

Assignment – 11

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244103007

assign_11

Euler Method:

| t | y_num | y_exact | trunc_err | rel_err |
|-----|--------|---------|------------|-----------|
| 0.5 | 2.948 | 2.9483 | -0.0021651 | 0.0076316 |
| 1 | 2.8965 | 2.897 | -0.0021841 | 0.015466 |
| 1.5 | 2.8455 | 2.8461 | -0.0022034 | 0.023509 |
| 2 | 2.7949 | 2.7958 | -0.0022231 | 0.031767 |
| 2.5 | 2.7447 | 2.7458 | -0.0022431 | 0.040249 |
| 3 | 2.695 | 2.6963 | -0.0022635 | 0.048961 |
| 3.5 | 2.6458 | 2.6473 | -0.0022843 | 0.057912 |
| 4 | 2.597 | 2.5987 | -0.0023055 | 0.067109 |
| 4.5 | 2.5486 | 2.5506 | -0.002327 | 0.076561 |
| 5 | 2.5007 | 2.5029 | -0.002349 | 0.086276 |
| 5.5 | 2.4533 | 2.4556 | -0.0023714 | 0.096264 |
| 6 | 2.4063 | 2.4089 | -0.0023942 | 0.10653 |
| 6.5 | 2.3598 | 2.3625 | -0.0024174 | 0.1171 |
| 7 | 2.3137 | 2.3166 | -0.0024412 | 0.12796 |
| 7.5 | 2.268 | 2.2712 | -0.0024654 | 0.13914 |
| 8 | 2.2229 | 2.2262 | -0.00249 | 0.15065 |
| 8.5 | 2.1781 | 2.1817 | -0.0025152 | 0.16249 |
| 9 | 2.1339 | 2.1376 | -0.0025409 | 0.17468 |
| 9.5 | 2.09 | 2.094 | -0.0025671 | 0.18724 |
| 10 | 2.0467 | 2.0508 | -0.0025939 | 0.20017 |

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|------|---------|---------|------------|---------|
| 10.5 | 2.0037 | 2.008 | -0.0026212 | 0.21349 |
| 11 | 1.9613 | 1.9657 | -0.0026492 | 0.22722 |
| 11.5 | 1.9193 | 1.9239 | -0.0026777 | 0.24137 |
| 12 | 1.8777 | 1.8825 | -0.0027068 | 0.25596 |
| 12.5 | 1.8366 | 1.8416 | -0.0027366 | 0.27101 |
| 13 | 1.7959 | 1.8011 | -0.0027671 | 0.28653 |
| 13.5 | 1.7557 | 1.7611 | -0.0027982 | 0.30254 |
| 14 | 1.716 | 1.7215 | -0.0028301 | 0.31907 |
| 14.5 | 1.6767 | 1.6823 | -0.0028627 | 0.33613 |
| 15 | 1.6378 | 1.6437 | -0.002896 | 0.35375 |
| 15.5 | 1.5994 | 1.6054 | -0.0029302 | 0.37194 |
| 16 | 1.5615 | 1.5676 | -0.0029651 | 0.39075 |
| 16.5 | 1.524 | 1.5303 | -0.003001 | 0.41019 |
| 17 | 1.487 | 1.4934 | -0.0030376 | 0.43028 |
| 17.5 | 1.4504 | 1.457 | -0.0030752 | 0.45107 |
| 18 | 1.4143 | 1.421 | -0.0031138 | 0.47257 |
| 18.5 | 1.3786 | 1.3854 | -0.0031533 | 0.49482 |
| 19 | 1.3434 | 1.3504 | -0.0031938 | 0.51787 |
| 19.5 | 1.3086 | 1.3157 | -0.0032354 | 0.54173 |
| 20 | 1.2743 | 1.2815 | -0.0032781 | 0.56646 |
| 20.5 | 1.2404 | 1.2478 | -0.003322 | 0.59208 |
| 21 | 1.207 | 1.2145 | -0.003367 | 0.61866 |
| 21.5 | 1.174 | 1.1817 | -0.0034133 | 0.64622 |
| 22 | 1.1415 | 1.1493 | -0.0034609 | 0.67482 |
| 22.5 | 1.1095 | 1.1174 | -0.0035098 | 0.70452 |
| 23 | 1.0779 | 1.0859 | -0.0035602 | 0.73536 |
| 23.5 | 1.0467 | 1.0548 | -0.003612 | 0.76741 |
| 24 | 1.016 | 1.0242 | -0.0036653 | 0.80073 |
| 24.5 | 0.98581 | 0.99411 | -0.0037203 | 0.83539 |
| 25 | 0.95602 | 0.96442 | -0.0037769 | 0.87145 |

