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About prediction of vehicle energy consumption for eco-routing

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***Abstract*— For a given route, vehicle and time of departure we consider the problem of energy consumption prediction. It is of prime importance in eco-routing applications since prediction errors reflect heavily on its performance. The problem is non-linear, nondeterministic and the information on which we can base these predictions is often sparse. We consider longitudinal vehicle consumption model of an ideal vehicle and reformulate it in closed form as a function of travel time and speed profile statistics. Microscopic traffic simulation of mid-sized European city is used to study the dependence of consumption on the departure time of day.**

I. INTRODUCTION

Eco-routing is a strategy for reducing vehicle operating costs [11], [10], [3], [4]. The idea is to minimize vehicle energy (or fuel) consumption by route selection: given some origin and destination, eco-routing plans such a route that energy (fuel) needed to finish the trip is minimal.

There is a considerable body of research on vehicle consumption estimation [9], [5], [2]. Most published models can be classified as either macroscopic or microscopic. Microscopic models are given by differential equations. In case of electric and hybrid vehicles they integrate instantaneous power over time to obtain trip energy. In the case of conventional vehicles they integrate instantaneous fuel intake to obtain the fuel consumption. Macroscopic models estimate the consumption using a closed set of summary variables. This often requires crude simplifications but the resulting model needs little information and is fast to compute.

We observed in a previous study [11] that current eco-routing methods often fail to save energy (or fuel). It was also observed that the key to obtain better performance is to have better consumption models. This is a difficult problem as there is a lack of information needed to predict consumption reliably. To make the situation worse, our simulations show extreme sensitivity of consumption to departure time (see Figure 1).

There is a natural trade-off between the quality of information provided to the consumption model and the estimation (resp. prediction) error. Microscopic and macroscopic models are at the two ends of this trade-off. Microscopic models yield good performance but require information that is not available at the route planning stage. Macroscopic models require little information but produce estimates with considerable errors.

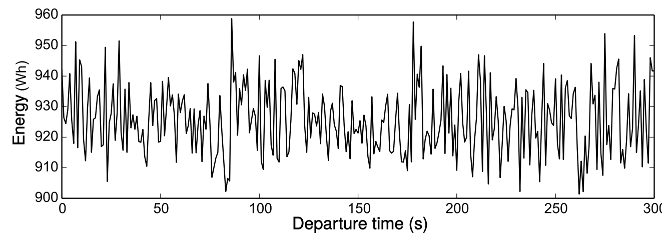

Fig. 1: Dependence of consumption on departure time for first five minutes past midnight on a random route

A closed-form reformulation of standard microscopic consumption model can solve this. However, this model would have to depend on parameters that can be reliably estimated from field data in order to be practical. Such a model is yet to be discovered. We present a closed-form reformulation of its simplified version that considers an idealized vehicle. This model is not suited to replace consumption models used in eco-routing, however it has simple analytical structure that can be used to study it. Three research questions are studied:

1) *Sensitivity of consumption to model parameters.* It is a common practice with macroscopic models to neglect some of the parameters or to use sample expectation (average value) instead. For example, speed profile is often assumed constant or piece-wise constant. We aim to assess errors introduced by such simplifications.
2) *Dependence of consumption on the departure time of day*. This allows to reason about the error introduced when the dependency is neglected (as it is done by current eco-routing methods).
3) *Dependence of the prediction error on characteristics of traffic information systems.* Current traffic information systems report average speed on the roads in the road-network. This information is based on measurements taken within last hour or so: it is not necessarily current and chosen averaging horizon can affect it. If it is too short then there might not be enough samples to generate reliable approximations. If it is too long, then the information might become insensitive to changing traffic situation. Hence, the goal is to see whether the way in which the traffic information systems process their data can have adverse effects on eco-routing performance.

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The paper is organized as follows: Section II presents the vehicle consumption model; Section III outlines the experiment design; Section IV presents simulation results and Section V discusses our findings. The conclusion is in Section VI.

