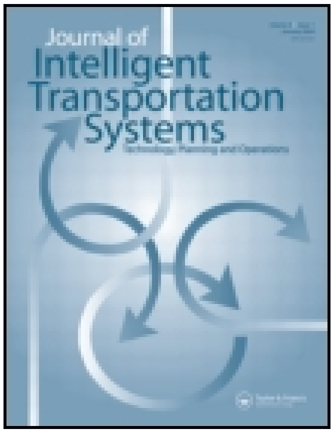

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Assessment of Alternative Polynomial Fuel Consumption Models for Use in Intelligent Transportation Systems Applications

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The objective of this article is to identify appropriate low-degree polynomial fuel consumption models for use in intelligent transportation systems (ITSs), eco-drive assist systems, and microscopic traffic simulation software. The different models that are assessed include models found in the literature and new models developed using a subsearch regression based on the Akaike information criterion. The models are evaluated based on model structures and their effectiveness in predicting instantaneous vehicle fuel consumption levels. Measurement data obtained from an engine dynamometer, a chassis dynamometer, and on-road testing are used to conduct the study. The study demonstrates that several low-degree polynomial fuel consumption models with a quadratic control term are appropriate for use in ITS applications ($R^2 > 0.9$).

Keywords Eco-Driving; Fuel Consumption Modeling; Intelligent Transportation Systems; Traffic Modeling

INTRODUCTION

The transportation sector is responsible for almost 15% of the anthropogenic greenhouse gas emissions (CO_2-equivalent) (Bernstein et al., 2007), and between 20 and 40% in major economies (International Energy Agency, 2008). In the United States, passenger automobiles make up 60% of the transportation energy (Davis, Diegel, & Boundy, 2008). Although transportation is not the largest source of greenhouse gas emissions, it is the fastest growing source and it is difficult to control. In Europe it is the only source of emissions that has increased since 1990 (Directorate-General for Energy and Transport of the European Commission, 2009). The emission of greenhouse gases, mainly CO_2 in transportation, is directly linked to the consumption of fossil fuels. Alternative fuel sources are a possible solution to reduce CO_2 emissions. However, in the short term these alternatives are not competitive with the conventional combustion engine. This is due to availability, cost, convenience, lack of technology, and accessibility. It is predicted that not even 25% of vehicles will be powered by electricity in 2050 (Dings, 2009). Probably the best way to lower fuel consumption is by changing the transportation mode, for example, using public transportation, cycling, or walking. Alternative ways include decreasing the resistance to motion of the vehicle by reducing its weight, rolling friction, and aerodynamic drag, and increasing the powertrain efficiency with, for example, more efficient engines or hybrid powertrains. Improving the driving behavior can yield significant fuel savings as well. Eco-driving entails adjusting driving behavior in order to minimize vehicle fuel consumption levels. Several studies show that eco-driving can reduce average fuel use (and thus CO_2 emissions) by 15 to 25% (CIECA, 2007). Consequently, eco-driving is both a short-term and cost-effective way to reduce greenhouse gas emissions.

In order to evaluate alternative eco-driving algorithms, and create training simulators and in-vehicle eco-drive assist systems, an appropriate fuel consumption model is required to

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predict the optimal driving strategy. This model can also be used to evaluate the system-wide impacts of eco-drive systems using microscopic traffic simulation software. Low-degree quasi-static polynomial fuel consumption models are widely adopted. For fuel consumption calculations, there is no need to model the engine dynamically; a quasi-static model suffices (Schmid & Bargende, 2011).

This article develops and assesses alternative low-degree polynomial fuel consumption models. These models are designed to provide simple computations and can be developed with a limited amount of field data, while still providing a reasonable level of accuracy.

POLYNOMIAL FUEL CONSUMPTION MODELS

A linear polynomial fuel consumption model is defined as follows:

$$\dot{m}_{\mathrm{f}} = \sum_{k=1}^{M} \alpha_k \omega^{p_k} u^{q_k},\ p, q \in \mathbb{N}^M, \alpha \in \mathbb{R}^M, \tag{1}$$

with $\dot{m}_{\mathrm{f}}$ (kg/s) the fuel mass flow rate, ω (rad/s) the engine rotation speed, u (—) a control input, α (—) model parameters, and p and q polynomial exponents. A polynomial fuel consumption model can also be nonlinear in the coefficients:

$$\dot{m}_{\mathrm{f}} = \sum_{k=1}^{M} \alpha_k \omega^{p_k} \cdot \sum_{l=1}^{N} \beta_l u^{q_l},\ p \in \mathbb{N}^M, q \in \mathbb{N}^N, \alpha \in \mathbb{R}^M, \beta \in \mathbb{R}^N, \tag{2}$$

with β_l (—) model parameters. The control input u can be the traction power of the wheels P (W), the engine brake torque T (Nm), or the engine load τ (—). The engine load is defined as the ratio of the torque to the maximum torque at a given engine speed:

$$\tau(T, \omega) = \frac{T}{T_{\max}(\omega)}. \tag{3}$$

If $\exists k : q_k = 2$ and $\forall k : q_k \leq 2$, then the consumption model is called quadratic. In what follows, extra assumptions for a quadratic fuel consumption model are made: $\exists k : q_k = 1$ and $\forall q_k > 0 : \alpha_k > 0$. A fuel consumption model is called affine if $\exists k : q_k = 1$ and $\forall k : q_k \leq 1$. One could also refer to this type of model as *linear*, that is, linear in the control input (linear $\leftrightarrow$ quadratic) as opposed to linear in the model parameters (linear $\leftrightarrow$ nonlinear).