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|------|---------|---------|------------|---------|
| 25.5 | 0.92669 | 0.93519 | -0.0038353 | 0.909 |
| 26 | 0.89781 | 0.9064 | -0.0038955 | 0.94811 |
| 26.5 | 0.86938 | 0.87806 | -0.0039577 | 0.98887 |
| 27 | 0.84141 | 0.85018 | -0.0040219 | 1.0314 |
| 27.5 | 0.81389 | 0.82274 | -0.0040882 | 1.0757 |
| 28 | 0.78683 | 0.79575 | -0.0041567 | 1.1221 |
| 28.5 | 0.76021 | 0.76922 | -0.0042276 | 1.1704 |
| 29 | 0.73406 | 0.74313 | -0.0043009 | 1.221 |
| 29.5 | 0.70835 | 0.7175 | -0.0043769 | 1.2739 |
| 30 | 0.68311 | 0.69231 | -0.0044556 | 1.3293 |
| 30.5 | 0.65831 | 0.66757 | -0.0045372 | 1.3874 |
| 31 | 0.63397 | 0.64329 | -0.0046219 | 1.4482 |
| 31.5 | 0.61008 | 0.61945 | -0.0047097 | 1.512 |
| 32 | 0.58665 | 0.59606 | -0.0048011 | 1.579 |
| 32.5 | 0.56367 | 0.57313 | -0.004896 | 1.6494 |
| 33 | 0.54115 | 0.55064 | -0.0049948 | 1.7235 |
| 33.5 | 0.51908 | 0.5286 | -0.0050977 | 1.8015 |
| 34 | 0.49747 | 0.50702 | -0.0052049 | 1.8836 |
| 34.5 | 0.47631 | 0.48588 | -0.0053168 | 1.9703 |
| 35 | 0.4556 | 0.46519 | -0.0054336 | 2.0617 |
| 35.5 | 0.43535 | 0.44496 | -0.0055557 | 2.1584 |
| 36 | 0.41556 | 0.42517 | -0.0056834 | 2.2607 |
| 36.5 | 0.39622 | 0.40583 | -0.0058172 | 2.3691 |
| 37 | 0.37734 | 0.38695 | -0.0059575 | 2.484 |
| 37.5 | 0.35891 | 0.36851 | -0.0061047 | 2.606 |
| 38 | 0.34093 | 0.35052 | -0.0062595 | 2.7358 |
| 38.5 | 0.32342 | 0.33299 | -0.0064224 | 2.874 |
| 39 | 0.30636 | 0.3159 | -0.006594 | 3.0213 |
| 39.5 | 0.28975 | 0.29926 | -0.0067751 | 3.1787 |
| 40 | 0.2736 | 0.28308 | -0.0069666 | 3.3471 |

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|------|-----------|-----------|------------|--------|
| 40.5 | 0.25791 | 0.26734 | -0.0071692 | 3.5275 |
| 41 | 0.24268 | 0.25206 | -0.0073841 | 3.7212 |
| 41.5 | 0.2279 | 0.23722 | -0.0076123 | 3.9295 |
| 42 | 0.21358 | 0.22283 | -0.0078553 | 4.154 |
| 42.5 | 0.19971 | 0.2089 | -0.0081144 | 4.3966 |
| 43 | 0.1863 | 0.19541 | -0.0083913 | 4.6592 |
| 43.5 | 0.17336 | 0.18237 | -0.008688 | 4.9442 |
| 44 | 0.16086 | 0.16979 | -0.0090066 | 5.2543 |
| 44.5 | 0.14883 | 0.15765 | -0.0093498 | 5.5928 |
| 45 | 0.13726 | 0.14596 | -0.0097204 | 5.9632 |
| 45.5 | 0.12614 | 0.13473 | -0.010122 | 6.37 |
| 46 | 0.11549 | 0.12394 | -0.010558 | 6.8183 |
| 46.5 | 0.10529 | 0.1136 | -0.011035 | 7.3142 |
| 47 | 0.095559 | 0.10372 | -0.011557 | 7.8651 |
| 47.5 | 0.086286 | 0.09428 | -0.012131 | 8.4797 |
| 48 | 0.077473 | 0.085294 | -0.012766 | 9.1689 |
| 48.5 | 0.069123 | 0.076757 | -0.013473 | 9.9458 |
| 49 | 0.061236 | 0.068671 | -0.014263 | 10.827 |
| 49.5 | 0.053812 | 0.061034 | -0.015154 | 11.833 |
| 50 | 0.046853 | 0.053848 | -0.016166 | 12.99 |
| 50.5 | 0.040359 | 0.047111 | -0.017325 | 14.332 |
| 51 | 0.034332 | 0.040825 | -0.018666 | 15.903 |
| 51.5 | 0.028773 | 0.034988 | -0.020239 | 17.762 |
| 52 | 0.023685 | 0.029601 | -0.022107 | 19.988 |
| 52.5 | 0.019068 | 0.024665 | -0.024367 | 22.693 |
| 53 | 0.014925 | 0.020178 | -0.027157 | 26.034 |
| 53.5 | 0.01126 | 0.016142 | -0.030695 | 30.243 |
| 54 | 0.0080767 | 0.012555 | -0.03534 | 35.672 |
| 54.5 | 0.0053806 | 0.0094189 | -0.041727 | 42.875 |
| 55 | 0.00318 | 0.0067323 | -0.051123 | 52.765 |

