AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

Department of Electrical and Electronic Engineering

Project Report

Course No: EEE-4106

Course Name: Control System Lab

Date of Submission: 23/02/2023

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Lab Section : A-2

Year : 4^{th}

Semester : 1^{st}

PID controller

- Systems that feed the error forward to the plant (or actual system) are called proportional control systems.
- Systems that feed the integral of the error to the plant are called integral control systems.
- Systems that feed the derivative of the error to the plant are called derivative control systems.
- PID stands for 'Proportional-plus-integral-plus-derivative'.
- Combination of PD controller and PI controller.
- Implemented using active components (for example, op-amps) as active networks.
- **PD controller:** The sum of a differentiator and a pure gain, also called an *ideal derivative compensator*. Its purpose is to improve the transient response of a system.
- **PI controller:** The sum of an integrator and a pure gain, also called an *ideal integral compensator*. Its purpose is to **reduce the steady-state error to zero**.
- Hence, the purpose of *PID controller* is to improve both the transient response and the steady-state response (zero steady-state error).

A PID controller is shown in Figure 1.

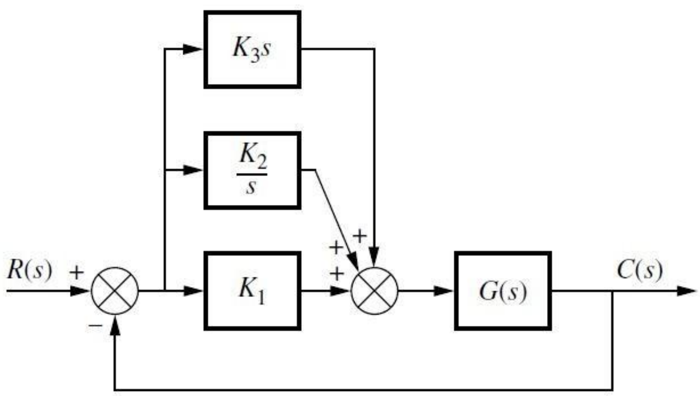


Figure 1: PID controller

Its transfer function is given by

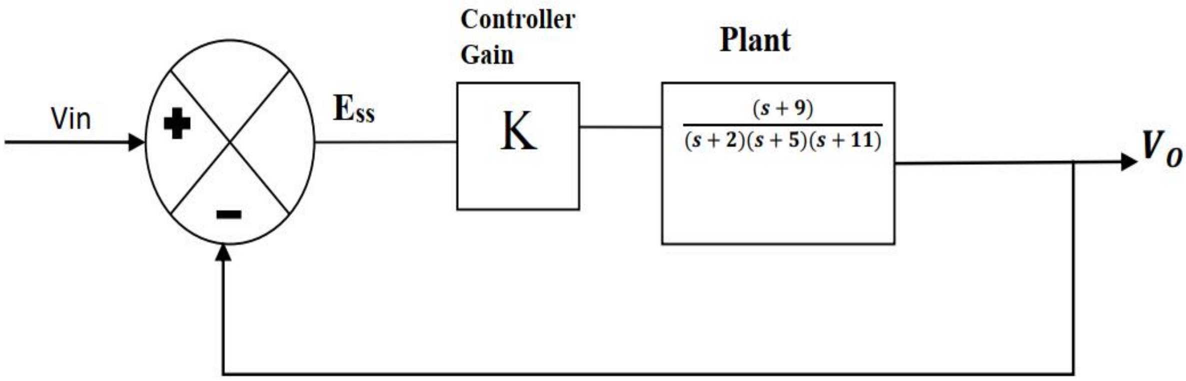
$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_1 s + K_2 + K_3 s^2}{s} = \frac{K_3\left(s^2 + \frac{K_1}{K_3}s + \frac{K_2}{K_3}\right)}{s} \qquad \textbf{(Equation (1))}$$

which has two zeros plus a pole at the origin. One zero and the pole at the origin can be designed as the ideal integral compensator; the other zero can be designed as the ideal derivative compensator.

- While improving both the steady-state response and transient response, both can be improved independently.

- For an approach, we can improve the steady-state error first and then follow with the design to improve the transient response. A disadvantage of this approach is that the improvement in transient response in some cases yields some decay in the improvement of the steady-state error, which was designed first.
- In other case, the improvement in transient response yields further improvement in steady-state errors. Thus, a system can be overdesigned with respect to steady-state errors (Overdesign is usually not a problem unless it affects cost or produces other design problems).
- Here, I am going to design controller for improving the transient response first and then design for reducing the steady-state error.
- First, the *PD controller* has to be designed to improve the transient response. Then, we have to design the *PI controller* to obtain the required steady-state error.

The given uncompensated sysyem is :



My ID is 190105046

So, A = $\frac{4}{2}$ = 2

$\therefore$ Next integer is 3. So A $=$ 3.

B = $\frac{6}{2}$ = 3

$\therefore$ Next integer is 4. So B $=$ 4

Given,

Peak time = $\frac{1}{A} = \frac{1}{3}$ times of the uncompensated system.

Persentage overshoot $(\%\ O.S) = B0\% = 40\%$

We know that,

$$\%\,O.S = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$\Rightarrow 0.4 = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$\Rightarrow \ln(0.4) = \frac{-\pi\zeta}{\sqrt{1-\zeta^2}}$$

$\therefore$ ζ = 0.279

Therefore,

$\cos\theta = \zeta$ = 0.279

$\Rightarrow \theta = \cos^{-1}(0.279)$

$\therefore$ θ = 73.80°

The characteristics equation is,

$$1 + \frac{K(s+9)}{(s+2)(s+5)(s+11)} = 0$$

Here,

Three open loop poles and one open loop zero.

So, No of poles, Np =3

No of zero, Nz =1

Now, Center of assymtotes,

$$\sigma_A = \frac{\sum p - \sum z}{Np - Nz}$$

$$= \frac{-2-5-11-(-9)}{3-1}$$

= - 4.5

Angle of assymptote,

$$\emptyset_A = \frac{2q+1}{Np - Nz} X\ 180°$$

= 90° , 270° $[\text{here q} = 0{,}1]$

The characteristics equation is,

$$1 + \frac{K(s+9)}{(s+2)(s+5)(s+11)} = 0$$

$$\Rightarrow (s+2)(s+5)(s+11) + K(s+9) = 0$$

$$\Rightarrow (s^3 + 18s^2 + 87s + 110) + K(s+9) = 0$$

$$\Rightarrow K = \frac{-(s3 + 18s2 + 87s + 110)}{(s+9)}$$

Now,

$$\frac{dk}{ds} = 0$$

$$\Rightarrow \frac{-\{(s+9)(3s^2+36s+87)\}-\{-(s^3+18s^2+87s+110)\}}{(s+9)^2} = 0$$

$$\Rightarrow -3s^3 - 36s^2 - 87s - 27s^2 - 324s - 783 + s^3 + 18s^2 + 87s + 110 = 0$$

$$\Rightarrow -2s^3 - 45s^2 - 324s - 673 = 0$$

$\therefore s = -3.55, -9.47 \pm 2.22j$ [By using calculator]

$\therefore$ Breakaway point, $s = -3.55$

Performance of the uncompensated system in MATLAB:

