

A review of vehicle fuel consumption models to evaluate eco-driving and eco-routing



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ABSTRACT

Fuel consumption models have been widely used to predict fuel consumption and evaluate new vehicle technologies. However, due to the uncertainty and high nonlinearity of fuel systems, it is difficult to develop an accurate fuel consumption model for real-time calculations. Additionally, whether the developed fuel consumption models are suitable for eco-routing and eco-driving systems is unknown. To address these issues, a systematic review of fuel consumption models and the factors that influence fuel economy is presented. First, the primary factors that affect fuel economy, including travel-related, weather-related, vehicle-related, roadway-related, traffic-related, and driver-related factors, are discussed. Then, state-of-the-art fuel consumption models developed after 2000 are summarized and classified into three broad types based on transparency, i.e., white-box, grey-box and black-box models. Consequently, the limitations and potential possibilities of fuel consumption modelling are highlighted in this review.

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1. Introduction

The automobile industry is currently experiencing pressures due to the oil shortage crisis and environmental protection requirements (Salvi and Subramanian, 2015). A considerable amount of fuel is consumed by cars each year, resulting in a large amount of exhaust emissions. In 2011, approximately 59% of oil was used for transportation (World Oil Outlook, 2014), which resulted in approximately 22% of anthropogenic carbon dioxide emissions (IEA, 2013). Therefore, automobile manufacturers are currently under pressure to provide more environmentally friendly and fuel-efficient vehicles to consumers.

There are various ways to improve vehicle fuel economy, including new engine technologies, new vehicle technologies, new energy as well as new planning and control technologies, as indicated in Fig. 1. However, new engine and vehicle technologies have limited effects on lowering fuel consumption; advanced engine and vehicle technologies have potential efficiency improvements of 4–10% and 2–8%, respectively (U.S. Environmental Protection Agency, 2015a). Furthermore, although new advanced energy vehicles represent the future development trend of automobiles, more challenges in terms of battery mileage, battery lifetime and battery charging time should be overcome (U.S. Environmental Protection Agency, 2015b). Another available method for improving fuel economy is eco-driving and eco-routing. Several studies have indicated that eco-driving can improve fuel economy by 15–25% (CIECA, 2007; Hellström et al., 2009; Kamal et al., 2011; Cheng et al.,

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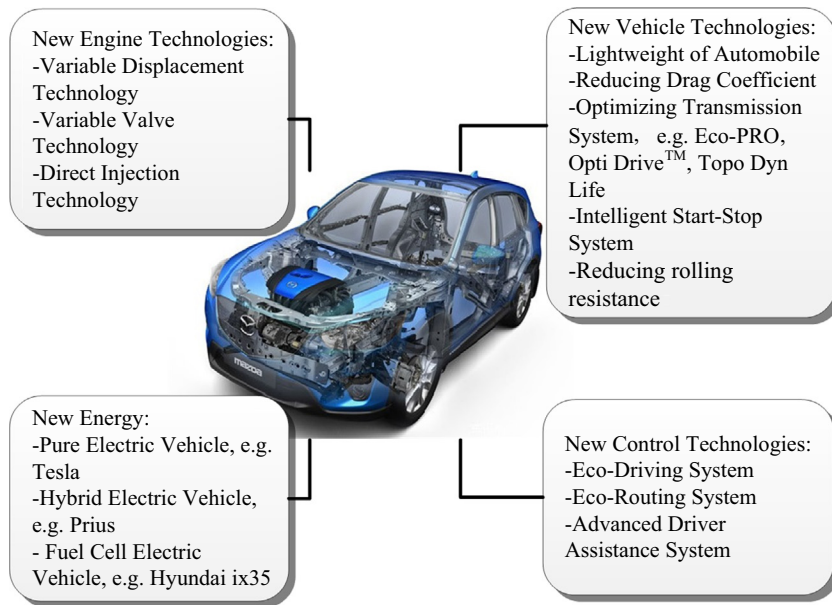


Fig. 1. New technologies for improving vehicle fuel economy.

2013; Casavola et al., 2010; Kundu et al., 2013; Ben Dhaou, 2011), and approximately 12–33% of fuel can be saved through eco-routing (Ben Dhaou, 2011; Navteq Green Streets, 2009). Therefore, eco-driving and eco-routing are effective ways to improve fuel economy in both the short-term and long-term.

To evaluate eco-driving and eco-routing algorithms, an appropriate fuel consumption model that can predict instantaneous fuel consumption second by second is needed. To identify the fuel consumption models that are best suited for eco-driving and eco-routing, a review of state-of-the-art fuel consumption models is necessary. Faris et al. performed a comprehensive review of state-of-the-art fuel consumption and emission models, such as the VT-Micro model (Ahn et al., 2002), power-based fuel consumption model (Post et al., 1984), and POLY model (Teng et al., 2002), and classified these models into five broad categories: (1) modelling based on the scale of the input variables, (2) modelling based on a formulation approach, (3) modelling based on the type of explanatory variables, (4) modelling based on state variable values and (5) modelling based on the number of dimensions (Faris et al., 2011). However, the authors only discussed the application of these models in evaluating the regional impacts of transportation projects. Whether these models are suitable for evaluating emerging eco-driving and eco-routing systems was not mentioned. Additionally, there are various factors that can influence fuel consumption, and few scholars have reviewed these factors thoroughly and systematically. Therefore, to help develop better fuel consumption models and find the fuel consumption models that are best suited for eco-driving and eco-routing systems, this paper discusses the main factors affecting fuel consumption and reviews classical fuel consumption models from the perspective of transparency. The paper classifies factors affecting fuel consumption into six broad categories, divides fuel consumption models into white-box, black-box and grey-box models, and addresses the following three problems:

- (1) The main factors affecting of fuel consumption, including travel-related, weather-related, vehicle-related, roadway-related, traffic-related and driver-related factors, are systematically analyzed and quantified, providing a systematic reference and direction for the development of future fuel consumption models.
- (2) The existing fuel consumption models are divided into white-box, black-box and grey-box models from the perspective of transparency. These three types of models are then compared in terms of advantages, disadvantages, accuracy, model structure and characteristics. This analysis will help engineers and researchers choose appropriate types of models and modelling methods while developing new fuel consumption models.
- (3) The models best suited for eco-driving and eco-routing systems are selected by comparing the model accuracy and structure of white-box, black-box and grey-box models. The limitations and potentials of fuel consumption modelling are highlighted by comparing the fuel consumption models based on their modelling methods and transparency. This analysis lays a good foundation for fuel consumption modelling and application.

This report is organized as follows: The primary factors that affect the fuel consumption are discussed in Section 2. Section 3 elucidates the classification method, dividing the models into three categories and presents the main models of each category. The different model categories are compared in terms of advantages, disadvantages, accuracy and model structure in Section 4. Conclusions and ideas for future studies are presented in Section 5.

2. Factors affecting vehicle fuel consumption

Numerous variables influence vehicle energy and emission rates. These variables can be classified into six broad categories: travel-related, weather-related, vehicle-related, roadway-related, traffic-related, and driver-related factors, as suggested by Ahn et al. (2002). The primary elements of each subcategory are shown in Fig. 2.