A classification of polynomial fuel consumption models is given in Table 1. Power-based, torque-based, and load-based models are defined. Note that in power-based models, ω and P do not occur together in the same term: $\forall k : p_k \cdot q_k = 0$.

Power-based models often do not take negative engine power into account:

$$\dot{m}_{\mathrm{f}} = \begin{cases} \sum_k \alpha_k \omega^{p_k} P^{q_k}, \text{ if } P > 0, \\ \sum_k \alpha_k \omega_{\min}^{p_k} 0^{q_k}, \text{ if } P \leq 0. \end{cases} \tag{4}$$

Table 1 Classification of polynomial fuel consumption models.

Type	Consumption model
Power-based (PB)	$\sum_k \alpha_k \omega^{p_k} P^{q_k}, \forall k : p_k \cdot q_k = 0$
Linear torque-based (LTB)	$\sum_k \alpha_k \omega^{p_k} T^{q_k}$
Nonlinear torque-based (NLTB)	$\sum_k \alpha_k \omega^{p_k} \cdot \sum_l \beta_l T^{q_l}$
Linear load-based (LLB)	$\sum_k \alpha_k \omega^{p_k} \tau^{q_k}$
Nonlinear load-based (NLLB)	$\sum_k \alpha_k \omega^{p_k} \cdot \sum_l \beta_l \tau^{q_l}$

Because of their simplicity, low-degree polynomial fuel consumption models are commonly used. Table 2 gives an overview of polynomial consumption models that are found in the literature. The equations can be found in the appendix. Model P2 is also used by Leung and Williams (2000). Model P3 is also used by Ross and An (1993). Model T6 is also used by Yule, Kohnen, and Nowak (1999). Model L1 is also used by Schwarzkopf and Leipnik (1977).

ASSESSMENT METHODOLOGY

The low-degree polynomial fuel consumption models that are considered for the assessment are first of all the models that are found in the literature; see Table 2. Second, some new fuel consumption models are developed based on a subsearch with the Akaike information criterion.

Model Structure

A priori assessment of the polynomial fuel consumption models is done based on the model structure itself. Three properties are evaluated: (1) existence of a well-defined optimal gear shift behavior; (2) existence of a well-defined optimal velocity control; and (3) the number of model parameters. Properties 1 and 2 are found with vehicles in the real world. Based on these three criteria, a first selection of models is made.

Table 2 Polynomial fuel consumption models found in the literature.

Type	Model	Source	p	q	Eq.
PB	P1	Chang and Morlok (2005)	[0]	[1]	26
	P2	Post et al. (1984)	[0 0]	[0 1]	27
	P3	Ahn et al. (2010)	[1 0]	[0 1]	28
LTB	T1	Hellström et al. (2009)	[1 2 1]	[0 0 1]	29
	T2	Stoicescu (1995)	[0 1 1 2]	[0 0 1 1]	30
	T3	Stoicescu (1995)	[0 1 2 3 1]	[0 0 0 0 1]	31
	T4	Wang and Zoerb (1989)	[1 2 3 1 2 3 1]	[0 0 0 1 1 1 2]	32
	T5	Passenberg et al. (2009)	[0 1 2 1 0 0]	[0 0 0 1 1 2]	33
NLTB	T6	Jahns et al. (1990)	[1 2 3]	[0 1 2]	34
NLLB	L1	Abenavoli et al. (1999)	[0 1 2]	[0 1 2]	35

Optimal Gear Shift Behavior

The overall driveline ratio i (1/m), which depends on the gear, dictates the combination of engine speed ω and torque T to produce a given force F (N) at the wheels for a given velocity v (m/s):

$$\begin{aligned} \omega &= i v, \\ T &= \tfrac{F}{i\eta}, \end{aligned} \tag{5}$$

with η (—) the efficiency of the transmission. Assuming that η is independent of the gear, gear shifting has no influence on the fuel consumption for a given set of v and F (e.g., given steady state velocity), if:

$$\begin{aligned} \forall k &: p_k = 0, \text{ if } u = P, \\ \forall k &: p_k = q_k, \text{ if } u = T. \end{aligned} \tag{6}$$

Even if η depends on the gear, the influence of the gear on the fuel consumption is low in the latter cases.

Optimal Velocity Control

Minimum-fuel velocity control of a vehicle minimizes the fuel consumption. Simple fuel consumption models can yield bang-off-bang control with a singular control for minimum-fuel driving (e.g., Stoicescu, 1995). This means that an optimal trajectory can only consist of the following controls: full throttle acceleration (bang), coasting (off), full brake deceleration (bang), and constant velocity (singular control). Saerens et al. (2009) demonstrate on an engine dynamometer that bang-off-bang control is not fuel-optimal. Furthermore, more complex fuel consumption models (that are more accurate) usually do not yield bang-off-bang control, (e.g., Abenavoli, Carlini, Kormanski, & Rudzinska, 1999). For use in eco-drive applications, one should discard consumption models that yield this kind of unrealistic minimum-fuel behavior. As yielded by Pontryagin's maximum principle, a polynomial fuel consumption model will imply bang-off-bang control when it is affine (Saerens, Diehl, & Van den Bulck, et al., 2010). This implies that the fuel mass flow rate should be superlinear in the control variable u to avoid bang-off-bang control.