II. MODELING

This study is based on the standard longitudinal consumption model [9]. To shortly review the theory, it considers one dimensional movement along the longitudinal axis of the vehicle. Losses due to turning maneuvers are neglected. The tractive force generated by the powertrain is based on Newton's second law of motion: $F_t = ma + F_{\text{res}}$, where F_t is the tractive force, F_{res} is the equivalent resistance force, m is the vehicle mass and a is the acceleration. Then, the tractive power is a mechanical power $P_t = F_t v$ where v denotes speed. The energy needed to finish the trip is the integral of instantaneous tractive power over time

$$E_w = \int_0^T P_t \, \mathrm{d}t = \int_0^T (F_{\text{res}} + ma)v \, \mathrm{d}t \tag{1}$$

We call this quantity the energy on wheels, E_w, to underline that this is the energy transferred from the vehicle to the environment. For a lossless vehicle (e.g. vehicle with perfect powertrain and perfect recuperation) this is the same as energy taken from its power source. The E_w is referred to in the text simply as the consumption. The F_{res} is a model of the equivalent resistance force caused mainly due to aerodynamic drag, rolling friction and climbing losses. The drag and the friction is modeled with second order polynomial of speed. Climbing losses are modeled with $mg\sin(\alpha)$, where α is the climbing angle. It accounts for the change in potential energy of the vehicle. Note that climbing losses can be negative on negative slopes (since sine function is odd).

$$F_{\text{res}} = c_a + c_b v + c_c v^2 + mg\sin(\alpha) \tag{2}$$

The coefficients c_a, c_b, c_c are coast-down parameters matched experimentally to the vehicle. Note that F_{res} is defined for $v > 0$. It is invalid when the vehicle is stationary. This does not affect our model as F_{res} is multiplied with speed in equation (1). If we expand equation (1) we obtain the classical formulation of the longitudinal consumption model for lossless vehicles

$$E_w = \int_0^T (mav + c_a v + c_b v^2 + c_c v^3 + mgv\sin(\alpha)) \, \mathrm{d}t \tag{3}$$

The goal is to reformulate (3) in closed form. First, observe that $dv = a\,dt$ which can be used to solve the first term

$$\int_0^T mav \, \mathrm{d}t = m \int_0^T v \, \mathrm{d}v = \frac{1}{2}mv^2(T) - \frac{1}{2}mv^2(0) \tag{4}$$

which can be interpreted as the difference between the kinetic energy the vehicle had when the trip started and when finished. In order to solve the second term in (3) notice that $ds = v\,dt$. It leads to

$$\int_0^T c_a v \, \mathrm{d}t = c_a D \tag{5}$$

where D is the trip length. The integral of the climbing losses can be simplified using similar approach and solved using $\sin(\alpha)\,ds = dh$, where dh is the altitude differential. It leads to

$$\int_0^T mgv\sin(\alpha) \, \mathrm{d}t = mg \int_0^D \mathrm{d}h = mg\Delta h \tag{6}$$

where Δh is the altitude difference between the origin and destination.

In order to solve the remaining terms we have to introduce further notation. Let us denote μ_i the i-th central moment of the speed $v(t)$ and μ'_i its i-th raw moment. First raw moment μ'_1 is the mean value. Higher raw moments can be expressed in terms of the central moments using inverse binomial transformation [14]. Second and third raw moments are

$$\mu'_2 = \mu_2 + \mu'^2_1 \tag{7}$$

$$\mu'_3 = \mu_3 + 3\mu_2\mu'_1 + \mu'^3_1 \tag{8}$$

Note that μ'_2 is the mean squared speed and μ'_3 is the mean cubed speed. The terms with v^2 and v^3 in (3) can be solved using (7) and (8)

$$\int_0^T c_b v^2 \, \mathrm{d}t = c_b T(\mu_2 + \mu'^2_1) \tag{9}$$

$$\int_0^T c_c v^3 \, \mathrm{d}t = c_c T(\mu_3 + 3\mu'_1\mu_2 + \mu'^3_1) \tag{10}$$

