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Finding sub-optimum signature matrices for overloaded code division multiple access systems

Mohsen Heidari Khoozani[1], Farokh Marvasti[1], Masoumeh Azghani[1], Mona Ghassemian[1,2]
[1]Advanced Communication Research Institute, Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran
[2]School of Computing and Math Sciences, University of Greenwich, London, UK
E-mail: mheidari@ee.sharif.edu

Abstract: The objective of this study is to design sub-optimal signature matrices for binary inputs for an overloaded code division multiple access (CDMA) system as developed by this author group. In this study, the authors propose to use the sum capacity, the bit error rate and distance criteria as objective functions for signature matrix optimisation. Three optimisation techniques, the genetic algorithm, the particle swarm optimisation and the conjugate gradient (CG) are exploited in this work. Since the optimisation computational complexity increases by matrix dimensions, it is practically impossible to directly optimise the large signature matrices. In order to address this problem, a method is proposed to enlarge small-scale signature matrices. It is also proved that the sum channel capacity of the enlarged matrix is scaled by its enlargement factor. The results indicate that this proposed signature matrices provide higher sum capacity than the commonly used signature matrices. The authors also address that this matrices are applicable for highly overloaded CDMA systems.

1 Introduction

Code division multiple access (CDMA) is a method for reliable data communication among multiple users and has become standard for third-generation wireless systems. The general model of a CDMA system is defined as [1]

$$Y = \boldsymbol{A}\boldsymbol{X} + \boldsymbol{N} \tag{1}$$

where $\boldsymbol{A}$ is an $m \times n$ signature matrix, m is the length of the signatures (chip rate) and n is the number of users. $\boldsymbol{X}$ is an $n \times 1$ user column vector and $\boldsymbol{N}$ is the additive white Gaussian noise (AWGN) vector $\boldsymbol{N} = [N_1, \ldots, N_m]^{\mathrm{T}}$ such that N_i's are independent and identically distributed (i.i.d.), random variables. For binary input CDMA systems, entries of $\boldsymbol{X}$ are binary (i.e. $\{\pm 1\}$) with a uniform distribution.

Owing to the bandwidth constraint in mobile communication systems, it is desirable to increase the number of users. An overloaded CDMA comes to life when the number of users exceeds the length of the signatures ($m > n$); in this situation orthogonal signature matrices, such as Hadamard codes, can no longer be used and a loading factor for such systems is defined as

$$\beta = \frac{n}{m} \tag{2}$$

Most of the published works in the evaluation of the sum channel capacity considered for large-scale CDMA systems (asymptotic results) [1, 2]. Although for the finite scale systems, the sum capacity region is not known, there are known lower and upper bounds [3–5]. Alishahi *et al.* [4] evaluated the upper and the lower bounds for the sum capacity for binary CDMA systems both in presence and absence of noise. Alishahi *et al.* [5] generalised their previous works [3, 4] to finite non-binary discrete input and matrix entries. A review of these papers has been done by Hosseini *et al.* in [6].

The total square correlation is used in [7] and [8] to obtain the Welch bound equality (WBE) sequences which maximises the sum channel capacity of real input synchronous CDMA systems. Furthermore, the total weighted square correlation was used in [9] as a measure to derive optimum signatures; we call these matrices weighted WBE sequences in this work.

Although in [7–9] the real input CDMA systems are considered, Pad *et al.* [3] presented optimum binary signature sets for binary inputs such as codes for overloaded wireless (COW) matrices. Furthermore, a new maximum-likelihood (ML) decoder was introduced in [3] for large-scale COW matrices. In our previous work [10] binary input synchronous CDMA systems are investigated with real-valued signature matrices and partial results for sub-optimal matrices based on the genetic algorithm (GA) are reported.

In this paper, we aim to derive optimum real and binary signature matrices for binary input synchronous CDMA systems. As a special case, the results of this paper can be used to find the optimum codes for overloaded wireless CDMA (COW)/codes for overloaded optimal CDMA (COO) matrices which are introduced in [3]. It is proposed to optimise the matrices with respect to different criteria such as the sum capacity, the bit error rate (BER) and distance criteria. The distance criteria are referred to three different criteria namely Minimum distance (MD), exponential distance (ED) and Q-function distance (QD).

The proposed criteria can also be applied to non-binary input CDMA systems. It is noted that we use criterion, measure and metric terms interchangeably, throughout this paper.

It is computationally costly to design large-scale matrices using direct optimisation. In order to overcome this difficulty, an enlargement method is exploited in this paper for obtaining large-scale sub-optimum matrices from the small-scale optimum ones. Also, it is proved that the sum capacity of the enlarged matrix is scaled by its enlargement factor. Moreover, we extend the ML decoder proposed in [3] for the enlarged matrices and it causes significant reduction in the computational complexity of decoding.

The rest of this paper is organised as follows: Section 2 discusses optimisation measures and methods for optimum signature matrices. Section 3 describes the optimisation techniques used in this paper. In Section 4, numerical and simulation results are presented. Section 5 proposes a method to derive sub-optimal large signature matrices. Finally, Section 6 concludes the paper and highlights the future works.

2 Signature matrix optimisation

In this section, different criteria for optimisation of the signature matrix are illustrated. These criteria are the sum capacity, the BER and distance criteria. The computational complexity of these measures are different from each other and will be discussed later.

2.1 Sum channel capacity

In a multiple access channel (MAC), there are several users sending information to a common receiver and the users should overcome not only the noise but also their mutual interference. Ahlswede [11, 12] and Liao [13] characterised the capacity region of an n-user discrete memoryless MAC as the closure of the convex hull of the rates $(R_1, R_2, \ldots, R_n)$ such that for every $\mathcal{J} \subseteq \{1, 2, 3, \ldots, n\}$

$$\sum_{j \in \mathcal{J}} R_j \leq I\big(X(\mathcal{J}); Y|X(\mathcal{J}^c)\big)$$

for some probability mass function (pmf) $p(x_1)\, p(x_2) \ldots p(x_n)$; where $X(\mathcal{J}) = \{X_i : i \in \mathcal{J}\}$.

The sum channel capacity is therefore defined as the sum of all the user rates that can be achieved and is equal to

$$\max_{p(x_1)p(x_2)\cdots p(x_n)} I\big(X_1, X_2, \ldots, X_n; Y\big) \tag{3}$$

Considering a synchronous CDMA channel as a special case of a multi-user MAC, the sum capacity for a given dimension $n \times m$ and noise with variance σ_{N} can be defined as

$$C\big(n, m, \sigma_{\mathrm{N}}\big) = \max_{\boldsymbol{A} \in \boldsymbol{R}_{m\times n}} C\big(n, m, \sigma_{\mathrm{N}}|\boldsymbol{A}\big) \tag{4}$$

where $C(n, m, \sigma_{\mathrm{N}}|\boldsymbol{A})$ is the sum channel capacity for a specific matrix $\boldsymbol{A}$. Considering that $\boldsymbol{A}$ is deterministic in (4), we have

$$\begin{aligned} C(n, m, \sigma_{\mathbf{N}}|\boldsymbol{A}) &= \max_{P(X)} I(\boldsymbol{X}; \boldsymbol{Y}|\boldsymbol{A}) \\ &= \max_{P(\boldsymbol{X})} h(\boldsymbol{Y}|\boldsymbol{A}) - h(\boldsymbol{N}) \end{aligned} \tag{5}$$

where $h(\boldsymbol{Y})$ is the differential entropy. Limiting $\boldsymbol{X}$ to be binary, we conjecture that $h(\boldsymbol{Y}|\boldsymbol{A})$ is maximised when $\boldsymbol{X}$ is uniformly distributed, as discussed in [14] and [4]. Since N_i's are i.i.d., $f_{\mathrm{N}}(\boldsymbol{N}) = \prod_{i=1}^{m} f_{n_i}(n_i)$. Thus the probability distribution function of $\boldsymbol{Y}$, when $\boldsymbol{X}$ has binary uniform distribution is

$$f_Y(\boldsymbol{Y}|\boldsymbol{A}) = \frac{1}{2^n} \times \sum_{\boldsymbol{X}_i \in \{\pm 1\}^{n\times 1}} \left[\left(\frac{1}{2\pi\sigma_{\mathrm{N}}^2} \right)^{(m/2)} \exp\left(\frac{-||\boldsymbol{Y} - \boldsymbol{A}\boldsymbol{X}_i||^2}{2\sigma_{\mathbf{N}}^2} \right) \right] \tag{6}$$

As a result of this, $h(\boldsymbol{Y}|\boldsymbol{A})$ is computable from (6) as

$$h_{\boldsymbol{Y}}(\boldsymbol{Y}|\boldsymbol{A}) = \underbrace{\int \cdots \int}_{m} f_Y(\boldsymbol{Y}|\boldsymbol{A}) \log_2\big(f_Y(\boldsymbol{Y}|\boldsymbol{A})\big) \mathrm{d}y_1 \ldots \mathrm{d}y_m$$

Consequently, $C(n, m, \sigma_{\mathrm{N}}|\boldsymbol{A})$ is derived from the mutual entropy in (5). For a more convenient comparison, the normalised per-user sum channel capacity is defined as

$$c\big(n, m, \sigma_{\mathrm{N}}|\boldsymbol{A}\big) = \frac{C\big(n, m, \sigma_{\mathrm{N}}|\boldsymbol{A}\big)}{n} \tag{7}$$

It must be mentioned that the computational complexity of the sum channel capacity for n users and n chips is of $O(m^n)$ which is a non-polynomial-hard algorithm.