Although all of these elements affect fuel consumption, it is believed that only a few factors have significant effects (>3%). The following sections will discuss the factors affecting the fuel consumption.

2.1. Travel-related factors

Travel-related factors include the distance and number of trips travelled within an analysis period. Ahn et al. investigated the effects of route choice on fuel usage based on the VT-Micro and CMEM models, and their results indicated that approximately 18–23% of fuel could be saved when drivers sacrificed 4.3 min of travel time by choosing a longer route that has better traffic conditions for the same origin–destination pair (Ahn and Rakha, 2008). Frey et al. validated the actual effects of different routes on fuel consumption by conducting field experiments under real-world driving cycles, and their results indicated that total fuel consumption could be reduced by approximately 14–41% when comparing the highest to lowest fuel consumption routes for a given origin–destination pair (Frey et al., 2008). Moreover, Boriboonsomsin et al. and Barth et al. developed a few eco-routing systems based on advanced navigation systems (Boriboonsomsin and Barth, 2009; Barth et al., 2007); these eco-routing systems were capable of not only providing the shortest-distance route and the shortest-duration route but also routes that minimize fuel consumption. The fuel economy improvements achieved when using the newly developed eco-routing systems ranged from 8.73% to 42.15% based on the different traffic conditions considered.

2.2. Weather-related factors

Weather-related factors include ambient temperature, humidity, and wind effects. These factors affect fuel consumption through vehicle attachments, e.g., the air conditioner and the water pump. According to the EPA, fuel economy improvements of up to 1% may be achieved if the attachments are properly used and more efficient pumps are adopted (U.S. Environmental Protection Agency, 2015c).

2.3. Vehicle-related factors

The most important vehicle-related factors are the engine, loading, vehicle speed and acceleration. The engine is the key factor that affects fuel economy. The size of an engine, its power and speed, the type of fuel it uses and whether a vehicle is equipped with an exhaust after-treatment system directly determines engine fuel consumption performance (Ben et al., 2013). Vehicle speed and acceleration are the most intuitive variables that have a significant effect on fuel consumption (Joumard et al., 1995; Ericsson, 2001; El Shawarby et al., 2005; Park et al., 2010; Zhou et al., 2013). Joumard et al. proposed a two-dimensional fuel consumption model based on vehicle speed and acceleration to study the effects of velocity and acceleration on passenger cars (Joumard et al., 1995). Ericsson et al. investigated the effects of independent driving pattern factors on fuel usage using factorial analysis and observed that among all influencing factors, four factors are associated with acceleration and two are associated with speed (Ericsson, 2001). Shawarby et al. evaluated the effects of vehicle cruise speed and acceleration levels on fuel consumption, and their results indicated that the fuel consumption and emission rates per maneuver decreased as the level of aggressiveness for acceleration maneuvers increased (El Shawarby et al., 2005).

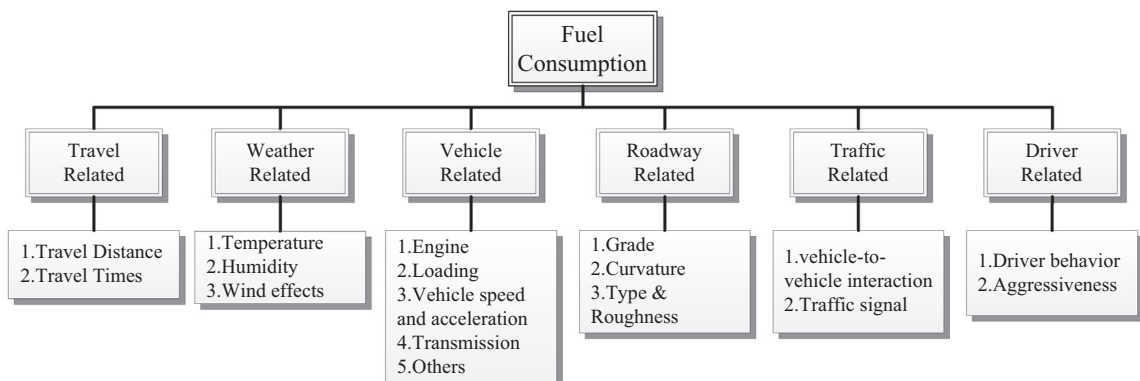


Fig. 2. A structure diagram of the factors affecting vehicle fuel consumption.

2.4. Roadway-related factors

Road-way-related factors primarily refer to the physical characteristics of a road, such as the roadway grade, surface roughness and horizontal curvature. Studies have indicated that the fuel economy on flat routes can be 5–20% superior to that on hilly routes (Kamal et al., 2011; Boriboonsomsin and Barth, 2009; Carrese et al., 2013; Wang et al., 2014; Renouf, 1979; Pan, 2005; Biggs, 1988). Kamal et al. developed an eco-driving system based on model predictive control (MPC) to run a vehicle on roads with up and down slopes (Kamal et al., 2011); their experiment on a virtually road indicated that fuel savings ranged from 5% to 7.04% when the eco-driving vehicle was compared with automatic speed control drive vehicle. This result was achieved by increasing the vehicle speed before entering the uphill slope and taking advantage of the downhill slope. A dynamic programming-based onboard look-ahead controller was proposed by Hellström et al. to minimize the trip time and fuel consumption of heavy trucks on hilly roads (CIECA, 2007). Consequently, a fuel reduction of approximately 3.5% was obtained over the 120 km route without an increase in the trip time. Similarly, Wang et al. optimized the fuel consumption of the up and down slopes using global and local optimization (Wang et al., 2014). The eco speed profile was obtained using a co-simulation involving MATLAB/Simulink and CarSim, and a fuel reduction of 5.5% was obtained.

The increased energy requirements due to a low-radius curve are approximately 3% for empty trucks and 9% for fully loaded trucks, as suggested by Renouf (Renouf, 1979; Pan, 2005). However, few studies addressing this issue have been found in the literature. Therefore, a considerable amount of work can be performed in future studies to quantify the effects of horizontal curvature on fuel consumption.

2.5. Traffic-related factors

Traffic-related factors include traffic flow and traffic signaling. Several studies have indicated that the use of traffic signal information has significant potential for saving fuel. Fuel consumption can be reduced by 22–50% depending on the application scenario considered (Biggs, 1988; Tielert et al., 2010; Asadi and Vahidi, 2011; Rakha et al., 2000; Pandian et al., 2009).

Tielert et al. studied the effect of traffic-light-to-vehicle communication on fuel consumption (Tielert et al., 2010); their result indicated that a fuel reduction of approximately 22% could be obtained by receiving phase-shifting information of the traffic lights and computing the optimal speed. Predictive cruise control (PCC) was used by Asadi et al. to quantify the effects of upcoming traffic signal information on fuel consumption (Asadi and Vahidi, 2011); their result indicated that approximately 47% of fuel consumption was saved when traffic signal information was utilized. Rakha et al. evaluated the effects of traffic signal control on fuel consumption under different operating scenarios using the VT-Micro model (Rakha et al., 2000). Greenwood et al. developed a calculation model to estimate the effects of traffic congestion on fuel consumption by modelling the acceleration noise (Greenwood et al., 2007). Widodo et al. demonstrated that fuel consumption under high vehicle densities and long traffic light cycle times could be lowered by inter-vehicle communication (Widodo et al., 2000). Sanchez et al. improved the fuel economy in urban circuits by 25% through interaction between the driver and traffic lights (Sanchez et al., 2006).