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|------|-------------|-----------|-----------|--------|
| 55.5 | 0.0014882 | 0.0044958 | -0.066499 | 66.897 |
| 56 | 0.00033091 | 0.0027093 | -0.097206 | 87.786 |
| 56.5 | -0.00021482 | 0.0013728 | -0.20615 | 115.65 |

Function evaluations: 113

Heun Method:

| t | y_num | y_exact | trunc_err | rel_err |
|-----|-----------|---------|----------------|------------|
| 0.5 | 2.9483+0i | 2.9483 | -0.00018042+0i | 3.3335e-05 |
| 1 | 2.897+0i | 2.897 | -0.000182+0i | 6.7853e-05 |
| 1.5 | 2.8461+0i | 2.8461 | -0.0001836+0i | 0.0001036 |
| 2 | 2.7958+0i | 2.7958 | -0.00018523+0i | 0.00014063 |
| 2.5 | 2.7458+0i | 2.7458 | -0.0001869+0i | 0.00017899 |
| 3 | 2.6963+0i | 2.6963 | -0.00018859+0i | 0.00021874 |
| 3.5 | 2.6473+0i | 2.6473 | -0.00019031+0i | 0.00025994 |
| 4 | 2.5987+0i | 2.5987 | -0.00019207+0i | 0.00030264 |
| 4.5 | 2.5506+0i | 2.5506 | -0.00019385+0i | 0.00034691 |
| 5 | 2.5029+0i | 2.5029 | -0.00019567+0i | 0.00039282 |
| 5.5 | 2.4557+0i | 2.4556 | -0.00019753+0i | 0.00044043 |
| 6 | 2.4089+0i | 2.4089 | -0.00019942+0i | 0.00048982 |
| 6.5 | 2.3625+0i | 2.3625 | -0.00020135+0i | 0.00054108 |
| 7 | 2.3167+0i | 2.3166 | -0.00020331+0i | 0.00059427 |
| 7.5 | 2.2712+0i | 2.2712 | -0.00020531+0i | 0.00064949 |
| 8 | 2.2262+0i | 2.2262 | -0.00020736+0i | 0.00070682 |
| 8.5 | 2.1817+0i | 2.1817 | -0.00020944+0i | 0.00076637 |
| 9 | 2.1376+0i | 2.1376 | -0.00021157+0i | 0.00082822 |
| 9.5 | 2.094+0i | 2.094 | -0.00021374+0i | 0.0008925 |
| 10 | 2.0508+0i | 2.0508 | -0.00021596+0i | 0.00095931 |

