AHSANULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

Department of Electrical and Electronic Engineering

Assignment

Course No : EEE 4105

Course Name: Control System

Project Name: Water Level Control System

Prepared By

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Student ID: 19.01.05.046

Section: A-2

Year: 4th

Semester: 1st

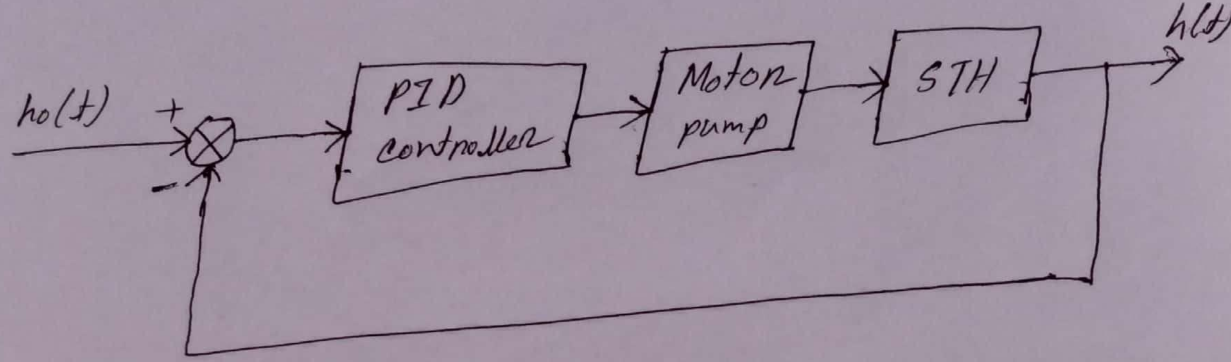
(i)

System: A system is a combination of elements acting together. For example: Electrical, Mechanical, chemical etc.

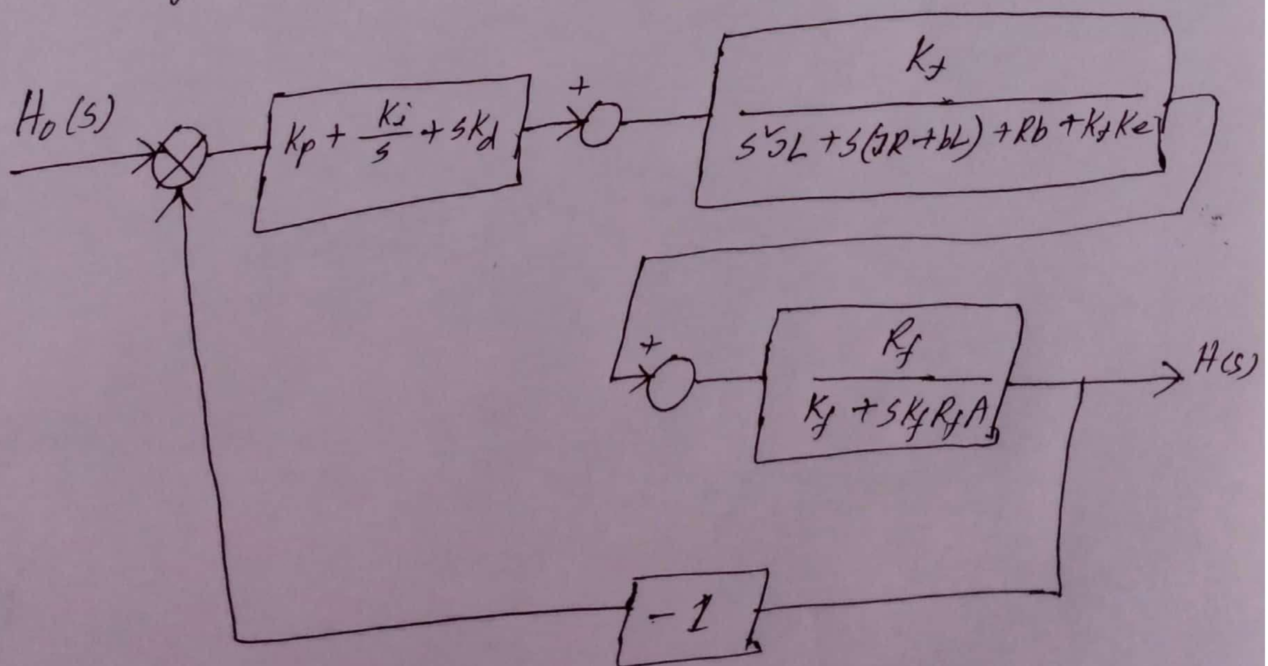
I have chosen "Water level Control System".
This system is chosen as it has practical applications in many industries such as agriculture, water treatment plants, flood control, irrigating system and so on. The system to be designed is a closed-loop control system for an automated water level control in a storage tank. That's why I chose a PID controlled "Water Level Control System" that can provide precise and stable control of water.

(ii)

The block model of my system:



Now, putting the value of PID controller, motor pump and STH.



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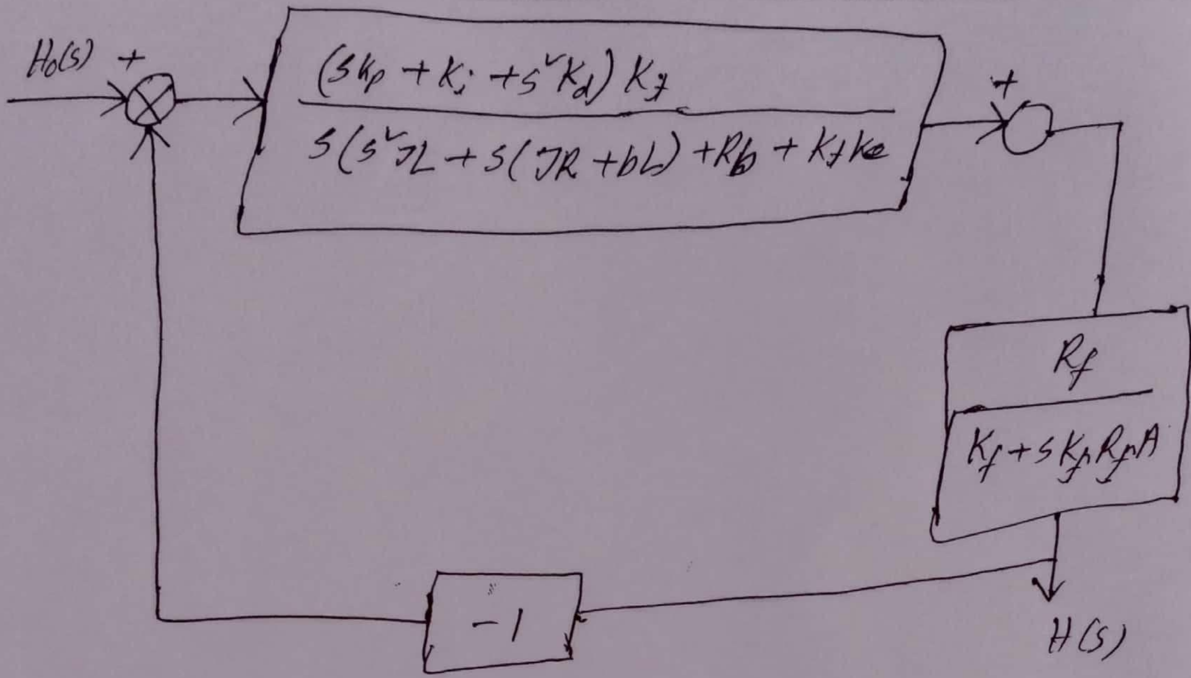
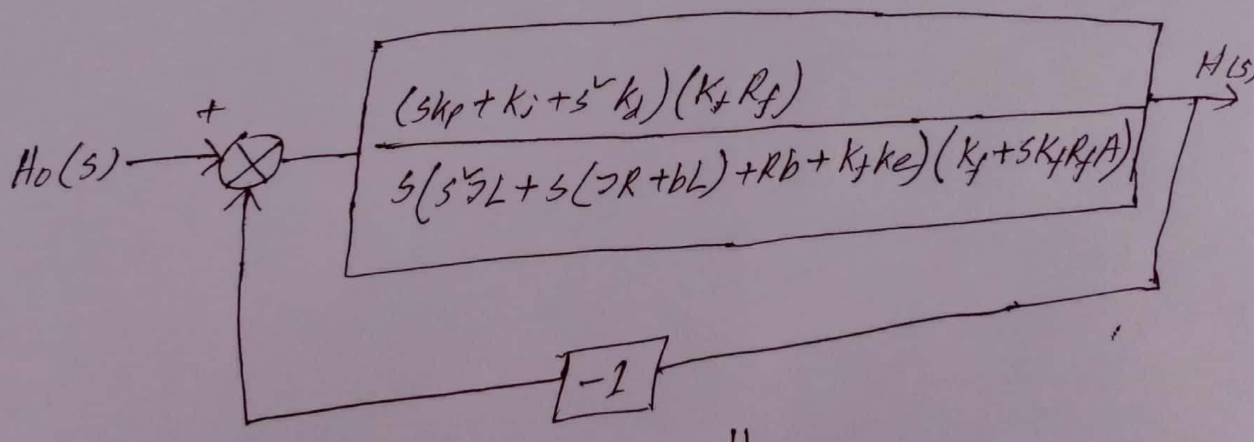
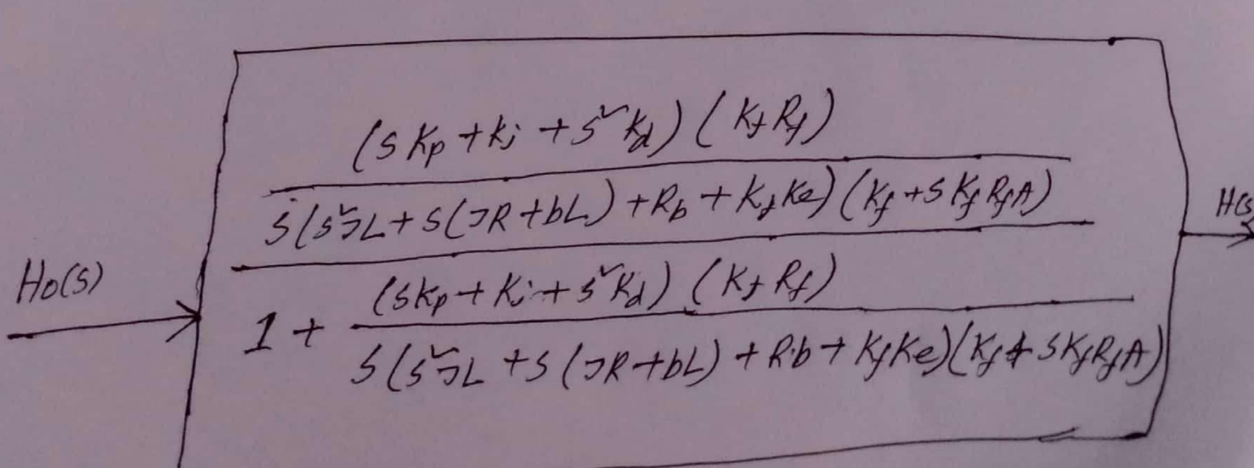