Furthermore, with the development of intelligent transportation systems (ITSs), intelligent vehicles can be used to assist the driver in reducing fuel consumption by communicating with other vehicles and traffic lights.

2.6. Driver-related factors

Driver-related factors primarily refer to driver behavior and aggressiveness, which are typically identified by speed and acceleration profiles. Compared with inexperienced drivers, experienced drivers can save fuel by skillfully adjusting their speed to avoid stops at traffic signals as well as hard acceleration and deceleration in other areas. Several studies have indicated that aggressive driving can cause 30–40% higher fuel consumption than calm driving (Sanchez et al., 2006; Ericsson, 2001; Evans, 1979; Berry, 2010; Van Mierlo et al., 2004). Furthermore, an experiment conducted by Taniguchi in the form of a driving contest on urban roadways demonstrated that fuel consumption could be reduced by up to 25% (Van Mierlo et al., 2004; Taniguchi, 2008).

To reduce fuel consumption from the driver's perspective, European scholars proposed a project known as FP7 ecoDriver (Cheng et al., 2013). This project aims to develop a new eco-driving assistance system to help drivers adopt the most energy-efficient driving style. It is expected that the ecoDriver project can deliver a 20% improvement in energy efficiency using autonomous methods (Cheng et al., 2013). Beusen et al. demonstrated an actual case study in Belgium on how eco-driving training reduced car fuel consumption by up to 6% (Beusen and et al., 2009). Zarkadoula et al. presented the results of an eco-driving pilot program in which the bus fuel consumption savings were approximately 4.35% when eco-driving was implemented by the drivers (Zarkadoula et al., 2007). Kamal et al. studied the on-board eco-driving system using MPC under varying road-traffic environments. Their results indicated that fuel economy was maximized when the number of times a car accelerated or braked and the magnitudes of those actions were minimized (Hellström et al., 2009).

Table 1 summarizes the effects of all of influencing factors on fuel consumption; the table indicates that only some of them have a significant effect (>3%) on fuel consumption. For a given vehicle model, certain factors, such as weather-related factor, have a small effect and can be ignored; however, other factors, particularly driver-related factors and the road slope within a given roadway-related category, cannot be neglected due to their considerable effects on fuel consumption. It should be noted that, these improvements are complementary instead of additive. When developing a new fuel consumption

Table 1

Summary of effects of all influencing factors on fuel consumption.

	Travel related	Weather related	Vehicle related	Roadway related	Traffic related	Driver related
Percentage (%)	8.73–42.15	1	Core	3–20	22–50	4.35–40
References	Ahn and Rakha (2008), Frey et al. (2008), Boriboonsomsin and Barth (2009), and Barth et al. (2007)	U.S. Environmental Protection Agency (2015c)	Ben et al. (2013), Joumard et al. (1995), Ericsson (2001), and El Shawarby et al. (2005)	Kamal et al. (2011), Boriboonsomsin and Barth (2009), Carrese et al. (2013), Wang et al. (2014), Renouf (1979), Pan (2005), and Biggs (1988)	Biggs (1988), Tielert et al. (2010), Asadi and Vahidi (2011), Rakha et al. (2000), and Pandian et al. (2009)	Kamal et al. (2011), Cheng et al. (2013), Ericsson (2001), Sanchez et al. (2006), Evans (1979), Berry (2010), Van Mierlo et al. (2004), and Taniguchi (2008)

model, the roadway-related and driver-related factors should be given first priority, followed by the travel and weather-related factors. Finally, traffic-related factors can be introduced by considering the communication between the driver, vehicle and traffic lights.

Additionally, three possibilities are highlighted by this analysis: improving fuel economy through eco-routing, eco-driving or communicating with other vehicles and traffic lights.

3. Vehicle fuel consumption models

Fuel consumption models can be classified into white-box models, grey-box models and black-box models in terms of their transparency. According to Madsen et al., a white-box model is typically derived from first principles; the mathematical framework of white-box models is highly deterministic and requires its developers to have a thorough understanding of the system considered and all influential sub-processes (Department of applied Mathematics and Computer Science, 2015; Cachón and Pucher, 2007). The opposite of the white-box model is the black-box model. Compared with white-box models, black-box models lack physics in their model structure, and only the input–output data of the system are used. Grey-box models lie between white-box and black-box models; this type of model is based on both insight into the system considered and experimental data (Kristensen et al., 2004; Department of applied Mathematics and Computer Science, 2015). The characteristics of these three types of models are presented in Fig. 3.

3.1. White-box fuel consumption models

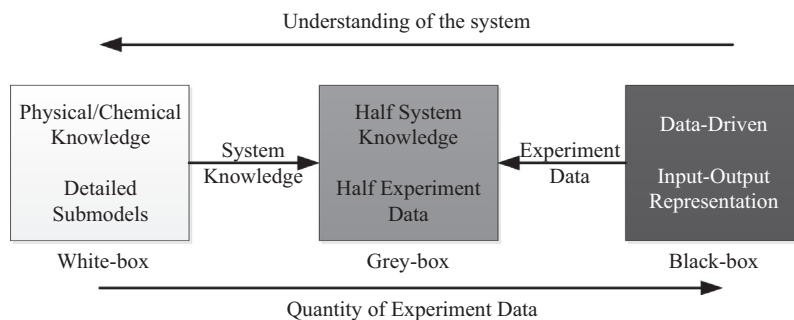
A so-called white-box fuel consumption model can be built based on an engine's physical or chemical processes, i.e., using mathematical formulas to describe the processes of engine intake, compression, combustion and exhaust. The key models of this type are as follows:

- (1). Carbon balance method
- (2). Mean value phenomenological model

The details of these two models will be discussed in the following section.

3.1.1. Carbon balance method

The basic principle of the carbon balance method is the conservation of mass, i.e., after combustion, the total carbon mass in the exhaust should be equal to the carbon quality of the fuel before combustion. The specific formulas used in China can be expressed as follows:

**Fig. 3.** Characteristics of white-, grey- and black-box models.

Gasoline vehicles:

$$FC = \frac{0.1154}{D} (0.886HC + 0.429CO + 0.273CO_2) \quad (1)$$

Diesel vehicles:

$$FC = \frac{0.1155}{D} (0.886HC + 0.429CO + 0.273CO_2) \quad (2)$$

where FC (L/100 km) is gas consumption; HC (g/km) is hydrocarbon emission; CO (g/km) is carbon monoxide emission; CO_2 (g/km) is carbon dioxide emission and D (kg/L) is the fuel density at 288 K.

Recently, Cachón et al. combined this method with a vehicle longitudinal dynamics model to predict the fuel consumption of a compressed natural gas (CNG) vehicle based on real-world measurements (Cachón and Pucher, 2007). The formula for the carbon balance method for CNG is:

$$FC = \frac{0.1336}{D} (0.749HC + 0.429CO + 0.273CO_2) \quad (3)$$

where FC (m³/100 km) is the fuel consumption; $D = 0.654$ kg/m³ is the CNG density at 288 K; and CO_2 , CO and HC are the exhausted emissions masses of carbon dioxide, carbon monoxide and hydrocarbons, respectively.