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|------|------------|---------|----------------|-----------|
| 10.5 | 2.0081+0i | 2.008 | -0.00021822+0i | 0.0010288 |
| 11 | 1.9658+0i | 1.9657 | -0.00022053+0i | 0.001101 |
| 11.5 | 1.9239+0i | 1.9239 | -0.00022289+0i | 0.0011762 |
| 12 | 1.8825+0i | 1.8825 | -0.0002253+0i | 0.0012543 |
| 12.5 | 1.8416+0i | 1.8416 | -0.00022776+0i | 0.0013357 |
| 13 | 1.8011+0i | 1.8011 | -0.00023028+0i | 0.0014205 |
| 13.5 | 1.7611+0i | 1.7611 | -0.00023285+0i | 0.0015087 |
| 14 | 1.7215+0i | 1.7215 | -0.00023548+0i | 0.0016007 |
| 14.5 | 1.6824+0i | 1.6823 | -0.00023817+0i | 0.0016966 |
| 15 | 1.6437+0i | 1.6437 | -0.00024093+0i | 0.0017965 |
| 15.5 | 1.6054+0i | 1.6054 | -0.00024375+0i | 0.0019007 |
| 16 | 1.5677+0i | 1.5676 | -0.00024663+0i | 0.0020095 |
| 16.5 | 1.5303+0i | 1.5303 | -0.00024959+0i | 0.002123 |
| 17 | 1.4934+0i | 1.4934 | -0.00025261+0i | 0.0022415 |
| 17.5 | 1.457+0i | 1.457 | -0.00025571+0i | 0.0023653 |
| 18 | 1.421+0i | 1.421 | -0.00025889+0i | 0.0024947 |
| 18.5 | 1.3855+0i | 1.3854 | -0.00026215+0i | 0.00263 |
| 19 | 1.3504+0i | 1.3504 | -0.00026549+0i | 0.0027715 |
| 19.5 | 1.3158+0i | 1.3157 | -0.00026892+0i | 0.0029195 |
| 20 | 1.2816+0i | 1.2815 | -0.00027243+0i | 0.0030745 |
| 20.5 | 1.2478+0i | 1.2478 | -0.00027604+0i | 0.0032369 |
| 21 | 1.2146+0i | 1.2145 | -0.00027975+0i | 0.003407 |
| 21.5 | 1.1817+0i | 1.1817 | -0.00028356+0i | 0.0035854 |
| 22 | 1.1493+0i | 1.1493 | -0.00028747+0i | 0.0037726 |
| 22.5 | 1.1174+0i | 1.1174 | -0.00029149+0i | 0.003969 |
| 23 | 1.0859+0i | 1.0859 | -0.00029563+0i | 0.0041752 |
| 23.5 | 1.0549+0i | 1.0548 | -0.00029988+0i | 0.004392 |
| 24 | 1.0243+0i | 1.0242 | -0.00030426+0i | 0.0046199 |
| 24.5 | 0.99416+0i | 0.99411 | -0.00030877+0i | 0.0048596 |
| 25 | 0.96447+0i | 0.96442 | -0.00031342+0i | 0.0051121 |

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| 25.5 | 0.93524+0i | 0.93519 | -0.0003182+0i | 0.0053779 |
| 26 | 0.90645+0i | 0.9064 | -0.00032314+0i | 0.0056582 |
| 26.5 | 0.87812+0i | 0.87806 | -0.00032823+0i | 0.0059539 |
| 27 | 0.85023+0i | 0.85018 | -0.00033348+0i | 0.0062661 |
| 27.5 | 0.8228+0i | 0.82274 | -0.00033891+0i | 0.0065958 |
| 28 | 0.79581+0i | 0.79575 | -0.00034451+0i | 0.0069445 |
| 28.5 | 0.76927+0i | 0.76922 | -0.0003503+0i | 0.0073134 |
| 29 | 0.74319+0i | 0.74313 | -0.00035629+0i | 0.0077041 |
| 29.5 | 0.71755+0i | 0.7175 | -0.00036249+0i | 0.0081182 |
| 30 | 0.69237+0i | 0.69231 | -0.00036891+0i | 0.0085575 |
| 30.5 | 0.66763+0i | 0.66757 | -0.00037556+0i | 0.009024 |
| 31 | 0.64335+0i | 0.64329 | -0.00038246+0i | 0.0095198 |
| 31.5 | 0.61951+0i | 0.61945 | -0.00038961+0i | 0.010047 |
| 32 | 0.59613+0i | 0.59606 | -0.00039703+0i | 0.010609 |
| 32.5 | 0.57319+0i | 0.57313 | -0.00040474+0i | 0.011208 |
| 33 | 0.5507+0i | 0.55064 | -0.00041276+0i | 0.011848 |
| 33.5 | 0.52867+0i | 0.5286 | -0.00042111+0i | 0.012532 |
| 34 | 0.50708+0i | 0.50702 | -0.00042979+0i | 0.013263 |
| 34.5 | 0.48595+0i | 0.48588 | -0.00043884+0i | 0.014047 |
| 35 | 0.46526+0i | 0.46519 | -0.00044829+0i | 0.014888 |
| 35.5 | 0.44503+0i | 0.44496 | -0.00045814+0i | 0.015791 |
| 36 | 0.42524+0i | 0.42517 | -0.00046844+0i | 0.016763 |
| 36.5 | 0.40591+0i | 0.40583 | -0.00047922+0i | 0.01781 |
| 37 | 0.38702+0i | 0.38695 | -0.0004905+0i | 0.018941 |
| 37.5 | 0.36858+0i | 0.36851 | -0.00050232+0i | 0.020163 |
| 38 | 0.3506+0i | 0.35052 | -0.00051473+0i | 0.021488 |
| 38.5 | 0.33306+0i | 0.33299 | -0.00052777+0i | 0.022924 |
| 39 | 0.31598+0i | 0.3159 | -0.00054148+0i | 0.024487 |
| 39.5 | 0.29934+0i | 0.29926 | -0.00055593+0i | 0.026189 |
| 40 | 0.28316+0i | 0.28308 | -0.00057117+0i | 0.028048 |