By combining results in (4), (5), (6), (9) and (10) with (3) we obtain closed form formulation of the consumption model. We can simplify by substituting $\mu'_1 = \frac{D}{T}$, $\mu_2 = \sigma^2$ and $\mu_3 = b\sigma^3$, where σ is standard deviation and b is skew.

$$\begin{aligned} E_w = {} & \frac{1}{2}mv^2(T) - \frac{1}{2}mv^2(0) + mg\Delta h + c_a D + \\ & c_b\frac{D^2}{T} + c_c\frac{D^3}{T^2} + \sigma^2(c_b T + 3c_c D) + b\sigma^3 c_c T \end{aligned} \tag{11}$$

The mathematical expression given in (11) gives the consumption model used in this study. It depends on three parameters: travel time T, speed profile variance σ^2 and skew b. Other parameters are known constants. The initial and final speeds $v(0)$, $v(T)$ are zero as long as the vehicle starts and ends its trip in stationary state. Parameters Δh and D are obtained from the map and the vehicle-specific coefficients m, c_a, c_b and c_c are known in advance.

III. METHODOLOGY

Answers to the research questions enumerated in Section I are based on the proposed consumption model and microscopic traffic simulations. Specifically, consumption of vehicles traveling along a set of 242 predetermined routes is measured at different times of day. The routes are based on shortest paths between randomly generated origins and destinations. The time of day spans twenty-four hour period with a step of thirty seconds. This results in 2880 departure times on 242 routes which translates to $696,960$ recorded speed profiles. SUMO microscopic simulator [8] and LuST traffic scenario developed by Codecà et al. [6] was used to conduct the simulations. The scenario features road network of real European city (Luxembourg city) with mobility patterns matched to a typical day there. The simulations took an equivalent of 59 days to finish. Feasible computation time was achieved by running the experiment on a computing cluster.

The probe vehicle used in the simulations had mass and coast-down parameters matched to a hatchback version of Renault Scénic: $m = 1,588$ kg, $c_a = 110.45$ N, $c_b = 1.5175$ N/(m/s), $c_c = 0.5119$ N/(m/s)^2. The altitude measurements are based on the digital elevation model of Europe (EU-DEM, see [1]). It has twenty-five meter resolution. Altitudes between the grid points were interpolated using cubic interpolation. All roads are assumed to have constant slope. This is because the elevation model follows the terrain shape and cannot distinguish bridges and overpasses.

IV. RESULTS

Results that relate to the first research question are presented in Table I and in Figure 3. The Table I lists performance of model (11) when correct values of its parameters are replaced with averages or with zeros. Each line is for one combination of such replacements. Only a selection of these combinations is showed. The other combinations can be computed using raw experiment results [12] and provided processing script. The "bias" column indicates tendency to overestimate or underestimate consumption. The "std. dev." column gives standard deviation of estimated-to-correct consumption ratio. It indicates dispersion of the prediction error in relative terms. The root mean square error gives the the prediction error in Watt-hours. The dependence of the bias, standard deviation and RMSE on the departure time is shown in figures 3a, 3b and 3c.

Results that relate to the second research question are presented in Figure 4. It shows the dependence of the consumption and of the model parameters on departure time. The plots are specific to one route chosen at random. Similar plots for all routes are available in the supplementary material [12]. The dependence of consumption on the departure time is shown in Figure 4a. The dependence of parameters T, σ^2, b on the departure time is in figures 4b, 4c, 4d. The thick black line is median of a subset of samples measured ± 15 minutes around the indicated time of departure. The gray area indicates dispersion: it shows the range between fifth and ninety-fifth percentile in the subset.