2.2 BER criterion

To compute the BER, a large array of bits (10^6 bits) must be produced, encoded, transmitted through the simulated channel and decoded at the receiver by an ML decoder. Then, the probability of error is statistically measured. We conjecture that the lower the BER of the signature matrix is, the higher its channel capacity will be; this conjecture is verified with simulation results. Hence, optimum matrices can be derived by minimising the BER measure.

The BER calculation requires much less computation than the sum channel capacity calculation. However, none of them are practical.

2.3 Distance criteria

In this section, we propose three different measures based on the output constellation points ($\boldsymbol{Z}(i)$), that is, the output points which are defined in the absence of noise

$$\boldsymbol{Z}(i) = \boldsymbol{A}\boldsymbol{X}(i) \quad \boldsymbol{X}(1), \boldsymbol{X}(2), \ldots, \boldsymbol{X}(2^n) \in \{\pm 1\}^{n\times 1} \tag{8}$$

The distance metrics are categorised in three measures namely, the MD, the QD and the ED criteria.

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2.3.1 MD criterion: One metric for measuring the performance of a signature matrix is the MD of the output constellation points. This measure guarantees an upper bound for the probability of error for high values of signal-to-noise ratio (SNR). We refer to this criteria as MD. The following equation explicitly defines MD criterion

$$\mathrm{MD} \triangleq \min_{i \neq j} ||\boldsymbol{Z}(i) - \boldsymbol{Z}(j)||$$

where $\|\boldsymbol{U}\|$ represents the Euclidean norm of the vector $\boldsymbol{U}$. The above equation can be rewritten as:

$$\mathrm{MD} = \min_{i \neq j} ||\boldsymbol{A}(\boldsymbol{X}(i) - \boldsymbol{X}(j)|| \tag{9}$$

Since $\boldsymbol{X}(i)$'s are symmetric, that is, $\boldsymbol{X}(i), -\boldsymbol{X}(i) \in \{\pm 1\}^{n \times 1}$ for every $i = 1, 2, \ldots 2^n$, the computational complexity is reduced by considering only one half of $\boldsymbol{X}(i)$'s in the distance criterion. Thus, the computational complexity of this metric is $O(m3^n)$. Note that MD must be maximised in order to find a sub-optimum matrix.

2.3.2 QD criterion: By taking a more analytical approach, suppose that x_i's (the ith element of the input vector $\boldsymbol{X} \in \{\pm 1\}^{n \times 1}$) are independent, hence the error probability of a block (with size of m) is derived as follows

$$P_{\mathrm{e}} = \frac{1}{2^n} \sum_{i=1}^{2^n} P\left(\bigcup_{\substack{k=1 \\ k \neq i}}^{2^n} ||\boldsymbol{Y}(i) - \boldsymbol{Z}(i)||^2 > ||\boldsymbol{Y}(i) - \boldsymbol{Z}(k)||^2 \right) \tag{10}$$

where $\boldsymbol{Y}(i) = \boldsymbol{Z}(i) + \boldsymbol{N}$ and $\boldsymbol{Z}(i) = \boldsymbol{A}\boldsymbol{X}(i)$ is the output vector in the noiseless channel. Since the noise vector elements are i.i.d. Gaussian random variables with variance σ_{N}, the upper bound for error probability can be calculated as

$$P_{\mathrm{e}} < \frac{1}{2^n} \sum_{i=1}^{2^n} \sum_{\substack{j=1 \\ j \neq i}}^{2^n} Q\left(\frac{||\mathbf{Z}(i) - \mathbf{Z}(j)||}{2\sigma_{\mathrm{N}}} \right) \tag{11}$$

where $Q(x)$ refers to cumulative distribution function of Gaussian distribution. The inequality (11) can be treated as a new measure. We refer to this criteria as QD measurement which is defined as

$$\mathrm{QD} \triangleq \sum_{i=1}^{2^n} \sum_{\substack{j=1 \\ j \neq i}}^{2^n} Q\left(\frac{||\mathbf{Z}(i) - \mathbf{Z}(j)||}{2\sigma} \right) \tag{12}$$

Minimising this criterion led to a sub-optimum signature matrix. The computational complexity of QD measure is $O(m3^n)$ times Q-function computational complexity.

2.3.3 ED criterion: The minimisation of QD demonstrates better results than MD in terms of the optimality of resultant signature matrix; however, it is more computationally intensive. To reduce the computational complexity of $Q(x)$, the following approximation can be used

$$Q(x) \simeq 0.7 \exp\left(-\left(\frac{x+1}{1.6} \right)^2 \right) \tag{13}$$

This new criteria is referred in this paper by ED which is

$$\mathrm{ED} \triangleq \sum_{i=1}^{2^n} \sum_{\substack{j=1 \\ j \neq i}}^{2^n} \exp\left[-\left(\frac{\left(||\mathbf{Z}(i) - \mathbf{Z}(j)||/\left(2\sigma_{\mathrm{N}}\right)\right) + 1}{1.6} \right)^2 \right] \tag{14}$$

Thus, this metric reduces the computational complexity of QD to $O(m3^n)$ times exponential function computational complexity. For high SNR values, (14) is simplified to a single exponential element which presents a similar behaviour to MD.

In this section, we proposed various measures to demonstrate the performance of the signature matrices from different aspects. Optimising these measures yields sub-optimum signature matrices. The next section presents a brief description of optimisation techniques used in this work.

3 Optimisation techniques

This section discusses the optimisation techniques used to find sub-optimum signature matrices based on the criteria proposed in this paper. We use conjugate gradient (CG) as a numerical optimisation algorithm. Since CG is not applicable for all proposed metrics, intelligent techniques such as the GA and the particle swarm optimisation (PSO) are also used. A brief discussion on these techniques are given in the following section.

3.1 Conjugate gradient

CG is originally designed for quadratic functions. Non-linear CG method [15] is a non-linear variant of the CG. CG is not suitable for all proposed criteria in this work. Therefore we apply it for the ED and the QD measures. Since the results obtained by CG method for the MD, the BER and the sum capacity measures are worse than that of the GA and the PSO, we do not include the simulations in our results. Section 4.1 shows that the optimisation results of the GA and the CG have minor differences for the MD, ED and QD measures. Thus we use the GA and PSO as a fair comparison of different criteria.

3.2 Genetic algorithm

The GA method [16] employs the principal of survival of the fittest in its search process to select and generates individuals (signature matrices) that are adapted to their environment (design objectives/constraints). Therefore over a number of generations (iterations), desirable traits (design characteristics) will evolve and remain in the genome composition of the population over traits with weaker undesirable characteristics. The GA is well suited and has been extensively applied to solve hard optimisation problems. It can handle both discrete and continuous variables with non-linear objective and constraint functions. In order to find sub-optimum signature matrices, we apply the GA to optimise the criteria discussed in Section 2 such as the sum channel capacity, the BER and the distance criteria. The parameters and options of the GA are presented in Table 1.

3.3 Particle swarm optimisation

The PSO algorithm [17, 18], similar to the GA method is a technique that optimises a problem by iteratively trying to improve a candidate solution, which results in an objective function. In the PSO, a set of randomly generated solutions (initial swarm) propagates in the design space towards the optimal solution over a number of iterations (moves) based on large amount of information about the design space that is assimilated and shared by all members of the swarm. The PSO algorithm considers some candidate solutions in the search domain. During each iteration, the cost function of each candidate solution is calculated. Each candidate solution can be considered as a particle moving towards the minimum value of the cost function. As the first step, PSO chooses the candidate solutions randomly inside the search space. It should be mentioned that the PSO does not have any prior information about the cost function; it does not know which particles are near or far from the global minimum of the cost function. What the PSO algorithm does is to evaluate the cost value of each particle and just work with the corresponding cost values. The position of a particle is composed of its candidate solution, cost and velocity. Moreover, it remembers the minimum cost (the best fitness) that it had during the operation of the algorithm (called the individual best fitness). The candidate solution corresponding to this fitness is referred to as the individual best candidate solution or the individual best position. At last, the PSO seeks for the minimum cost among all the particles in the swarm (named the global best fitness). The simulation parameters and options set for the PSO algorithm are listed in Table 2. We will compare the GA and the PSO methods in terms of the convergence rate in the following section.