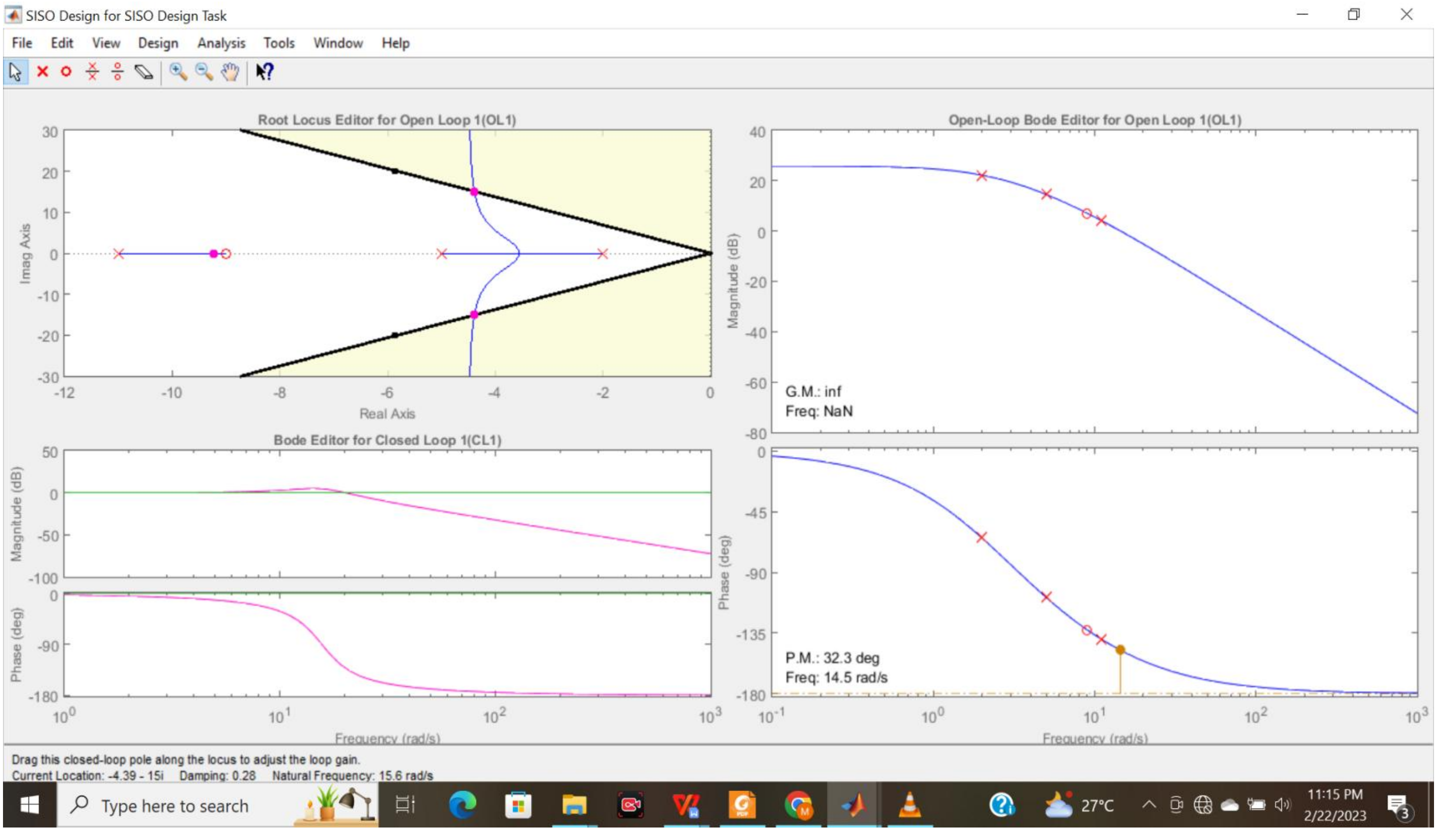


Figure 2: Root locus for uncompensated system

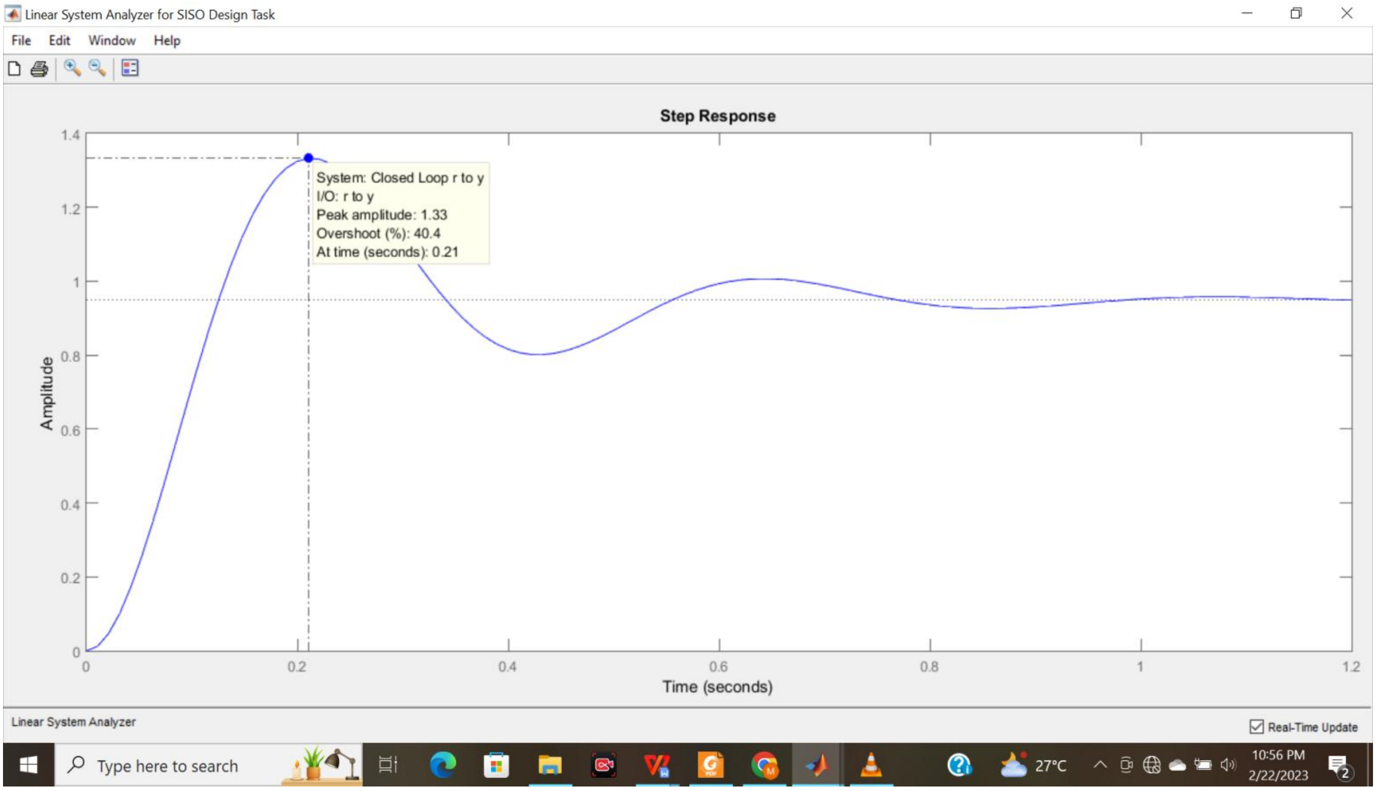


Figure 3: Step response of the uncompensated system

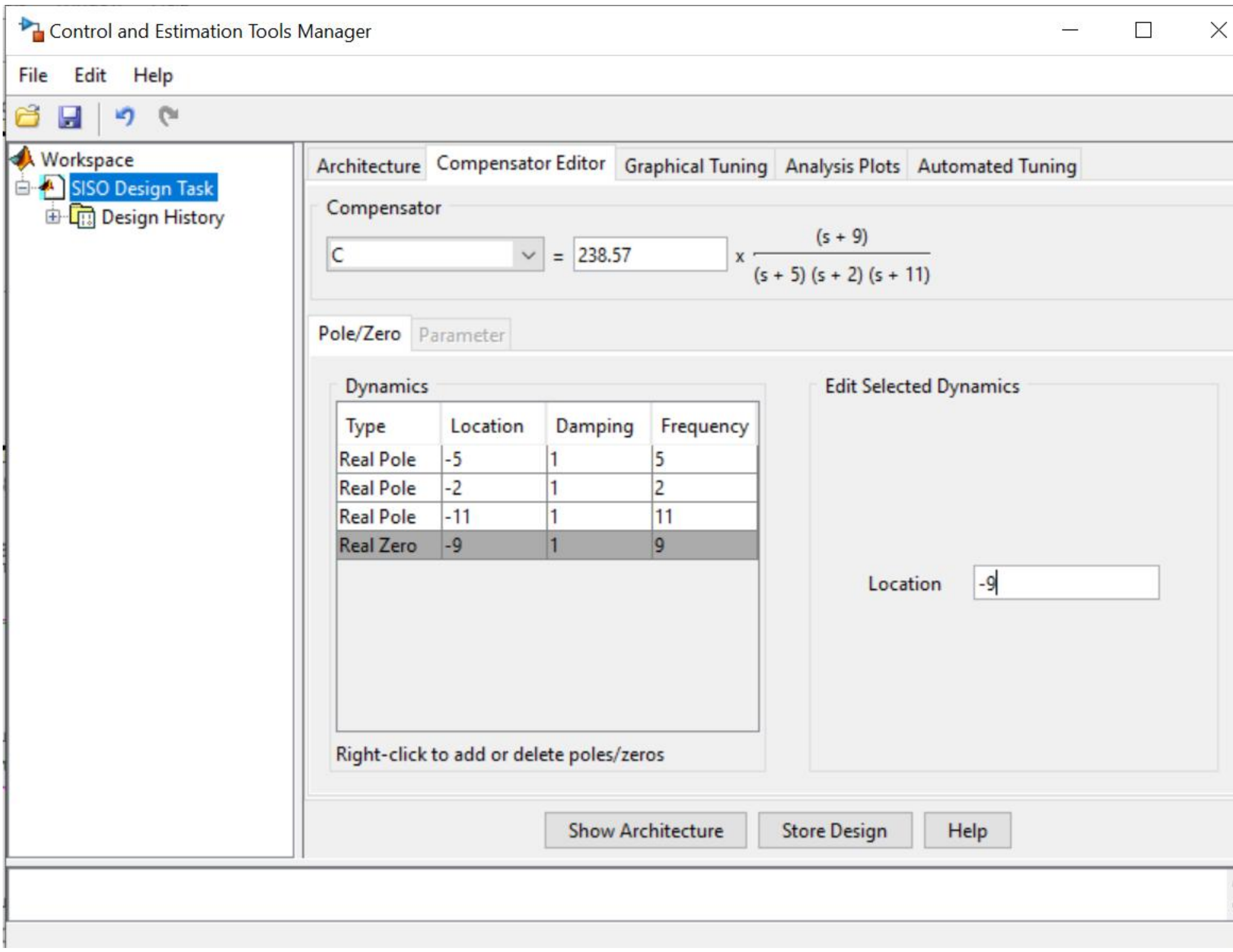


Figure 4: Gain of the uncompensated system

Find the dominant poles at -4.39-15j with gain of 238.57.

Peak time is observed as 0.21 seconds.Theoretically, the peak time can be calculated from the imaginary part of the dominant pole (that is, 15*j*). We know that the dominant poles are given by the roots $-\zeta\omega_n \pm j\omega_d, where\ w_{d} = \omega_n\sqrt{1-\zeta^2}$

Now the peak time , $T_p = \frac{\pi}{\omega_d} = \frac{\pi}{15} = 0.20944\ seconds$

A PD controller has to be designed to reduce the peak time to one-thirds of that of the uncompensated system, that is, (1/ 3) × 0.21 seconds = **0.07 seconds** (theoretical value = 0.0698 seconds). The dominant poles for the desired peak time of 0.0698 seconds (theoretical value) can be obtained by theoretical means. From the peak time relation, the imaginary part of the desired dominant pole is obtained as

$$\omega_d = \frac{\pi}{0.0698} = 45.0086$$

The real part of desire dominent pole can be obtained,

$$\zeta\omega_n = \frac{45.0086}{tan(73.80)} = 13.076$$