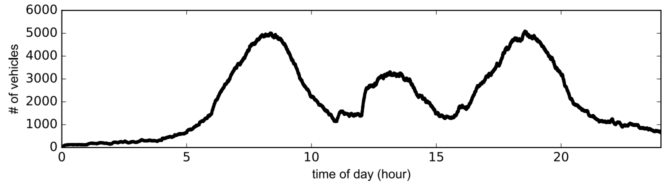


Fig. 2: Number of vehicles in the traffic scenario through the day

Results that relate to the third research question are presented in Figure 5. It shows the dependence of prediction error (RMSE) on the length of the averaging horizon used to compute mean speeds in the city. Each curve is for different age of the information: blue is for the case when the it is real-time, the others are for ten, twenty and thirty minute delays. The results relate to the same route as the data in Figure 4. The supplementary materials [12] contain such analysis for all routes.

V. DISCUSSION

The consumption prediction problem is challenging. The consumption is nonlinear function of many variables (the speed, slope, clutch state, transmission state, engine temperature, outside temperature, tire pressure and others). Further, real drivers do not behave deterministically. Even if we assume otherwise, the consumptions are chaotic [15]. Chaos was reportedly described by Lorenz as a system in which "the present determines the future, but the approximate present does not approximately determine the future" [7]. The simulations show such behavior. For example, Figure 1 shows a consumption on a random route for departure times with one second step for five minutes. It is based on three hundred simulations. The vehicle started its journey at different departure time in each simulation and its progression to the destination was recorded. The consumption is based on the model (11).

The consumption of a realistic vehicle relates to consumption of a lossless vehicle considered here. The model (11) gives the energy that must be transferred from the vehicle out to the environment to finish the trip. It can be interpreted as lower bound on a consumption of real vehicle. The consumption of a real vehicle is typically modeled as

$$E = \frac{E_w}{\overline{\eta}} + E_b \tag{12}$$

where $\overline{\eta} \in (0, 1)$ is average powertrain efficiency and $E_b > 0$ is the energy dissipated as heat in friction brakes. The powertrain efficiency is often modeled numerically. Simplified analytical models such as Willan's model [9] are sometimes used. The energy lost as heat in friction brakes depend on the driving style of the driver, on the traffic around the vehicle and other factors that are difficult to express analytically. Consequently, it is difficult to find simple analytical model for E. This is why it is interesting to consider the model (11). It has simple algebraic structure that allows analytical reasoning and can be considered a sufficient approximation in some cases.

TABLE I:
RELAXATIONS ON CONSUMPTION

T	σ^2	b	bias	std. dev.	corr.	RMSE
✓	✓	✓	0.0 %	0.0 %	1.000	0.00 Wh
✓	✓	∼	-0.3 %	2.5 %	0.999	18.34 Wh
✓	✓	–	-0.3 %	4.0 %	0.997	29.23 Wh
✓	∼	∼	-0.3 %	4.6 %	0.996	31.76 Wh
✓	∼	–	-0.4 %	5.3 %	0.995	38.57 Wh
✓	–	–	-32.6 %	14.3 %	0.929	294.57 Wh
∼	∼	∼	-1.2 %	2.9 %	0.998	23.81 Wh
∼	∼	–	-1.3 %	4.1 %	0.996	33.26 Wh
∼	–	–	-33.5 %	13.9 %	0.930	301.02 Wh

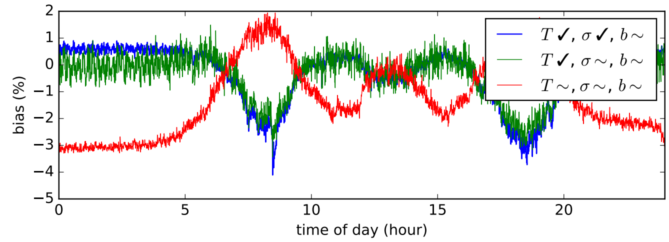
✓correct value, ∼ full-day average, – zero