The carbon balance method has a simple prediction principle, excellent prediction performance and simple model structure. This model can be used for both offline fuel consumption analysis and onboard real-world fuel consumption estimation. However, the fuel consumption is calculated using the amounts of HC , CO , and CO_2 , and additional sensors are needed if this model is used for estimating onboard fuel consumption. Furthermore, the prediction accuracy is sensitive to the performance of the corresponding sensors.

The carbon balance method describes fuel consumption from the perspective of chemical combustion whereas the mean value phenomenological model was developed based on physical working principles.

3.1.2. Mean value phenomenological model

Heywood developed a mean value phenomenological model based on knowledge of internal combustion to predict the amount of consumed fuel, the exhausts produced and the torque (Heywood, 1988). This model consists of four primary subsystems: an intake manifold system, a fuel delivery system, a torque production system and an exhaust system.

- (1) By controlling the throttle valve, engine speed and manifold pressure, the intake manifold subsystem determines the amount of air taken into the manifold, \dot{m}_{ai} , and the cylinders, \dot{m}_{ao} .
- (2) In the fuel delivery subsystem, the engine control unit (ECU) controls the stoichiometric air–fuel ratio AFR_{st} . Thus, the fuel mass flow, \dot{m}_f (kg/s), into the cylinder can be calculated as follows:

$$\dot{m}_f = \frac{\dot{m}_{ao}}{AFR_{st}} \quad (4)$$

- (3) Fuel consumption is measured by the timing of the control signal from the ECU to the injectors. The amount of fuel injected, m_f (kg), is determined by the duration of the injection.

This mean value phenomenological model is generally used for conventional engine design and performance analysis. Recently, scholars have also used the model in the control field (Saerens et al., 2009; Moskwa and Hedrick, 1992; Chaumerliac et al., 1994; Weeks and Moskwa, 1995; Merker et al., 2006). For example, Saerens et al. used this model as the dynamic optimization objective function to minimize the fuel consumption of a gasoline engine (Saerens et al., 2009). This approach provides a statistical probability of 99% that the measured fuel consumption is within a deviation of $\pm 1\%$ of the actual consumption (Saerens et al., 2009). However, certain modelling parameters, such as the frontal flow area of the throttle body, manifold pressure ratio and manifold pressure, are generally inaccessible for most researchers. Therefore, this model is not practical for most modelers and users.

In conclusion, developing white-box models requires a thorough understanding of the entire engine system and all of its influential sub-processes. The number of parameters that need to be determined in a white-box fuel consumption models is typically large. Moreover, in certain cases, such models will be excessively complex or even impossible to obtain in a reasonable time frame due to the complex nature of the fuel system.

3.2. Black-box fuel consumption models

In black-box fuel consumption models, the entire vehicle or its engine alone is considered a black box. Based on the scale of inputs to black-box models, there are three different types of models:

- (1) engine-based black-box fuel consumption models,

- (2) vehicle-based black-box fuel consumption models,
- (3) modal-based black-box fuel consumption models.

The input variables of engine-based black-box fuel consumption models are engine-level variables, e.g., engine speed, engine torque, and engine output power. Vehicle-level variables, such as the vehicle's instantaneous speed and acceleration, average speed and acceleration, are inputs to vehicle-based black-box fuel consumption models. Modal-based black box fuel consumption models use basic operation modes as inputs, i.e., idling, cruising, decelerating and accelerating.

3.2.1. Engine-based black-box fuel consumption models

Fig. 4 illustrates the general procedure for estimating the fuel consumption using engine-based black-box fuel consumption models.

Saerens et al. assessed polynomial fuel consumption models to identify the appropriate low-degree fuel consumption models used in driver assistance systems (Saerens et al., 2013). Most of the evaluated models used engine speed, engine torque and engine load as inputs; therefore, they are classified as engine-based black-box fuel consumption models. According to Saerens et al., the engine-based black-box fuel consumption can be either linear or non-linear with respect to its coefficients, as follows (Saerens et al., 2013):

$$\dot{m}_f = \sum_{k=1}^M \alpha_k \omega^{p_k} u^{q_k} \quad p, q \in N^M, \alpha \in R^M \quad (5)$$

$$\dot{m}_f = \sum_{k=1}^M \alpha_k \omega^{p_k} \sum_{l=1}^N \beta_l u^{q_k} \quad p \in N^M, q \in N^N, \alpha \in R^M, \beta \in R^N \quad (6)$$

where \dot{m}_f (kg/s) is the fuel mass flow rate; ω (rad/s) is the engine rotation speed; u (–) is a control input; α (–) and β (–) are the model parameters; and p and q are the polynomial exponents. The control input u can be the traction power of the wheels P (W), brake torque T (Nm), or engine load τ (–).

It should be note that only when the engine brake torque or the engine load is used as the control input can this type of model be considered an engine-based black-box fuel consumption model. Hellström et al. developed an engine-based black-box fuel consumption model using engine torque and engine rotation speed as inputs (Hellström et al., 2009). Using this model with the trip time as the object function of a dynamic programming algorithm, the authors attempted to minimize the trip time and the fuel consumption of heavy trucks when traveling on hilly roads. The model they used can be expressed as follows:

$$\dot{m}_f = \alpha_1 \omega + \alpha_2 \omega^2 + \alpha_3 \omega T \quad (7)$$

A summary of the engine-based black-box models found in the literature is presented in Table 2.

The model structures in Hellström et al. (2009) and Table 2 are extremely simple, thus making them easy to develop. However, these models were assessed by Saerens et al. based on (1) measurements performed on a universal engine dynamometer, (2) measurements performed on a chassis dynamometer, and (3) on-road measurements (Saerens et al., 2013). The evaluation was performed based on fit quality. The authors' results indicated that these models behaved the best with engine dynamometer measurements ($99.01\% \leq R^2 \leq 99.68\%$), worse with chassis dynamometer measurements ($89.43\% \leq R^2 \leq 93.43\%$) and the worst with on-road measurements ($81.18\% \leq R^2 \leq 90.77\%$) in most cases. This finding indicates that even though these models exhibit satisfactory performances under engine or chassis dynamometer measurements, they are incapable of accurately predicting fuel consumption under on-road measurements. These models can be applied in situations that do not require high prediction accuracy. The models' accuracy should be improved if they are to be used in eco-driving and eco-routing systems.

3.2.2. Vehicle-based black-box fuel consumption models

If the inputs of a fuel consumption model are vehicle-level inputs, the model should be regarded as a vehicle-based black-box model. As depicted in Fig. 5, the vehicle itself is considered a black box.

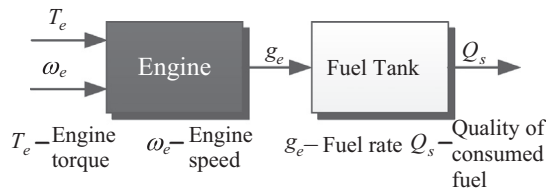
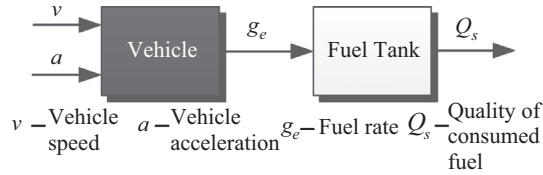


Fig. 4. Engine-based black-box fuel consumption models.