3.4 Convergence evaluation of the GA and the PSO

In this section, we investigate the convergence behaviour of both optimisation algorithms. The convergence issue of the PSO and the GA methods are investigated in [19, 20]. The

Table 1 Simulation parameters set for the GA

Population	
size	20 individual matrices
type	double vector
creation function	uniform in the first run, best results of previous runs afterwards
lower bound	−1
upper bound	+1
New generation	
Elite count	2
crossover fraction	0.8
migration direction	forward
mitigation factor	0.2
Stopping criteria	
number of iterations	100
function tolerance	10^{-6}

Table 2 Simulation parameters set for the PSO

Particle's position	
number of particles	20 individual vectors
initial position	random with uniform distribution
lower and upper bounds	[1, +1]
particle velocity	initiated randomly with uniform distribution
Stopping criterion	
number of iterations	100

simulation results which are used to exhibit the convergence of the algorithms are shown in Figs. 1*a* and *b*. Fig. 1*a* depicts the best fitness values against iteration numbers for both algorithms using MD as an objective function for $n = 5$ and $m = 4$. In this figure the best fitness value is unchanged after 28 iterations for PSO and after 53 iterations for GA, which demonstrates that the PSO converges faster than the GA. In addition, Fig. 1*b* shows that the PSO algorithm performs a search over larger space in comparison with the GA. For the other measures, different dimensions and various SNR values, we observed similar results.

Note that for terminating the GA method the number of iterations is limited to 100, as noted in Table 1. Also, when the cumulative change in the cost function value is less than function tolerance over the population at each iteration, the algorithm stops. In this work, we set function tolerance to 10^{-6}, as shown in Table 1. For the PSO method, the stopping criterion is the number of iterations which is set to 100 iterations, as noted in Table 2. These parameters are set by trial and error. The above mentioned optimisation techniques have been applied in various dimensions for different criteria. Our observation was that the corresponding cost functions converge to their minimum values and remain unchanged before 100 iterations. Therefore the maximum number of iterations is set to 100 in our evaluation.

4 Numerical and simulation results

This section presents numerical and simulation results for the GA and the PSO method for the various criteria. Sub-optimum signature matrices based on various criteria using the GA and the PSO algorithms are presented in the Appendix.

4.1 Results of GA for real-valued signature matrices

In this section, we apply the GA method based on the measures discussed in Section 2, namely the BER, the sum channel capacity and the distance criteria. First, we will compare different distance metrics with each other and then, we will show the simulation and numerical results for other various metrics.

4.1.1 Comparison of various distance criteria: In order to compare the matrices obtained by different distance measures in terms of the sum capacity, we obtain sub-optimal matrices for an arbitrary E_b/N_0 value and compare the normalised sum capacity of such matrices for the given E_b/N_0 value. Fig. 2 illustrates the normalised sum capacity of the optimised matrices using different distance criteria for various E_b/N_0 values.

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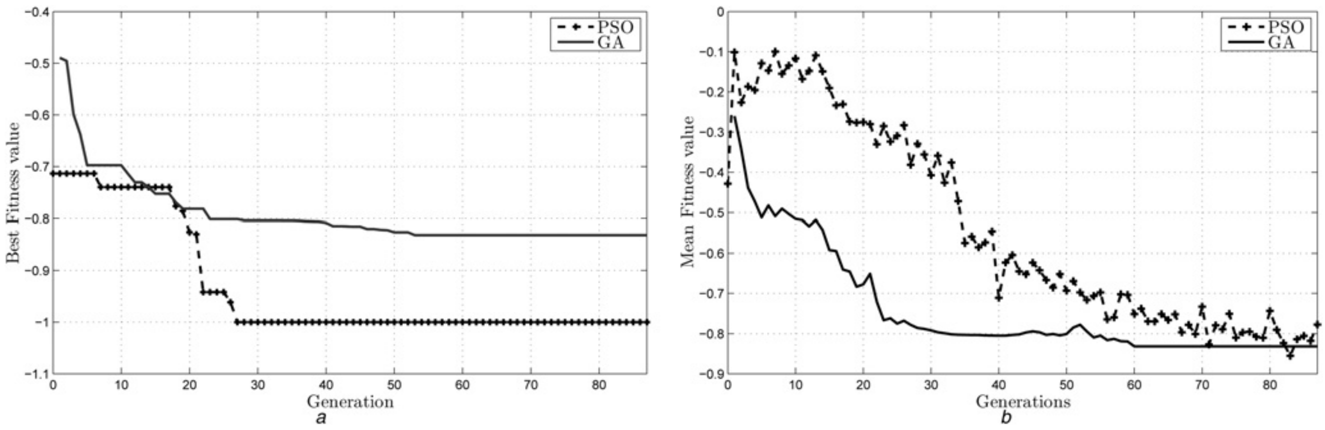

Fig. 1 *Best value and the mean value of populations against iterations for the MD metric [β = (5/4)]*
a Best fitness
b Mean fitness

The matrices exhibit a normalised sum capacity close to the extreme upper bound, that is, 1 bps per user. The minor difference between the curve of ED and QD measures justifies the approximation of QD in (12) and (13). As is expected in Section 2.3, the MD criteria capacity performance is close to ED and QD methods for high E_b/N_0 values. Since the ED criterion has less complexity than the QD measure, we select the ED criterion for the comparison with other metrics such as the BER and the sum channel capacity. Fig. 2 also shows the results of the CG for the ED and the QD measures. It is concluded that the CG method has similar performance as the GA for these measures in terms of optimality. Although, the CG converges faster than the GA, we use the GA for a fair comparison of different criteria. Our further investigations observed in Section 4.4 show the similarity between the results gained for these measures for both the GA and the PSO. Therefore for simplicity, we do not show the results of PSO in Fig. 2.

4.1.2 Comparison of different criteria: Fig. 3 demonstrates the normalised sum capacity of the proposed matrices optimised by the BER, the sum channel capacity and the ED criteria for different E_b/N_0 values. In addition, we compare these results with the WBE codes which are introduced in [7] and [8]. Since WBE is optimum for Gaussian input distribution; there is no guarantee to be optimum for binary input vectors. Among the proposed criteria, the results of the ED are close to that of BER criterion; this verifies that our approximation in (11) and (14) is accurate. As discussed earlier, to compute the BER criterion, a large array of bits are needed which makes the computation of this criterion more complex as opposed to the ED criterion.

Fig. 4 compares weighted WBE sequences [9] and our proposed sub-optimum matrices based on ED criterion. It exhibits the normalised sum capacity against different E_b/N_0 values for $\beta = (6/2)$ and $\beta = (6/3)$. The normalised sum

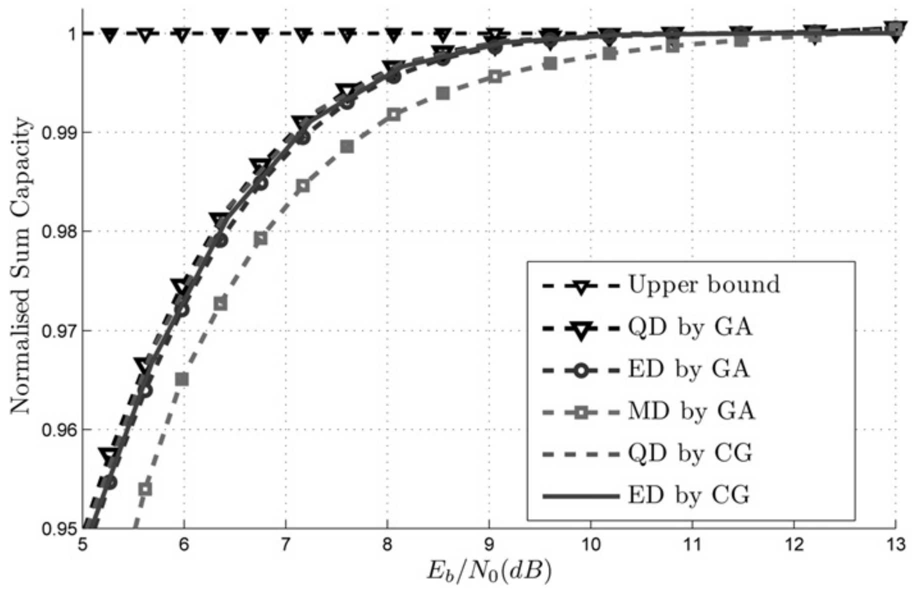

Fig. 2 *Normalised sum capacity of sub-optimum matrices based on distance criteria against different E_b/N_0 values [β = (4/3), using the GA and the CG]*