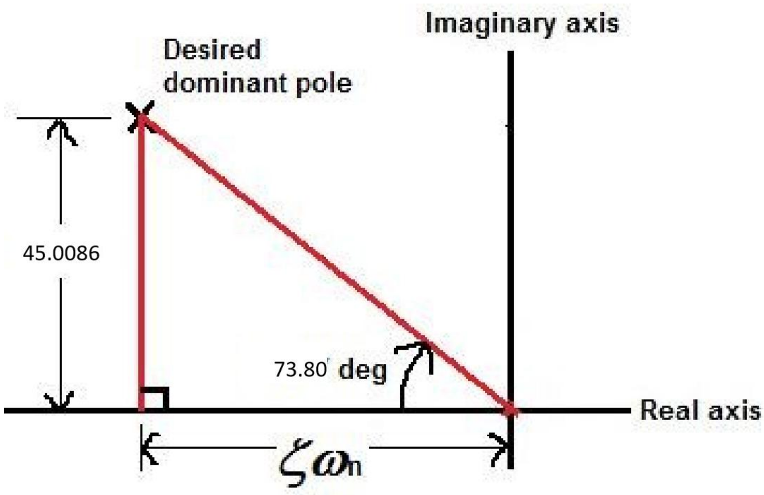


Figure 5: Right angle triangle formed in s-plane to find the dominant pole

Therefore, the dominant poles for the desired transient response specification of peak time = 0.0698 seconds are **–13.076 ±45.0086*j***. Now, we start the design of PD controller by finding the location of the controller's zero using the root locus property as shown in Figure 6

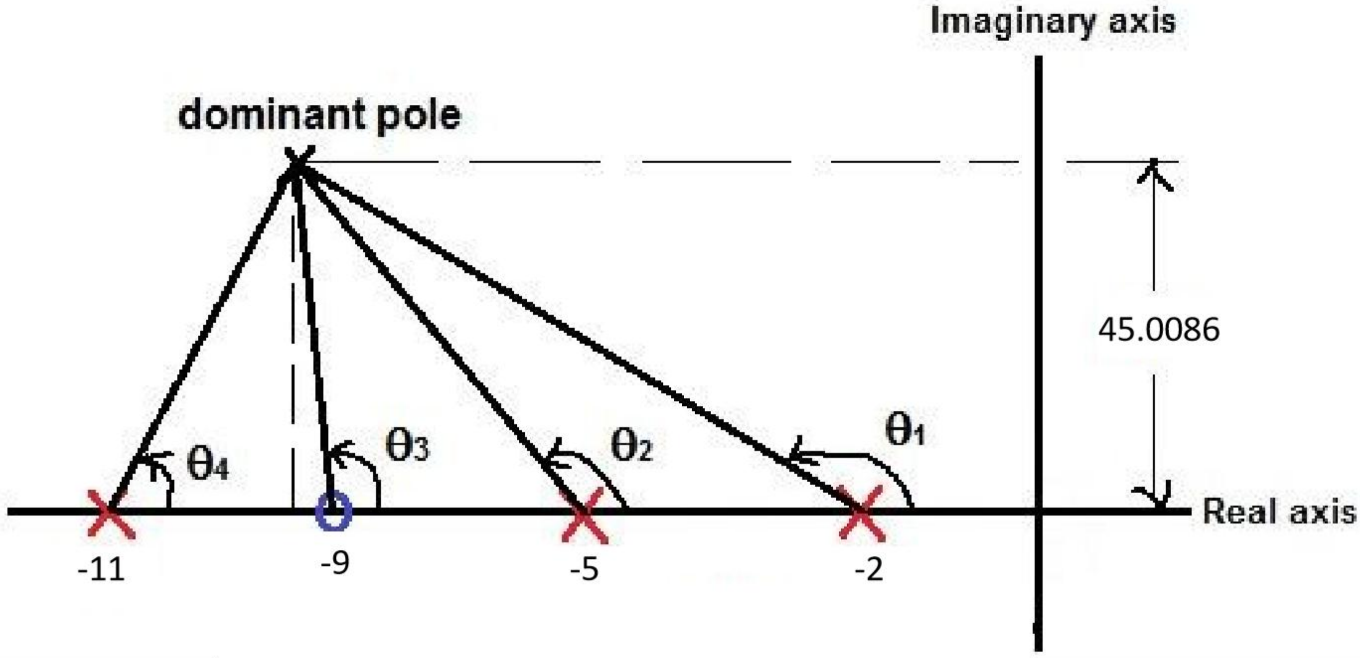


Figure 6: Angles formed between the dominant pole and all other poles and zeros

The angles formed between the dominant pole and all other poles and zeros can be obtained as follows:

$$\theta_1 = 180° - [tan^{-1}(\frac{45.0086}{13.076-2})] \ = \ 103.83°$$

$$\theta_2 = 180° - [tan^{-1}(\frac{45.0086}{13.076-5})] \ = \ 100.173°$$

$$\theta_3 = 180° - [tan^{-1}(\frac{45.0086}{13.076-9})] \ = \ 95.175°$$

$$\theta_4 = 180° - [tan^{-1}(\frac{45.0086}{13.076-11})] \ = \ 92.64°$$

Angle contribution = 180° – (sum of angles from the dominant pole to all other poles)

+ (sum of angles from the dominant pole to all other zeros)

$= 180° \ - (\theta_1 + \theta_2 + \theta_4) \ + \theta_3$

$=180° - (103.83° \ + 100.173° + 92.64°) + 95.175°$

$=21.468°$

Here, the angle contribution is for zero location, so, the angle can be taken as positive.