Ho(s)
+
(sKp + Ki + s²Kd) Kf
s(s²JL + s(JR + bL) + Rb + KfKe
+
Rf
Kf + sKfRfA
-1
H(s)


+
Ho(s)
(sKp + Ki + s²Kd)(KfRf)
s(s²JL + s(JR + bL) + Rb + KfKe)(Kf + sKfRfA)
H(s)
-1

$\Downarrow$


Ho(s)
(sKp + Ki + s²Kd)(KfRf)
s(s²JL + s(JR + bL) + Rb + KfKe)(Kf + sKfRfA)
1 + (sKp + Ki + s²Kd)(KfRf)
s(s²JL + s(JR + bL) + Rb + KfKe)(Kf + sKfRfA)
H(s)

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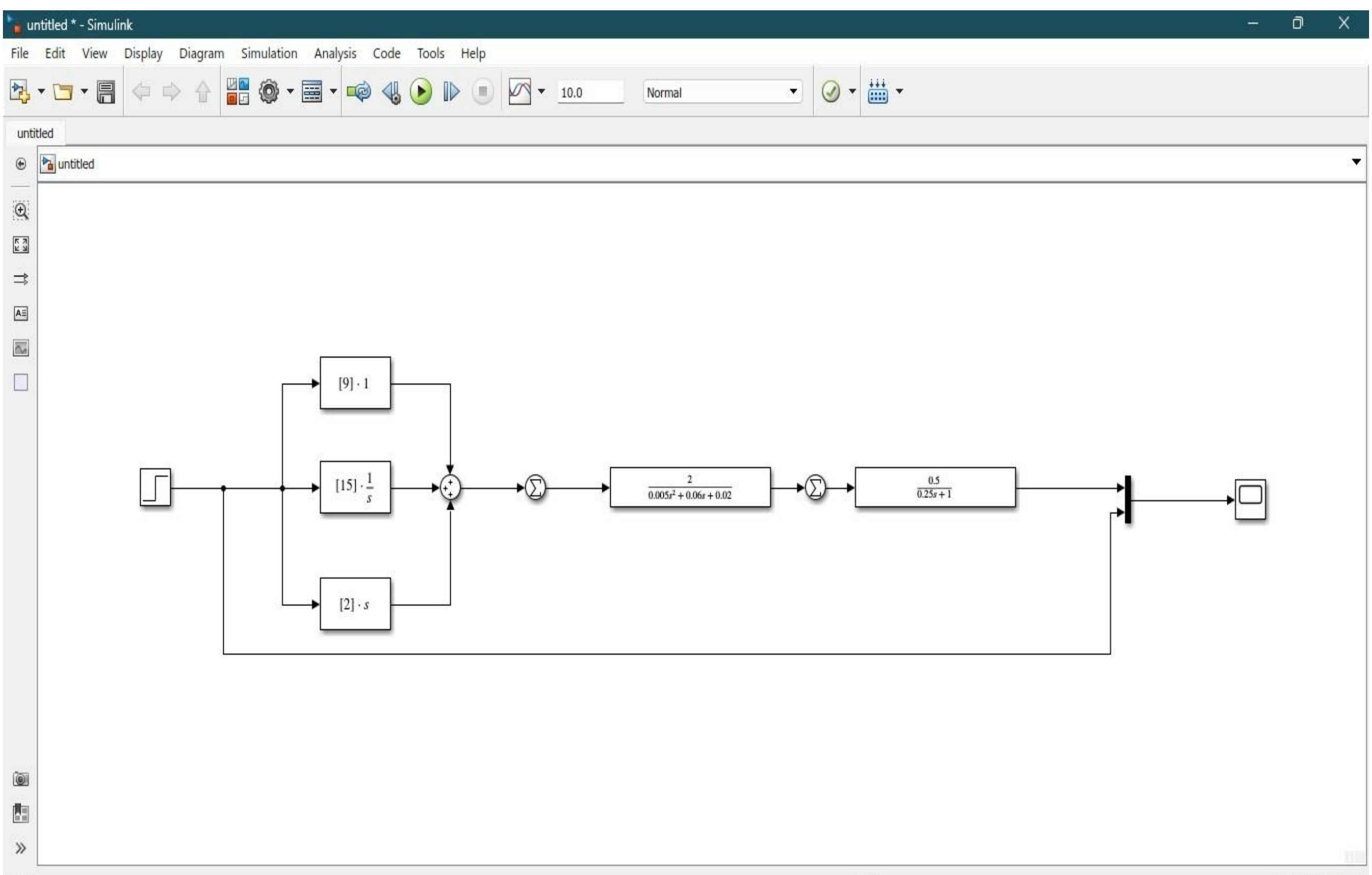
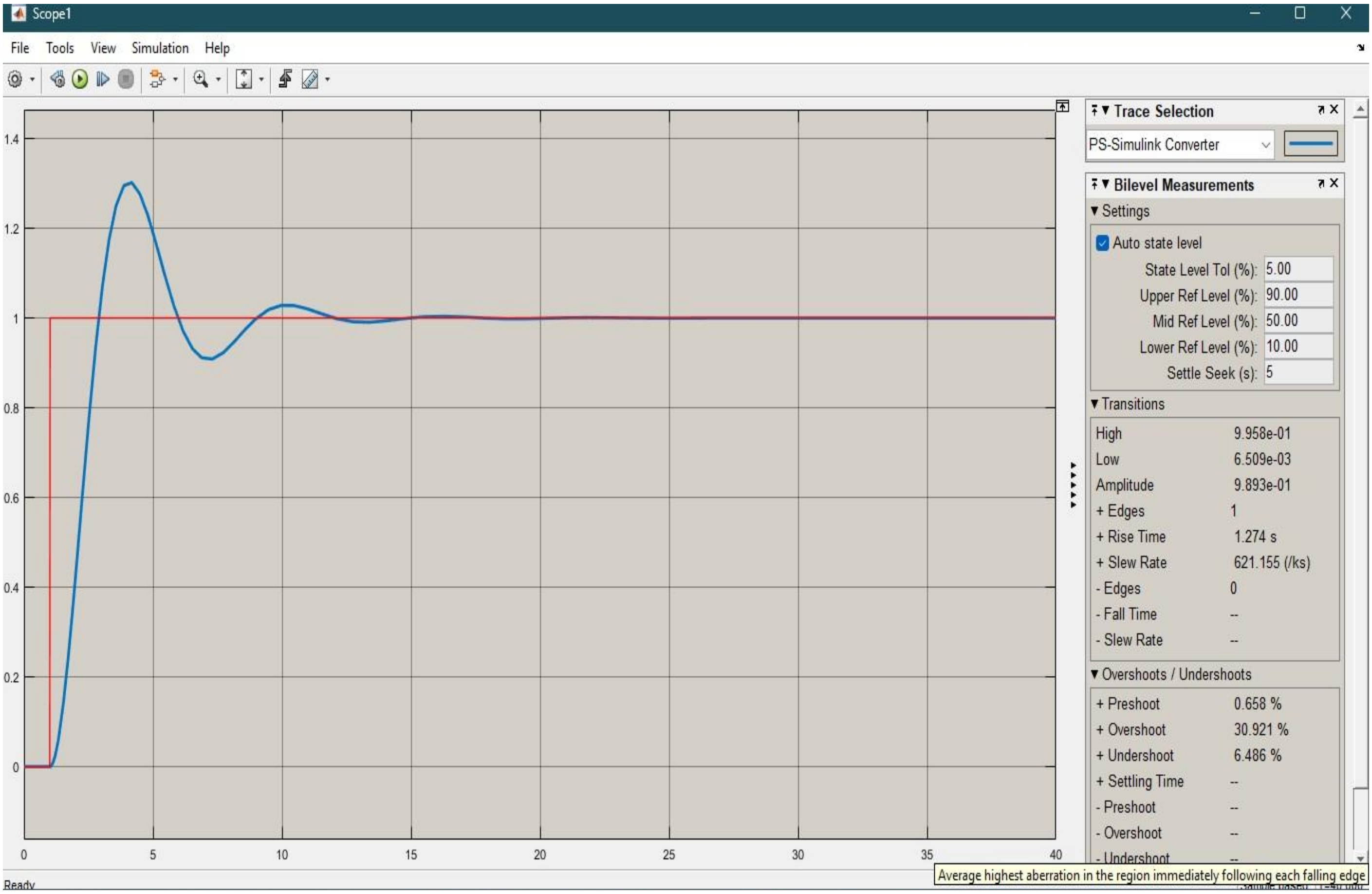
$$H_0(s) \longrightarrow \boxed{\dfrac{s(K_p + K_i + s^2 K_d)(K_f R_f)}{s(s^2 JL + s(JR + bL) + Rb + K_f K_e)(K_d + sK_f R_f A) + (sK_p + K_i + s^2 K_d)(K_f R_f)}} \longrightarrow H(s)$$




