AC can be replaced by S in the 'S' productions for the cases S → ACD/AAC, and AAC can be replaced by S for the production S → AACD. Introduce three non-terminals E, F, and G for C_aA, C_aD, and C_bD, respectively. Including these changes, the modifi ed grammar becomes

S → SD/SD/CD/AS/AC/C_aC/a
A → EC_b/C_aCb
C → C_aC/a
D → $FC_a/GC_b/C_aC_a/C_bC_b$
C_a → a
C_b → b
E → C_aA
F → C_aD
G → C_bD

Now, the grammar is in CNF.

22. Convert the following grammar into GNF.

S → A
A → aBa/a
B → bAb/b

Solution: The grammar is not in CNF. So, it has to be converted into CNF. Introduce two nonterminals C_a, C_b and two production rules C_a → a, C_b → b.

S → A
A →C_aBC_a/a
B → C_bAC_b/b
C_a → a
C_b → b

Introduce two non-terminals X and Y and two production rules X → BC_a and Y → AC_b. The production rule becomes

S → A
A → C_aX/a
B → C_bY/b
C_a → a
C_b → b
X → BC_a
Y → AC_b

Step I: In the grammar, there is no null production and no unit production. The grammar also is in CNF.

Step II: In the grammar, there are seven non-terminals S, A, B, C_a, C_b, X, and Y. Rename the non-terminals as A_1, A_2, A_3, A_4, A_5, A_6, and A_7, respectively. After renaming the non-terminals, the modifi ed grammar will be

$A_1 \rightarrow A_2$
$A_2 \rightarrow A_4A_6$

$$A_3 \rightarrow A_5A_7/b$$
$$A_4 \rightarrow \text{a}$$
$$A_5 \rightarrow \text{b}$$
$$A_6 \rightarrow A_3A_4$$
$$A7 \rightarrow A_2A_5$$

Step III: In the previous production, $A_6 \rightarrow A_3A_4$ and $A_7 \rightarrow A_2A_5$ are not in the form $A_i \rightarrow A_jV$, where $i \leq j$.

Using Lemma I, replace $A_3 \rightarrow A_5A_7/b$ in the production $A_6 \rightarrow A_3A_4$. The rule becomes

$$A_6 \rightarrow A_5A_7A_4/bA_4$$

Still $A_6 \rightarrow A_5A_7A_4$ is not in the form $A_i \rightarrow A_jV$, where $i \leq j$. Using Lemma I, replace $A_5 \rightarrow$ b in the production. The modified production is $A_6 \rightarrow bA_7A_4/bA_4$, which is in GNF.

Step IV: Using Lemma I, replace $A_2 \rightarrow A_4A_6/a$ in the production $A_7 \rightarrow A_2A_5$. The production rule becomes $A_7 \rightarrow A_4A_6A_5/aA_5$.

Still $A_7 \rightarrow A_4A_6A_5$ is not in the form $A_i \rightarrow A_jV$, where $i \leq j$. Using Lemma I, replace $A_4 \rightarrow$ a in the production.

The modified production is $A_7 \rightarrow aA_6A_5/aA_5$, which is in GNF.

Lemma II can be applied on the productions $A_2 \rightarrow A_4A_6/a$ and $A_3 \rightarrow A_5A_7/b$.

Applying Lemma II on $A_2 \rightarrow A_4A_6/a$, we get

$$A_2 \rightarrow \text{a/aX}$$
$$\text{X} \rightarrow A_6/A_6X$$

X $\rightarrow A_6/A_6X$ are not in GNF. Replacing $A_6 \rightarrow bA_7A_4/bA_4$ in the production, the productions X $\rightarrow bA_7A_4/bA_7A_4X/bA_4/bA_4X$ are in GNF.

Applying Lemma II on $A_3 \rightarrow A_5A_7/b$, we get

$$A_3 \rightarrow \text{b/bY}$$
$$\text{Y} \rightarrow A_7/A_7Y$$

Y $\rightarrow A_7/A_7Y$ are not in GNF. Replacing $A_7 \rightarrow aA_6A_5/aA_5$ in the production, the productions Y $\rightarrow aA_6A_5/aA_6A_5Y/aA_5/aA_5Y$ are in GNF.

$A_1 \rightarrow A_2$ is not in GNF. Replacing $A_2 \rightarrow$ a/aX in the production, we get $A_1 \rightarrow$ a/aX, which is in GNF.

The grammar converted into GNF is

$$A_1 \rightarrow \text{aX/a}$$
$$A_2 \rightarrow \text{aX/a}$$
$$A_3 \rightarrow \text{bY/b}$$
$$A_4 \rightarrow \text{a}$$
$$A_5 \rightarrow \text{b}$$
$$A_6 \rightarrow bA_7A_4/bA_4$$
$$A_7 \rightarrow aA_6A_5/aA_5$$
$$\text{X} \rightarrow bA_7A_4/bA_7A_4X/bA_4/bA_4X$$
$$\text{Y} \rightarrow aA_6A_5/aA_6A_5Y/aA_5/aA_5Y$$