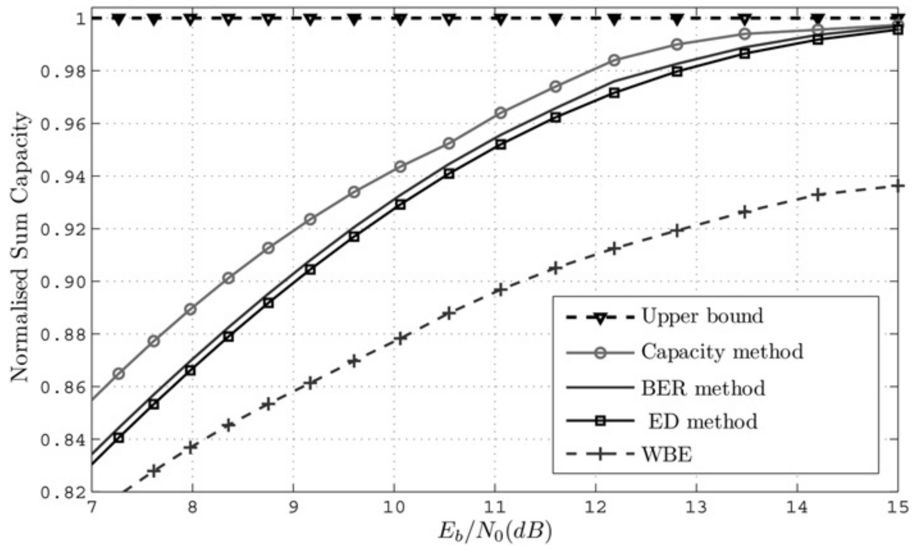

Fig. 3 *Normalised sum capacity of sub-optimum matrices based on different criteria for various E_b/N_0 values [β = (5/2), using the GA]*

capacity of the weighted sequences are much lower than that of our proposed matrices. Since weighted WBE sequences are designed for real input systems, they are no longer optimum for binary input CDMA systems.

Fig. 5 shows the normalised sum capacity of the optimised matrices based on the sum capacity, ED and BER measures against the loading factor [$\beta = (n/m)$]. In this figure, we select a fixed value for E_b/N_0 (8 dB) to compute the sum capacity. The performance of the sum capacity metric degrades slower than the other methods.

4.2 Results of the GA for binary valued signature matrices

Although, our proposed methods are applied to real-valued matrices, we can apply them to find sub-optimal binary (±1) matrices. Binary matrices are much simpler, in implementation, than real-valued matrices. We consider a 4×5 binary matrix [$\boldsymbol{A}(5)$ in Table 4], which is optimised by the sum capacity metric, and compare it to another binary matrix derived from the ED criterion [$\boldsymbol{A}(3)$ in Table 5]. Fig. 6 shows normalised sum capacity of these two binary matrices along with a real-valued sub-optimum matrix derived from the sum capacity criterion [$\boldsymbol{A}(4)$ in Table 4]. This figure shows that the binary matrices developed by the sum capacity criterion can be close to the real-valued matrices, however, the binary matrix derived from the ED method is not that much good.

4.3 Results for the PSO

In this section, we present the simulation and numerical results of the PSO method. Owing to the poor result of this algorithm as opposed to the GA method for binary matrices, we only show the results for real-valued signature matrices.

Fig. 7 shows the performance of the sub-optimised matrices derived by the PSO method for the case when $\beta = (5/2)$. The curves of the BER, ED and the capacity criteria are near the upper bound (1 bps per user) for high SNR value. In this figure, the results of the BER and the ED methods are very close to the capacity method unlike the GA as depicted in Fig. 3.

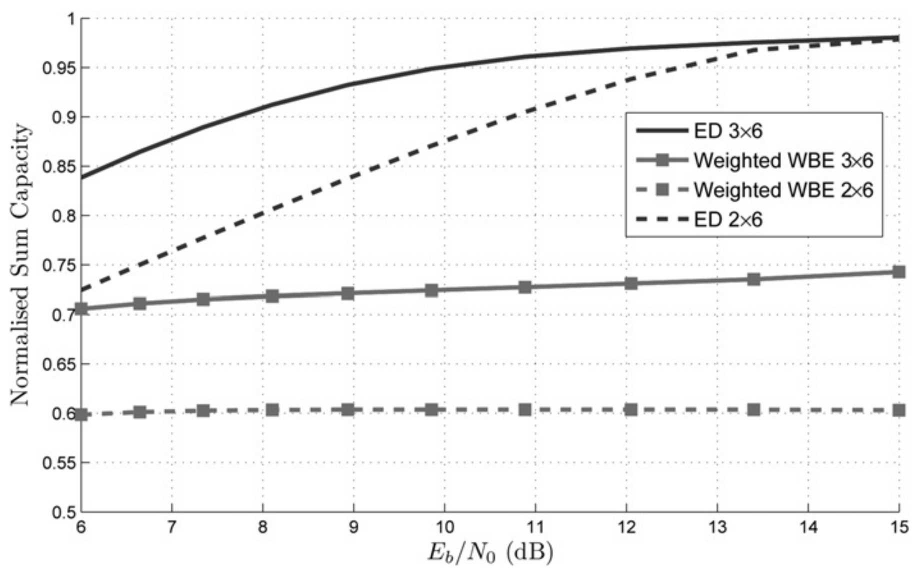

Fig. 4 *Normalised sum capacity of weighted WBE sequences and our sub-optimum matrices based on ED metric for β = (6/2) and β = (6/3)*

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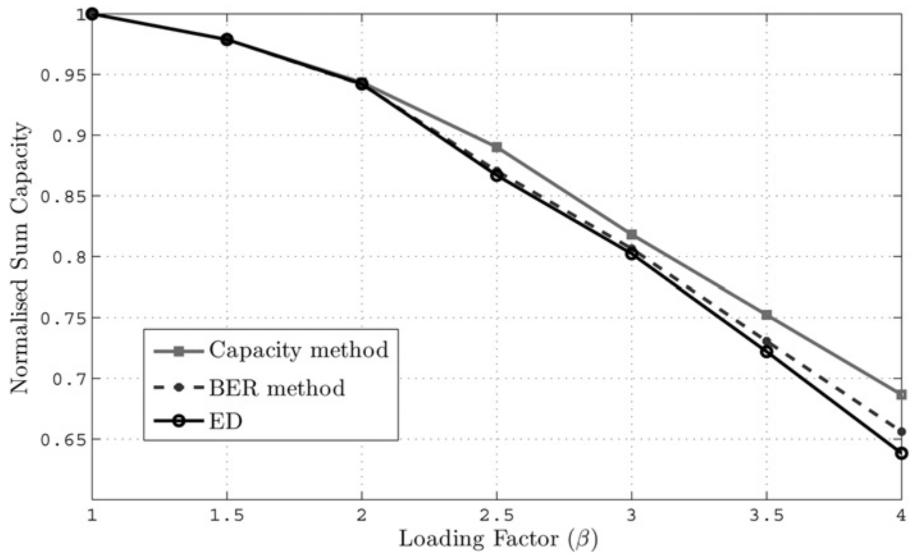

Fig. 5 *Normalised sum capacity against loading factor ($E_b/N_0 = 8$ dB, using the GA)*

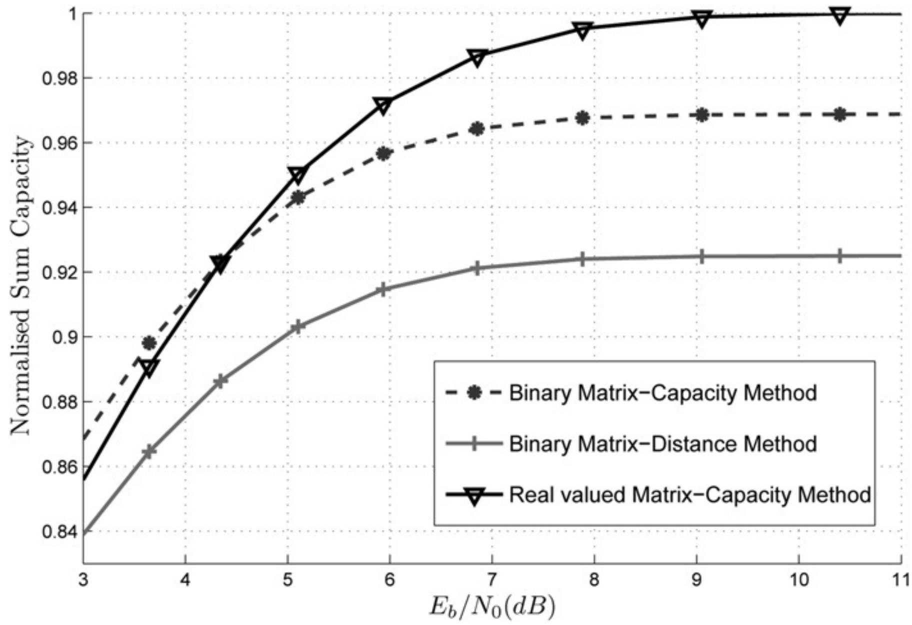

Fig. 6 *Normalised sum capacity of binary matrices and real-valued matrices [$\beta = (5/4)$, using the GA]*

Fig. 8 shows the normalised capacity against the loading factor for various criteria. As expected, the performances of all the cases decrease with increasing β. Similar to the GA method, the sum capacity and the BER measures are the best criteria for large β.