Scope1
File Tools View Simulation Help
Trace Selection
PS-Simulink Converter
Bilevel Measurements
Settings
Auto state level
State Level Tol (%): 5.00
Upper Ref Level (%): 90.00
Mid Ref Level (%): 50.00
Lower Ref Level (%): 10.00
Settle Seek (s): 5
Transitions
High 9.958e-01
Low 6.509e-03
Amplitude 9.893e-01
+ Edges 1
+ Rise Time 1.274 s
+ Slew Rate 621.155 (/ks)
- Edges 0
- Fall Time --
- Slew Rate --
Overshoots / Undershoots
+ Preshoot 0.658 %
+ Overshoot 30.921 %
+ Undershoot 6.486 %
+ Settling Time --
- Preshoot --
- Overshoot --
- Undershoot --
Average highest aberration in the region immediately following each falling edge
1.4
1.2
1
0.8
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5
10
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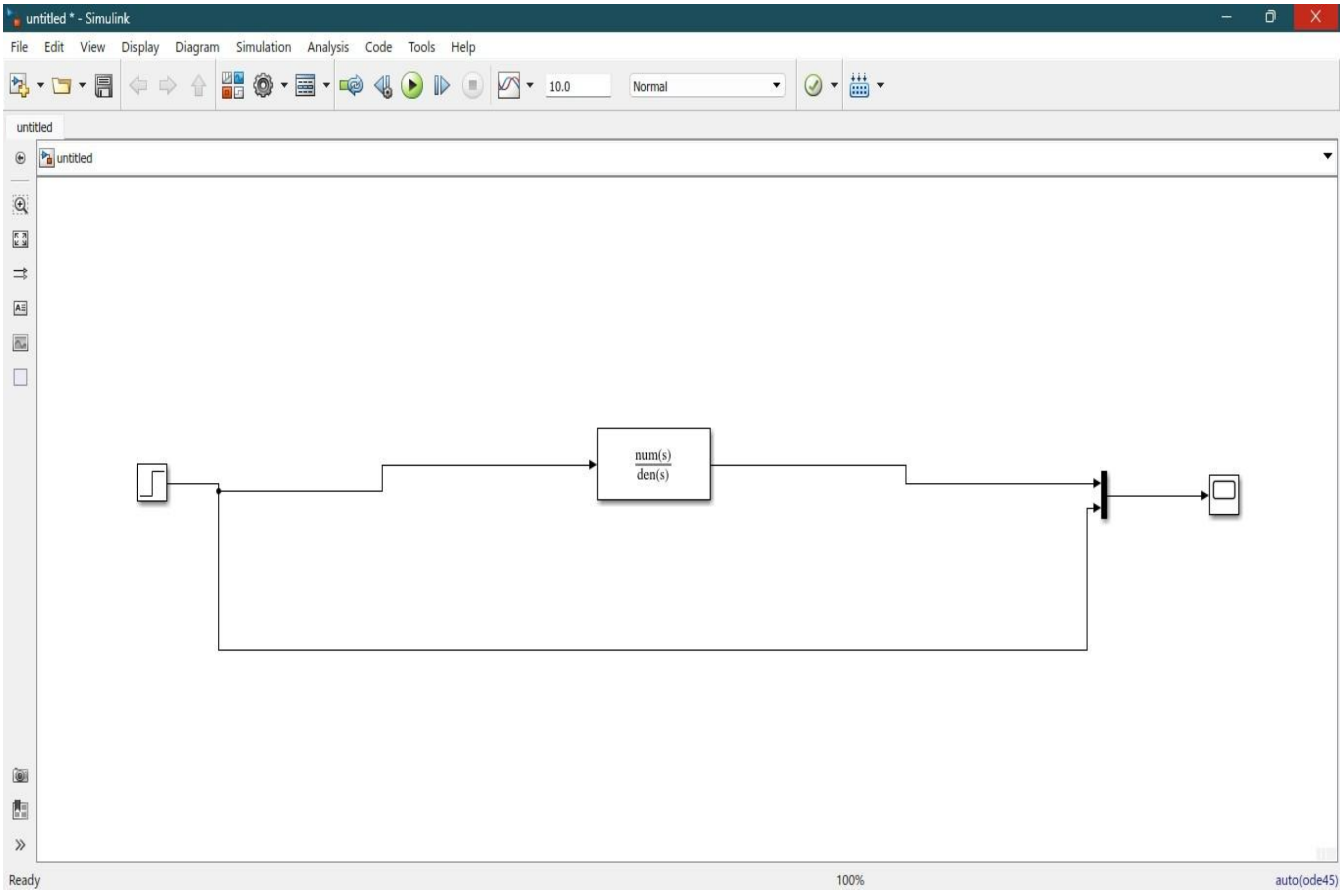
40
Ready


untitled * - Simulink
File Edit View Display Diagram Simulation Analysis Code Tools Help
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Ready
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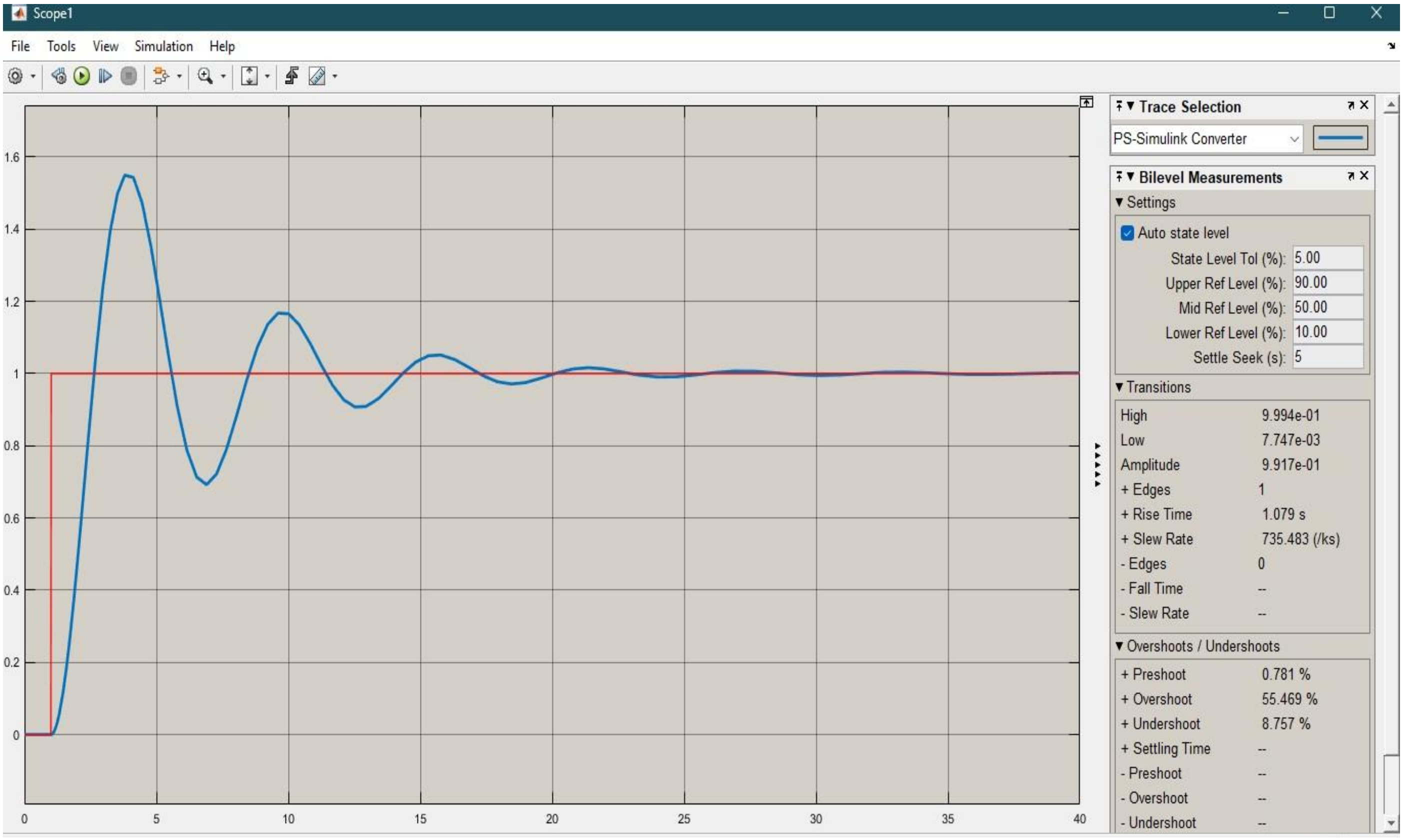


