4^{th} class

Fuzzy logic

المنطق المضبب

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2. **Fuzzy Controllers, By: Leonid Reznik, 1997.**
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Lecture 1: Introduction:

Why do we need this new theory

In the field of system science many complex problems are difficult to deal with by the conventional approach (e.g. mathematical equation), because nonlinear, time varying behavior and imprecise measurement information. Nevertheless, human operators can handle these complex problems by their practical experience. They only need imprecise system states and a set of imprecise linguistic sets and fuzzy logic can be used to del with such complex systems.

Advantages of fuzzy logic:

1. It offers machine intelligence with such human traits as the ability to make decisions based on shades of grey, instead of black and white inputs.
2. It provide an appropriate technique for describing the behavior of systems which are too imprecisely or ill defined to be amenable to formal mathematical analysis.
3. It allows non liners input/output relationship to be expressed by a set of qualitative if then rules.
4. Uses natural language
5. It is easy to setup
6. Provide accurate responses to ambiguous data
7. Works well with other techniques
8. Trade off fuzzy logic
9. Must understand and be able to define problem
10. Must evaluate and find true results

Applications where fuzzy logic is used:

1. Products
    a. Washing machine
    b. Microwave oven
    c. Rice cookers
    d. Word translators

2. Systems
    a. Elevators
    b. Train
    c. Traffic control
3. Software
    a. Medical diagnosis
    b. Securities
    c. Data compression

Lecture2: Fuzzy Sets:

Fuzzy Sets Definition

A fuzzy set is one to which objects can belong to different degrees, called grades of membership. The fuzzy set as a number ranging from zero (absolutely false) to 1 (absolutely true). Before dealing with fuzzy systems preliminary definitions should be known. In these sections we will illustrate some important definitions and concepts.

Definition 1

Let C be an ordinary set called the universe of discourse that may be either discrete or continuous and let *U* be the function taking values from the interval [0,1]. The fuzzy subset C in the universe of discourse C maybe represented by the function:

$$U : C \rightarrow [0,1]$$

where *U* is called the membership function of fuzzy set *C*. Thus the fuzzy subset $\underline{C}$ in C is represented as a set of ordered pairs of an element c and its membership function:

$$C = \{(U-(c)/c | c \equiv C\}$$

When a membership function takes only the values 1 or 0, the fuzzy set becomes the ordinary set. The value of the membership function for a given element of a fuzzy set is called the membership grade.

Definition 2

The support of fuzzy set $\underline{C}$ is an ordinary set defined as:

$$su(\underline{C}) = \{c : U(c) \neq 0\}$$

Definition 3

The α -cut of the fuzzy set $\underline{C}$ for the given α is an ordinary set:

$$\underline{C}(\alpha) = \{c : U(c) \geq \alpha\}$$

Example: Let X will be a universe of discourse defined as X={0,1,2,3,4,5} and let LV be the fuzzy subset defined on X corresponding to 'x is large'. LV={0 / 0 , 0 / 1 , 0. 1 / 2 , 0.3 / 3 , 0.8 / 4 , 1 / 5} then:

su (LV) = {2,3,4,5}

LV (α =O.3) = {3,4,5}

Two things should be noted. First, based on definition 1, the membership function is equivalent to the fuzzy set determined by this membership function . Second, fuzzy sets should not be regarded as a some kind of probability. Although probabilities, as well as membership functions, take values from the closed interval [0,1], and both concepts are used to express uncertainty, there is a significant difference between these two concepts. One obvious distinction is that the summation of probabilities on a finite universal set must equal 1, but there is no such requirement for membership grades.

