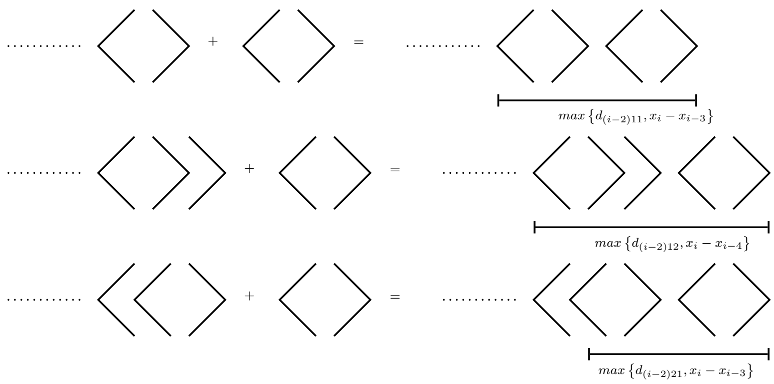

Fig. 5 Creating d_{i11} from sub-assignments

connected. Therefore it only takes two points $x_{i-k-u-w}$ and x_{i-k+1} to be connected for symmetrical connectivity in $x_1...x_i$ so we conclude that $d_{ik} = max\{x_{i-k+1} - x_{i-k-u-w}, x_i - x_{i-k}\}$ because the equation $x_{i-k+1} - x_{i-k-u-w} \geqslant x_{i-k+1} - x_{i-k-u+1}$ always holds true even if $w = 0$. Therefore by assuming that $w \geqslant 1$, $d_{ik} = max\{d_{(i-k-u)w}, x_{i-k+1} - x_{i-k-u-w}, x_i - x_{i-k}\}$ and if $w = 0$ it is obvious that $d_{ik} = max\{x_{i-k+1} - x_1, x_i - x_{i-k}\}$.

Observation 2 *Suppose that k consecutive antennas are directed in the same direction (Figure 3). Note that at least one $\succ$ antenna should exist to the right of these points which is connected to the leftmost one and subsequently all of them. Therefore only the leftmost antenna suffices to cover all of them. Also, the rightmost antenna in this set suffices to cover all the antennas which are covered by this set. In conclusion, in each set of k consecutive antennas with the same direction, except for the first and the last one, we can direct each antenna to an arbitrary direction as shown in Figure 4. Therefore, we can have an optimal assignment for the antennas in which, at most two consecutive antennas have the same direction.*

Now we explain how we can break the problem to sub-problems. We show the minimum radius for an assignment for $x_1...x_i$ by d_{i11} when the two rightmost antennas in this set have $\prec\succ$ directions. We also show the minimum radius for an assignment for $x_1...x_i$ by d_{i12} and d_{i21} when the three rightmost antennas have $\prec\succ\succ$ and $\prec\prec\succ$ directions respectively. We add the antennas with $\succ$, $\prec\succ$, $\prec\prec\succ$ and $\prec\succ\succ$ directions to the each assignment and transform it to another. Therefore we find the sub-assignments to each of these assignments when new antennas are added. Figures 5, 6 and 7 show the sub-assignments for d_{i11}, d_{i12} and d_{i21} respectively. Also Equations 1, 2 and 3 show the recursive formulas for d_{i11}, d_{i12} and d_{i21} from sub-assignments respectively.

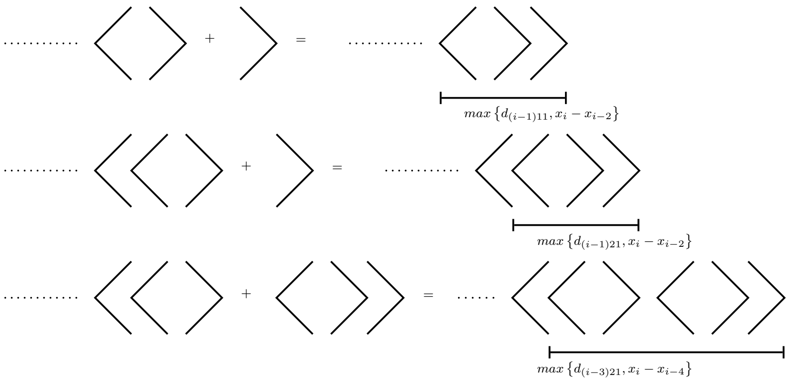

Fig. 6 Creating d_{i12} from sub-assignments

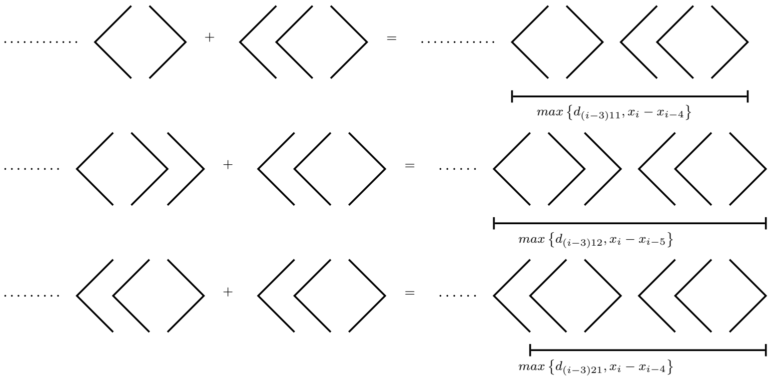

Fig. 7 Creating d_{i21} from sub-assignments

$$d_{i11} = \left\{ \begin{array}{cc} -\infty & i < 3 \\ min \left\{ \begin{array}{c} max\left\{d_{(i-2)11}, x_i - x_{i-3}\right\}, \\ max\left\{d_{(i-2)12}, x_i - x_{i-4}\right\}, \\ max\left\{d_{(i-2)21}, x_i - x_{i-3}\right\} \end{array} \right\} & i \geqslant 3 \end{array} \right\} \tag{1}$$

$$d_{i12} = \left\{ \begin{array}{cc} -\infty & i < 2 \\ min \left\{ \begin{array}{c} max\left\{d_{(i-1)11}, x_i - x_{i-2}\right\}, \\ max\left\{d_{(i-1)21}, x_i - x_{i-2}\right\}, \\ max\left\{d_{(i-3)21}, x_i - x_{i-4}\right\} \end{array} \right\} & i \geqslant 2 \end{array} \right\} \tag{2}$$

$$d_{i21} = \left\{ \begin{array}{cc} -\infty & i < 3 \\ min \left\{ \begin{array}{c} max\left\{d_{(i-3)11}, x_i - x_{i-4}\right\}, \\ max\left\{d_{(i-3)12}, x_i - x_{i-5}\right\}, \\ max\left\{d_{(i-3)21}, x_i - x_{i-4}\right\} \end{array} \right\} & i \geqslant 4 \end{array} \right\} \tag{3}$$

Theorem 3 *Given a set V of points on a line, using dynamic programming, there is an algorithm with $O(n)$ runtime complexity which assigns the directions and radius to the antennas positioned on each point in a way that the resulting connectivity graph is symmetrically connected.*

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