(iii)

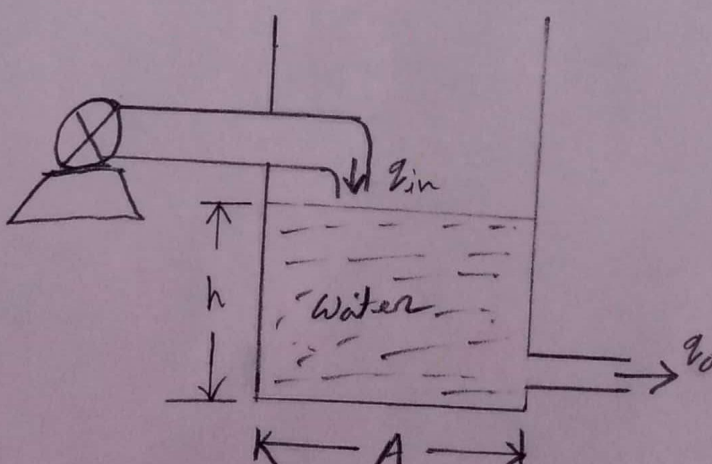


Fig: Single Tank container filled to level h

Here,

Water entering the container $= q_{in}$

Water leaving " " $= q_{out}$

Cross sectional Area $= A$

Water level $= h$

Now, using the balance of the flows into and out of the tank, the height 'h' is related to q_{in} and q_{out}

$$q_{in} - q_{out} = A \frac{dh}{dt} \quad \text{---(1)}$$

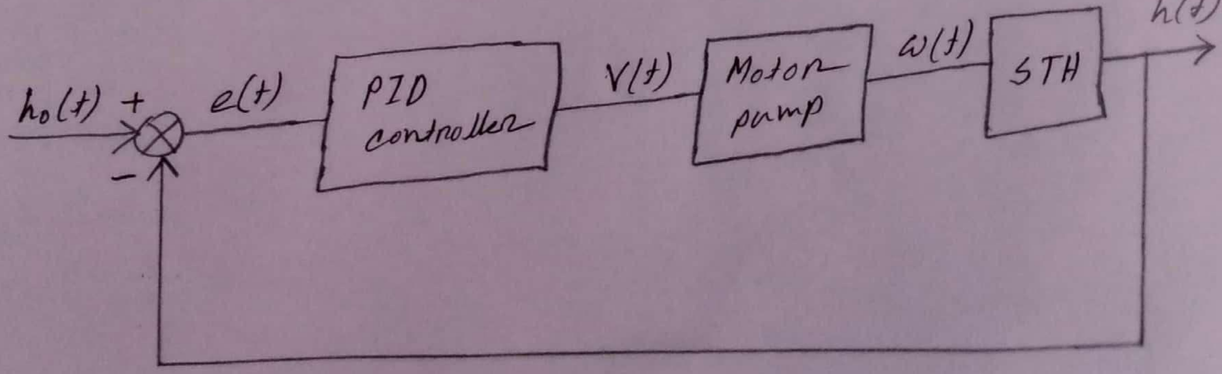
The out flow from the tank and the height can also be related assuming linear resistance to flow for simplicity of analysis and it gives as,

$$q_{out} = \frac{h}{R_f} \quad \text{---(2)}$$ where, R_f = Flow resistance

We assume a simple linear relationship between the speed and the incoming flow rate to the given as,

$$\omega(t) = k_f\, q_{in}(t) \quad \text{---(3)}$$

The block diagram of water controlling system,



Here, STH means Speed to Height transformation.

The DC motor equations relating the motor speed to the physical parameter of the motor can be written as,

$$J\ddot{\theta} + b\dot{\theta} = k_t i \quad \text{---} (4)$$

$$\Rightarrow L\frac{di}{dt} + Ri = e = V - k_e\dot{\theta} \quad \text{---} (5)$$

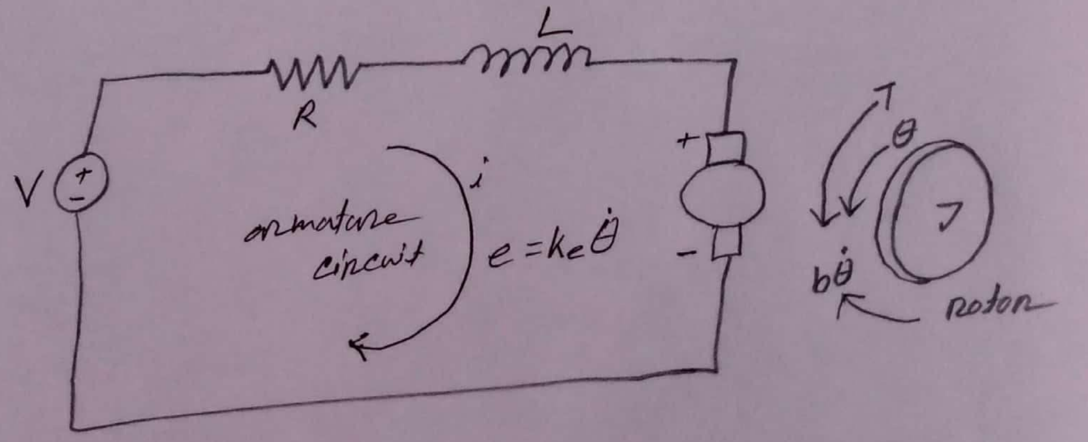
Here,

k_e = electromotive force constant

k_t = motor torque constant.

$$\dot{\theta} = \frac{d\theta}{dt} = \omega$$

Equivalent circuit of the DC motor,



Using Laplace transformation,

$$s^2 J\theta(s) + sb\theta(s) = k_t I(s)$$

$$\Rightarrow sLI(s) + RI(s) = V(s) - sk_e\theta(s) \quad \text{---} (6)$$

Now, the motor rotational speed output $\omega(s)$ is related to the armature voltage input, $V(s)$,

$$P_m(s) = \frac{\omega(s)}{V(s)} = \frac{k_t}{s^2 JL + s(JR+bL) + Rb + k_t k_e} \quad ---(7)$$

In general design of the PID, the error $e(t)$ is related to the output $V(t)$,

$$V(t) = K_p e(t) + k_i \int e(t) + k_d \frac{de(t)}{dt} \quad ---(8)$$

Using Laplace domain the input-output relation for the controller is,

$$C(s) = \frac{V(s)}{e(s)} = K_p + \frac{K_i}{s} + s K_d \quad ---(9)$$

Combining eqn (1), (2) and (3), $w(t)$ is related to $h(t)$,

$$R_f w(t) - K_f h = K_f R_f A \frac{dh}{dt} \quad ---(10)$$

Eqn (10) in Laplace domain, we have.

$$R_f w(s) - K_f h(s) = K_f R_f A s h(s) \quad ---(11)$$

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So, the transfer function is,

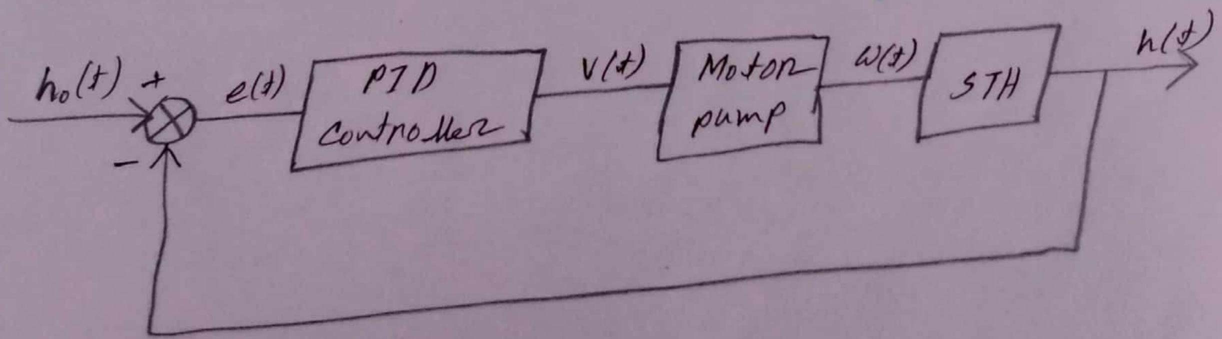
$$G_2(s) = \frac{h(s)}{\omega(s)} = \frac{R_f}{K_f + sK_f R_f A} \quad \text{---}(12)$$

The overall transfer function of the forward path,

$$F(s) = \frac{h(s)}{e(s)} = C(s)\, P_m(s)\, G_2(s)$$

$$= \left(K_p + \frac{K_i}{s} + sK_d\right)\left(\frac{K_t}{s^2 JL + s(JR+bL) + Rb + K_t K_e}\right) \times \left(\frac{R_f}{K_f + sK_f R_f A}\right) \quad \text{---}(13)$$