23. Convert the following grammar to GNF.
i) S $\rightarrow aSa/aSb/\in$
ii) S $\rightarrow aSB/aSbS/\in$ [WBUT 2010]

Solution: (It can be done by Lemma I and II, but it is done here in a simpler way.)
i) S $\rightarrow aSa/aSb/\in$
Introduce two productions Ca → a and Cb → b, and the modified productions are

$$\text{S} \rightarrow aSC_a/aSC_b/\in$$
$$C_a \rightarrow \text{a}$$
$$C_b \rightarrow \text{b}$$

ii) S $\rightarrow aSB/aSbS/\in$ Introduce a production $C_b \rightarrow$ b, and the modified productions are

$$\text{S} \rightarrow aSB/aSCbS/\in$$
$$C_b \rightarrow \text{b}$$

24. Convert the following grammar into GNF.

$$A_1 \rightarrow A_2A_3$$
$$A_2 \rightarrow A_3A_1/b$$
$$A_3 \rightarrow A_1A_2/a$$

[Cochin University 2009]

Solution: In the grammar, the production $A_3 \rightarrow A_1A_2$ is not in the form $A_i \rightarrow A_jV$ where $i \leq j$. Using Lemma 1, replace $A_1 \rightarrow A_2A_3$ in the production $A_3 \rightarrow A_1A_2$. The rule becomes

$$A_3 \rightarrow A_2A_3A_2/a$$

Still it is not in the form $A_i \rightarrow A_jV$, where $i \leq j$. Replacing $A_2 \rightarrow A_3A_1/b$ in $A_3 \rightarrow A_2A_3A_2/a$, we get $A_3 \rightarrow A_3A_1A_3A_2/bA_3A_2/a$.

Now, the modified grammar is

$$A_1 \rightarrow A_2A_3$$
$$A_2 \rightarrow A_3A_1/b$$
$$A_3 \rightarrow A_3A_1A_3A_2/bA_3A_2/a$$

The production $A_3 \rightarrow bA_3A_2/a$ is in the format A $\rightarrow \beta_i$, and the production $A_3 \rightarrow A_3A_1A_3A_2$ is in the format of A $\rightarrow A\alpha_j$. So, we can introduce a new non-terminal B3, and the modified A3 production is (according to Lemma II)

$$A_3 \rightarrow bA_3A_2/a$$
$$A_3 \rightarrow bA_3A_2B_3$$
$$A_3 \rightarrow aB_3$$

And the B3 productions will be

$$B_3 \rightarrow A_1A_3A_2$$
$$B_3 \rightarrow A_1A_3A_2$$

All A3 productions are in GNF. Replacing A_3 by all A_3 productions in A_2, we get the following productions.

$A_2 \to bA_3A_2A_1/aA_1/bA_3A_2B_3A_1/aB_3A_1/b$

Now, all A_2 productions are in GNF. Replacing A_2 by all A_2 productions in A_2, we get the following productions.

$A_1 \to bA_3A_2A_1A_3/aA_1A_3/bA_3A_2B_3A_1A_3/aB_3A_1A_3/bA_3$

Now, all A_1 productions are in GNF. Replacing A_1 by all A_1 productions in two B_3 productions, we get the following productions.

$B_3 \to bA_3A_2A_1A_3A_3A_2/aA_1A_3A_3A_2/bA_3A_2B_3A_1A_3A_3A_2/aB_3A_1A_3A_3A_2/bA_3A_3A_2$

$B_3 \to bA_3A_2A_1A_3A_3A_2B_3/aA_1A_3A_3A_2B_3/bA_3A_2B_3A_1A_3A_3A_2B_3/aB_3A_1A_3A_3A_2B_3/bA_3A_3A_2B_3$

Now, the grammar becomes

$A_1 \to bA_3A_2A_1A_3/aA_1A_3/bA_3A_2B_3A_1A_3/aB_3A_1A_3/bA_3$

$A_2 \to bA_3A_2A_1/aA_1/bA_3A_2B_3A_1/aB_3A_1/b$

$A_3 \to bA_3A_2B_3/bA_3A_2/aB_3/a$

$B_3 \to bA_3A_2A_1A_3A_3A_2/aA_1A_3A_3A_2/bA_3A_2B_3A_1A_3A_3A_2/aB_3A_1A_3A_3A_2/bA_3A_3A_2$

$B_3 \to bA_3A_2A_1A_3A_3A_2B_3/aA_1A_3A_3A_2B_3/bA_3A_2B_3A_1A_3A_3A_2B_3/aB_3A_1A_3A_3A_2B_3/bA_3A_3A_2B_3$

25. Remove the left recursion from the given grammar.

A → Ba/b

B → Bc/Ad/e

Solution: The grammar has indirect left recursion. To remove the left recursion, rename A as A_1 and B as A_2. The modified production rules become

$$A_1 \to A_2a/b$$

$$A_2 \to A_2c/A_1d/e$$

For i = 1 and j = 1, there is no production in the form $A_1 \to A_1\alpha$.

For i = 2 and j = 1, there is a production in the form $A_2 \to A_1\alpha$. The production is $A_2 \to A_1d$. According to the algorithm for removal of indirect left recursion, the production becomes

$$A_2 \to A_2ad/bd$$

Now, the A2 production is $A_2 \to A_2c/A_2ad/bd/e$

$A_2 \to A_2c$ has immediate left recursion. After removing the left recursion, the production becomes

$$A_2 \to bdA'_2eA'_2A'_2 \to cA_2/\in$$

$A_2 \to A_2ad$ has immediate left recursion. After removing the left recursion, the production becomes

$$A_2 \to bdA''_2/eA''_2A''_2 \to adA$$