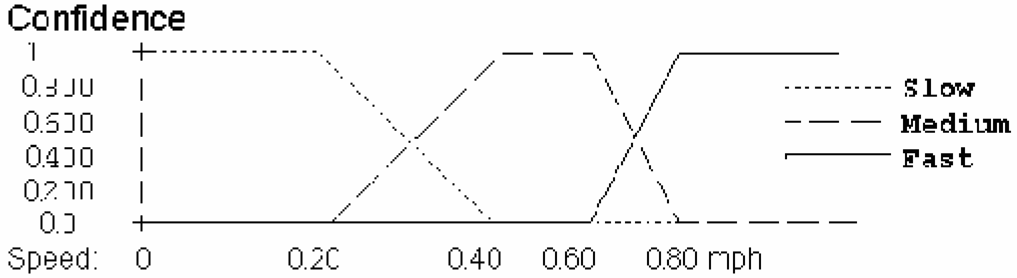
Comparison Between the Crisp Set and Fuzzy Set

In mathematics, the concept of set is very simple, but very important. ***A set*** is simply a collection of things. The things can be most anything you want - numbers, names of autos, you name it. Things either belong to the set or don't belong, while the idea of a fuzzy set is one to which objects can belong to different degrees, called grades of membership. The fuzzy set as a number ranging from zero (absolutely false) to 1 (absolutely true). A fuzzy sets of descriptive words, where the grade of membership represents our confidence that the descriptor is true of whatever we are considering. Usually, we will use the term confidence rather than grade of membership. This permits us to use ordinary language in describing things in a precise way. As an example, consider a fuzzy set **speed**, with members **Slow**, **Medium** and **Fast**.

Table : Fuzzy Set Speed

	Confidonco n:		
Velocity, mph	Slow	Meciun	Fas:
0.10	0.1000	0.0	0.0
0.20	0.500	0.500	0.0
0.30	0.0	0.1000	0.0
0.40	0.0	0.1000	0.0
0.50	0.0	0.500	0.500
0.6U	0.U	0.U	0.1UUU
0.70	0.0	0.0	0.1000

Of course, we have to define in some way how to go back and forth between the description of speed in numbers and the description of speed in words. This is done through defining *membership functions*, which look something like this:



Other useful fuzzy sets are for example: fuzzy set **color** might have members **Red**, **Green** and **Blue**. Fuzzy set **car** might have members **Ford, Chevrolet**, **Toyota** and **Buick**. In fuzzy sets like speed and color it is very likely that two or more descriptors might have non-zero confidences at the same time; that is fine, since we are seldom certain that one descriptor is entirely right and the others are entirely wrong. These are ambiguities, and as mentioned above they lend strength to the reasoning process. But in other fuzzy sets like car, it is not possible for a car to be both a Buick and a Chevrolet. In this case, if more than one member has a non-zero confidence, we have a contradiction. Contradictions need to be resolved before leaving our program, often by examining more data.

Lecture 3:Operations on fuzzy sets and comparison to the crisp operations

Basic Operators On Fuzzy Sets

There are three basic operations in conventional logic: complement, union, and intersection, an d these can be extended to fuzzy sets. It can be mentioned that all those operations which are extension of crisp concepts reduce to their usual meaning when the fuzzy subsets have membership degrees that are drawn from {0,1}. The basic operations are defined as functions that satisfy certain axiomatic requirements and operate on membership grades of fuzzy set. The results of these operations are membership functions of new fuzzy sets representing the concept of fuzzy complement, union, and intersection.

Fuzzy Complement (Not)

The complement ('not') of a fuzzy set *C* with a membership function *U* has a membership function given by:

$$\overline{U}(c) = 1\text{-}U(c),$$

Fuzzy complement function can be defined as a function com : $[0,1] \to [0,1]$.

The following requirements need to be fulfilled:

<u>Axiom 1</u>: com (com(1))= 1 and com(l)=0, for any fuzzy compliment .

<u>Axiom 2:</u> For all a, b ≡ [0, 1], if a < b then com(a) > com(b)

In most cases of practical significance, the class of function that can be Defined as fuzzy complement must be narrowed. Thus, two additional axioms are introduced:

<u>Axiom 3:</u> com is a continuous function.

<u>Axiom 4</u> : com is involutes, which means that

com(com(a)) = a , for all a ≡ [0, 1].