The block diagram of water controlling system:



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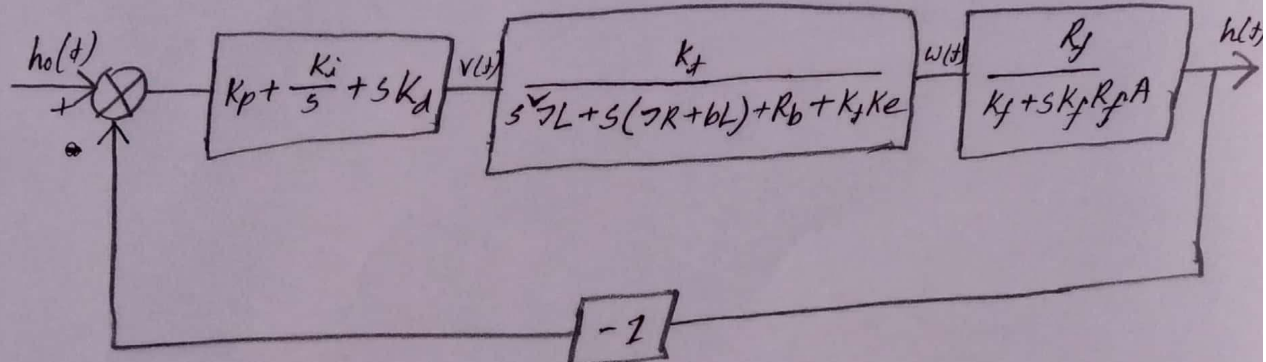
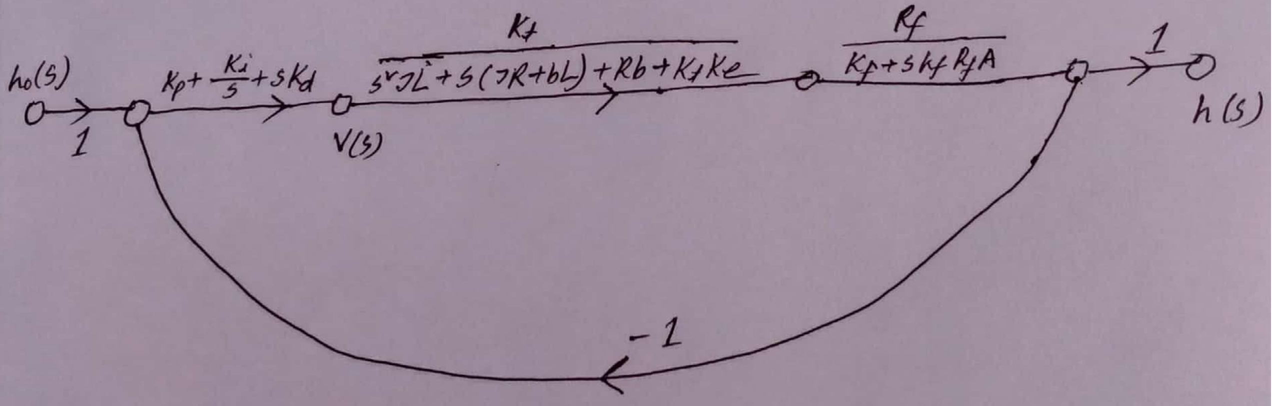
After putting the value of PID controller, Motor pump, and STH. Then the block diagram will be



So the signal flow graph (SFG) of the system will be,



Now,

$$P_1 = \left(K_p + \frac{K_i}{s} + sK_d\right)\left(\frac{K_t}{s^2JL + s(JR+bL) + Rb + K_tK_e}\right)\left(\frac{R_f}{K_f + sK_fR_fA}\right)$$

$$= \left(\frac{sK_p + K_i + s^2K_d}{s}\right)\left(\frac{K_t R_f}{(s^2JL + s(JR+bL) + Rb + K_tK_e)(K_f + sK_fR_fA)}\right)$$

$$= \frac{(sK_p + K_i + s^2K_d)(K_tR_f)}{s(s^2JL + s(JR+bL) + Rb + K_tK_e)(K_f + sK_fR_fA)}$$

$$\Delta = 1 + \frac{(sK_p + K_i + s^2 K_d)(K_f R_f)}{s(s^2 JL + s(JR + bL) + Rb + K_t K_e)(K_f + sK_f R_f A)}$$

$$= \frac{s(s^2 JL + s(JR + bL) + Rb + K_t K_e)(K_f + sK_f R_f A) + (sK_p + K_i + s^2 K_d)(K_f R_f)}{s(s^2 JL + s(JR + bL) + Rb + K_t K_e)(K_f + sK_f R_f A)}$$

$$\Delta_1 = 1$$

Now the transfer function will be,

$$T.F = \frac{h(s)}{h_0(s)} = \frac{P_1 \Delta_1}{\Delta}$$

$$= \frac{(sk_p + k_i + s^2 k_d)(K_t R_f)}{s(s^2 JL + s(JR + bL) + Rb + K_t K_e)(K_d + sK_f R_f A) + (sk_p + k_i + s^2 k_d) \cdot (K_f R_f)} \quad \text{---} (14)$$

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(iv)

let,

$J = 0.01 \ kg\,m^2$ | $R = 1\ \Omega$

$b = 0.1\ N{\cdot}m{\cdot}s$ | $L = 0.5\,H$

$k_t = 0.1\ N{\cdot}m/A$ | $A = 0.5\,m^2$

$K_e = 0.01\ V/rad/s$ | $R_f = 0.5\ s/m^2$

$k_f = 1$

$k_p = 9$

$K_i = 15$

$K_d = 2$

From eq^n (14),

$$T.F = \frac{H(s)}{H_o(s)} = \frac{0.1s^2 + 0.45s + 0.75}{0.00125s^4 + 0.02s^3 + 0.0015s^2 + 0.101s}$$

$$= \frac{0.1s^{-2} + 0.45s^{-3} + 0.75s^{-4}}{0.00125 + 0.02s^{-1} + 0.0015s^{-2} + 0.101s^{-3}}$$

[Multiply by s^{-4}]

$$\frac{H(s)}{H_o(s)} = \frac{80s^{-2} + 360s^{-3} + 600s^{-4}}{1 + 16s^{-1} + 1.2s^{-2} + 80.8s^{-3}}$$

[Multiply by 800]

$$\Rightarrow H(s) = 80s^{-2}H_o(s) + 360s^{-3}H_o(s) + 600s^{-4}H_o(s) - 16s^{-1}H(s) - 1.2s^{-2}H(s) + 80.8s^{-3}H(s)$$

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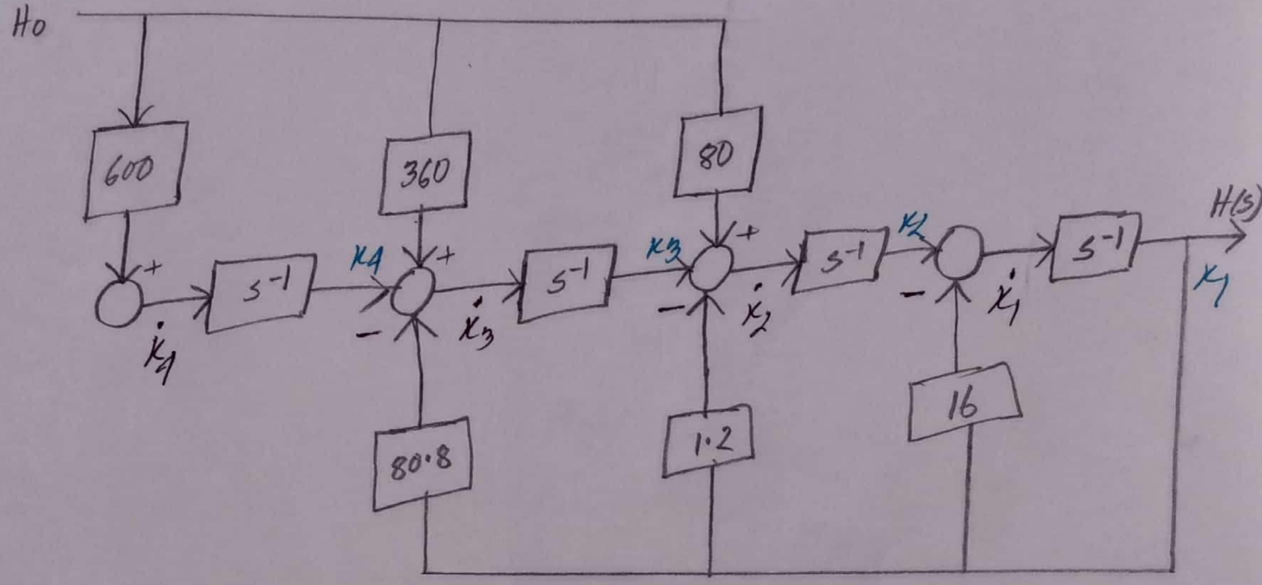
Now,



$$\dot{x}_1 = -16x_1 + x_2$$

$$\dot{x}_2 = -1.2x_1 + x_3 + 80H_o$$

$$\dot{x}_3 = -80.8x_1 + x_4 + 360H_o$$

$$\dot{x}_4 = 600H_o$$

Now,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -16 & 1 & 0 & 0 \\ -1.2 & 0 & 1 & 0 \\ -80.8 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 80 \\ 360 \\ 600 \end{bmatrix} H_o(t)$$

$$Y = x_1$$

$$\therefore y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

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(V)