Table 2

Engine-based black box fuel consumption models: Engine torque/load as input.

Model Source	Model Structure
Passenberg et al. (2009)	$\dot{m}_f = \alpha_0 + \alpha_1 \omega + \alpha_2 \omega^2 + \alpha_3 \omega T + \alpha_4 T + \alpha_5 T^2$
Benedetto et al. (2013)	$\dot{m}_f = \alpha_0 \omega + \alpha_1 \omega^2 + \alpha_2 \omega^3 + \alpha_3 \omega T + \alpha_4 \omega^2 T + \alpha_5 \omega T^2$
	$\dot{m}_f = (\alpha_0 + \alpha_1 \omega^2)(\beta_0 + \beta_1 T + \beta_2 T^2)$
	$\dot{m}_f = (\alpha_0 + \alpha_1 \omega^2 + \alpha_2 \omega^3)(\beta_0 + \beta_1 \tau + \beta_2 \tau^2)$
	$\dot{m}_f = \alpha_0 \omega + \alpha_1 \omega \tau + \alpha_2 \omega^2 \tau + \alpha_3 \omega^3 \tau + \alpha_4 \omega \tau^2 + \alpha_5 \omega^2 \tau^2$

**Fig. 5.** Vehicle-based black-box fuel consumption models.

When the traction power of the wheels is used as the control input in Eqs. (5) and (6), the polynomial models should be regarded as a vehicle-based black-box model. Post et al. developed a vehicle-based fuel consumption model to estimate fuel consumption for trips of any length (Post et al., 1984). This model was built based on chassis dynamometer tests of 177 different spark ignition and diesel vehicles, and it used the instantaneous power demand experienced by a vehicle as input. Post et al. argued that the fuel consumption of a motor vehicle has a linear relationship with the requested power (Post et al., 1984), which can be formulated as follows:

$$FC = \begin{cases} \alpha + \beta \cdot Z_{tot} & \text{for } Z_{tot} \geq 0 \\ \alpha & \text{for } Z_{tot} < 0 \end{cases} \quad (8)$$

where FC (ml/min) is the fuel consumption; α (ml/min) and β (ml/(min·kW)) are the vehicle parameters; and Z_{tot} (kW) is the instantaneous total power derived from the vehicle's mass, drag, velocity, acceleration and road gradient.

The idling ($Z_{tot} < 0$) and nonidling ($Z_{tot} \geq 0$) modes were modelled separately, which allows this model to have a simple structure and a relatively high prediction accuracy (the errors for aggregate fuel consumption estimates are within 2%). Additionally, the effects of road gradients on fuel consumption are also considered in this model. The limitation of this model is that its accuracy decreases for microtrips (less than 100 m) particularly at low speed (Post et al., 1984). The same model framework was used by Piccoli et al. Benedetto et al. (2013), who used traffic data from mobile phones to estimate fuel consumption and emissions, and Leung and Williams (2000), who used data derived from in-use cars to predict the fuel consumption of spark ignition vehicles.

The CMEM model developed by An et al. takes acceleration, air/fuel equivalence ratio, fuel rate, speed, road grade, and accessory use as inputs (An et al., 1997). Modelling data were collected by the researchers using a dynamometer by testing 300 real world vehicles. The CMEM model consists of six main modules that predict the engine power, engine speed, air/fuel ratio, fuel use, engine-out emissions, and catalyst pass fraction. The fuel use module is the core of the CMEM model. It is a function of vehicle power demand, engine speed, and air/fuel ratio:

$$\dot{m}_f = \phi \left(k\omega + \frac{P}{\eta} \right) \frac{1}{44} \quad (9)$$

where \dot{m}_f (g/s) is the fuel rate; ϕ (–) is the air /fuel equivalence ratio; ω (rpm) is the engine speed; k (–) is a dimensionless coefficient; P (kW) is vehicle power demand; and η (–) is the transmission efficiency.

The CMEM model describes the process of vehicle fuel consumption. It is capable of estimating the second-by-second fuel consumption of different vehicle/technology categories. The model provides a clearer physical explanation than other black-box fuel consumption models. Table 3 shows a few typical vehicle-based black-box models developed by other researchers.

Another typical vehicle-based black-box fuel consumption model called VT-Micro was proposed by Rakha and Ahn (Ahn et al., 2002). The VT-Micro model was developed based on the chassis dynamometer measurements of five light-duty automobiles and three light-duty trucks; these fuel consumption rates were collected at Oak Ridge National Laboratory (ORNL). The VT-Micro model uses instantaneous speed and acceleration profiles as inputs to predict the fuel consumption of individual vehicles. The VT-Micro model was intended to show the significant effect of speed and acceleration on the fuel consumption rate, and its general framework is as follows:

$$\ln(MOE_e) = \begin{cases} \sum_{i=0}^3 \sum_{j=0}^3 (L_{ij}^e s^i a^j) & a \geq 0 \\ \sum_{i=0}^3 \sum_{j=0}^3 (M_{ij}^e s^i a^j) & a < 0 \end{cases} \quad (10)$$

Table 3

Vehicle-based black-box fuel consumption model: Requested power as input.

Model source	Model structure
Chang and Morlok (2005)	$\dot{m}_f = \alpha_0 P$
Ahn et al. (2010)	$\dot{m}_f = \alpha_0 \omega + \alpha_1 P$
Rakha et al. (2011)	$\dot{m}_f = \begin{cases} \alpha_0 + \alpha_1 P + \alpha_2 P^2 & P \geq 0 \\ \alpha_0 & P < 0 \end{cases}$
Saerens et al. (2013)	$\dot{m}_f = \begin{cases} \alpha_0 \omega_e + \alpha_1 P + \alpha_2 P^2 & P \geq 0 \\ \alpha_0 \omega_{idle} & P < 0 \end{cases}$
	$\dot{m}_f = \alpha_0 + \alpha_1 \omega + \alpha_2 \omega^2 + \alpha_3 P + \alpha_4 P^2$
	$\dot{m}_f = \alpha_0 \omega + \alpha_1 P + \alpha_2 P^2$

where MOE_e (ml/s) is the instantaneous fuel consumption or emission rate; L_{ij}^e (–) is the model regression coefficient for the MOE_e at a speed power “ i ” and an acceleration power “ j ” for positive accelerations; M_{ij}^e (–) is the model regression coefficient for the MOE_e at a speed power “ i ” and an acceleration power “ j ” for negative accelerations; s (km/h) is the instantaneous speed; and a (km/h/s) is the instantaneous acceleration.

The VT-Micro model is capable of precisely predicting real-world fuel consumption. It has two primary advantages. The first advantage is that the models for positive and negative acceleration regimes are developed separately, which ensures that the VT-Micro model accounts for the differences in the fuel consumption rate sensitivity to speed between acceleration and deceleration operation modes. The other advantage is the use of natural logarithms to ensure that non-negative fuel consumption rates are produced by the models (Faris et al., 2011). If a user wants to estimate the impact of grade using the VT-Micro model, the user can use a modified acceleration value to address the impact of grade because vehicle acceleration can be estimated by vehicle power and roadway grade.