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|------|--------------|-----------|----------------|----------|
| 40.5 | 0.26742+0i | 0.26734 | -0.00058727+0i | 0.030083 |
| 41 | 0.25214+0i | 0.25206 | -0.0006043+0i | 0.032315 |
| 41.5 | 0.2373+0i | 0.23722 | -0.00062235+0i | 0.034771 |
| 42 | 0.22292+0i | 0.22283 | -0.00064151+0i | 0.03748 |
| 42.5 | 0.20898+0i | 0.2089 | -0.00066188+0i | 0.040478 |
| 43 | 0.19549+0i | 0.19541 | -0.00068359+0i | 0.043806 |
| 43.5 | 0.18246+0i | 0.18237 | -0.00070678+0i | 0.047511 |
| 44 | 0.16987+0i | 0.16979 | -0.00073159+0i | 0.051653 |
| 44.5 | 0.15774+0i | 0.15765 | -0.00075821+0i | 0.056301 |
| 45 | 0.14605+0i | 0.14596 | -0.00078683+0i | 0.061538 |
| 45.5 | 0.13482+0i | 0.13473 | -0.0008177+0i | 0.067466 |
| 46 | 0.12403+0i | 0.12394 | -0.00085109+0i | 0.074209 |
| 46.5 | 0.1137+0i | 0.1136 | -0.00088733+0i | 0.08192 |
| 47 | 0.10381+0i | 0.10372 | -0.00092678+0i | 0.090789 |
| 47.5 | 0.094375+0i | 0.09428 | -0.0009699+0i | 0.10106 |
| 48 | 0.08539+0i | 0.085294 | -0.0010172+0i | 0.11302 |
| 48.5 | 0.076855+0i | 0.076757 | -0.0010694+0i | 0.12708 |
| 49 | 0.068769+0i | 0.068671 | -0.0011272+0i | 0.14374 |
| 49.5 | 0.061134+0i | 0.061034 | -0.0011917+0i | 0.16366 |
| 50 | 0.053949+0i | 0.053848 | -0.0012639+0i | 0.18775 |
| 50.5 | 0.047213+0i | 0.047111 | -0.0013454+0i | 0.21724 |
| 51 | 0.040928+0i | 0.040825 | -0.0014382+0i | 0.25384 |
| 51.5 | 0.035093+0i | 0.034988 | -0.0015447+0i | 0.3 |
| 52 | 0.029708+0i | 0.029601 | -0.0016682+0i | 0.35932 |
| 52.5 | 0.024773+0i | 0.024665 | -0.0018131+0i | 0.43725 |
| 53 | 0.020288+0i | 0.020178 | -0.0019855+0i | 0.54237 |
| 53.5 | 0.016253+0i | 0.016142 | -0.002194+0i | 0.68882 |
| 54 | 0.012669+0i | 0.012555 | -0.0024512+0i | 0.90123 |
| 54.5 | 0.0095343+0i | 0.0094189 | -0.0027764+0i | 1.2256 |
| 55 | 0.0068506+0i | 0.0067323 | -0.0032004+0i | 1.7563 |

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|------|-------------------------|------------|----------------------|--------|
| 55.5 | 0.0046177+0i | 0.0044958 | -0.0037756+0i | 2.7122 |
| 56 | 0.0028367+0i | 0.0027093 | -0.0045987+0i | 4.7013 |
| 56.5 | 0.0015098+0i | 0.0013728 | -0.0058674+0i | 9.9822 |
| 57 | 0.0006487+0i | 0.00048624 | -0.0080425+0i | 33.412 |
| 57.5 | 0.00026666-0.00016113i | 4.9714e-05 | -0.01227+0i | 543.58 |
| 58 | 2.4912e-06+0.00014416i | 6.319e-05 | -0.017055-0.0047525i | 247.54 |
| 58.5 | -0.00017584+0.00026231i | 0.00052667 | -0.018561+0.018243i | 142.38 |

Function evaluations: 351

Step Size Comparison:

| h | Euler_fevals | Heun_fevals |
|-----|--------------|-------------|
| 0.1 | 574 | 1734 |
| 0.3 | 190 | 579 |
| 0.5 | 113 | 351 |
| 0.7 | 80 | 252 |
| 0.8 | 70 | 219 |