Axiom 1 assures that fuzzy complement does not violate the complement for ordinary sets. Moreover an increase in the degree of membership in a fuzzy set should cause a decrease in the degree of membership of its complement. This is provided by Axiom 2. Axioms 1 and 2 are called the axiomatic skeleton for fuzzy complement. The basic complement operator is given below: For example let

LV = {0 / 0 , 0 / 1 , 0. 1 / 2 , 0.3 / 3 , 0.8 / 4 , 1 / 5}, be a fuzzy set then, the fuzzy complement is given as LV = {0 / 0 , 1 / 1 , 0. 9 / 2 , 0.7 / 3 , 0.2 / 4 , 0 / 5}. The bellow Figure illustrate A fuzzy sets and its fuzzy logic complement $\overline{A}$.

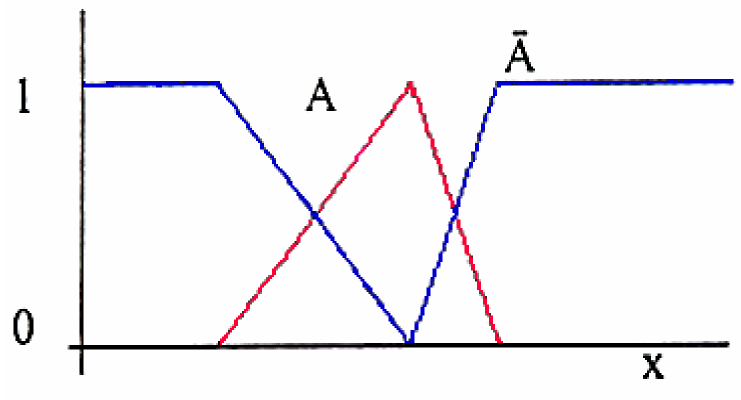


Figure: Not Operations in Fuzzy Logic

Fuzzy Union (OR)

The union of fuzzy sets U1 and U2 which are defined on the universe U ,is in general a function U of the form

$$u:[0,1] \text{ x } [0,1] \rightarrow [0,1].$$

For each element x in the universal set, this function takes as its argument the pair consisting of the element's membership grades in set A and in set B and yields the membership grade of the element in the set constituting the union of A and B. Thus:

$$U(x) = U[U_A, U_b(x)]$$

Where U(x) is membership function of fuzzy union A Y B . This function has to satisfy the following requirements:

Axiom 1 U(0,0) = 0 and U(0,1) = U(1,O) = u(l,l) = 1

Axiom 2 U(a,b) = U(b,a).

Axiom 3 If a $\leq$ b and c $\leq$ d, then U(a,c) $\leq$ U(b,d) (U is monotonic)

Axiom 4 U(U(a,b),c) = U(a,U(b,c)) (U is associative)

The first axiom ensures that the fuzzy union is a generalization of the classical union for ordinary sets. This axiom is analogous to the first axiom for fuzzy compliment. The second axiom indicates indifference to the order in which the sets are combined in fuzzy union. The third axiom is the natural requirement that a decrease in the degree of membership in fuzzy sets can not produce an increase in the degree of membership of their union. Finally, the fourth axiom allows one to extend the fuzzy union operation for any number of fuzzy sets. As used in the discussion of the fuzzy complement two additional axioms are formulated. These are:

Axiom 5 U is continuous function.

Axiom 6 U(a,a) = a

The axiom of continuity prevents a situation, where a small change in the membership grade in fuzzy sets produces a large change in the membership grade in their union. Axiom 6 ensures that the union of any fuzzy set with itself yields precisely the same set. Based on the above definitions, many union operators can be formulated. The most important and most used in approximate reasoning are:

Max union	a Y b = max(a, b)
Algebraic sum	a Y b = a + b - ab
Bounded sum	a Y b = min(l,a + b)
Drastic sum	a Y b= a when b = 0
	a Y b= b when a = 0
	a Y b= 0 when a,b $\geq$ 0
Disjoint sum	a Y b = max(min(a, 1 - b), min(l - a, b))

For example: bellow Figure shows OR operator in fuzzy domain (using Maximum operator) of two fuzzy sets A and B to yield the fuzzy set C.