From eqn (13),

$$F(s) = \left(K_p + \frac{K_i}{s} + sK_d\right)\left(\frac{K_t}{s^2JL + s(JR + bL) + Rb + K_tK_e}\right) \times \left(\frac{R_f}{K_f + sK_fR_fA}\right)$$

Now, assumed the parameter values for the motor the tank and the STH system:

$J = 0.01\ kg\,m^2$	$K_e = 0.01\ V/rad/s$
$b = 0.1\ Nms$	$R = 1\ \Omega$
$K_t = 2\ N.m/A$	$L = 0.5\ H$
$A = 0.5\ m^2$	$R_f = 0.5\ s/m^2$

$$\therefore F(s) = \left(K_p + \frac{K_i}{s} + sK_d\right)\left(\frac{2}{0.005s^2 + 0.06s + 0.02}\right)\left(\frac{0.05}{1 + 0.25s}\right)$$

$$= \underbrace{\left(K_p + \frac{K_i}{s} + sK_d\right)}_{\text{PID controller}}\left(\frac{1}{0.00125s^3 + 0.02s^2 + 0.0015s + 0.02}\right)$$

Initially consider an uncompensated system,

So, $$G(s)H(s) = \frac{K}{0.00125s^3 + 0.02s^2 + 0.0015s + 0.02} \quad \text{---} (15)$$

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(V) Continue

Now,

$$G(s)H(s) = \frac{K}{0.00125s^3 + 0.02s^2 + 0.0015s + 0.02}$$

Now, the characteristics eqn,

$$1 + G(s)H(s) = 0$$

$$\Rightarrow 1 + \frac{K}{0.00125s^3 + 0.02s^2 + 0.0015s + 0.02} = 0$$

$$\Rightarrow 0.00125s^3 + 0.02s^2 + 0.0015s + 0.02 + K = 0$$

Now, Routhian Array,

s^3	0.00125	0.0015
s^2	0.02	$0.02+K$
s^1	$\frac{3\times10^{-5} - 2.5\times10^{-5} - 0.00125K}{0.02}$	0
s^0	$0.02+K$	0

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Here,

$$\frac{3\times10^{-5} - 2.5\times10^{-3} - 0.00125K}{0.02} = 0$$

$$\Rightarrow 3\times10^{-5} - 2.5\times10^{-3} - 0.00125K = 0$$

$$\therefore K = -1.976$$

Here, all the elements in the first column are always positive if the value of $K < -1.976$ positive, negative and zero can be depending upon the value of K.

if $K < -1.976$ the all the terms in the first column will be positive and since there are no sign changes, then the system will be stable.

if $k > -1.976$ then s' term in the first column is negative. There are two sign changes, indicating that the system has two right half plane poles.

Which makes the system unstable.

if $K = -1.976$ the entire row is zero. So,

s^3	0.00125	0.0015
s^2	0.02	$0.02 - 1.976$
s^1	$\cancel{0} = 0.04$	0
s^0	$0.02 - 1.976$	

$\rightarrow 0.02s^2 - 1.956 = a(s)$

$\frac{d(s)}{ds} = 0.04$

Now,

$$0.02s^2 - 1.956 = 0$$

$$\Rightarrow 0.02s^2 = 1.956$$

$$\therefore s = \pm 9.89j$$

Here, roots are not repeated and placed on the imaginary axis. So the system is marginal stable.

(vi)

From eq^n (15),

$$G(s)\,H(s) = \frac{K}{0.00125s^3 + 0.02s^2 + 0.0015s + 0.02}$$

$$= \frac{K}{(s+15.99)(s+6.23 \pm 1.0j)}$$

Here,

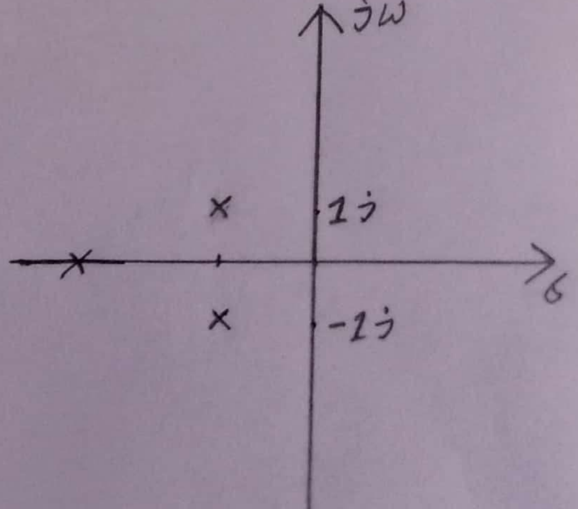
3 open-loop poles at,

$$s = -15.99$$
$$s = -6.23 + j1$$
$$s = -6.23 - j1$$

and, no open-loop zeros.

So, No of poles, $n_p = 3$

No of zeros, $n_z = 0$



Now the characteristics equation,

$$1 + G(s)H(s) = 0$$

$$\Rightarrow 1 + \frac{K}{0.00125s^3 + 0.02s^2 + 0.0015s + 0.02} = 0$$

$$\Rightarrow 0.00125s^3 + 0.02s^2 + 0.0015s + 0.02 + K = 0 \quad \text{---} \quad (16)$$

$$\Rightarrow K = -(0.00125s^3 + 0.02s^2 + 0.0015s + 0.02)$$

$$\therefore \frac{dK}{ds} = 0$$

$$\Rightarrow \frac{d}{ds}\left\{-(0.00125s^3 + 0.02s^2 + 0.0015s + 0.02)\right\} = 0$$

$$\Rightarrow -0.00375s^2 - 0.04s - 0.0015 = 0$$

$$\Rightarrow 0.00375s^2 + 0.04s + 0.0015 = 0$$

$$\therefore s = -0.0376, -10.629$$

Centre of asymptotes,

$$\sigma_A = \frac{\Sigma p - \Sigma z}{n_p - n_z} = \frac{-15.99 - 6.23 + j1 - 6.23 - j1.0}{3 - 0}$$

$$= -9.48$$

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Angle of asymptotes,

$$\varphi_A = \frac{2q+1}{n_p - n_Z} \times 180^\circ$$

$q = 0$ to $(n_p - n_Z - 1)$
$= 0$ to $(3-0-1)$
$= 0, 1, 2$

$$\therefore \varphi_0 = \frac{2\times 0 + 1}{3-0} \times 180^\circ = 60^\circ$$

$$\varphi_1 = \frac{2\times 1 + 1}{3-0} \times 180^\circ = 180^\circ$$

$$\varphi_2 = \frac{2\times 2 + 1}{3-0} \times 180^\circ = 300^\circ$$

Now, from eqn (16),

$$0.00125 s^3 + 0.02 s^2 + 0.0015 s + 0.02 + K = 0$$

Routhian Array,

s^3	0.00125	0.0015
s^2	0.02	$(0.02+K)$
s^1	$\frac{3\times 10^{-5} - 2.5\times 10^{-3} - 0.00125K}{0.02}$	0
s^0	$0.02+K$	0