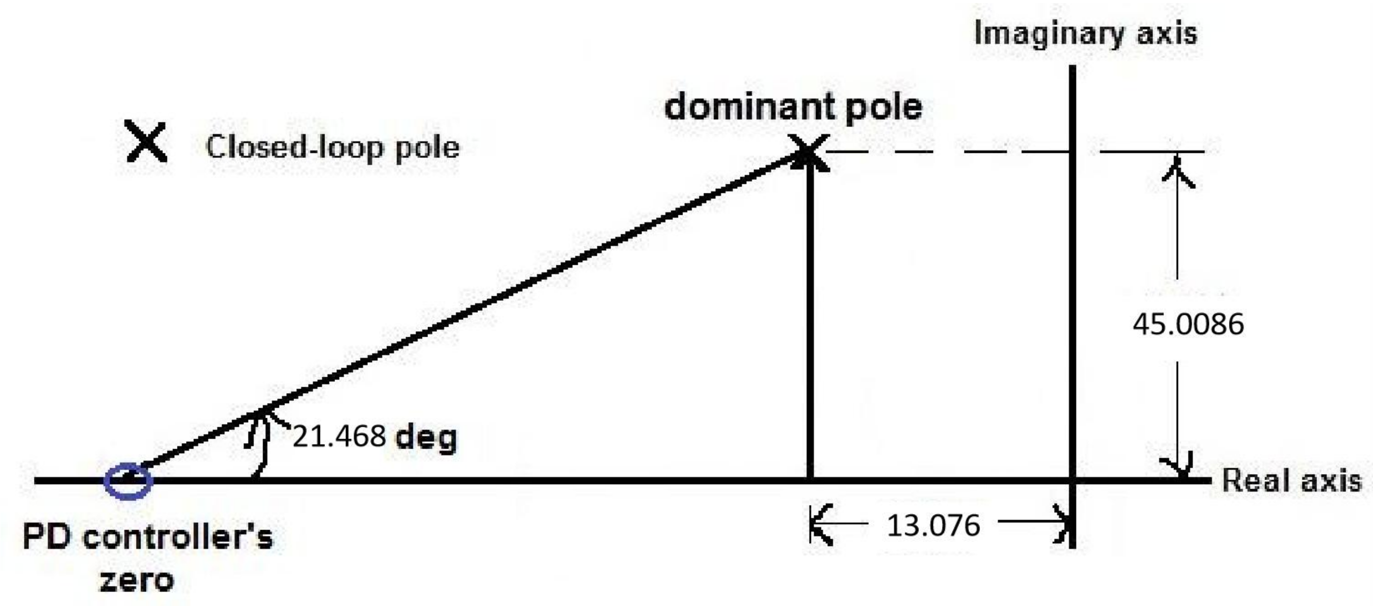


Figure 7: Calculating the PD compensator zero (z_c)

Using the geometry shown in Figure 7,

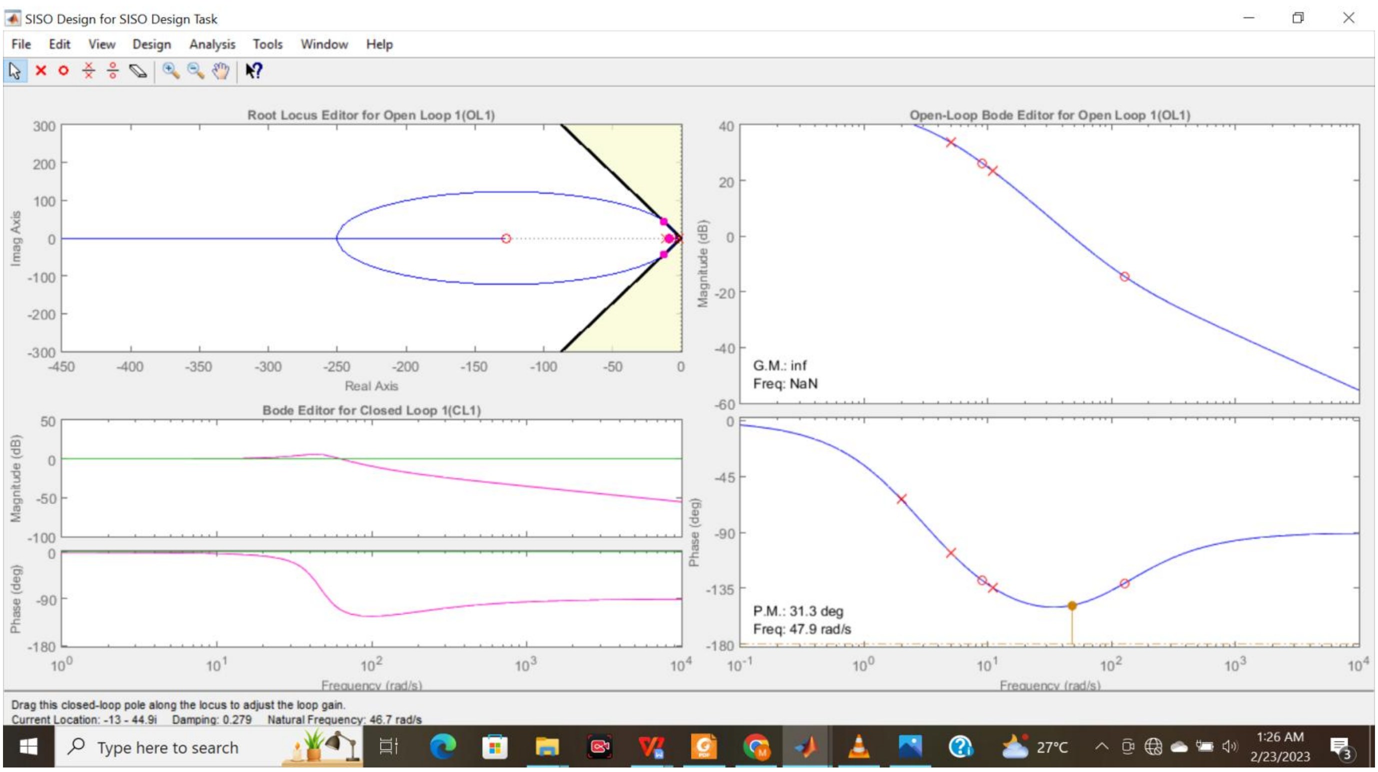
$$\frac{45.0086}{Z_c - 13.076} = \tan(21.468)$$

$$\therefore\ Z_c = -127.51$$

Therefore, the transfer function of the designed PD controller is given by

$$G_{PD}(s) = K\,(s + 127.51)$$

Now, the loop gain *K* for the PD-compensated system can be determined by either manual calculations or from root locus graph of the PD-compensated system. The complete root locus of the PD- compensated system $G_{PD}(s)$ $G(s)$ is shown in Figure 8