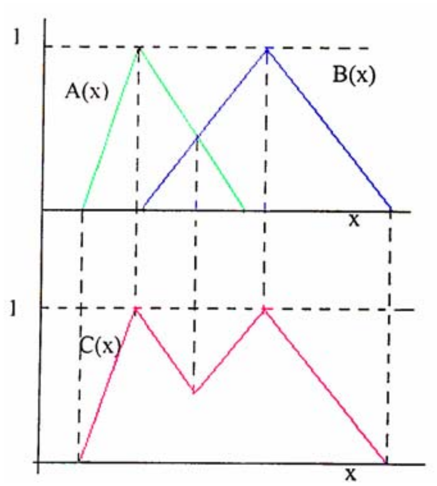


Figure: ORing Operations in Fuzzy Logic
Using maximum Operation

Fuzzy Intersection (AND)

The intersection of fuzzy sets U_1 and U_2 which are defined on the universe of discourse U , as a function i such that :

[0, 1] x [0, 1] $\rightarrow$ [0, 1].

For each element x in the universal set, this function takes as its argument the pair consisting of the element's membership grades in set A and in set B, and yields the membership grade of the element in the set constituting the intersection of A and B. Thus:

$$U(x) = i[U_a, U_b(x)]$$

where U(x) is membership function of fuzzy union $A \cap B$. This function has to satisfy the following conditions:

<u>Axiom 1:</u> i(0,0) = 0 and i(0, 1) = i(1,0) = i(l, 1) = 1

<u>Axiom 2</u>: i(a,b) = i(b,a).

<u>Axiom 3</u>: If a $\leq$ b and c $\leq$ d, then i(a,c) i(b,d) $\leq$ (U is monotonic)

<u>Axiom 4</u>: i(i(a,b),c) = i(a,i(b,c)) (i is associative)

<u>Axiom 5</u>: i is continuous function.

<u>Axiom 6</u>: i(a,a) = a

The completion of the two last axioms is unnecessary. The incentive for the above conditions is the same as for fuzzy union. Moreover, all axioms, except the first, are identical to axioms formulated for fuzzy union. The first axiom states that fuzzy intersection has to reduce to ordinary set intersection when the grade of membership is restricted to 0 and 1. Unlimited number of functions can be used as fuzzy intersection, though only a few of them have been employed in approximate reasoning. Some of them are illustrate below:

Min Intersection	$a \cap b$ = min(a, b)
Algebraic product	$a \cap b$ = - a b
Bounded product	$a \cap b$ = max(0, a + b - 1)
Drastic product	$a \cap b$= a when b = 1
	$a \cap b$= b when a = 1
	$a \cap b$= 0 when a, b < 1

For example: Figure 3.3 shows AND operator in fuzzy domain (using Minimum operator) of two fuzzy sets A and B to yield the fuzzy set C. Using maximum Operation

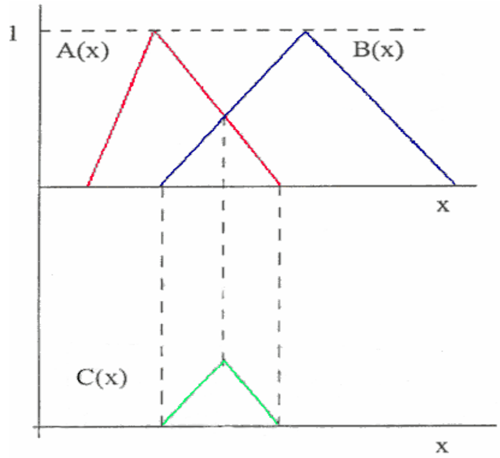


Figure: ANDing Operations in Fuzzy Logic
Using minimum Operation

Lecture 4:Fuzzy Logics:

Definition of fuzzy logic

It is an extension of Boolean logic dealing with concept of partial truth . Whereas classical logic holds that everything can be expressed in binary terms (0,1), black or white, yes or no , on or off) fuzzy logic replaces Boolean truth values with degree of truth.