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Here,

$$\frac{3\times10^{-5} - 2.5\times10^{-3} - 0.00125K}{0.02} = 0$$

$\therefore K = 4.166$

Here,

$$0.02s^2 + 0.02 + K = 0$$

$$\Rightarrow 0.02s^2 + 0.02 + 4.166 = 0$$

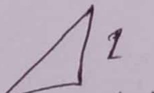
$\therefore s = \pm j14.47$

Angle of departure at the complex pole,

$$\sum \angle Z - \sum \angle P = -180^\circ$$

$$\Rightarrow 0 - (\theta_d + 90^\circ + 5.85) = -180^\circ$$

$\therefore \theta_d = 84.15^\circ$



$\tan\theta = \frac{1}{9.76}$

$\theta = 5.85$

Gain at, operating point,

$$\left| \frac{K}{(s+15.99)(s+6.23\pm 1.0j)} \right|_{s=4.85+6.18j} = 1$$

$\therefore K = 510.37$

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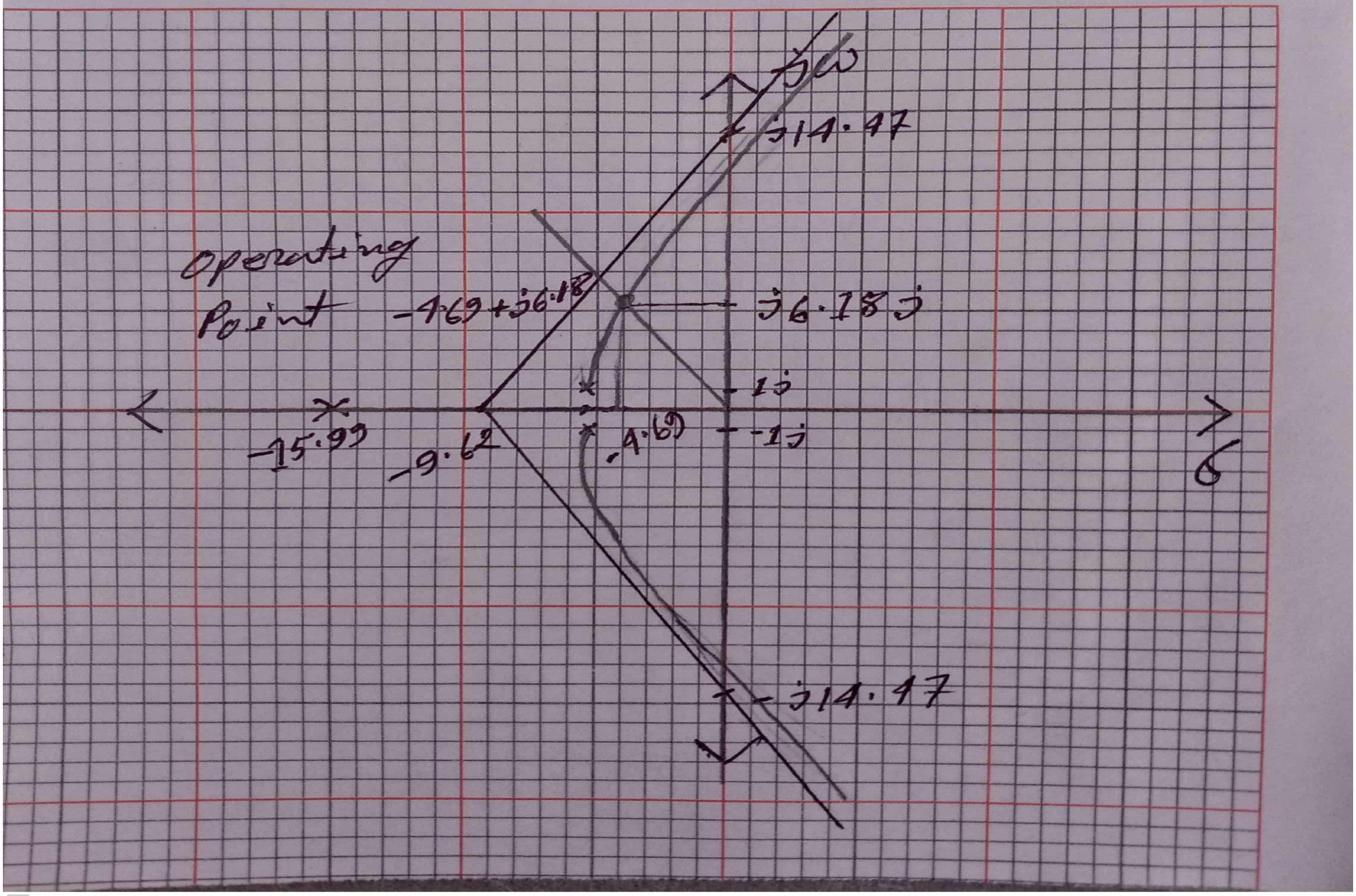

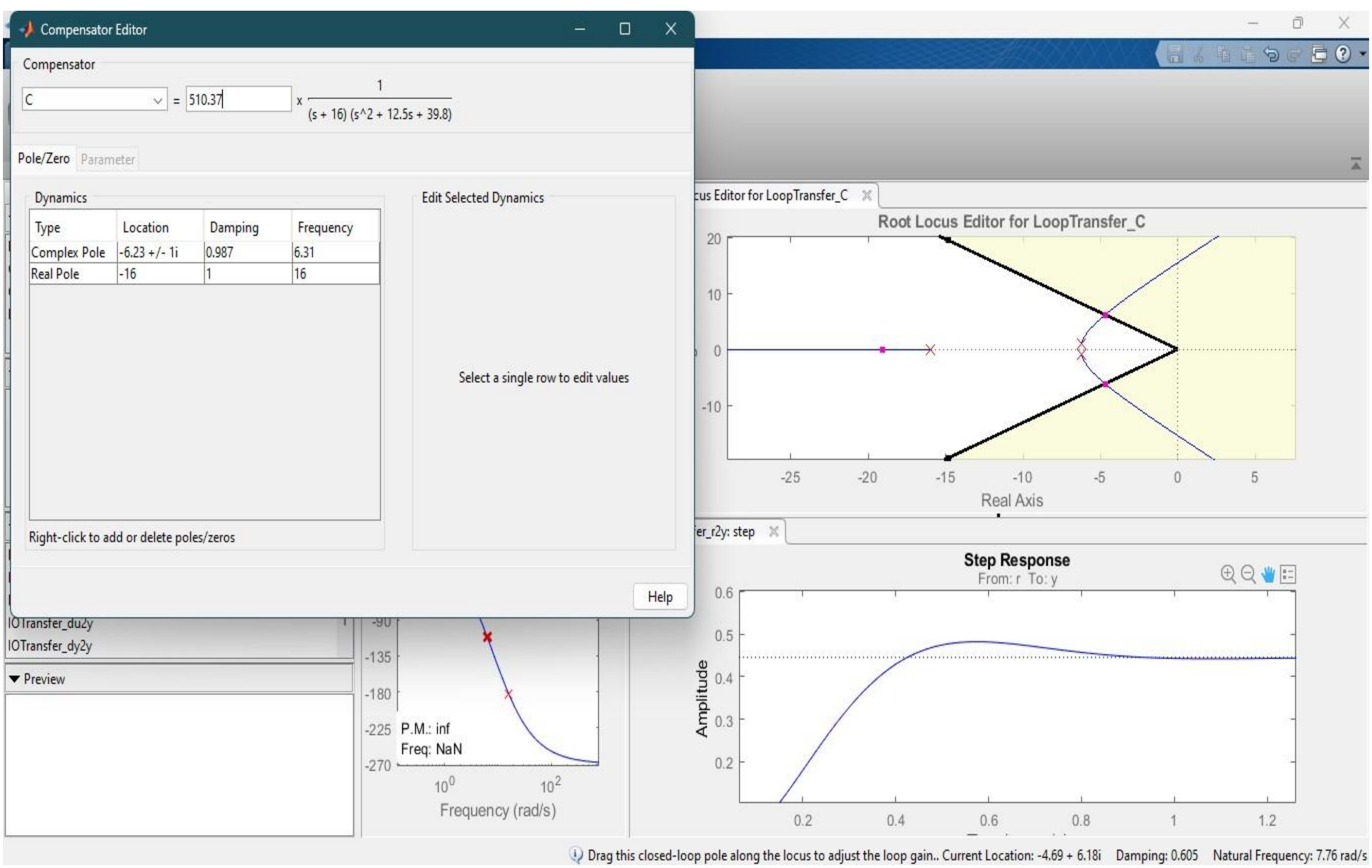

(Vii)

For PID compensator, first of all I will design a PD compensator,

From the graph,

$$\omega_n \zeta = 4.69$$

$$\Rightarrow \omega_n = \frac{4.69}{0.607} \quad \left[\begin{matrix}\text{As assumed} \\ \zeta = 0.607\end{matrix}\right]$$

$$= 7.72$$

Now,

$$T_s = \frac{4}{\zeta \omega_n}$$

$$= \frac{4}{0.607 \times 7.72}$$

$$= 0.853 \text{ sec}$$

$$K_p = \lim_{s \to 0} K G(s) H(s)$$

$$= \lim_{s \to 0} \frac{K}{(s+15.99)(s+6.23 \pm 7j)}$$

$$= \frac{510.37}{15.99 \times (6.23 \pm j)}$$

$$= 4.995$$

$$\therefore E_{ss} = \frac{1}{1+K_p}$$

$$= \frac{1}{1+4.995}$$

$$= 0.167$$

So the error for the uncompensated system is 0.167

Now, we know,

$$\text{Peak time, } T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$= \frac{\pi}{7.72\sqrt{1-(0.67)^2}}$$

$$= 0.512$$

The imaginary part of the compensated domiant

$$\omega_d = \frac{\pi}{T_p}$$

$$= \frac{\pi}{\frac{1}{4}\times 0.512}$$

$$= 24.54.$$

[Assume, peak time is $\frac{1}{4}$ times of the uncompensated system

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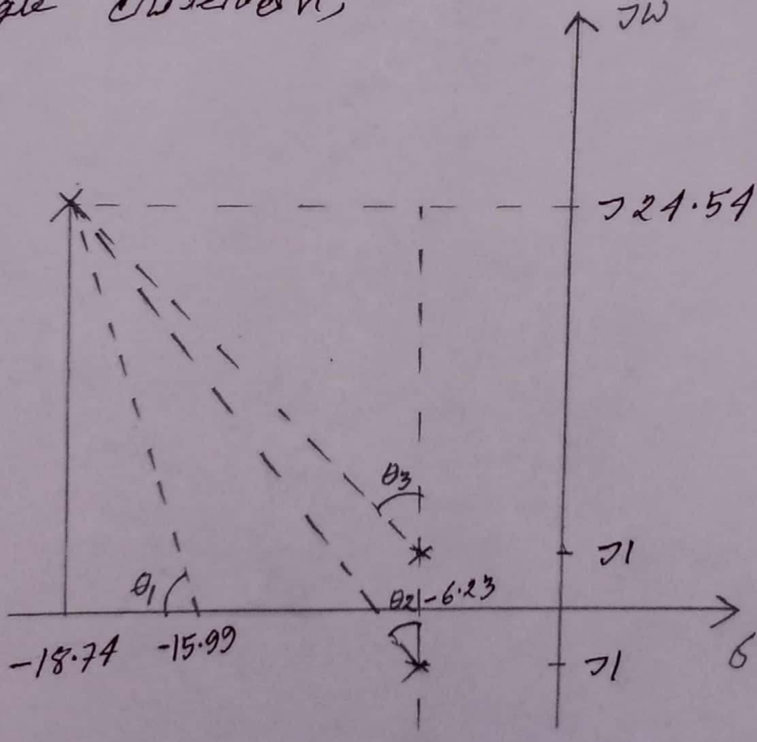
The real part of compensated dominant pole is,