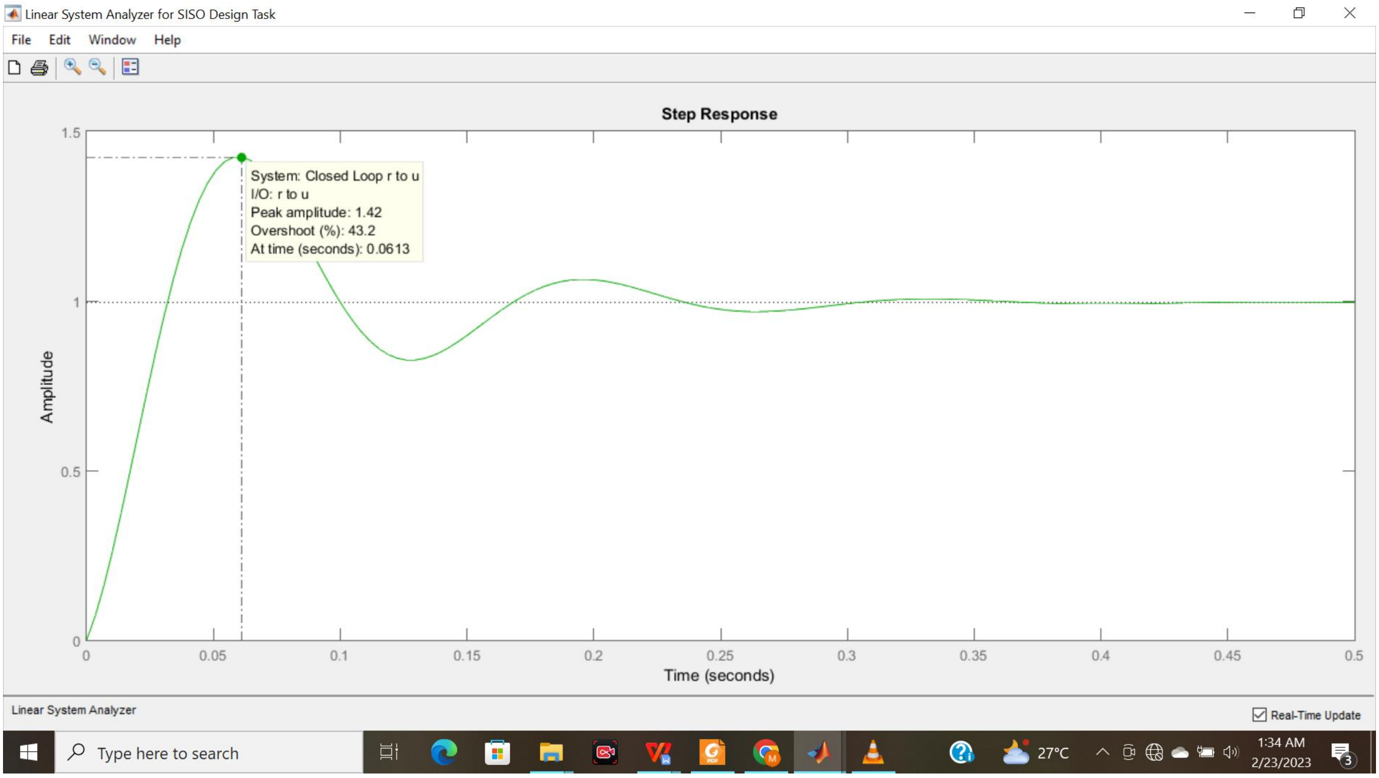


Figure 9: Step response of the PD-compensated system

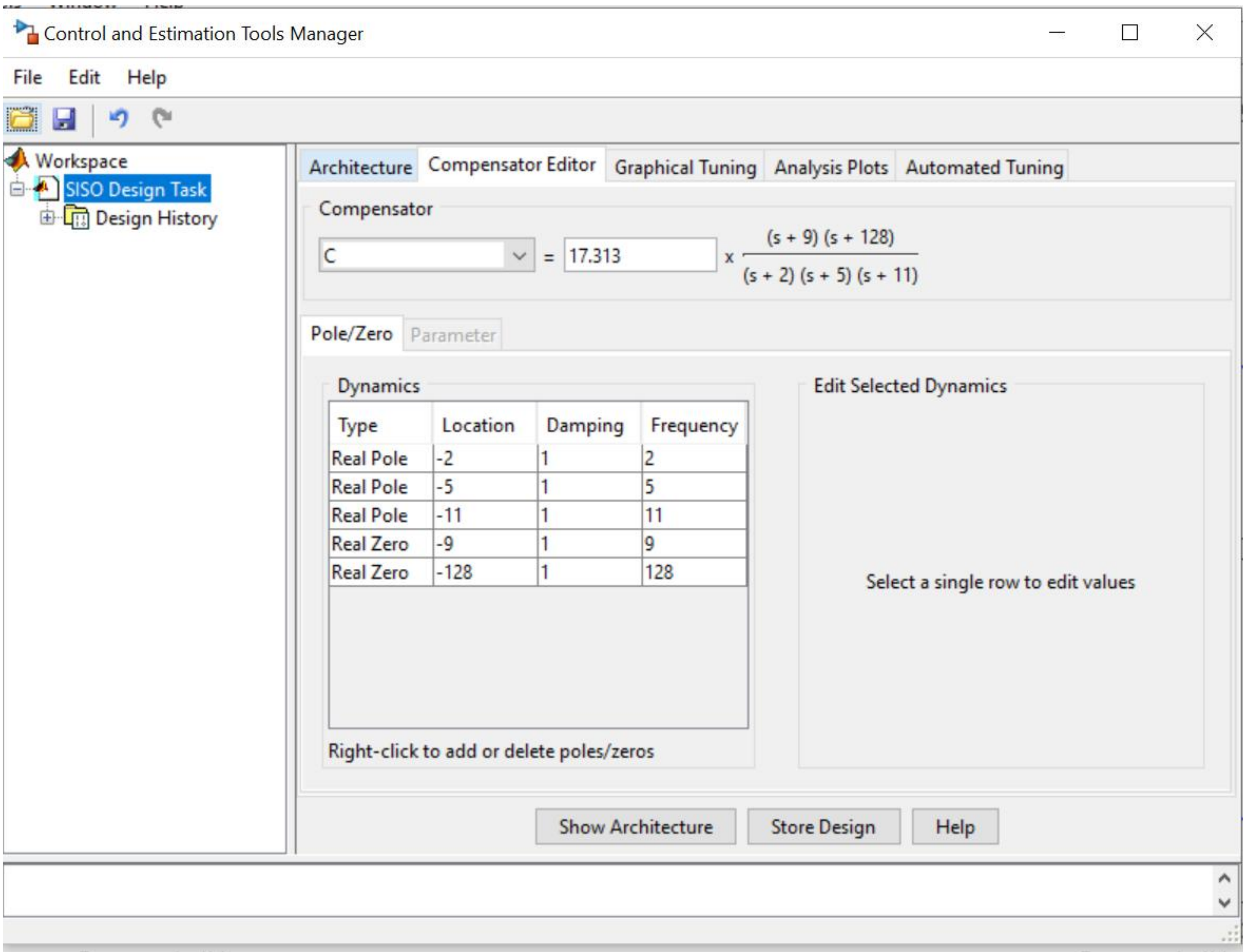


Figure 10: Gain of the PD-compensated system

Determination of loop gain *K* for the PD-compensated system:

The characteristic equation for a system is given by 1+$G(s)$ $H(s)$ = 0. Since the given system in Figure 1 is unity feedback system, $H(s)$ = 1.

Therefore, the characteristic equation is simply 1+$G(s)$ = 0

G(s) = $\frac{K(s+9)(s+127.51)}{(s+2)(s+5)(s+11)}$

For characteristic equation,

$1 + \frac{K(s+9)(s+127.51)}{(s+2)(s+5)(s+11)} = 0$

$\Rightarrow K = \frac{-((s+2)(s+5)(s+11))}{(s+9)(s+127.51)} |s = -13.076 + 45.0086j$

$\Rightarrow$ K= 17.285

the simulation that the PD-compensated system satisfies the peak time requirement better than the desired one and operating with overshoot of nearly 43.2% (somewhat acceptable) and the result is satisfactory.

Now,we need to design the ideal integral compensator (or PI controller) to reduce the steady-state error to zero for a step input. Any ideal integral compensator zero will work, as long as the **zero is placed close to the origin**. Here, I am choosing the ideal integral compensator zero to be 0.1, and thus the transfer function of the PI controller is given by,

$$G_{pI}(s) = \frac{s + 0.1}{s}$$

Now, the loop gain *K* for the PID-compensated system (combination of PD and PI) can be determined by either manual calculations or from root locus graph of the PID-compensated system.