Number of Parameters

A small number of parameters reduces the model complexity. Fuel consumption models with few parameters can be easily calibrated using publicly available data, without the need for field measurements (Rakha, Ahn, Moran, Saerens, & Van den Bulck, 2011). Relevant data that are publicly available are (1) the engine size, (2) the city fuel economy, and (3) the highway fuel economy. Thus, fuel consumption models with three or fewer parameters can be calibrated without the need for field or in-laboratory measurements.

Quality of the Model Fit

A posteriori assessment of the polynomial fuel consumption models is done based on the quality of the model fit. The model parameters α of the linear models are identified with a linear least squares fit with the Moore–Penrose pseudo-inverse (Wetherill, 1986). The nonlinear models use a nonlinear regression based on iterations using the Newton method (Seber & Wild, 1989). This iteration needs a starting value. The solution is very sensitive to this starting value, which is obtained by trial and error.

The quality of the fit is evaluated and a model selection is performed. This can be done based on the Akaike information criterion AIC (Akaike, 1974) or the Bayesian information criterion BIC (Gelfand & Dey, 1994). In this work, AIC is chosen because it identifies the best model from a set of candidate models. BIC tries to find the *true* model (Burnham & Anderson, 2004). The fuel consumption of a combustion engine is by no doubt more complex than can be completely described by a low-degree polynomial. Thus, it is assumed that the true model is not in the set of candidate models. Furthermore, for any model selection criterion to be consistent, it must have suboptimality compared to AIC in terms of mean squared error (Yang, 2005).

The coefficient of determination R^2 (—) and the root mean squared error RMSE (kg/s) are also given in the evaluation (Gunst & Mason, 1980).

Available Measurement Data

Three sets of measurement data for the assessment of polynomial fuel consumption models are used: (1) measurements on a universal engine dynamometer; (2) measurements on a chassis dynamometer; and (3) on-road measurements. All measurements are done with gasoline engines.

Engine Dynamometer

Engine speed ω, engine torque T, and the fuel mass flow rate $\dot{m}_f$ are measured on a universal engine dynamometer at the KULeuven in Belgium. In 500 steady-state hot-stabilized working points, measurements are taken on a 2004 1.6-L port-fuel-injected 4-cylinder gasoline engine. The engine speed ranges from 900 to 4000 rpm and the torque from zero to maximum torque. The fuel mass flow rate is obtained with a second-order model of the fuel injector dynamics, resulting in 1% accuracy. Values of the vehicle power P for power-based consumption models are obtained by simulating a vehicle.

Chassis Dynamometer

Vehicle velocity v and fuel mass flow rate $\dot{m}_f$ are measured on six vehicles. The latter is done by measuring CO_2, CO, and HC in the exhaust. Cold-start fuel consumption is converted to hot-stabilized consumption using a linear model (Rakha et al., 2003):

$$\dot{m}_{\mathrm{f,hs}} = \frac{\dot{m}_{\mathrm{f,cs}}}{1+\alpha\left(1-\frac{\min(t,t_{\mathrm{hs}})}{t_{\mathrm{hs}}}\right)}, \tag{7}$$

with $\dot{m}_{\mathrm{f,hs}}$ (kg/s) the hot-stabilized fuel mass flow rate, $\dot{m}_{\mathrm{f,cs}}$ (kg/s) the fuel mass flow rate with a cold start, $t_{\mathrm{hs}} = 200$ s the engine warmup time, and $\alpha = 0.28$ the extra relative initial cold start consumption. Values of the traction force F are calculated with basic longitudinal dynamics. The freeway high speed (FWHS) and arterial/collectors LOS A-B (ARTA) cycles are used for the identification of the consumption model parameters. These two cycles cover a decent range of engine working points.

On-Road Tests

A global positioning system (GPS) is used to log each second the velocity v, distance s (m), and altitude h (m). The road slope θ (rad) is derived from the distance and altitude:

$$\theta = \tan^{-1}\frac{\Delta h}{\Delta s}. \tag{8}$$

Then θ is smoothed. An on-board diagnostics (OBD) reader logs the engine speed ω, the fuel mass flow rate $\dot{m}_f$, and the coolant temperature T_{c} (°C). Only hot-stabilized measurements are used ($T_{\mathrm{c}} > 80$C). Values of the traction force F and engine torque T are calculated with basic longitudinal dynamics. Eight vehicles were driven in Blacksburg, VA, partly on a highway and partly on a signalized arterial through the downtown. This results in about 2500 to 3000 data points for each vehicle. Measurements were always taken in dry weather with calm wind.

Consumption Model Assessment

The data from the engine dynamometer are gathered in a controlled environment at freely chosen steady-state operating points of the engine. These data are used to develop several new polynomial consumption models with a subsearch: New models are generated based on simplifications of a global model that has many parameters and includes all relevant consumption phenomena.