4.4 Comparison of GA and PSO results

In this section, we evaluate the sensitivity of the proposed criteria with respect to the loading factor and optimisation algorithms (GA and PSO). Fig. 9*a* demonstrates a comparison between the GA and the PSO for all of the criteria when $\beta = (5/2)$. This figure shows the normalised sum capacity of the GA minus the PSO algorithms for various criteria. Also, Fig. 9*b* presents the same results when $\beta = (4/3)$. A comparison between these two figures shows that for low loading factors (β) and small values of E_b/N_0, the PSO method performs better than the GA. On the other hand, for high values of β, the GA performs better. In addition, these two figures show that for the ED and BER criteria, the choice of the GA and the PSO algorithms does not make any difference.

Note that as discussed in Section 3, the PSO algorithm is about five times faster than the GA algorithm. Although, in general, the GA algorithm performs slightly better than the PSO method for real signature matrices, it is the only choice for binary matrices.

5 Large signature matrices

In this section, we propose a theorem to construct large signature matrices from small ones without reducing per-user sum channel capacity.

The computational complexity of deriving sub-optimum signature matrices for larger value of n and m is much more than that of smaller ones. Therefore by the proposed

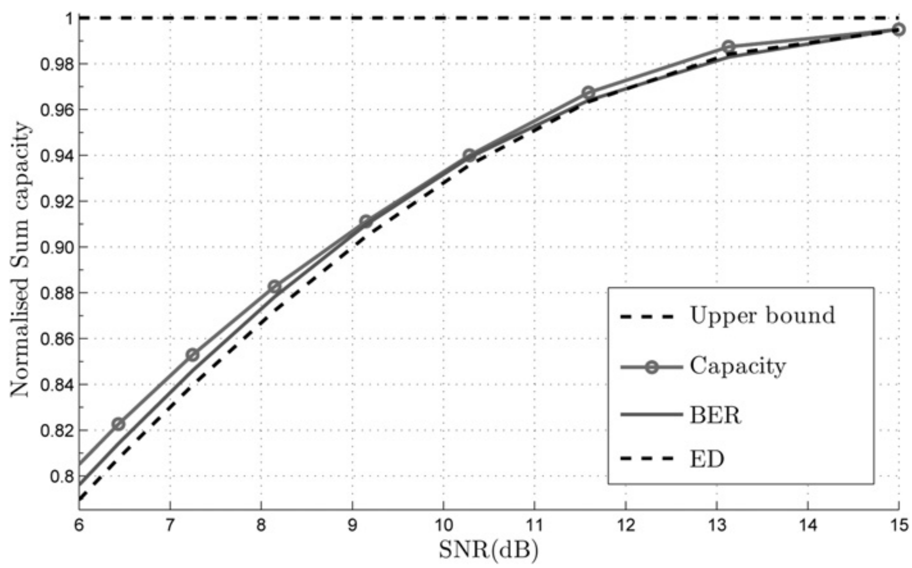

Fig. 7 *Normalised sum capacity of sub-optimum matrices based on various criteria with respect to E_b/N_0 values [$\beta = (5/2)$, using the PSO]*

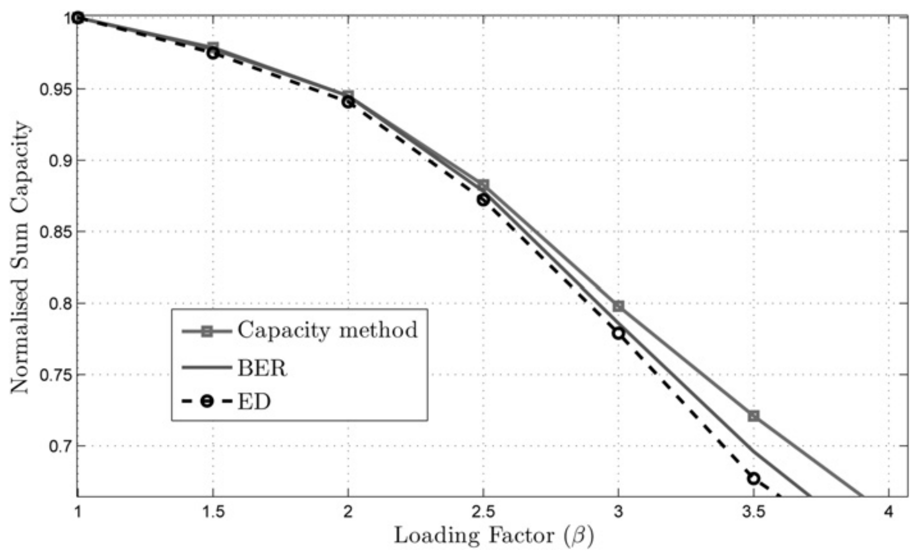

Fig. 8 *Normalised sum capacity of sub-optimum matrices against loading factor ($E_b/N_0 = 8$ dB, using the GA)*

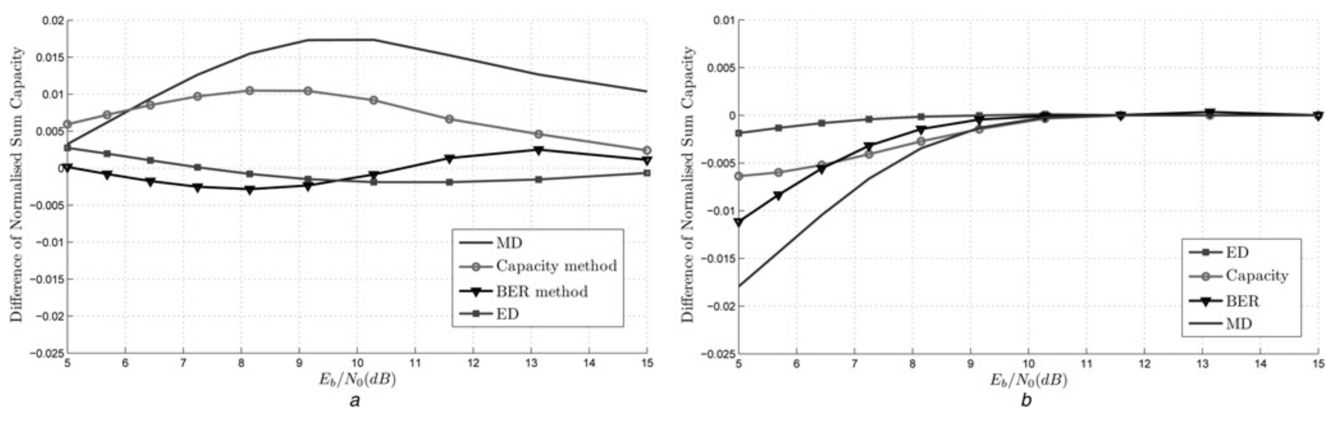

Fig. 9 *Normalised sum capacity derived from the GA minus that from the PSO for various measures*

a $\beta = (5/2)$
b $\beta = (4/3)$

theorem, we can create larger sub-optimal signature matrices from smaller ones. Instead of directly optimising the matrices using the proposed methods discussed in Section 2, we derive an optimum signature matrix for a certain small value of n and m. Then, we use 'Kronecker product' to enlarge the small sub-optimum matrix to obtain the larger one. We refer to this matrix as an 'enlarged matrix'.

The following theorem provides a method to enlarge a small signature matrix without changing the loading factor (β).