The results in Table I suggest that slight bias (-0.3%) and 4% dispersion is introduced by neglecting the skew. If the average skew is used instead, then the standard deviation drops to 2.5%. This can be acceptable in some applications. However, if the variance parameter is neglected, then unacceptable bias (-32.6%) and significant random content ($\approx 14\%$ std. dev.) is introduced. This is likely unacceptable for most applications. Nevertheless, variance does not show high sensitivity: it can be replaced with full-day average for the price of $\approx 2\%$ increment in standard deviation. This can be considered a good trade-off: the full-day average of the variance parameter is much easier to predict as the traffic patterns show approximate day-to-day periodicity. Note that the consumption is time-independent in this case as all parameters are replaced with full-day averages.

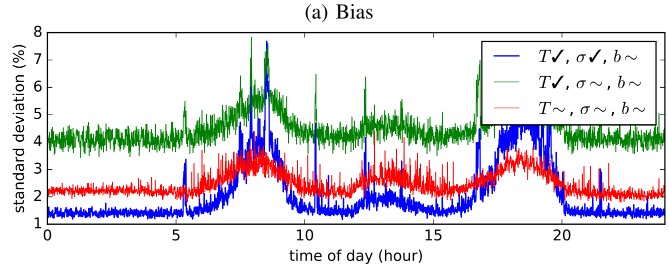
Last line in Table I relates to a case that is interesting on its own: it is the only case realizable with current traffic information systems. The only information these systems provide is the average speed (which is proportional to average travel time). If the correct value of the travel time is replaced with the average travel time and both variance and skew is neglected, then data suggest severe underestimation of consumption and strong random content. This lowers correlation which impedes the ability of eco-routing to find minimum consumption route

Bias dependency on departure time in Figure 3a shows correlation with the number of vehicles in the city (compare to Figure 2). In two out of three cases the correlation is negative. The peaks during the morning and afternoon congestion are clearly visible. This suggest that it might be possible to correct for bias since the traffic shows approximate daily periodicity. The standard deviation dependency on departure time (Figure 3b) is also positively correlated with the number of vehicles in the city. In the case when correct travel time and correct variance is used the dispersion is minimal ($\approx 1.5\%$ std. dev.), except for the congestion periods where it grows sharply. Curiously, the model that replaces values of all three parameters with averages (red curve) performs better than the one which uses the correct travel time (green curve). This suggests that if we want to predict correct value for these parameters then we should consider doing it for all three.

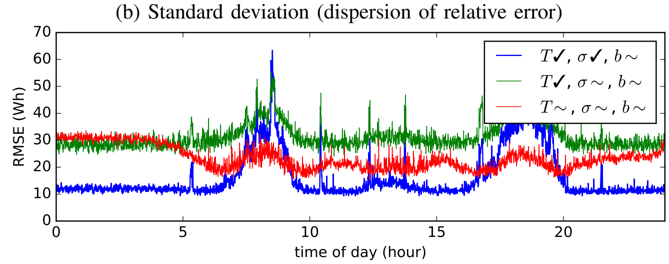
The dependence of consumption on departure time in



(a) Bias



(b) Standard deviation (dispersion of relative error)

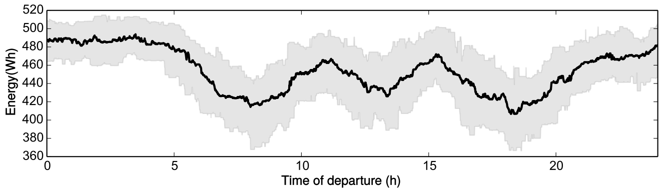


(c) Root mean squared error

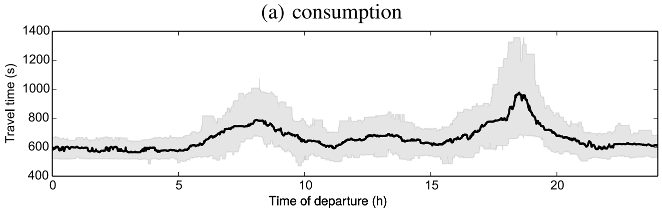
Fig. 3: Evolution of prediction performance metric during the day

