Corrections

Corrections to "Abelian Group Codes for Channel Coding and Source Coding"

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THE group capacity of a discrete memoryless channel $(\mathcal{X}, \mathcal{Y}, W_{Y|X})$ is characterized with maximal probability of error in of [1, Sec. II]. There is a mistake in the proof of achievability as given in Section VII.A. It is correctly shown on page 2408-2409 that

$$\lim_{n\to\infty} \max_a \mathbb{E}[P(E(a))] = 0$$

if for all $\hat{\theta} \neq \boldsymbol{s}$,

$$R\frac{\sum_{(p,s)\in\mathcal{S}(G)}(s-\hat{\theta}_{p,s})w_{p,s}\log p}{\sum_{(p,s)\in\mathcal{S}(G)} s w_{p,s}\log q} < \log|H_{\eta^*+\hat{\theta}}| - H(X_{\eta^*,b}|Y,[X_{\eta^*,b}]_{\hat{\theta}}) - O(\epsilon).$$

However it is incorrectly claimed that the achievability conditions are: for all $\hat{\theta} \neq \boldsymbol{s}$,

$$R \leq \frac{1}{1-\omega_{\hat{\theta}}} I(X_{\eta^*,b}; Y|[X_{\eta^*,b}]_{\hat{\theta}}).$$

Our original objective was to characterize the average error group capacity of a discrete memoryless channel. The average error is more widely used than the maximal error. Although we had the proof of achievability for the average error case, we could not prove the converse. So we settled for characterizing the maximal error group capacity. In light of the above error, we have the following resolution.

We go back and consider the more widely used average error group capacity and provide a converse by slightly modifying the existing converse for maximal error group capacity. The incorrect achievability argument (given in the paper for maximal error) becomes perfectly valid for average error case. We refer the reader to the following 3 corrections.

1) The codebook $\mathbb{C}$ of Section II.5 is redefined as a shifted subgroup of G^n whose size is equal to Θ and the mappings are defined as $\text{Enc} : \{1, 2, \cdots, \Theta\} \to \mathbb{C}$, and $\text{Dec} : \mathcal{Y}^n \to \{1, 2, \cdots, \Theta\}$, such that

$$\frac{1}{\Theta}\sum_{m=1}^{\Theta}\sum_{x\in\mathcal{X}^n}\mathbb{1}_{\{x=\text{Enc}(m)\}}\sum_{y\in\mathcal{Y}^n}\mathbb{1}_{\{m\neq\text{Dec}(y)\}}W^n(y|x) \leq \tau.$$

Next, we start with the second paragraph of Section VI.A. We refer the reader to the following argument.

2) For a given channel $(\mathcal{X} = G, \mathcal{Y}, W_{Y|X})$, suppose that rate R is achievable using group codes. Consider an arbitrary $\epsilon > 0$. This implies that there exists a shifted group code $\mathbb{C}$ with parameters (n, k, w) that yields an average error probability τ such that $\tau \leq \epsilon$ and $\frac{k}{n}\sum_{(p,s)\in\mathcal{S}(G)} s w_{p,s}\log p \geq R-\epsilon$. Using a uniform distribution on a, we let X_i denote the random channel input at the ith channel use induced by this code.

Using the fact that $\tilde{a}$ is uniformly distributed over its range, for $i \in \Gamma_{\eta,b}$, in the code $\mathbb{C}_1(\hat{\theta}, \hat{a})$, the channel input $X_i(\hat{\theta}, \hat{a})$ at the ith channel use has the following distribution

$$P(X_i(\hat{\theta},\hat{a}) = \beta) = \prod_{(p,r,m)\in\mathcal{G}(G)} p^{-|r-\boldsymbol{\theta}(\eta+\hat{\theta})_{(p,r,m)}|^+} = \frac{1}{|H_{\eta+\hat{\theta}}|},$$

if $\beta_{(p,r,m)} \in \sum_{s=1}^{r_p}\hat{a}_{p,s}g^{(i)}_{(p,s)\to(r,m)} + b_{p,r,m} + p^{\boldsymbol{\theta}(\eta+\hat{\theta})_{(p,r,m)}}\mathbb{Z}_{p^r}$ for all $(p, r, m) \in \mathcal{G}(G)$, and $P(X_i(\hat{\theta}, \hat{a}) = \beta) = 0$ otherwise.

Let A denote the random message of the group transmission system of the code $\mathbb{C}$. Let $\hat{A}$ denote the part of the random message such that $\hat{A}_{p,s} \in \mathbb{Z}^{k w_{p,s}}_{p^{\hat{\theta}_{p,s}}}$, for all $(p, s) \in \mathcal{S}(G)$. Now consider a genie-aided receiver which gets access to $\hat{A}$ and performs maximum likelihood decoding. Clearly the average probability of error for this decoder must be not greater than ϵ. Using Fano's inequality, and the standard information theoretic arguments, we have for every $\hat{\theta}$ with $0 \leq \hat{\theta}_{p,s} \leq s$,

$$\begin{aligned}(1-\omega_{\hat{\theta}})(R-\epsilon)(1-\tau) - \frac{1}{n} &\leq \frac{1}{n}\sum_{\hat{a}} P(\hat{a}) I(X^n(\hat{\theta},\hat{a}); Y^n|\hat{a}) \\ &\leq \sum_{\hat{a}} P(\hat{a})\frac{1}{n}\sum_{i=1}^{n} I(X_i(\hat{\theta},\hat{a}); Y_i)\end{aligned}$$

This is equation (21) in the paper. The rest of Section VI.A continues.

3) Replace "..maximal.." with "...average.." on the 3rd line after the equation array on the left column on page 2409.

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REFERENCES

[1] A. G. Sahebi and S. S. Pradhan, "Abelian group codes for channel coding and source coding," *IEEE Trans. Inf. Theory*, vol. 61, no. 5, pp. 2399–2414, May 2015.