Theorem 1: Suppose that $\boldsymbol{A}$ is a real $m \times n$ signature matrix for a CDMA system with binary or non-binary inputs and $\boldsymbol{G}$ is a $k \times k$ reversible matrix such that its columns are normalised. Also, assume $C(kn, km, \sigma_N|\boldsymbol{A})$ is the sum channel capacity assigned to matrix $\boldsymbol{A}$ in the presence of AWGN noise with variance σ_N. Denote $\otimes$ as the Kronecker product and let $\boldsymbol{B} = \boldsymbol{G} \otimes \boldsymbol{A}$, then

$$C\big(kn, km, \sigma_N|\boldsymbol{B}\big) \le kC\big(n, m, \sigma_N|\boldsymbol{A}\big) \tag{15}$$

The equality holds if and only if $\boldsymbol{G}$ is unitary.

Proof: Considering (1) as the model of CDMA systems and $\boldsymbol{B}$ as the signature matrix, we have

$$\boldsymbol{Y}_{km\times 1} = \boldsymbol{B}\boldsymbol{X}_{kn\times 1} + \boldsymbol{N}_{km\times 1} \tag{16}$$

where $\boldsymbol{N}$ is the noise vector with variance σ_N. For an arbitrary vector $\boldsymbol{U}$, define $\boldsymbol{U}_i$ as the ith segment of $\boldsymbol{U}$ containing m entries; thus $\boldsymbol{U}^T = [U_1, U_2, \ldots, U_k]$. Multiply both sides of (16) by $\boldsymbol{D} = \boldsymbol{G}^{-1} \otimes \boldsymbol{I}_m$ and define $\boldsymbol{Z} = \boldsymbol{D}\boldsymbol{Y}$, hence

$$\boldsymbol{Z} = \big(\boldsymbol{G}^{-1} \otimes \boldsymbol{I}_m\big)(\mathbf{G} \otimes \mathbf{A})\boldsymbol{X} + \big(\boldsymbol{G}^{-1} \otimes \boldsymbol{I}_m\big)\boldsymbol{N}$$

From the Kronecker product properties, we have

$$\big(\boldsymbol{G}^{-1} \otimes \boldsymbol{I}_m\big)(\boldsymbol{G} \otimes \boldsymbol{A}) = \boldsymbol{G}^{-1}\boldsymbol{G} \otimes \boldsymbol{I}_m\boldsymbol{A} = \boldsymbol{I}_k \otimes \boldsymbol{A}$$

Denote $\boldsymbol{M} = \boldsymbol{D}\boldsymbol{N}$. Thus for each i, the entries of $\boldsymbol{M}_i$ are independent Gaussian random vectors with variance $\sigma_{M,i} = \|\boldsymbol{G}^{-1}(i, :)\|_2\sigma_N$. From $h(\boldsymbol{Z}|\boldsymbol{A})$, we can calculate $h(\boldsymbol{Y}|\boldsymbol{B})$ as follows

$$h(\boldsymbol{Y}|\boldsymbol{B}) = h(\boldsymbol{Z}|\boldsymbol{A}) - \log_2|\det \boldsymbol{D}| \tag{17}$$

From the Kronecker product properties

$$\det\big(\boldsymbol{G}^{-1} \otimes \boldsymbol{I}_m\big) = \big(\det \boldsymbol{G}^{-1}\big)^m \big(\det \boldsymbol{I}_m\big)^k \tag{18}$$

Considering (17) and (18), we have

$$h(\boldsymbol{Y}|\boldsymbol{A}) = h(\boldsymbol{Z}|\boldsymbol{B}) + m\log_2|\det \boldsymbol{G}|$$

Similarly, for $\boldsymbol{N}$ and $\boldsymbol{M}$, we have

$$h(\boldsymbol{N}) = h(\boldsymbol{M}) + m\log_2|\det \boldsymbol{G}| \tag{19}$$

Note that $\boldsymbol{N}$ and $\boldsymbol{M}$ are independent of $\boldsymbol{A}$ and $\boldsymbol{B}$. From (5), we can write

$$C\big(kn, km, \sigma_N|\boldsymbol{B}\big) = \max_{P(X)} h(\boldsymbol{Z}|\boldsymbol{B}) - h(\boldsymbol{M}) \tag{20}$$

Since $\boldsymbol{Z} = [Z_1, Z_2, \ldots, Z_k]^T$, we have the following upper bound for $h(Z)$

$$h(\boldsymbol{Z}|\boldsymbol{A}) \le \sum_{i=1}^{k} h\big(Z_i|\boldsymbol{A}\big) \tag{21}$$

Considering the relation between $\boldsymbol{Y}$ and $\boldsymbol{Z}$ and by using (5), we can derive the following equation

$$\max_{P(X)} h\big(Z_i|\boldsymbol{A}\big) = C\big(n, m, \sigma_{M,i}|\boldsymbol{A}\big) + \frac{m}{2}\log_2 2\pi e\sigma_N^2 - m\log_2\big\|\boldsymbol{G}^{-1}(i, :)\big\|$$

where $\boldsymbol{G}^{-1}(i, :)$ is the ith column of $\boldsymbol{G}^{-1}$. From (5) for $\boldsymbol{Y}$, we have

$$C\big(kn, km, \sigma_N|\boldsymbol{B}\big) \le \sum_{j=1}^{k} C\big(n, m, \big(||\boldsymbol{G}^{-1}(i, :)||\sigma_N\big)|\boldsymbol{A}\big) + m\log_2\frac{|\det \boldsymbol{G}|}{\prod_{i=1}^{k}\big\|\boldsymbol{G}^{-1}(i, :)\big\|}$$

If $\boldsymbol{G}$ is unitary, the equality holds and we have

$$C\big(kn, km, \sigma_N|\boldsymbol{B}\big) = kC\big(n, m, \sigma_N|\boldsymbol{A}\big)$$

Assume, $\boldsymbol{G}$ is not unitary. Since the columns of $\boldsymbol{G}$ are normalised, $|\det \boldsymbol{G}| < \prod_{i=1}^{k}||\boldsymbol{G}(:, i)|| = 1$ and $\|\boldsymbol{G}^{-1}(i, :)\| \ge 1$. Thus, owing to the reverse relationship between the sum-capacity and noise power, $C(n, m, (\|\mathbf{G}^{-1}(i, :)\|\sigma_N)|\boldsymbol{A}) \le C(n, m, \sigma_N)|\boldsymbol{A})$ and the following in equality holds

$$C\big(kn, km, \sigma_N|\boldsymbol{B}\big) < kC\big(n, m, \sigma_N|\boldsymbol{A}\big)$$

Therefore $C(kn, km, \sigma_N|\boldsymbol{B})$ is maximised when $\boldsymbol{G}$ is unitary.

According to this theorem, the best choice of $\boldsymbol{G}$ for enlarging a signature matrix is a unitary matrix. In practice, we use a binary Hadamard matrix with normalised columns as $\boldsymbol{G}$. Let $\boldsymbol{H}_n$ be a Hadamard matrix of order n, then $\boldsymbol{H}_2 \otimes \boldsymbol{H}_n$ is a Hadamard matrix of order $2n$ [21]. Using this fact, we can enlarge a signature matrix by a factor of 2^k where $k = 1, 2, \ldots$ and construct Hadamard matrices only from $\boldsymbol{H}_2$. The following provides an example of how to enlarge a signature matrix using Theorem 1.

Example 1: Assume $\boldsymbol{A}$ to be a 4×5 signature matrix

$$\boldsymbol{A} = \frac{1}{\sqrt{4}}\begin{bmatrix} +1 & +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 & +1 \\ +1 & +1 & -1 & -1 & +1 \\ +1 & -1 & -1 & +1 & -1 \end{bmatrix}$$

Let $\boldsymbol{A}$ be multiplied by $\big(1/\sqrt{4}\big)\boldsymbol{H}_4$ using Kronecker product and call the resultant matrix $\boldsymbol{B}$. Thus $\boldsymbol{B}$ is a 16×20 matrix; Table 3 shows $4 \times \boldsymbol{B}$.

Fig. 10 shows the normalised sum capacity of matrix $\boldsymbol{A}$ and $\boldsymbol{B}$. According to the figure, the curves (the normalised sum capacity) overlap. Since $\boldsymbol{B}$ is four times bigger than $\boldsymbol{A}$, the sum capacity of $\boldsymbol{B}$ is four times of that of $\boldsymbol{A}$, that is, $C(16, 20, \sigma_N|\boldsymbol{B}) = 4 \times C(4, 5, \sigma_N|\boldsymbol{A})$.

The enlarged matrices can be decoded using a simple ML decoder with significant reduction in the complexity of decoding. The following section describes the procedure of this decoder.