Figure 4a shows 150 Wh peak-to-peak difference with mean consumption around 440 Wh. This translates to roughly $\pm 20\%$ peak-to-peak around the mean. The median consumption ranges from 420 Wh during the congestion hours to 480 Wh in the night. While this particular route shows correlation of consumption to the number of vehicles in the city this is not universally true for all the routes. It depends on congestion of the roads on which the vehicle travels. Arterial roads are typically subject to congestion while local roads are not. The correlation is always nonpositive as more vehicles on the road correlate with lower consumption. This is not in agreement with the consensus in the community and it suggests that the model (11) is not suitable for use in eco-routing as is. This is because (11) models a lossless vehicle: it has no losses due to braking and no losses when stationary. Simple approximations that incorporate these losses are possible. For example, first order approximation $E_b = \beta T$ of the braking energy can be used to inhibit this problem.

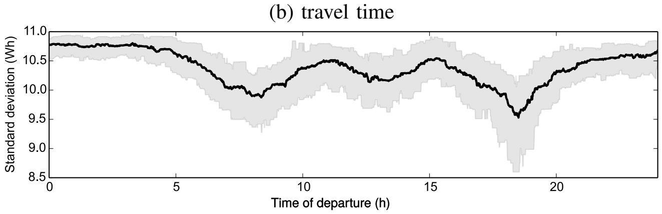
Simulations have sometimes shown periodical behavior in the night. The period in which the consumption pattern repeats itself is the superperiod of the sequence of traffic lights along the route. It is not likely that such behavior would manifest in reality because the simulation is deterministic while real-life driving is not. Nonetheless, it suggests that there are departure times at which lower consumption can be achieved, typically because of a sequence of green lights on the route. However, the driver must keep pace with this sequence which can prove difficult in heavier traffic.



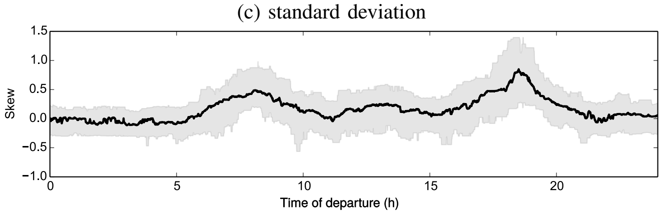
(a) consumption



(b) travel time



(c) standard deviation



(d) skew

Fig. 4: Midnight to midnight evolution of consumption and model parameters

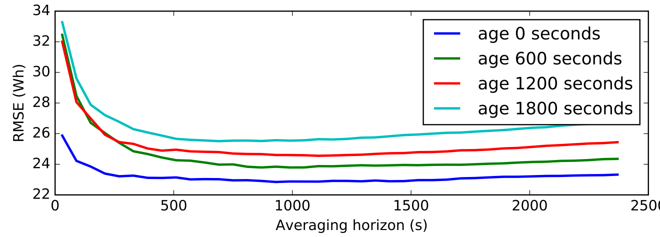


Fig. 5: RMSE vs horizon

The dependence of parameters T, σ^2 and b on departure time is in figures 4b, 4c and 4d. The travel time (Figure 4b) varies between 520 and 640 seconds in the night. There the dispersion is dominated by the traffic lights since the traffic is at its minimum. During the morning and afternoon congestion the median travel time rises sharply and both upper and lower boundaries widen. Data suggest that the rise in travel time is disproportional to the rise in number of vehicles on the roads. This suggests nonlinear positive correlation. Negative correlation was not observed. All-day peak-to-peak travel time ranges between 500 and 1400 seconds. During the afternoon congestion alone this ranges between 600 to 1400 seconds. This suggest that travel time prediction is likely unreliable in congested traffic: the travel time can be as short as its all-day minimum or almost its triple.