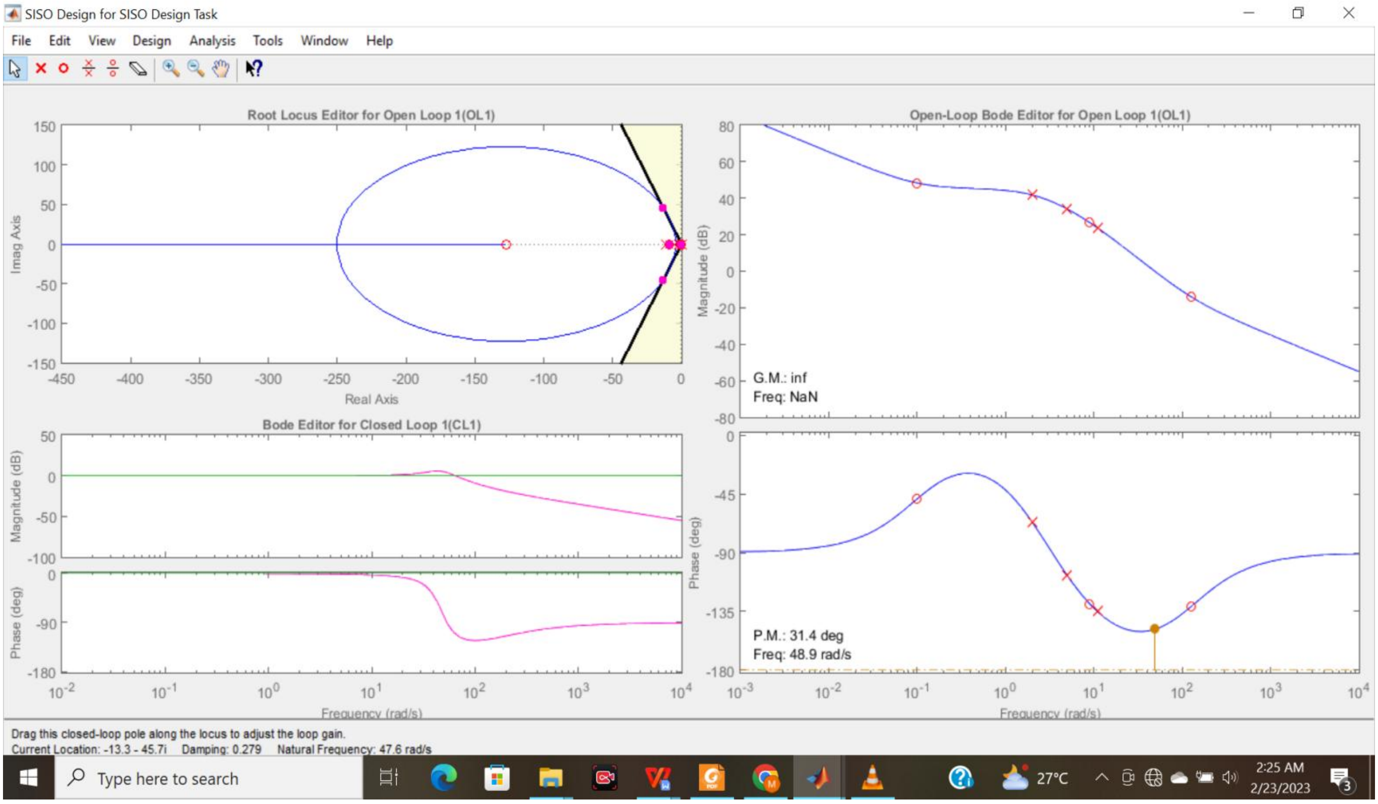


Figure 11: Root locus for PID-compensated system

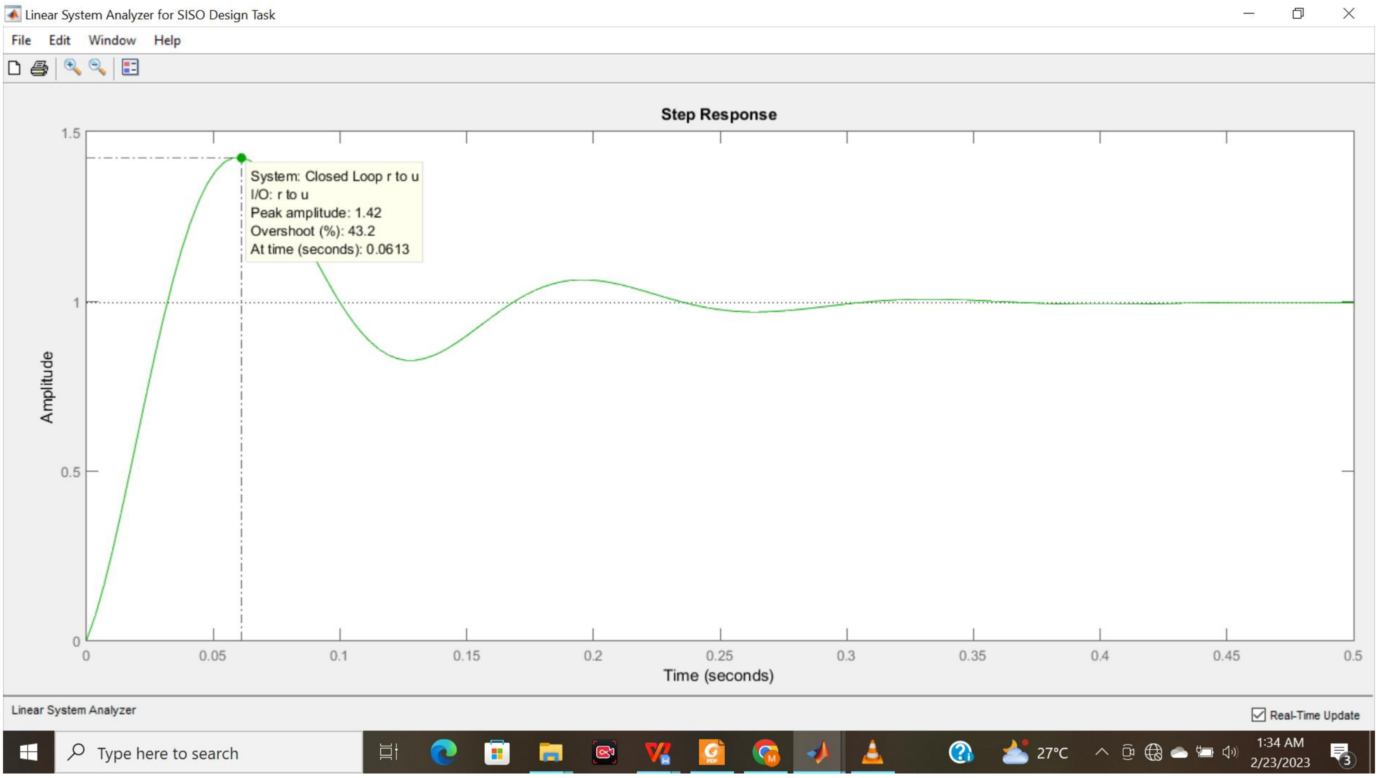


Figure 12: Step response of the PID-compensated system

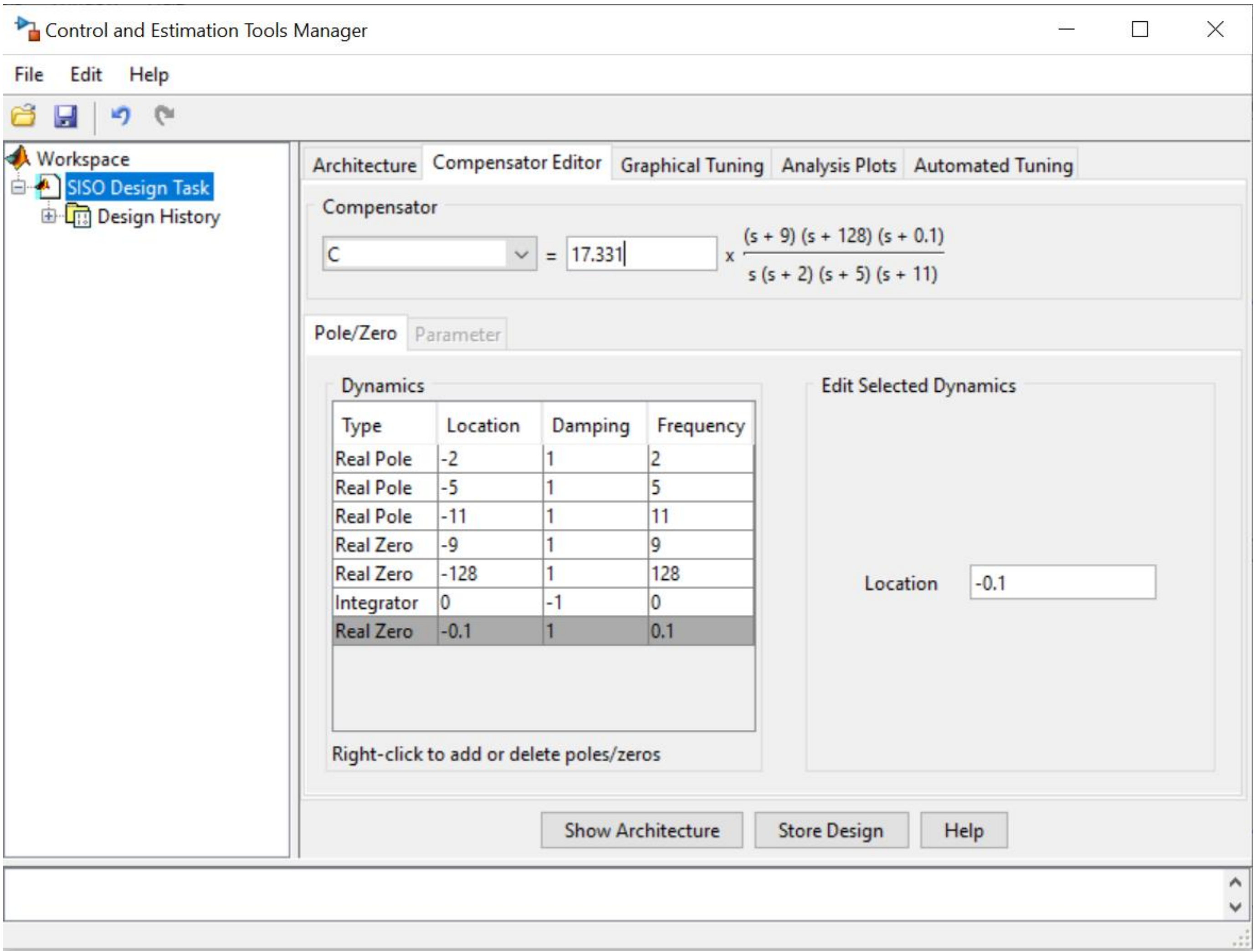


Figure 13: Gain of the PID-compensated system

Manual calculations for finding the loop gain *K* for the PID-compensated system:

Here,

G(s) = $\frac{K(s+9)(s+127.51)(s+0.1)}{s(s+2)(s+5)(s+11)}$

For characteristic equation,

$1 + \frac{K(s+9)(s+127.51)(s+0.1)}{s(s+2)(s+5)(s+11)} = 0$