Park et al. extended the VT-Micro model by applying it to a high-speed field to estimate the fuel consumption of heavy duty trucks, light-duty trucks and light-duty vehicles at speeds in excess of 80 mph (Park et al., 2010). Lei et al. combined the VT-Micro model with a polynomial model and produced the microscopic emission and fuel consumption (MEF) model Wei et al., 2010. The MEF considers not only the current acceleration but also the history of acceleration over the nine seconds prior to the current time point. The instantaneous acceleration in the VT-Micro model is replaced with the composited acceleration, as follows:

$$\bar{a}(t) = \alpha \cdot a(t) + (1 - \alpha) \sum_{i=1}^9 a(t - i)/9 \quad (11)$$

where $\bar{a}(t)$ is the composite acceleration; and α is the acceleration impact factor (AIF), $0 < \alpha < 1$.

The EPA's Motor Vehicle Emission Simulator (MOVES) is a state-of-the-art emission modelling system that estimates emissions for mobile sources at the national, county, and project levels for criteria air pollutants, greenhouse gases, and air toxics (<https://www3.epa.gov/otaq/models/moves/index.htm>). The current version of MOVES is MOVES2014a. MOVES is a vehicle-based black-box fuel consumption model. Its inputs are the vehicle's instantaneous speed and acceleration and its outputs are the rates of NO_x , PM, CO, CO_2 , SO_2 , and NH_3 emission. MOVES exhibits a good performance in estimating vehicle fuel consumption and emission rates, but it is relatively time-consuming (Faris et al., 2011). Because eco-routing and eco-driving systems needed to output the optimal solution in real-time, MOVES does not meet the requirement of fast computing.

The European Environment Agency's (EEA's) COPERT is a computer program that can compute emissions from road transport. The basic operating principle of COPERT is that the average EMFAC for a certain pollutant and a given type of vehicle varies according to the average speed during a trip (Faris et al., 2011). COPERT takes vehicle average speed as its input, and its output is the emission rate. The fuel consumption model in eco-routing and eco-driving systems needs to predict the instantaneous fuel consumption such that the objective function values of the eco-routing and eco-driving systems can be precisely calculated. Therefore, COPERT is not suitable for application in eco-routing and eco-driving systems.

It should be noted that there is a particular type of fuel consumption model known as neural network fuel consumption models. Depending on the scale of the data used to train the models, the neural network models can be either an engine-based or a vehicle-based black-box fuel consumption models. For example, Amer et al. developed a three-layer vehicle-based black-box model based on vehicle speed (Amer et al., 2014), whereas Parlak et al. proposed an engine-based black box fuel consumption model using the back propagation learning algorithm based on engine speed (Parlak et al., 2006).

3.2.3. Modal-based black-box fuel consumption models

The approach for predicting fuel consumption using modal-based black-box fuel consumption models is illustrated in Fig. 6.

One of the most classic modal-based black-box fuel consumption models is the elemental model. This type of model was developed to isolate the individual components of a trip and account for the fuel consumption in terms of the basic elements. Akcelik proposed an elemental model based on the basic operation modes of the vehicle (Post et al., 1984; Akcelik, 1982), which can be expressed as follows:

$$FC = (f_1 L + f_2 t_s + f_3 h)/T \quad (12)$$

where FC (ml/min) is the vehicle fuel consumption rate; f_1 (ml/km) is the cruise fuel consumption rate; f_2 (ml/min) is the idle fuel consumption rate; f_3 (ml/stop) is the fuel consumption per complete stop; L (km) is the trip cruise distance; t_s (min) is the trip stopped delay time; h (–) is the number of stops in the trip; and T (min) is the trip time.

However, this model only considers the cruise, idle, and stop modes and does not consider the acceleration or deceleration modes. Hung et al. developed a modal-based black-box fuel consumption model to predict vehicle fuel consumption based on the instantaneous speed and the driving modes (Hung et al., 2005). Its modelling data included the instantaneous speed and the fuel consumption of four different vehicles, and these data were collected in the urban areas of Hong Kong. The model structures can be expressed by the following equations:

Idling mode:

$$F_I(t_I) = \begin{cases} a \cdot e^{-bt_I} & t_I < T \\ c & t_I \geq T \end{cases} \quad (13)$$

Non-idling modes:

$$f(x) = \sum_{j=1}^N w(x_j) e_j / \sum_{j=1}^N w(x_j) \quad (14)$$

where $F_I(t_I)$ (mg/s) is the fuel consumption rate at time t_I ; a (–) and b (–) are the regression coefficients; t_I (s) is the idle time; T (s) is a certain threshold value that varies among vehicles; x is the distance between the data point and the center point of the corresponding speed class; $w(x_j) = 1 - x_j$ is the weighting function; and e_j (mg/s) is the fuel consumption rate of the data point located at x_j .

Unlike the element model proposed by Akcelik, this model considers all four basic driving modes. The non-idling modes and the idling mode were modelled separately, which allowed this model to have a relatively high prediction accuracy (average absolute percentage error within 15%). Specifically, piecewise interpolation functions were proposed for each non-idling mode, and a negative exponential function was used to estimate the fuel consumption of the idling mode. However, some parameters, such as the threshold of the idle time T and the weighting function, are difficult to determine. This model can be used with traffic simulation models but is not suitable for eco-driving or eco-routing systems because too many weighting functions need to be determined for the non-idling modes, which will slow down the calculation speed of eco systems.

A black-box fuel consumption model is typically based on experimental data and data processing technologies. The model does not offer much physical explanation, which makes it a purely mathematical model. Moreover, certain drawbacks exist in black-box fuel consumption modelling: the black-box model is a completely data-driven model, and all of the coefficients, regardless of whether the model is an engine-based model, a vehicle-based model or a modal-based model, must be determined based on a large amount of data using multiple linear or nonlinear regression methods. As experienced by most black-box modelers, it is labor-intensive and time-consuming to collect such large amounts of field data.

3.3. Grey-box fuel consumption models

In terms of understanding the internal workings of a system, the grey-box model lies between the black-box and the white-box models. Grey-box models imply a partial understanding of the internal system. The main models in the grey-box fuel consumption model subcategory as follows:

- (1) Vehicle transient emissions simulation software (VeTESS).
- (2) Transient fuel consumption model for non-road vehicles.

To calculate the fuel consumption of real traffic transient vehicle operations, Pelkmans et al. developed a vehicle-level simulation tool called VeTESS within the European project Decade (Pelkmans et al., 2004). VeTESS was built based on the measurements of both the chassis dynamometer and real-world traffic. The steady-state emission/fuel consumption rate, jump fraction, time constant, transient emissions/fuel consumption, engine speed, engine torque and change in torque

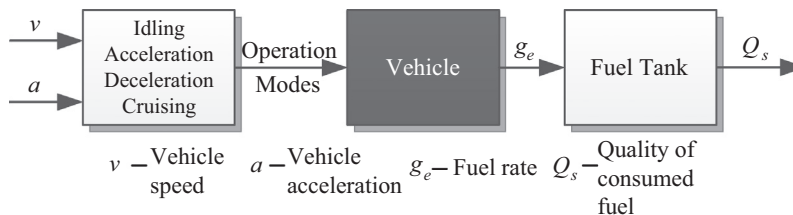


Fig. 6. Modal-based black-box fuel consumption model.

are used in the modelling process of VeTESS. The VeTESS assumes that when a vehicle operates under transient conditions, the engine moves through a series of quasi-steady states, which are described by a combination of engine speed and output torque, and the fuel consumption associated with each of these quasi-steady state conditions can be determined in the engine map. It should be noted that the engine map is corrected based on the traditional engine map by introducing the torque change to consider the dynamic behavior of the engine.