An inference from a fuel consumption model to some aspect of real-world driving is only justified if the model has been shown to adequately fit relevant data. Therefore, a first evaluation of existing and newly developed low-degree polynomial fuel consumption models is based on the model structure and the quality of the fit with the engine dynamometer data. This first evaluation yields a number of appropriate consumption models that are then further evaluated based on the quality of the fit with the data from a chassis dynamometer and the on-road measurements.

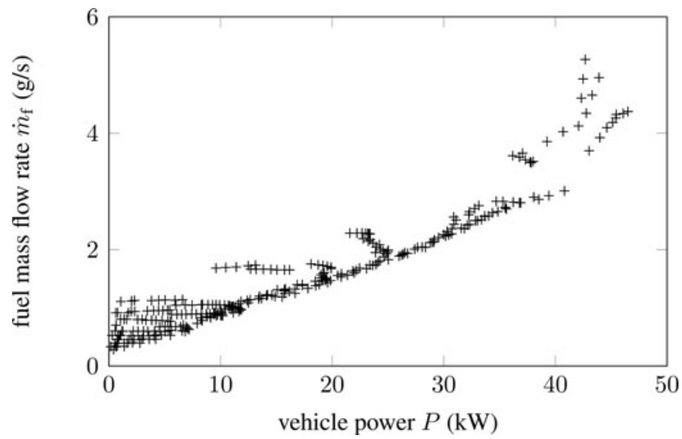

Figure 1 Measurement data from the engine dynamometer.

DEVELOPMENT OF NEW CONSUMPTION MODELS

Besides the polynomial fuel consumption models that are found in the literature, several new models are considered in the assessment. These new models are obtained from a *global model*, using a subsearch regression (Burnham & Anderson, 1998). Six models are developed: (1) a power-based model without engine speed terms; (2) a power-based model with engine speed terms; (3) a linear torque-based model; (4) a nonlinear torque-based model; (5) a linear load-based model; and (6) a nonlinear load-based model. The global model has a relatively large amount of parameters and can capture all necessary fuel consumption effects. This model is then simplified using a subsearch regression based on the Akaike information criterion AIC:

$$\mathrm{AIC} = N\ln\left(\frac{\mathrm{RSS}}{N}\right) + 2\,(K+1)\,, \tag{9}$$

with N the number of data points, RSS the residual sum of squares, and K the number of parameters in the polynomial fuel consumption model. AIC is a measure of the accuracy of the regression, considering the quality of model fit and the number of model parameters. This measure is only useful to compare different models that are developed using the same data set.

Power-Based Models

Figure 1 shows the fuel mass flow rate as a function of the vehicle power as it is measured on the engine dynamometer. Two important properties are observed: (1) an offset at zero power, and (2) a convex shape (upward curvature). A simple power-based model that can capture these effects is a quadratic model:

$$\dot{m}_{\mathrm{f}} = \alpha_1 + \alpha_2 P + \alpha_3 P^2. \tag{10}$$

Table 3 illustrates the model subsearch starting from a base model. For illustration, the base model is chosen to be more complex than the model of Eq. 10. All the possible

Table 3 Subsearch for a power-based consumption model.

Model	AIC
$\dot{m}_f = \alpha_1 + \alpha_2 P + \alpha_3 P^2 + \alpha_4 P^3$	−5054.4
$\dot{m}_f = \alpha_1 P + \alpha_2 P^2 + \alpha_3 P^3$	−5056.4
$\dot{m}_f = \alpha_1 + \alpha_2 P^2 + \alpha_3 P^3$	−5143.8
$\dot{m}_f = \alpha_1 + \alpha_2 P + \alpha_3 P^3$	−5193.7
$\dot{m}_f = \alpha_1 + \alpha_2 P + \alpha_3 P^2$	−5194.4
$\dot{m}_f = \alpha_1 P + \alpha_2 P^2$	−5000.9
$\dot{m}_f = \alpha_1 + \alpha_2 P^2$	−4998.6
$\dot{m}_f = \alpha_1 + \alpha_2 P$	−5182.6
$\dot{m}_f = \alpha_1 P$	−4937.4
$\dot{m}_f = \alpha_1$	−4327.6

simplifications are considered by discarding each term. The simplification that has the lowest AIC value is the best simplification. The subsearch yields the following sequence of models:

1. $\dot{m}_\text{f} = \alpha_1 + \alpha_2 P + \alpha_3 P^2 + \alpha_4 P^3$.
2. $\dot{m}_\text{f} = \alpha_1 + \alpha_2 P + \alpha_3 P^2$.
3. $\dot{m}_\text{f} = \alpha_1 + \alpha_2 P^2$.
4. $\dot{m}_\text{f} = \alpha_1 P^2$.

All models have an upward curvature. The fourth model in the sequence has no offset at zero power and is therefore not considered. The second model has the lowest AIC of the remaining models and thus is selected as the best model. It is named model P4 in the rest of the article.