Where dose fuzzy logic come from

Fuzzy logic was introduced by professor Lotfi Zadeh in 1965, while contemplating how computers could be programmed for handwriting recognition. Tis theory has its roots in the previous history of science particularly in logic science

Comparison between binary logic & fuzzy logic:

Fuzzy logic is a mathematical technique dealing with imprecise data and problems that have many solutions rather than one. Although it is implemented in digital computer which make yes-No decisions, fuzzy logic works with ranges of values, solving problems in a way that more resembles human logic.

Fuzzy logic is used for solving problems with expert systems and real time systems that must react to an imperfect environment of variable. For example it smoothes the edges of image processing

Example:

Consider the following scenario : Bob is in a house with two adjacent rooms (the kitchen and dining room). In many cases , bob's status within the set of things " in the kitchen" is completely plain: he's either " in the kitchen " or "not in the kitchen". What about when Bob stands in the doorway He may considered " partially in the kitchen". Quantifying this partial state yields a fuzzy set membership.

Lecture 11:Big picture of fuzzy logic system structure

Basic Concepts Of fuzzy Logic:

Fuzzy inference is actual process of mapping from a given input to an output using fuzzy logic. Fuzzy logic starts with the concept of a fuzzy set. A fuzzy set is a set without a crisp, clearly defined boundary. It can contain elements with only a partial degree of membership. The fuzzy inference process consists of following steps:

- Fuzzy Set Definition
- fuzzification
- Rule Base
- Fuzzy Operators
- Implication
- Aggregation
- Defuzzification

Fuzzy Set Definition

The universe of discourse is identified and fuzzy sets are defined. the definition of these sets requires expert knowledge of the control system. The most common shape of membership functions is triangular, although trapezoidal and bell curves are also used, but the shape is generally less important than the number of curves and their placement. From three to seven curves are generally appropriate to cover the required range of an input value (universe of discourse). The shape of the membership function is also determined at this step. The fuzzy set definition is more important than the shape of the membership function.

Fuzzification

The input data (crisp data) is fuzzified according to the membership functions, which maps a crisp value x into the corresponding amount of membership. The fuzzified input is the degree to which each part of the antecedent has been satisfied for each rule. We will limit the following analysis only to shapes that are symmetric around acenter value, and a monotonic on each side of the center. These can be expressed as:

$$\mu_A(x) = H(\frac{x-b}{a})$$

Rule Base

Fuzzy rule-based control systems are similar to expert systems in that the rules embody human expert knowledge about the control operation that is being mechanized. The way of implements control rules depends on whether or not the process can be controlled by a human operator. If the operator's knowledge and experience can be explained in words, then linguistic rules can be written immediately. If the operator's skill can keep the process under control, but this skill cannot be easily expressed in words, then control rules may be formulated based on the observation of operator's actions in terms of the input-output operating data. However, when the process is exceedingly complex, it may not be controllable by human expert. In this case, a fuzzy model of the process is built and the control rules are derived theoretically. It should be noted however, that this approach is quite complicated and has not yet been fully developed. Therefore the FLC is ideal for complex ill – defined systems that can be controlled by a skilled human operator without the knowledge of their underlying dynamics. In such cases an FL controller is quite easy to design and implementation is less time consuming than for a conventional controller.

Determination of control rules remains a problem. Generally there are three possible methods described as follows.

1. The method based on a fuzzy model of the process.

The relation between inputs and outputs can be expressed using linguistic description. This description is referred to as the fuzzy model of the system and takes the form of conditional statements. Based on this model the optimal set of control rules can be obtained.

2. The method based on the operator's experience and/or the control engineer's knowledge. These control rules are similar to rules formed by the human decision process. This case is used in formulation of rules for the purpose of this controller this makes the rule formulation straightforward. In a man – machine interaction rules can also be formed according to the operator's control actions.

3. the method based on learning A FLC can be developed which can learn from good or a bad experience. Such a controller was first proposed by Mamandi in 1979.