The inputs to the VeTESS are the vehicle speed and road gradient, and its outputs are emissions and fuel consumption rates. Based on the force acting upon the vehicle, VeTESS determines the engine speed and the torque through inverse simulation. Subsequently, the fuel consumption is determined by the supplemented engine map. This model can predict the dynamic fuel consumption of a specific vehicle as long as the speed profile is known. The VeTESS produces highly accurate estimates of fuel consumption (generally within 5%), increasing the calculated fuel consumption by approximately 6% for diesel vehicles and 10% for gasoline vehicles by introducing the transient corrections (Pelkmans et al., 2004). However, the VeTESS considers only one vehicle and one journey at a time (Pelkmans et al., 2004), and the jump fraction, change in torque, and transient fuel consumption are difficult to measure, which makes the VeTESS unavailable for users in certain instances. The VeTESS is best suited for analysis on the level of individual vehicles and journeys, and it is also suitable for use in eco-driving and eco-routing systems.

Based on studies on the effects of dynamic conditions on fuel consumption, Lindgren proposed a transient grey-box fuel consumption model for non-road mobile machinery (Lindgren and Hansson, 2004; Lindgren, 2005). The modelling data included the engine chassis measurements of synthetic transient cycles and a 20-mode steady-state test cycle. Similarly to the VeTESS, the change in both the engine speed and the torque are considered in this model. The entire model consists of two different parts, a semi-static part and a transient correction part, which can be represented as follows:

$$Z_t = Z_s(n, T) \times (1 + R_t(n, T, dn, dT)) \quad (15)$$

where Z_t (g/h) is the transient fuel consumption; Z_s (g/h) is the semi-static fuel consumption; R_t (–) is the transient correction function; n (rpm) and T (Nm) are the engine speed and torque, respectively; and dn and dT are the rate of change in the engine speed and torque, respectively.

The semi-static part is a traditional engine map that uses engine speed and torque as inputs. When calculating the fuel consumption of this part, the transient effects are neglected, and only the steady-state fuel consumption is considered. Three types of transient effects due to the change in the engine speed and the torque are evaluated in the transient correction part. First, the effects of the transient speed and the torque are calculated independently, and the effect is then evaluated as a synergism of the two, as follows:

$$R_t = q_n(T, dn) + q_T(n, dT) - cq_n(T, dn) \cdot q_T(n, dT) \quad (16)$$

where q_n (–) is a dimensionless correction factor due to transients in the engine speed; q_T (–) is a dimensionless correction factor due to transients in the engine torque; and c is a model-specific coefficient. Both q_n and q_T are obtained by looking up a certain correction matrix using the two-dimensional interpolation technique.

This grey-box model exhibits high accuracy; increasing the prediction accuracy by 25% by introducing transient correction factors. Additionally, the changes in the engine torque and speed are calculated instead of measured, which ensures that all of the modelling data are amenable to the measurement. Therefore, this model is suitable for use in the eco-routing and eco-driving systems. One of the obvious limitations is that the transient correction factors are difficult to determine.

A grey-box fuel consumption model can be viewed as a combination of a white-box and a black-box fuel consumption model. Specifically, compared with the white-box model, the grey-box fuel consumption model does not need to know exactly how an engine works, which makes the model easier to develop. Compared with the black-box model, the grey-box model has a certain physical meaning rather than being a purely mathematical model determined by its input and output. Its model structure is simpler than that of the white-box model, but more complex than that of the black-box model.

3.4. Hybrid fuel consumption model

Because different modelling methods have different characteristics, some scholars have attempted to combine different modelling methods to model fuel consumption. The following model is an example of the combination of a white-box and a grey-box model.

Chiara et al. developed an instantaneous hybrid fuel consumption model for diesel engines (Chiara et al., 2011). The model inputs are the torque requested and the engine speed and its outputs are the instantaneous engine brake torque and the fuel consumption under warmed-up conditions. This transient fuel consumption model consists of four primary modules: steady-state fuelling, simplified air path dynamics, fuelling transient correction and torque prediction. The steady-state fuelling module is a traditional engine map built based on the engine chassis dynamometer measurements of a turbocharged diesel engine. The steady-state module provides a steady-state fuel consumption prediction by looking up the map. Thus, the steady-state fuelling module is a type of grey-box model.

Under transient conditions, the fuel consumption prediction provided by the steady-state fuelling module is modified by the transient correction module and the simplified air path dynamics module. Parameters such as the intake manifold pressure, mass flow rate of the gas going into the cylinders and temperature of the gas input of the intake manifold are used as

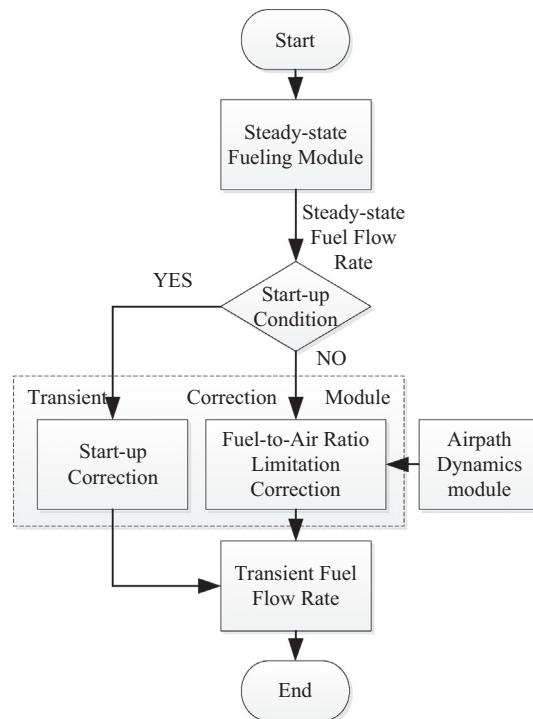


Fig. 7. Transient correction flow chart.

the inputs of the transient correction module. As illustrated in Fig. 7, two types of corrections are proposed: the start-up correction and the fuel to air ratio (FAR) limitation correction.

The air-path submodule is modelled based on physical principles and can be regarded as a white-box model. Thus, this instantaneous fuel consumption model features a synergy between white-box and grey-box modelling.

This hybrid fuel consumption model has a high prediction accuracy and a complex model structure. The parameters used in the steady-state module are accessible for most modelers, whereas the parameters used in the transient correction module are not. Therefore, this model is not practical for most users and is not recommended to be used in eco-driving or eco-routing systems.

4. Comparison and discussion

To determine the limitations of the existing fuel consumption models and determine the most suitable models to be used in eco-routing and eco-driving systems, comparisons between the white-box, grey-box, and black-box fuel consumption models are presented with respect to the influencing factors, advantages, disadvantages and accuracy.