The standard deviation of the speed profile in Figure 4c correlates negatively with the number of active vehicles. This indicates that the speed profile variance is lower in heavier traffic. This is likely due to lower travel speed and prolonged full-stop periods. Lower dispersion was observed at night than during the day. During the day, on the other hand, the boundaries widen significantly when the traffic gets dense.

The speed profile skew (Figure 4d) is close to zero in the night. This is because on this specific route the vehicles travel mostly undisturbed at that time. Slightly positive skew appears during the day. This implies that the vehicle tends to travel at speeds lower than the mean speed. When the road is congested the vehicle needs to slow down, however since the city is not congested homogeneously there is a portion of the route where the vehicle travels at a speed close to free-flow speed. Such scenarios can result in positive skew.

Figure 5 shows only weak sensitivity of the RMSE error to the way in which mean speed information is processed. The RMSE magnitude is specific to each route, however, the relationship between averaging window size, age of the information and RMSE is similar for all routes. The RMSE is high when short averaging windows are used (less than 5-10 minutes). This is because the average is not based on sufficient information to stabilize. The error drops when longer windows are used. On averaging windows longer than approximately twenty minutes the RMSE tends to grow on arterial roads, presumably because the traffic state changes within the window. Nevertheless the growth is insignificant in most cases. On local roads, however, the error can further decrease with averaging windows longer than twenty minutes. See the supplementary material for such cases [12]. The age of the information does not affect RMSE significantly either: even the information is outdated the error rises only slightly. This suggest that averaging horizon used by traffic information systems can be chosen in a relatively wide range without severe degradation of eco-routing performance.

VI. Conclusion

The problem of consumption prediction for given route, vehicle and departure time is considered in this paper. Existing consumption estimation methods can be applied to the prediction problem, but they are not necessarily a good fit. Microscopic estimation models require recorded speed profile which is not available when predicting consumption and performance of current macroscopic methods is not satisfactory. This paper introduces macroscopic, closed-form reformulation of a standard microscopic consumption model for a lossless vehicle (see Section II). This model has three unknowns: travel time, speed profile variance and speed profile skew. Current macroscopic models are often based on empirical constructs and regression. The model presented here is on the other hand analytical and exact with respect to widely accepted microscopic model. While not necessarily suitable for consumption prediction of realistic vehicles, this model allows to gain new insights that can lead to development of new consumption predictors for realistic vehicles.

Three research questions are studied with this model:

1) Sensitivity of consumption to model parameters.
2) Dependence of consumption on departure time
3) Prediction error dependency on characteristics of traffic information systems.

For the first question it was observed that if the skew parameter is fixed to zero then the prediction error might be tolerable for some applications. However, neglecting travel time or speed profile variance would introduce unacceptable errors. If all three parameters are replaced with their full-day averages then only negligible bias and weak dispersion is introduced. This can be considered a good trade-off as the model is no longer time-dependent. If the travel time is replaced with the expected travel time and other parameters are fixed to zero then significant errors manifest. Unfortunately, this is the only case realizable with current traffic information systems: they provide only average speed on roads in the road-network. Our results suggest that good macroscopic consumption models need to consider speed profile variance as well. Recently, some authors addressed this issue using heuristics, see [13].

For the second question, the consumption shows strong dependence on departure time. Chaotic consumption manifested in deterministic simulation (see Figure 1 for example). This supports conclusions made by other authors [15].

The average speed information provided by the traffic information systems is based on averaging in typically unknown time-horizon. Further, the information loses its validity as it ages. In order to answer the third question we observed errors due to aged average speed information when based on averaging horizons of different length. The results show only weak sensitivity of the prediction error to the way in which traffic information system process their data.

VII. Supplementary Material

All experiment data are published together with this paper, see [12]. The dataset includes raw experiment results, processing scripts and processed results. The source code for generation of all the figures presented here is also included. It serves as a written record of the data processing steps used to generate them.