$\Rightarrow K = \frac{-(s(s+2)(s+5)(s+11))}{(s+0.1)(s+9)(s+127.51)} \Big|_{s = -13.076 + 45.0086j}$

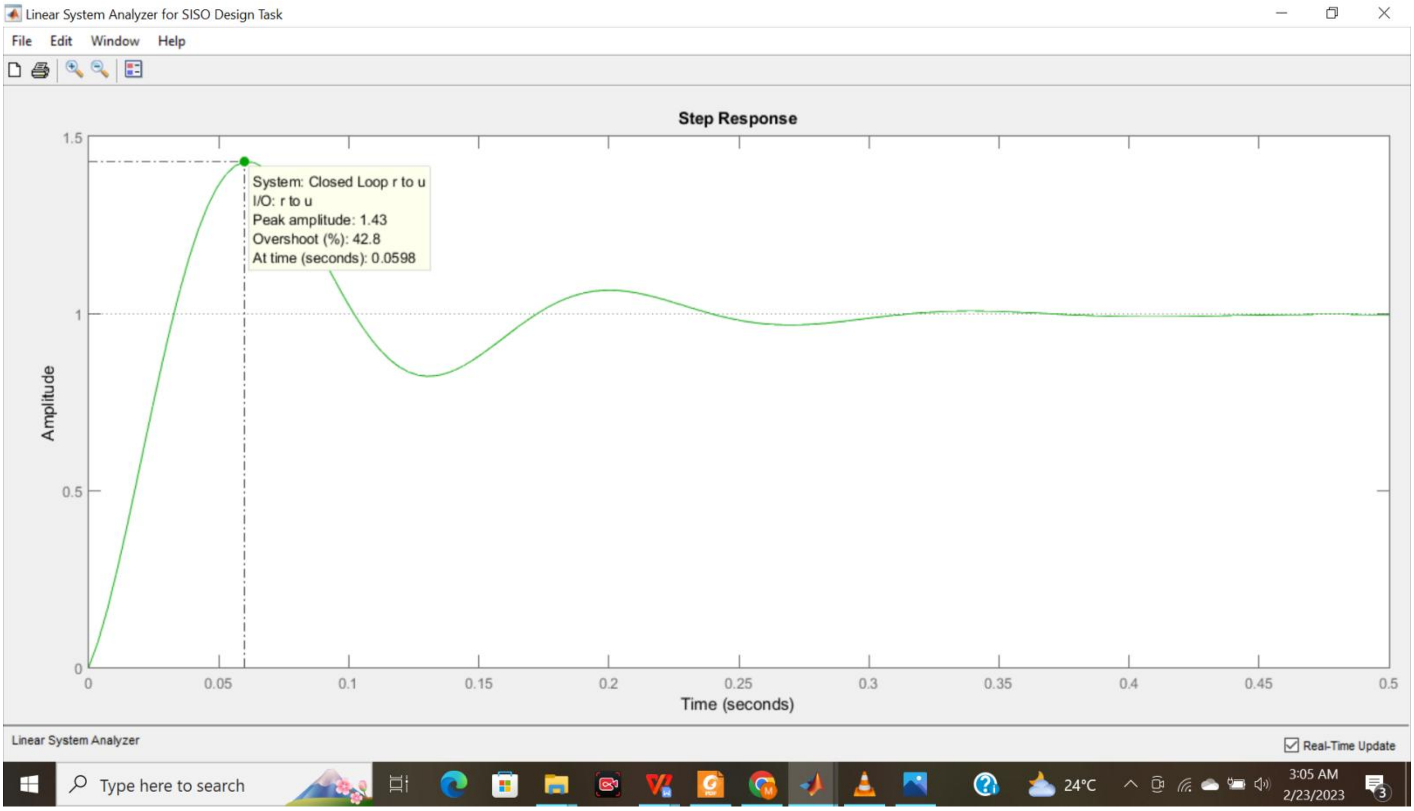
$\Rightarrow$ K= 17.31

The transfer function of the PID controller can be obtained as,

$$G_{PID}(s) = \frac{K(s+127.51)(s+0.1)}{s}\ ,\ K = 17.331$$

$$= \frac{17.331(s+127.51)(s+0.1)}{s}$$

Now, we have to check the performance of PID controller design with loop gain of 17.331 in MATLAB simulation. The step response of the PID-compensated system is shown in Figure 14.



Simulink Simulation:

For uncompensated sysyem,

Let,C1=C2=C3=C4=1F

R8=R3=R5=1Ω

K=$\frac{R8C4}{R1C1R3C2R5C3}$

$\Rightarrow R_1 = \frac{1}{238.57}$

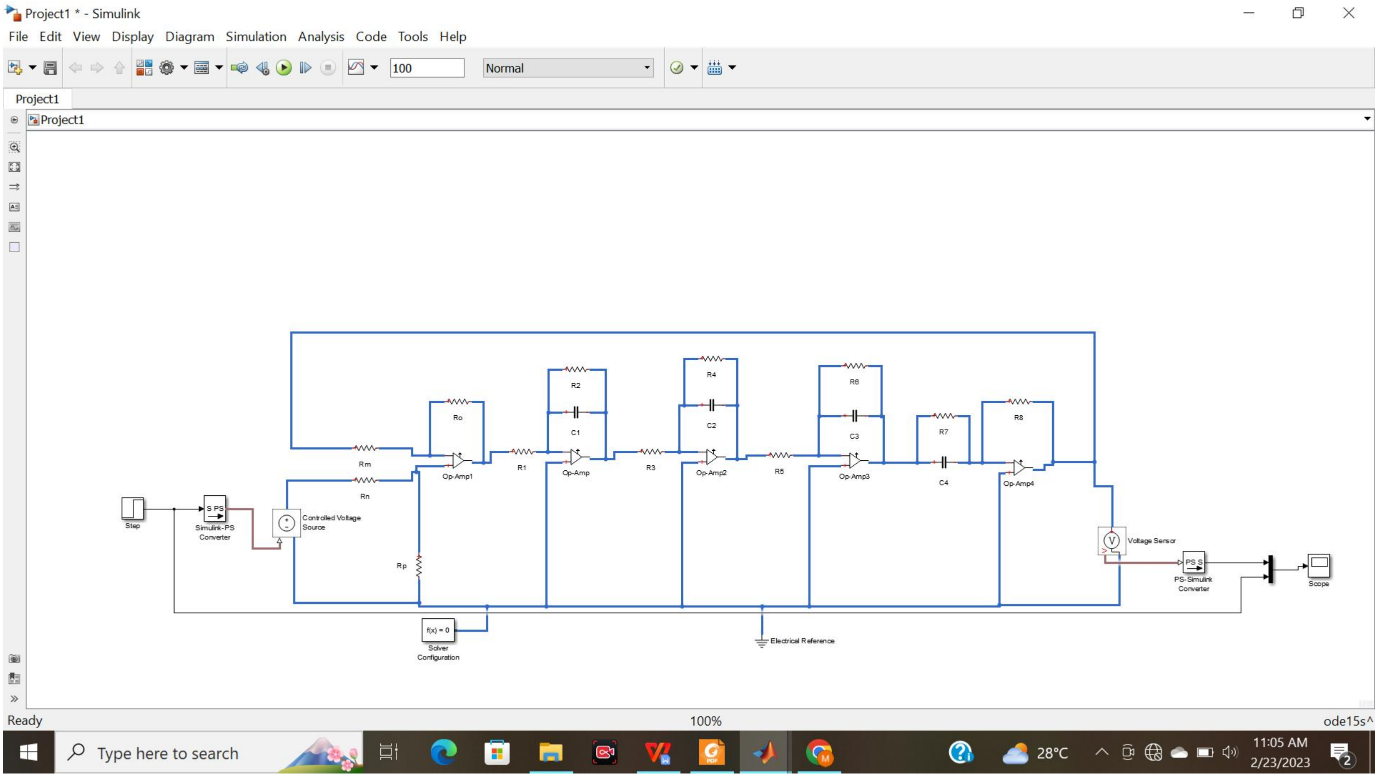
$\therefore\ \mathrm{R}_1 = \ 4.2X10\text{^} - 3\ \Omega$

$R_2 = \frac{1}{2} = 0.5\Omega$

$R_4 = \frac{1}{5} = 0.2\Omega$

$R_6 = \frac{1}{11} = 0.0909\Omega$

$R_7 = \frac{1}{9} = 0.11\Omega$


Project1 * - Simulink
File Edit View Display Diagram Simulation Analysis Code Tools Help
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Rm
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C3
R7
C4
R8
Op-Amp1
Op-Amp
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Step
Simulink-PS Converter
Controlled Voltage Source
Voltage Sensor
PS-Simulink Converter
Scope
f(x) = 0
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Scope
File Tools View Simulation Help
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Transitions
High 948.865 mV
Low 6.635 mV
Amplitude 942.230 mV
+ Edges 1
+ Rise Time 84.455 ms
+ Slew Rate 8.925 V/s
- Edges 0
- Fall Time --
- Slew Rate --
Overshoots / Undershoots
+ Preshoot 0.704 %
+ Overshoot 40.141 %
+ Undershoot 7.299 %
+ Settling Time --
- Preshoot --
- Overshoot --
- Undershoot --
- Settling Time --
Cycles
Cursor Measurements
Settings
Screen cursors
Horizontal
Vertical
Waveform cursors
PS-Simulink Converter
PS-Simulink Converter
Lock cursor spacing
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ΔT 500.000 ms ΔV 911.005 m
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For compensated sysyem,

Let,C1=C2=C3=C4=C5=C6=C7=1F

R8=R3=R5=R10=R12=R13=R14=1Ω

$$K=\frac{R8C4R10R12R15}{R1C1R3C2R5C3R13R14}$$

$$\Rightarrow R_1 = \frac{1}{17.331}$$

$$\therefore\ R_1 = 0.0577\ \Omega$$

R_2,R_4,R_6, R_7 *same as uncompensated system*

$$R_9 = \frac{1}{0.1} = 10\Omega$$

$$R_{11} = \frac{1}{127.51} = 7.843X10^{\wedge} - 3\Omega$$

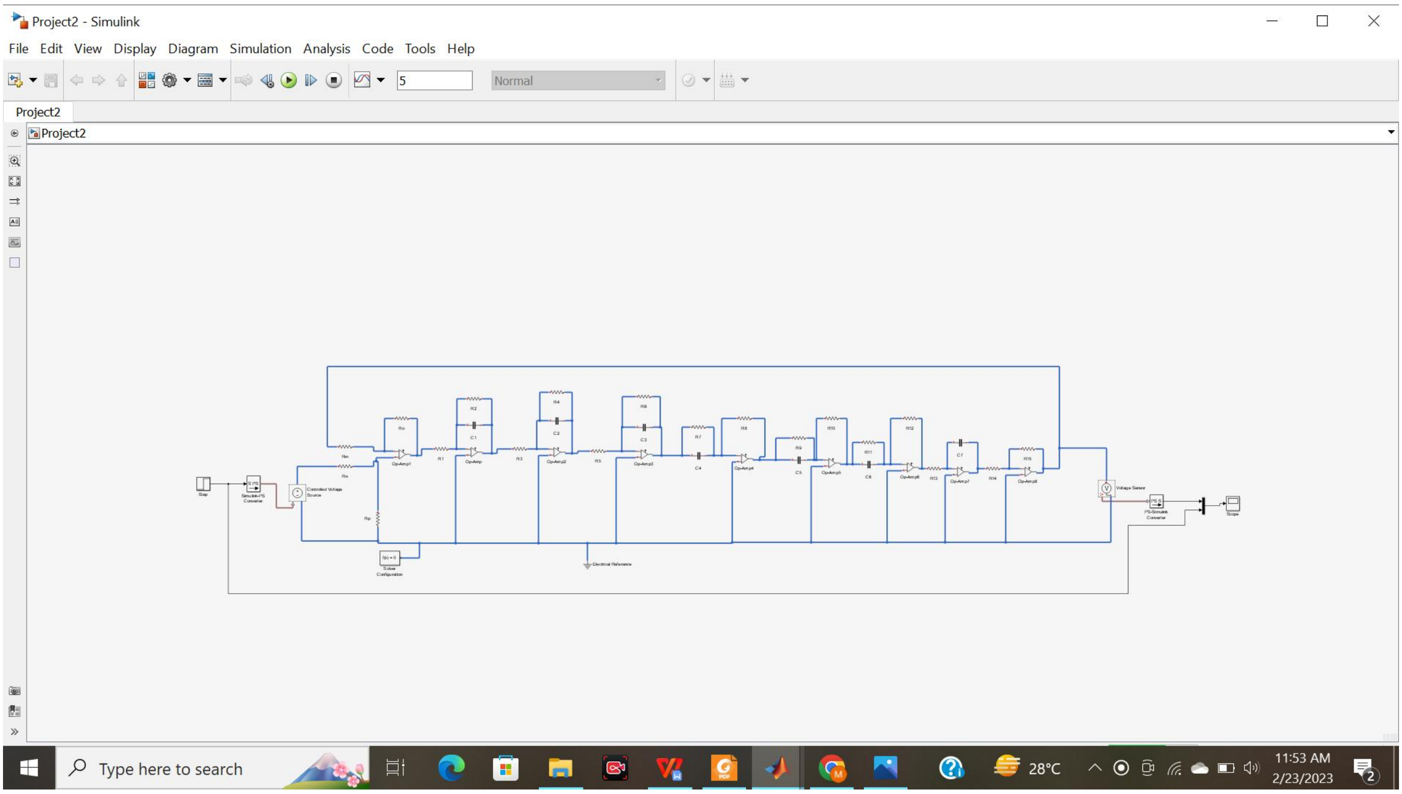

Fig: circuit diagram of compensated sysyem

Fig: Wave-form of compensated sysyem