The influencing factors considered by the aforementioned models, the advantages and disadvantages and other characteristics of these models are presented in Tables 4–6, respectively. As indicated in Table 4, the grey-box, black-engine and black-vehicle models consider the most factors. Because white-box models describe fuel consumption from the perspective of an engine's physical or chemical processes, compared with the other types of models, this type of model, particularly the mean value phenomenological model, is most suitable for use in the research and development of a new engine model. The carbon balance method is best suited for measuring fuel consumption when the driving cycle is determined. With respect to computing vehicle instantaneous fuel consumption, black-box models are recommended. The important influencing factors including roadway-related, traffic-related, and driver-related factors are considered by black-box models. These models are computationally faster than white-box and the grey-box models. Among them, power-based models are suggested for use in simple applications or applications that do not allow for field or in-laboratory measurements such as microscopic traffic simulation software; linear torque-based models are recommended in more complex applications that allow for measurement data such as eco-drive assist systems (Saerens et al., 2013). Modal-based black-box fuel consumption models consider travel-related and traffic-related factors only; thus, they are more suitable for estimating the aggregate fuel consumption of some cities. Grey-box models consider most of the six broad categories of influencing factors. They are computationally fast and precise and are recommended for estimating vehicle instantaneous fuel consumption, which requires high prediction accuracy.

Eco-routing and eco-driving systems have strict requirements on fuel consumption models selection. The fuel consumption model is used to calculate the object function values in eco-routing and eco-driving systems. This type of model should

Table 4

Influencing factors considered by the fuel consumption models.

	Travel Related	Weather related	Vehicle related	Roadway related	Traffic Related	Driver related
White-model	✓	×	✓	×	×	×
Black-engine	✓	×	✓	✓	×	✓
Black-vehicle	✓	×	✓	✓	×	✓
Black-modal	✓	×	×	×	✓	×
Grey-model	✓	×	✓	✓	×	✓

Table 5

Advantages and disadvantages.

Model type	Advantages	Disadvantages
White-model	1. Highest prediction accuracy.	1. Sophisticated model structures. 2. Too many model parameters need to be determined.
Black-engine	1. Simple model structures. 2. Few model input parameters.	1. 1.Cannot reflect real-world fuel consumption as data are collected using dynamometer. 2. 2.Difficult and time-consuming to collect experimental data
Black-vehicle	1. Simple model structures. 2. Few model input parameters.	1. Most cannot reflect real-world fuel consumption except for the VT-Micro and MEF models.
Black-modal	1. Simple model structures. 2. 2.Reflects vehicle's actual operation condition.	1. Fuel consumption is only related to the four basic modes instead of the actual engine load.
Grey-model	1. Less accurate than white-box model, but more accurate than black-box model. 2. Reflects real-world fuel consumption. 3. 3.Considers effects of the transient conditions on the fuel use.	1. More complicated than black-box models but less complicated than the white-box models. 2. Certain variables, such as the transients in engine speed and torque used in Lindgren (2005) are difficult to measure.

Table 6

Simple comparison of the white-, grey- and black-box fuel consumption models.

	White-box models	Grey-box models	Black-box models
System knowledge	Fully understand	Relatively clear	Unclear
Model structure	Complicated	Medium	Simple
Model parameters	Multiple	Few	Few
Model accuracy	High	Moderate	Low
Experimental data	Small	Medium	Large
Physical explanation	Explicit	Relatively clear	Unclear

be accurate enough to predict fuel consumption under any vehicle condition. In addition, the models should be simple enough to ensure that the entire eco-routing and eco-driving systems can output the optimal solution in real-time.

As indicated in [Tables 5 and 6](#), it is difficult for a model to simultaneously have a high accuracy and a simple structure. White-box models require the lowest amount of experimental data and their accuracy is quite high; however, their structures are extremely complex, which will increase the computation time if used in eco-routing and eco-driving systems. Therefore, white-box models are not suitable for using in eco-routing and eco-driving systems. Black-box models have simpler structures and lower prediction accuracy than white-box models do. However, black-box models provide little physical explanation, and a large amount of experimental data is needed to determine the model coefficients. Saerens et al. suggested that only the linear torque-based model, a type of engine-based black-box fuel consumption model is suitable for use in complex applications such as eco-driving and eco-routing systems, whereas the power-based model, a vehicle-based black-box model, is only suitable for simple applications or applications that do not allow for field or in-laboratory measurements ([Saerens et al., 2013](#)). Grey-box models strike a balance between accuracy and simplicity; they require less data and can predict fuel consumption under both steady-state and transient conditions. The structure of grey-box models is simpler than that of white-box models, which allows for faster computation. The prediction accuracy of grey-box models is higher than that of black-box models, which ensures that eco-routing and eco-driving systems can output more accurate optimal solutions. Therefore, grey-box and linear torque-based black-box models are recommended for use in eco-routing and eco-driving systems.

5. Conclusions

The primary factors that affect fuel consumption were classified into six categories, i.e., travel-related, weather-related, vehicle-related, roadway-related, traffic-related, and driver-related factors. The key elements in each category were discussed and further demonstrated using studies performed by other scholars. It was found that for a given vehicle model,

roadway-related, driver-related and traffic-related factors have the most significant effects on fuel consumption followed by travel-related factors; weather-related factors have the weakest impact on fuel consumption.

Currently available, fuel consumption models were thus classified as white-box, black-box or grey-box fuel consumption models with respect to transparency. The black-box fuel consumption models were further divided into three subcategories based on the scale of the input variables, i.e., engine-based, vehicle-based and modal-based black-box fuel consumption models. The key relevant models in each of these categories were presented. An example of the combination of the white-box and grey-box models was also provided.

Although few white-box fuel consumption models were identified in the literature, it was found that their structures are typically complicated, except for the structure of carbon balance method. These models generally consist of specialized physical or chemical knowledge and detailed submodels of a fuel system. White-box models have the highest prediction accuracy and the most complicated model structure. These models are not suitable for use in eco-routing and eco-driving systems due to their complicated structures.

Black-box fuel consumption models are purely mathematical models based on a large amount of experimental data. The most common inputs to the black-box fuel consumption models are the engine speed, engine torque, requested power, vehicle speed, acceleration, and operation modes. Most existing fuel consumption models can be classified into this category. These models have the least complicated model structure and the lowest prediction accuracy. Power-based models, which fall under that categories of vehicle-based and modal-based black-box models, are not suitable for applications in eco-routing and eco-driving systems. While Saerens et al. suggested that the linear torque-based model, a type of engine-based black-box fuel consumption model, is suitable for use in eco-driving and eco-routing systems,

Grey-box models strike a balance between model accuracy and complexity. They have more complicated frameworks than black-box models do but are simpler than white-box models. Their accuracy lies between that of white-box models and that of black-box models. The grey-box fuel consumption models introduced in this article can estimate fuel consumption under both steady- state and transient conditions. These advantages ensure that grey-box models can calculate the objective function values in real-time and output the optimal solutions precisely. Therefore, grey-box models are more suitable for applications in eco-routing and eco-driving systems than white-box models and certain black-box models.

A few possibilities in fuel consumption modelling are also highlighted in this review, i.e., improving fuel economy by eco-routing, eco-driving and communicating with other vehicles and traffic lights.

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