A power-based model with engine speed terms can also be considered. The proposed base model is the following:

$$\dot{m}_\text{f} = \alpha_1 + \alpha_2\omega + \alpha_3\omega^2 + \alpha_4\omega^3 + \alpha_5 P + \alpha_6 P^2 + \alpha_7 P^3. \quad (11)$$

The first four terms take into account the engine friction, and the remaining three the efficiency of the production of internal power. Three different sources of friction can be identified: (1) dry (constant or coulomb) friction from certain engine peripherals or external loads (α_1); (2) viscous friction from engine parts separated by an oil filter, oil pump, and water pump ($\alpha_2\omega$); and (3) aerodynamic friction from gasses flowing through the engine ($\alpha_3\omega^2$). Since several existing polynomial fuel consumption models use a cubic friction term ($\alpha_4\omega^3$), for example. models T3–T5 in Table 1, this is added to the base model too.

A subsearch starting from the presented base model yields the following model based on the lowest AIC:

$$\text{P6}: \dot{m}_\text{f} = \alpha_1 + \alpha_2\omega + \alpha_3\omega^2 + \alpha_4 P + \alpha_5 P^2. \quad (12)$$

This model is named model P6. The best model with three or less parameters is the following:

$$\text{P5}: \dot{m}_\text{f} = \alpha_1\omega + \alpha_2 P + \alpha_3 P^2. \quad (13)$$

It is named model P5 and is also considered for further evaluation.

Torque- and Load-Based Models

Both torque- and load-based models are directly linked to the engine (not using the vehicle power P). For these models it is desired that they capture certain observations in engine maps. Figure 2 shows the brake-specific fuel consumption BSFC (g/kWh) as it is measured on the engine dynamometer. Important observations in the map are: (1) There is a point of minimum BSFC around 2000 rpm and almost full torque; and (2) iso-lines are convex in the mid- and low-torque range. Figure 3 shows the measured brake consumption per rotation BCPR (kg/rad). An important observation is that the iso-lines are slightly concave and have a maximum around 2000 rpm. Following the same reasoning used in the proposition of the last power-based model, the following base models are proposed (linear torque-based, nonlinear torque-based, linear load-based, and nonlinear load-based, respectively):

$$\begin{aligned}\dot{m}_\text{f} = {} & \alpha_1 + \alpha_2\omega + \alpha_3\omega^2 + \alpha_4\omega^3 + \alpha_5 T + \alpha_6\omega T \\ & + \alpha_7\omega^2 T + \alpha_8\omega^3 T + \alpha_9 T^2 + \alpha_{10}\omega T^2 \\ & + \alpha_{11}\omega^2 T^2 + \alpha_{12}\omega^3 T^2 + \alpha_{13} T^3 + \alpha_{14}\omega T^3 \\ & + \alpha_{15}\omega^2 T^3 + \alpha_{16}\omega^3 T^3, \end{aligned} \quad (14)$$

$$\begin{aligned}\dot{m}_\text{f} = {} & \left(\alpha_1 + \alpha_2\omega + \alpha_3\omega^2 + \alpha_4\omega^3\right) \cdot \\ & \left(\beta_1 + \beta_2 T + \beta_3 T^2 + \beta_4 T^3\right), \end{aligned} \quad (15)$$

$$\begin{aligned}\dot{m}_\text{f} = {} & \alpha_1 + \alpha_2\omega + \alpha_3\omega^2 + \alpha_4\omega^3 + \alpha_5\tau + \alpha_6\omega\tau \\ & + \alpha_7\omega^2\tau + \alpha_8\omega^3\tau + \alpha_9\tau^2 + \alpha_{10}\omega\tau^2 + \alpha_{11}\omega^2\tau^2 \\ & + \alpha_{12}\omega^3\tau^2 + \alpha_{13}\tau^3 + \alpha_{14}\omega\tau^3 \\ & + \alpha_{15}\omega^2\tau^3 + \alpha_{16}\omega^3\tau^3, \end{aligned} \quad (16)$$

$$\begin{aligned}\dot{m}_\text{f} = {} & \left(\alpha_1 + \alpha_2\omega + \alpha_3\omega^2 + \alpha_4\omega^3\right) \cdot \\ & \left(\beta_1 + \beta_2\tau + \beta_3\tau^2 + \beta_4\tau^3\right). \end{aligned} \quad (17)$$

Starting from the base models and only considering models that yield the mentioned observations in the maps, the following models are generated with a subsearch (linear torque-based, nonlinear torque-based, linear load-based, and nonlinear load-based, respectively):

$$\begin{aligned}\text{T7}: \dot{m}_\text{f} = {} & \alpha_1\omega + \alpha_2\omega^2 + \alpha_3\omega^3 + \alpha_4\omega T \\ & + \alpha_5\omega^2 T + \alpha_6 T^2 + \alpha_7\omega T^2 + \alpha_8\omega^2 T^2, \end{aligned} \quad (18)$$

$$\text{T9}: \dot{m}_\text{f} = \left(\alpha_1 + \alpha_2\omega^2\right) \cdot \left(\beta_1 + \beta_2 T + \beta_3 T^2\right), \quad (19)$$