Implication

The implication method is defined as shaping of the consequent (a fuzzy set) based on the antecedent (a single number). The input for the implication process is a single number given by the antecedent, and the output is a fuzzy set.

1-mini-operation implicator

$$y_j = \min\{\mu_{A_{jN}}(x_1)......\mu_{A_{jN}}(x_N) \tag{3.8}$$

2-product-operation implicator

$$y_j = \mu_{A_{j1}}(x_1)......\mu_{A_{jN}}(x) \tag{3.9}$$

3-interactive-or (i-or) implicator (*)

$$y_j = \mu_{A_{j1}}(x_1) * ...... * \mu_{A_{jN}}(x_N)$$

$$= \frac{\mu_{A_{j1}} ......\mu_{A_{jN}}}{(1-\mu_{A_{j1}})......(1-\mu_{A_{jN}}) + (\mu_{A_{j1}} + ... + \mu_{A_{jN}})} \qquad (3.10)$$

Aggregation

Aggregation is the method for unifying the outputs of each rule by joining the parallel threads. In this process all the output fuzzy sets from each rule are united into one fuzzy set. The SUM (simple sum of each rule's output) type of aggregation is used in this controller.

Defuzzification

A task of defuzzification is to map afuzzy output to crisp output of the system.A number of defuzzification strategies exist, and it is not hard to invent more. Each provide a means to choose a single output (which denoted as z_j) based on the implied fuzzy sets. The input for defuzzification is the fuzzy set (the aggregate output fuzzy set) and the output is a crisp number. Defuzzification can be obtained using three known ways:

- The mean of maximum method
- The maximizing decision
- The center of gravity method

Centroidal defuzzification method (center of gravity method) is perhaps the most popular method is used for defuzzification. A crisp output z_j is chosen using the center of area and area of each implied fuzzy set, and is given by:

$$z_k = \frac{\sum_{j=1} w_{jk} y_j}{\sum_{j=1} y_j} \qquad (3.11)$$

z_k is determined by means of a gravity center of the area under the membership function curve of the fuzzy output and y_j is a membership grade w_{jk} . Figure 3.4 shows how the fuzzy implication works and the center of gravity defuzzification.

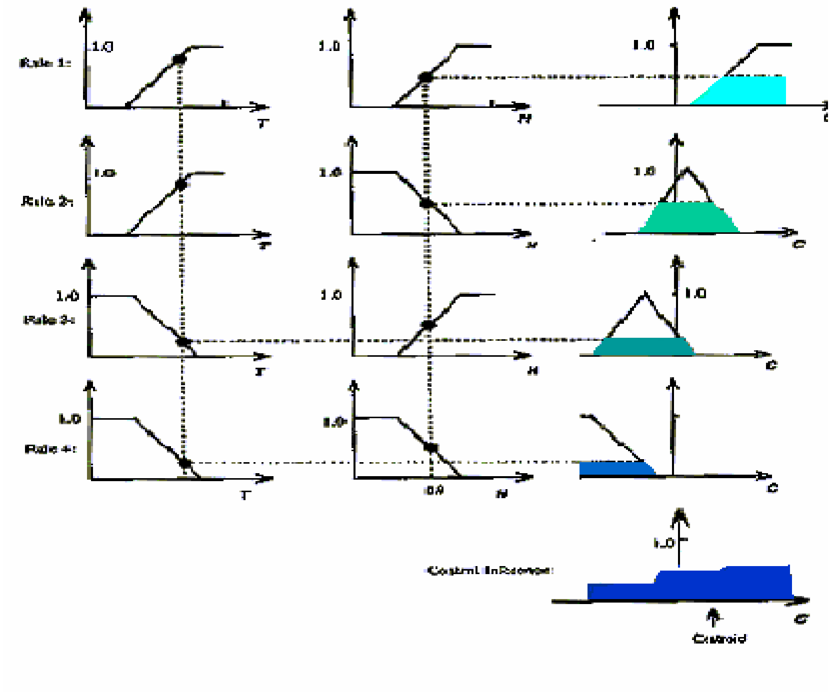


Figure : Fuzzy Centroidal defuzzification