References

[1] European Environment Agency. EU digital elevation model. `http://www.eea.europa.eu/data-and-maps/data/eu-dem`. Accessed: 2016-01-22.

[2] Kyoungho Ahn, Hesham Rakha, Antonio Trani, and Michel Van Aerde. Estimating vehicle fuel consumption and emissions based on instantaneous speed and acceleration levels. *Journal of transportation engineering*, 128(2):182–190, 2002.

[3] Matthew Barth, Kanok Boriboonsomsin, and Alex Vu. Environmentally-friendly navigation. In *Intelligent Transportation Systems Conference, 2007. ITSC 2007. IEEE*, pages 684–689. IEEE, 2007.

[4] Kanok Boriboonsomsin, Matthew J Barth, Weihua Zhu, and Alexander Vu. Eco-routing navigation system based on multisource historical and real-time traffic information. *Intelligent Transportation Systems, IEEE Transactions on*, 13(4):1694–1704, 2012.

[5] Alesaandn Cappiello, Ismail Chabini, Edward K Nam, Alessandro Lue, and Maya Abou Zeid. A statistical model of vehicle emissions and fuel consumption. In *Intelligent Transportation Systems, 2002. Proceedings. The IEEE 5th International Conference on*, pages 801–809. IEEE, 2002.

[6] Lara Codeca, Raphaël Frank, and Thomas Engel. Luxembourg sumo traffic (lust) scenario: 24 hours of mobility for vehicular networking research. In *Proceedings of the 7th IEEE Vehicular Networking Conference*, pages 1–8, 2015.

[7] C. M. Danforth. Chaos in an atmosphere hanging on a wall. `http://mpe2013.org/2013/03/17/chaos-in-an-atmosphere-hanging-on-a-wall/`. Accessed: 2016-06-06.

[8] Krajzewicz Daniel, Jakob Erdmann, and Bieker Laura Behrisch, Michael and. Recent development and applications of SUMO - Simulation of Urban MObility. *International Journal On Advances in Systems and Measurements*, 5(3&4):128–138, December 2012.

[9] Lino Guzzella and Antonio Sciarretta. *Vehicle propulsion systems: introduction to modeling and optimization*. Springer, 2005.

[10] Tomáš Juřík, Arben Cela, Redha Hamouche, Rene Natowicz, Abdellatif Reama, Silviu-Iulian Niculescu, and Jérôme Julien. Energy optimal real-time navigation system. *Intelligent Transportation Systems Magazine, IEEE*, 6(3):66–79, 2014.

[11] M. Kubička, J. Klusáček, A. Sciarretta, A. Cela, H. Mounier, L. Thibault, and S. I. Niculescu. Performance of current eco-routing methods. In *IEEE Intelligent Vehicles symposium (IV), 2016*, June 2016.

[12] Matěj Kubička, Antonio Sciarretta, Arben Cela, Hugues Mounier, Laurent Thibault, and Silviu-Iulian Niculescu. About prediction of vehicle energy consumption for eco-routing: supplementary materials. `http://dx.doi.org/10.5281/zenodo.61623`, September 2016. DOI: 10.5281/zenodo.61623.

[13] Giovanni De Nunzio, Laurent Thibault, and Antonio Sciarretta. A model-based eco-routing strategy for electric vehicles in large urban networks. In *IEEE Intelligent Transportation Systems Conference (ITSC), 2016*, November 2016.

[14] Athanasios Papoulis and S Unnikrishna Pillai. Probability, random variables, and stochastic processes. *McGraw-Hill*, 1985.

[15] Leonid A Safonov, Elad Tomer, Vadim V Strygin, Yosef Ashkenazy, and Shlomo Havlin. Multifractal chaotic attractors in a system of delay-differential equations modeling road traffic. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 12(4):1006–1014, 2002.