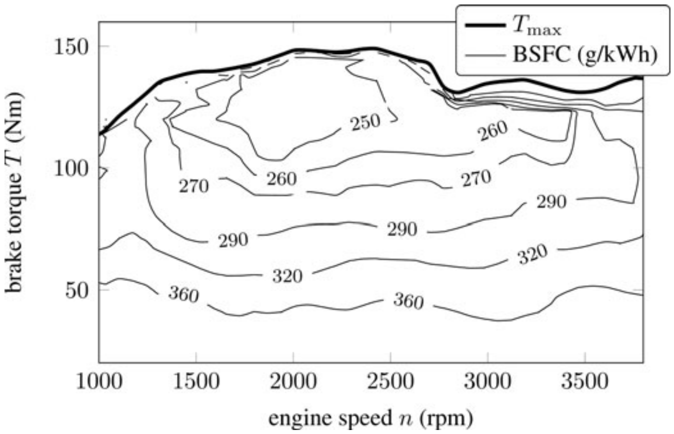

Figure 2 Contour plot of the measured brake specific fuel consumption BSFC on the engine dynamometer.

$$\mathrm{L2}: \dot{m}_{\mathrm{f}} = \alpha_1\omega + \alpha_2\omega\tau + \alpha_3\omega^2\tau + \alpha_4\omega^3\tau + \alpha_5\omega\tau^2 + \alpha_6\omega^2\tau^2, \tag{20}$$

$$\mathrm{L4}: \dot{m}_{\mathrm{f}} = \left(\alpha_1 + \alpha_2\omega^2 + \alpha_3\omega^3\right)\cdot\left(\beta_1 + \beta_2\tau + \beta_3\tau^2\right). \tag{21}$$

These models are named T7, T9, L2, and L4 respectively. Note that none of the models has a cubic control term (T^3 or τ^3). Cubic terms hardly improve the model fit and cause unnatural shapes in both BSFC and BCPR. Before the presented model search was conducted, the author developed a linear torque-based model,

$$\mathrm{T8}: \dot{m}_{\mathrm{f}} = \alpha_1\omega + \alpha_2\omega^2 + \alpha_3\omega^3 + \alpha_4\omega T + \alpha_5\omega^2 T + \alpha_6\omega T^2, \tag{22}$$

and a linear load-based model,

$$\mathrm{L3}: \dot{m}_{\mathrm{f}} = \alpha_1\omega + \alpha_2\omega^2 + \alpha_3\omega^3 + \alpha_4\omega\tau + \alpha_5\omega^2\tau + \alpha_6\omega\tau^2. \tag{23}$$

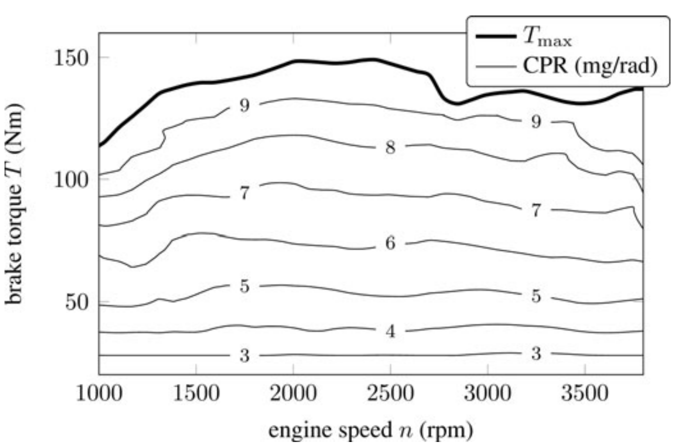

Figure 3 Contour plot of the measured brake consumption per rotation BCPR on the engine dynamometer.

Table 4 Novel polynomial fuel consumption models.

Type	Model	p	q	Eq.
PB	P4	[0 0 0]	[0 1 2]	10
	P5	[1 0 0]	[0 1 2]	13
	P6	[0 1 2 0 0]	[0 0 0 1 2]	12
LTB	T7	[1 2 3 1 2 0 1 2]	[0 0 0 1 1 2 2 2]	18
	T8	[1 2 3 1 2 1]	[0 0 0 1 1 2]	22
NLTB	T9	[0 2]	[0 1 2]	19
LLB	L2	[1 1 2 3 1 2]	[0 1 1 1 2 2]	20
	L3	[1 2 3 1 2 1]	[0 0 0 1 1 2]	23
NLLB	L4	[1 2 3]	[0 1 2]	21

These models are named model T8 and L3, and are also considered for further evaluation. An overview of all the new models is given in Table 4.

EVALUATION OF THE CONSUMPTION MODELS

Evaluation Based on Model Structure

A first selection of consumption models is done based on the model structure as explained in the Consumption Model Assessment section. Table 5 shows the evaluation of the models found in the literature and the models developed in the Development of New Consumption Models section. A + is positive, meaning that the property is well represented. A – is negative. The models with three or more positives are selected for further assessment (P4–P6, T1, T4–T9, and L1–L4). Model P3 is not considered. Adding a quadratic term to this model results in model P5, which has a lower AIC than P3 and not more than three parameters.

