In polar coordinates,

$$x = r \cos \varphi \quad \text{and} \quad y = r \sin \varphi \,,$$

such that

$$\frac{x\,y}{x^2 + y^2} = \frac{r^2 \cos \varphi \sin \varphi}{r^2} = \cos \varphi \sin \varphi$$

with $\varphi = \arctan(y/x)$. So we can replace

$$\frac{x\,y}{x^2 + y^2} \rightarrow \sin(2 \arctan(y/x))/2 \,.$$