5.1 New ML decoder algorithm

In this section, we introduce a new ML decoder for enlarged signature matrices that reduces the decoding complexity, dramatically. This decoder was originally introduced in [3] for binary matrices and its non-binary version is discussed in [5]. In this paper, we extend it to real signature matrices.

Assume that we are given a $km \times kn$ enlarged signature matrix ($\boldsymbol{B}$) which is created from an $m \times n$ signature matrix ($\boldsymbol{A}$), that is, $\boldsymbol{B} = \boldsymbol{G} \otimes \boldsymbol{B}$. This decoder divides the decoding of $\boldsymbol{B}$ into k decoding of $\boldsymbol{A}$ matrices. Let $\boldsymbol{Y}$ be the received vector, $\boldsymbol{Y} = \boldsymbol{BX} + \boldsymbol{N}$, then The following is the step-by-step procedure for this decoder:

1. Define $\boldsymbol{Z} = \boldsymbol{G}^{-1} \otimes \boldsymbol{I_m Y}$ and split it into $\boldsymbol{Z}(i)$, $i = 1, 2, \ldots, k$ such that $\boldsymbol{Z}^{\mathrm{T}} = [Z^{\mathrm{T}}(1), Z^{\mathrm{T}}(2), \ldots, Z^{\mathrm{T}}(k)]$.
2. For $i = 1, 2, \ldots, k$, decode $\boldsymbol{Z}(i)$ and obtain $\hat{\boldsymbol{X}}(i)$.

If $\hat{\boldsymbol{X}}(i)^{\mathrm{T}} = \left[\hat{\boldsymbol{X}}_1^{\mathrm{T}}(i), \hat{\boldsymbol{X}}_2^{\mathrm{T}}(i)\right]$ and $\boldsymbol{A}_{m\times n} = \left[\tilde{\boldsymbol{A}}_{m\times m} | \boldsymbol{A}'\right]$, then $\hat{\boldsymbol{X}}_1^{\mathrm{T}}(i), \hat{\boldsymbol{X}}_2^{\mathrm{T}}(i)$ can be found as shown below:

$\hat{\boldsymbol{X}}_2(i)$ is a vector which minimises

$$\left\| \left(\tilde{\boldsymbol{A}}^{-1}\mathbf{Z}(i) - \tilde{\boldsymbol{A}}^{-1}\boldsymbol{A}'\hat{\boldsymbol{X}}_2(i) - \mathrm{sign}\left(\tilde{\boldsymbol{A}}^{-1}\mathbf{Z}(i) - \tilde{\boldsymbol{A}}^{-1}\boldsymbol{A}'\hat{\boldsymbol{X}}_2(i)\right)\right\|$$

and $\hat{\boldsymbol{X}}_1(i) = \mathrm{sign}(\tilde{\boldsymbol{A}}^{-1}\mathbf{Z}(i) - \tilde{\boldsymbol{A}}^{-1}\boldsymbol{A}'\hat{\boldsymbol{X}}_2(i))$.

3. Join $\hat{\boldsymbol{X}}(i)$'s to construct $\hat{\boldsymbol{X}}$ such that $\hat{\boldsymbol{X}}^{\mathrm{T}} = \left[\hat{X}^{\mathrm{T}}(1), \hat{X}^{\mathrm{T}}(2), \ldots, \hat{X}^{\mathrm{T}}(k)\right]$.

Note that step 2 uses a decoder which is discussed in [3]. The proposed decoder can be used for signature matrices, which are enlarged by Kronecker product, as discussed in theorem 1. For an enlarged $km \times kn$ signature matrix, this algorithm performs k simple ML decoding of size $m \times n$ instead of one complex $km \times kn$ ML decoding. Since the enlarged signature matrix can be divided by k identical $m \times n$ matrices, the result of this decoding procedure is the same as an ML decoder. The following provides an example to clarify this algorithm.

Example 2: Suppose that the enlarged signature matrix is $\boldsymbol{D} = \left(1/\sqrt{2}\right)\boldsymbol{H}_2 \otimes \boldsymbol{A}_{4\times 5}$, where $\boldsymbol{A}_{4\times 5}$ is $\boldsymbol{A}$ (4) from Table 2 in the Appendix; consequently

$$\boldsymbol{D} = \frac{1}{\sqrt{2}}\begin{bmatrix} \boldsymbol{A} & \boldsymbol{A} \\ \boldsymbol{A} & -\boldsymbol{A} \end{bmatrix}$$

Table 3 Matrix $4 \times \boldsymbol{B}$ where + is +1 and − is −1.

+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
+	−	+	−	+	+	−	+	−	+	+	−	+	−	+	+	−	+	−	+
+	+	−	−	+	+	+	−	−	+	+	+	−	−	+	+	+	−	−	+
+	−	−	+	−	+	−	−	+	−	+	−	−	+	−	+	−	−	+	−
+	+	+	+	+	−	−	−	−	−	+	+	+	+	+	−	−	−	−	−
+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−
+	+	−	−	+	−	−	+	+	−	+	+	−	−	+	−	−	+	+	−
+	−	−	+	−	−	+	+	−	+	+	−	−	+	−	−	+	+	−	+
+	+	+	+	+	+	+	+	+	+	−	−	−	−	−	−	−	−	−	−
+	−	+	−	+	+	−	+	−	+	−	+	−	+	−	−	+	−	+	−
+	+	−	−	+	+	+	−	−	+	−	−	+	+	−	−	−	+	+	−
+	−	−	+	−	+	−	−	+	−	−	+	+	−	+	−	+	+	−	+
+	+	+	+	+	−	−	−	−	−	−	−	−	−	−	+	+	+	+	+
+	−	+	−	+	−	+	−	+	−	−	+	−	+	−	+	−	+	−	+
+	+	−	−	+	−	−	+	+	−	−	−	+	+	−	+	+	−	−	+
+	−	−	+	−	−	+	+	−	+	−	+	+	−	+	+	−	−	+	−

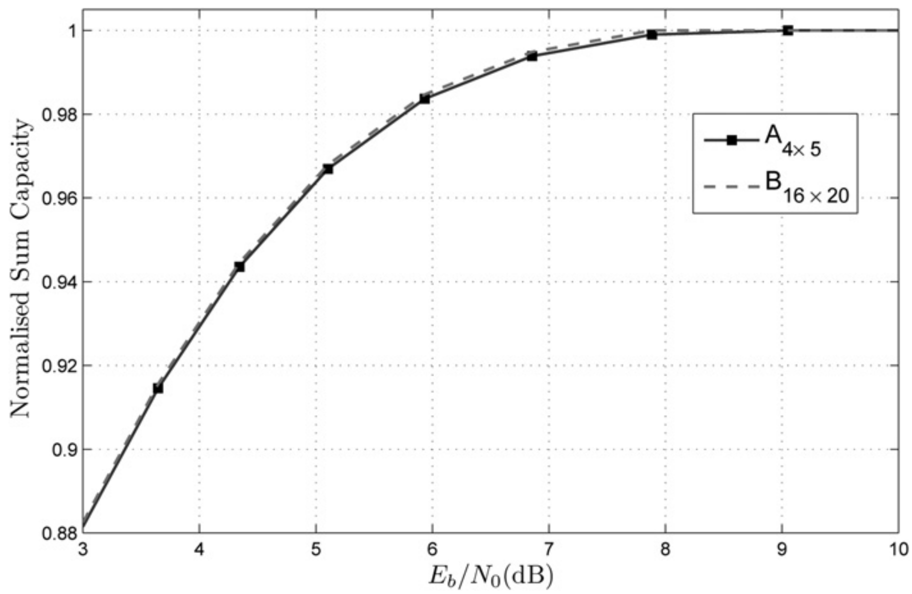

Fig. 10 *Normalised sum capacity of matrix* $\boldsymbol{A}$ *and* $\boldsymbol{B}$ *(Example 1)*

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Assume that $\boldsymbol{X} = [1\ 1\ -1\ -1\ -1\ -1\ -1\ 1\ 1\ -1]^{\mathrm{T}}$, then

$$\boldsymbol{Y} = \boldsymbol{DX} + \boldsymbol{N} = [-1.4586 \quad -0.5227 \quad -0.8251 \quad -1.3148 \quad 0.9584 \quad -0.1522 \quad 3.7170 \quad 2.0180]^{\mathrm{T}}$$

where $\boldsymbol{N}$ is a 8×1 AWGN vector. For the decoding at the receiver, we split $\boldsymbol{Y}$ into two equal length vectors $\boldsymbol{Y}(1)$ and $\boldsymbol{Y}(2)$ such that $\boldsymbol{Y} = [\boldsymbol{Y}(1)^{\mathrm{T}}, \boldsymbol{Y}(2)^{\mathrm{T}}]^{\mathrm{T}}$. We then have

$$\boldsymbol{Y}(1) = \left[-1.4586 \quad -0.5227 \quad -0.8251 \quad -1.3148\right]^{\mathrm{T}}$$
$$\boldsymbol{Y}(2) = \left[0.9584 \quad -0.1522 \quad 3.7170 \quad 2.0180\right]^{\mathrm{T}}$$

Define

$$\boldsymbol{Z}(1) = \frac{1}{\sqrt{2}}\left[Y^{(1)} + Y^{(2)}\right]$$
$$\boldsymbol{Z}(2) = \frac{1}{\sqrt{2}}\left[Y^{(1)} - Y^{(2)}\right]$$

The decoding of $\boldsymbol{Z}(1)$, $\boldsymbol{Z}(2)$ yields $\boldsymbol{X}(1) = [1\ 1\ -1\ -1\ -1]^{\mathrm{T}}$ and $\boldsymbol{X}(2) = [-1\ -1\ 1\ 1\ -1]^{\mathrm{T}}$, where $\boldsymbol{X} = [\boldsymbol{X}(1)^{\mathrm{T}}, \boldsymbol{X}(2)^{\mathrm{T}}]^{\mathrm{T}}$.