Evaluation Based on Fit Quality

To allow for a simple comparison, a relative measure for the AIC is defined:

$$\Lambda = 1 - \frac{\mathrm{AIC} - \mathrm{AIC}_{\min}}{\mathrm{AIC}_{\max} - \mathrm{AIC}_{\min}}, \tag{24}$$

with $\mathrm{AIC}_{\min}$ and $\mathrm{AIC}_{\max}$ the minimum and maximum AIC value. $\Lambda = 1$ for the best model and $\Lambda = 0$ for the worst model.

Table 6 shows the evaluation of the selected models based on the fit quality with the engine dynamometer measurements. Model L2 is the best model. Model P4 is clearly outperformed by the rest. The difference between the torque-based and load-based models is small. The same can be said for the difference between linear and nonlinear models. Since it is difficult to identify the parameters of nonlinear models, they are not considered in the further assessment.

The chassis dynamometer and on-road data include negative control inputs ($u < 0$). Figure 4 shows a typical fuel consumption measurement as a function of the vehicle power P. It can

Table 5 Evaluation of models based on model structure.

Model	Eq.	Shifting	Bang-off-bang	Parameters
P1	26	−	−	+
P2	27	−	−	+
P3	28	+	−	+
P4	10	−	+	+
P5	13	+	+	+
P6	12	+	+	−
T1	29	+	−	+
T2	30	+	−	−
T3	31	+	−	−
T4	32	+	+	−
T5	33	+	+	−
T6	34	+	+	−
T7	18	+	+	−
T8	22	+	+	−
T9	12	+	+	−
L1	35	+	+	−
L2	20	+	+	−
L3	23	+	+	−
L4	21	+	+	−

Note. Shifting: The model can decently capture the influence of gear shifting. Bang-bang: The model does not result in bang-bang control. Parameters: The model has three or fewer parameters.

be seen that the fuel consumption rate does not drop below the idle fuel mass flow rate. Therefore, the power-based models are adapted as follows:

$$\dot{m}_{\mathrm{f}} = \max\left(\dot{m}_{\mathrm{f,i}}, \sum_k \alpha_k \omega^{p_k} P^{q_k}\right), \tag{25}$$

with $\dot{m}_{\mathrm{f,i}}$ (kg/s) the idle fuel mass flow rate.

Table 7 shows the evaluation of the selected models based on the fit quality with the chassis dynamometer measurements (average values are shown). Model T1 is the worst, model T4 the best. The difference between models P6, T4–L3, is rather small. The overall fit quality is clearly worse compared to the engine dynamometer measurements. This is due to modeling errors;

Table 6 Evaluation of models based on the fit quality with engine dynamometer measurements.

Model	R^2 (%)	RMSE (mg/s)	Δ (%)
P4	94.22	13.93	0.0
P5	98.96	5.90	60.1
P6	99.33	4.75	74.8
T1	99.06	5.62	63.5
T4	99.44	4.33	80.8
T5	99.48	4.18	83.5
T6	99.43	4.38	80.3
T7	99.64	3.49	95.6
T8	99.32	4.77	74.3
T9	99.43	4.37	80.6
L1	99.01	5.77	61.0
L2	99.68	3.30	100.0
L3	99.54	3.92	88.0
L4	99.54	3.93	87.8

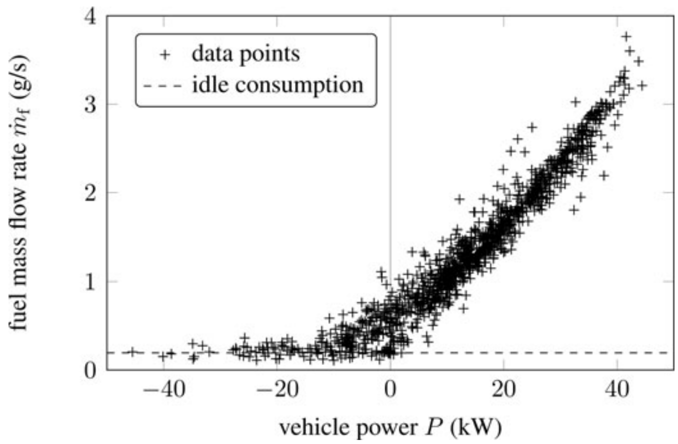

Figure 4 Example of chassis dynamometer measurement data.

the engine torque is not directly measured, but estimated based on the vehicle velocity and assumed longitudinal dynamics.

Table 8 shows the evaluation of the selected models based on the fit quality with the on-road measurements (average values are shown). This evaluation is based on a cross validation: half of the data is used for identification of the parameters and the rest for validation. The data are split based in 10 s batches of data. Thus, in each subsequent batch of 20 s (20 data points), the first 10 are used for identification and the other 10 for validation. A cross-validation is relevant for an eco-drive assist tool where a fuel consumption model, which is based on previous measurements, is used to predict optimal driving behavior. Models T4–L3 are clearly better than the rest. Model P4 is the worst, and model L3 the best. Again, the difference between models T4–L3 is rather small.

Model Transferability

The data, coming from 15 different gasoline engines, yield quite consistent results. Model P4 is the worst model. The difference between models T4–L3 is rather small. It seems that any of these models yields a relatively good fit. Which one is the best depends on the specific engine.

Table 7 Evaluation of models based on the fit quality with chassis dynamometer measurements.

Model	R^2 (%)	RMSE (mg/s)	Δ (%)
P4	90.06	15.97	25.1
P5	91.07	15.44	37.3
P6	92.78	13.81	74.2
T1	89.43	16.56	12.8
T4	93.43	13.12	95.1
T5	93.36	13.18	93.0
T7	93.35	13.19	93.1
T8	93.32	13.20	92.2
L2	92.87	13.58	83.6
L3	93.29	13.23	91.7

Table 8 Evaluation of models based on the fit quality with on-road measurements.

Model	R^2 (%)	RMSE (mg/s)	Δ (%)
P4	78.87	46.18	0.0
P5	85.63	37.47	41.4
P6	84.41	37.93	49.6
T1	81.18	36.52	56.8
T4	89.00	32.55	85.4
T5	88.55	33.42	78.8
T7	88.88	32.55	84.8
T8	88.22	33.82	74.9
L2	88.91	32.63	84.6
L3	90.77	31.74	86.1

To illustrate the model transferability, extra measurements are taken on a 2002 1.4-L common-rail 4-cylinder diesel engine on a dynamometer. Model P4 yields $R^2 = 96.6\%$, model P5 $R^2 = 97.8\%$, and model P6 $R^2 = 98.8\%$. Models T4–L3 range between 99.2 and 99.5%.

CONCLUSIONS

This article assesses different low-degree polynomial fuel consumption models. Three types of models are defined, based on the control input u of the model: (a) power-based ($u = P$, traction power); (b) torque-based ($u = T$, engine torque); and (c) load-based ($u = \tau = T/T_{\max}(\omega)$, engine load).

The first set of models is extracted from the literature. A second set of models is developed using a subsearch regression based on measurement data from an engine dynamometer.

First a selection of models is made based on the model structure. In this case, three criteria are used: (a) optimal gear shift behavior, (b) optimal velocity control, and (c) the number of model parameters. The parameters of a three-parameter or less fuel consumption model can be identified using publicly available data, without the need to collect field or in-laboratory data.

The selected models are further evaluated based on the quality of a least-squares fit with three sets of data: (a) measurements on a universal engine dynamometer, (b) measurements on a chassis dynamometer, and (c) on-road measurements. The fit quality is based on (a) the Akaike Information criterion, (b) the coefficient of determination, and (c) the root mean squared error.

The three data sets yield rather consistent results:

1. Model P5 is the best three-parameter model.
2. Models T4–L3 perform similarly.
3. A good polynomial fuel consumption model has a quadratic control term (u^2).

For simple applications or applications that do not allow for field or in-laboratory measurements (such as microscopic traffic simulation software), the power-based models P4 and P5 are advised. Model P5 provides better predictions, but does require that gear shifting be captured. Alternatively, model P4 is ideal in the case where gear shifting is not modeled, as is the case in the state-of-the-practice microscopic traffic simulation software. For more complex applications that allow for measurement data (such as eco-drive assist systems), a linear torque-based model (e.g., T8) is advised. Nonlinear models and load-based models complicate the modeling, but do not yield significantly better models.

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APPENDIX

$$Model P1: \quad \dot{m}_{\mathrm{f}} = \alpha_1 P. \tag{26}$$

$$Model P2: \quad \dot{m}_{\mathrm{f}} = \alpha_1 + \alpha_2 P. \tag{27}$$

$$Model P3: \quad \dot{m}_{\mathrm{f}} = \alpha_1 \omega + \alpha_2 P. \tag{28}$$

$$Model T1: \quad \dot{m}_{\mathrm{f}} = \alpha_1 \omega + \alpha_2 \omega^2 + \alpha_3 \omega T. \tag{29}$$

$$Model T2: \quad \dot{m}_{\mathrm{f}} = \alpha_1 + \alpha_2 \omega + \alpha_3 \omega T + \alpha_4 \omega^2 T. \tag{30}$$

$$Model T3: \quad \dot{m}_{\mathrm{f}} = \alpha_1 + \alpha_2 \omega + \alpha_3 \omega^2 + \alpha_4 \omega^3 + \alpha_5 \omega T. \tag{31}$$

$$Model T4: \quad \dot{m}_{\mathrm{f}} = \alpha_1 \omega + \alpha_2 \omega^2 + \alpha_3 \omega^3 + \alpha_4 \omega T + \alpha_5 \omega^2 T + \alpha_6 \omega^3 T + \alpha_7 \omega T^2. \tag{32}$$

$$Model T5: \quad \dot{m}_{\mathrm{f}} = \alpha_1 + \alpha_2 \omega + \alpha_3 \omega^2 + \alpha_4 \omega T + \alpha_5 T + \alpha_6 T^2. \tag{33}$$

$$Model T6: \quad \dot{m}_{\mathrm{f}} = \left(\alpha_1 \omega + \alpha_2 \omega^2 + \alpha_3 \omega^3\right) \cdot \left(\beta_1 + \beta_2 T + \beta_3 T^2\right). \tag{34}$$

$$Model L1: \quad \dot{m}_{\mathrm{f}} = \left(\alpha_1 + \alpha_2 \omega + \alpha_3 \omega^2\right) \cdot \left(\beta_1 + \beta_2 \tau + \beta_3 \tau^2\right). \tag{35}$$