In general, for a $km \times kn$ signature matrix that is enlarged from an $m \times n$ matrix, the usual ML decoding needs $2^{km \times kn}$ Euclidean distance measurements whereas our approach needs $k2^{n-m}$ Euclidean distance measurements. As an example, suppose that we have a 64×80 signature matrix that is enlarged from a 4×5 signature matrix. The ML decoding for this matrix needs $2^{64} \times 2^{80} = 2^{142}$ Euclidean distance computations whereas for the proposed decoder, we only need $16 \times 2 = 32$ Euclidean distance measurements.

6 Conclusion and future work

In this paper, we have defined and derived the sum capacity for a specific signature matrix and a given E_b/N_0 for overloaded CDMA systems. To the best of our knowledge, deriving optimal real-valued signature matrices for binary input overloaded CDMA systems had not been investigated before. In order to find sub-optimum matrices, a number of optimisation criteria have been introduced; namely, the sum capacity, the BER and the distance measures. We have modelled these criteria in this work and shown that they differ from each other based on our simulation evaluations. Our simulation results demonstrate that our proposed ED criterion (i.e. one of the distance measures) provides the best performance with respect to the complexity and optimality accuracy. To derive the sub-optimum signature matrices, we have applied the CG, the GA and the PSO algorithms.

In addition to the real-valued matrices, we have also applied our methods to binary signature matrices. For large-scale systems, instead of direct optimisation of signature matrices, we are proposed to enlarge small sub-optimal matrices using Kronecker products. We have shown that the capacity of the enlarged matrices is increased by the enlargement factor k. We are proposed to employ simple ML decoding for such enlarged matrices, which significantly reduces the implementation complexity while maintaining optimality.

We have applied the CG, the GA and the PSO algorithms for the optimisation in this work. Other intelligent optimisation techniques such as 'simulated annealing' method may be used to improve the sub-optimum results. Also, further work is required for the derivation of sub-optimum matrices for non-binary discrete valued signatures to maintain lower implementation complexity in comparison with real-valued signatures.

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9 Appendix

Tables 4–7

Table 4 Sub-optimum matrices based on the sum capacity criterion using the GA (note: A_2 is optimised for $E_b/N_0 = 11$ dB and other matrices are optimum for $E_b/N_0 = 8$ dB)

Capacity criteria		
2×5	$A(1) = \begin{bmatrix} 0.1235 & 0.3177 & 0.7605 & 0.8739 & 0.4069 \\ 0.3723 & 0.9240 & 0.5021 & 0.0154 & -0.2553 \end{bmatrix}$	$A(2) = \begin{bmatrix} -0.3724 & 0.7299 & -0.0115 & 1.0000 & 0.5408 \\ -0.5254 & 0.2538 & 0.9584 & -0.3117 & -0.7224 \end{bmatrix}$
3×4	$A(3) = \begin{bmatrix} 0.9764 & 0.3895 & 0.7448 & -0.9375 \\ -1.0000 & 0.1711 & 0.4241 & 0.6451 \\ 0.8529 & 0.6424 & 0.0930 & 1.0000 \end{bmatrix}$	
4×5	$A(4) = \begin{bmatrix} 1 & 1 & 0.969 & 0.468 & 1 \\ 0.424 & -1 & 0.5 & -0.871 & 0.5 \\ 1 & 0.015 & -0.906 & -0.75 & 0.719 \\ 0.430 & 0.995 & -0.938 & 0.984 & 0.984 \end{bmatrix}$	$A(5) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 \end{bmatrix}$

Table 5 Sub-optimum matrices based on the ED criterion using the GA method (the matrices are optimum for $E_b/N_0 = 8$ dB)

ED criterion	
$A(1) = \begin{bmatrix} 0.0591 & 0.8787 & -0.6226 & 0.4163 & 0.2166 \\ -0.9198 & 0.1760 & 0.1907 & 0.6094 & 0.8851 \end{bmatrix}$	$A(2) = \begin{bmatrix} 0.9572 & 0.4704 & 0.5922 & 0.1288 \\ -1.0000 & 0.8393 & 0.3621 & 0.7090 \\ 0.3995 & 0.6776 & -0.7468 & -0.1777 \end{bmatrix}$
$A(3) = \begin{bmatrix} -1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 \\ -1 & -1 & -1 & -1 & -1 \end{bmatrix}$	

Table 6 Sub-optimum matrices based on the QD, MD and BER criteria using the GA method (the matrices are optimum for $E_b/N_0 = 8$ dB)

Criterion	3×4	2×5
QD	$\begin{bmatrix} 0.4520 & -0.3740 & 0.9029 & 0.1059 \\ -0.7780 & 0.3048 & 0.9585 & -0.6561 \\ 0.9163 & 0.4018 & 0.3265 & -0.0717 \end{bmatrix}$	$\begin{bmatrix} 0.5315 & 0.9989 & -0.9456 & 0.5273 & 0.4257 \\ 0.4364 & 0.3203 & 0.5859 & -0.9514 & 0.7039 \end{bmatrix}$
MD	$\begin{bmatrix} 0.5924 & 0.1238 & 0.4630 & -0.4371 \\ 0.0557 & 0.4388 & 0.6436 & 0.5020 \\ 0.9595 & 0.4137 & 0.0075 & 0.4325 \end{bmatrix}$	
BER	$\begin{bmatrix} 0.2502 & 0.4917 & 0.1048 & -0.9300 \\ 0.6206 & 0.9009 & -0.9958 & 0.4022 \\ 0.9903 & 0.2592 & 0.4383 & 0.9961 \end{bmatrix}$	

Table 7 Sub-optimum matrices based on various criteria using the PSO algorithm (the matrices are optimum for $E_b/N_0 = 8$ dB)

Criterion	2×5	3×4
capacity	$\begin{bmatrix} 1.0000 & 0 & 1.0000 & 1.0000 & -0.3120 \\ 0.9419 & 1.0000 & -0.6067 & 0.0812 & 0.6859 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1.0000 & 0.3137 \\ 0 & 1.0000 & 1.0000 & 0 \\ 1.0000 & 0 & 1.0000 & 0 \end{bmatrix}$
ED	$\begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 0.5432 & -0.0269 \\ 0.5206 & 0.1099 & -0.2031 & 1.0000 & 1.0000 \end{bmatrix}$	$\begin{bmatrix} 1.0000 & 0 & 0.0665 & 1.0000 \\ 0 & 1.0000 & 0 & 1.0000 \\ 0 & 0 & 1.0000 & 1.0000 \end{bmatrix}$
MD	$\begin{bmatrix} 0.3045 & 0.6719 & 1.0000 & 0.2925 & -0.0804 \\ 1.0000 & 0.2708 & 0.0711 & -0.7045 & 1.0000 \end{bmatrix}$	$\begin{bmatrix} 1.0000 & 0 & 1.0000 & 0.0483 \\ 1.0000 & 1.0000 & 0.0574 & 0 \\ 1.0000 & 0.0701 & 0 & 1.0000 \end{bmatrix}$
BER	$\begin{bmatrix} 1.0000 & -0.7644 & 0 & 1.0000 & 0.4113 \\ 1.0000 & 1.0000 & 0.5402 & -0.4707 & 1.0000 \end{bmatrix}$	$\begin{bmatrix} 1.0000 & 1.0000 & -0.2516 & -0.9898 \\ 1.0000 & -0.1536 & 1.0000 & -0.0209 \\ 1.0000 & 0 & -0.6976 & 1.0000 \end{bmatrix}$

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