$$\sigma = \frac{\omega_d}{\tan(127.37^\circ)}$$

$$= \frac{24.54}{\tan(127.37^\circ)}$$

$$= -18.74$$

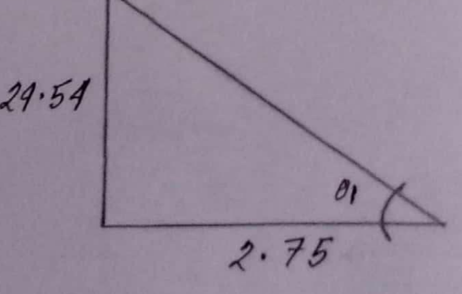
Now, Angle criterion,



For θ_1:

$$\tan\theta_1 = \frac{24.54}{2.75}$$

$$\therefore \theta_1 = 83.6^\circ$$



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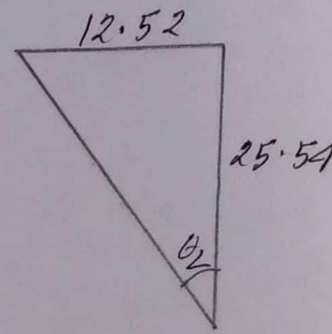
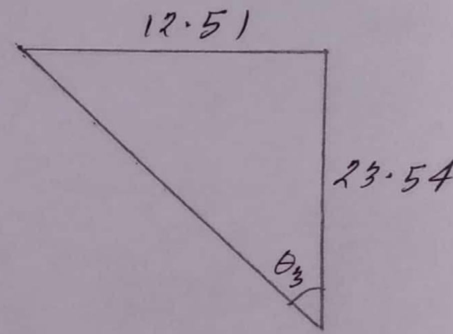


For θ_2:

$$\tan\theta_2 = \frac{12.52}{25.54}$$

$$\theta = 26.11^\circ$$



For θ_3:

$$\tan\theta_3 = \frac{12.52}{23.54}$$

$$\therefore \theta_3 = 28^\circ$$

Now, Applying the angle criterion,

$$\sum \angle Z - \sum \angle P = -180^\circ$$

$$\Rightarrow \varphi - (83.6 + 26.11 + 28) = -180^\circ$$

$$\therefore \varphi = 42.29^\circ$$

Assume that, the compensator zero is located at $-Z_c$ as shown in figure,

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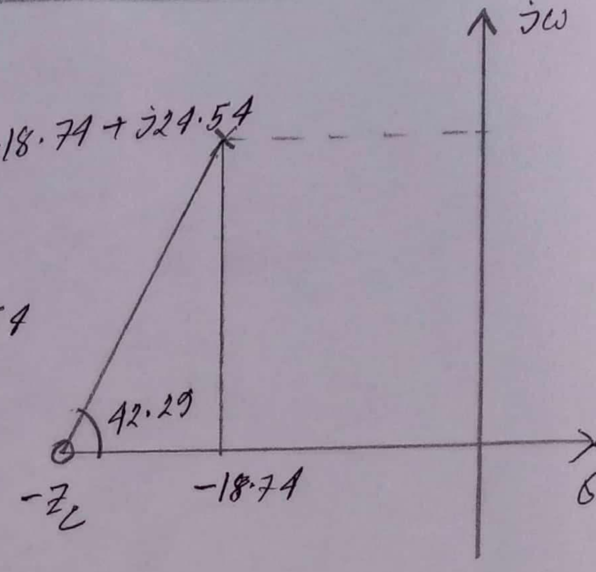
From the figure,

$$\tan(42.29) = \frac{24.54}{Z_c - 18.74}$$

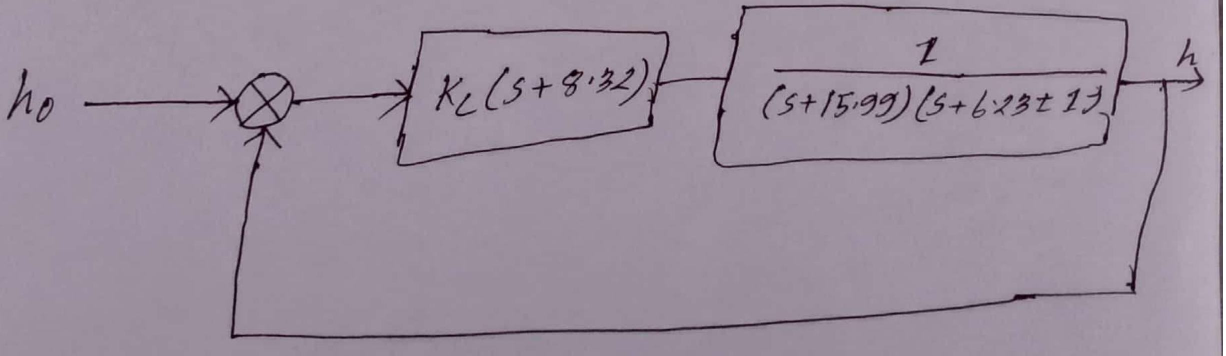
$$\Rightarrow 0.9 Z_c - (18.74 \times 0.9) = 24.54$$

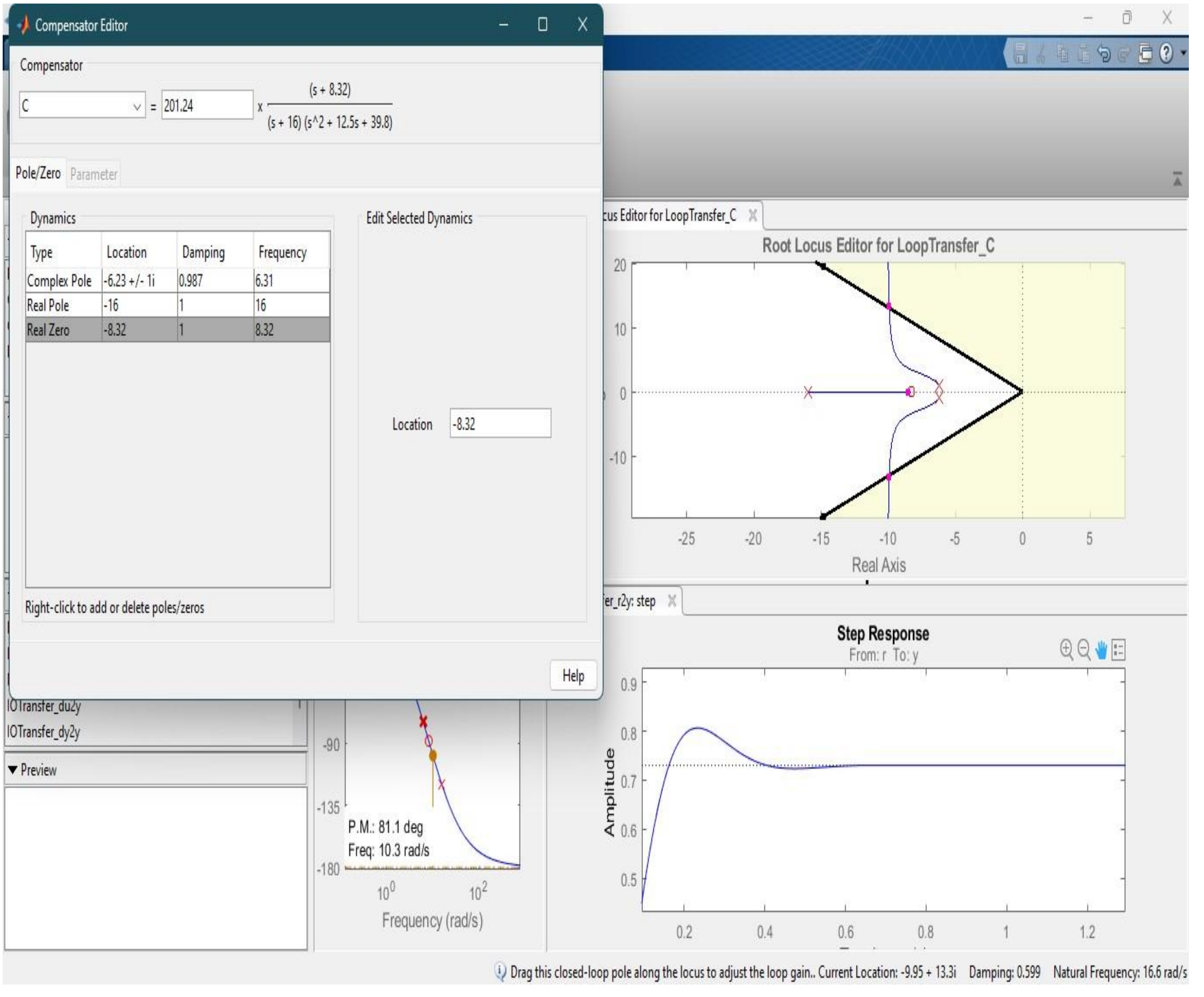
$$Z_c = 8.32$$



So, the PD controller is $G_{PD}(s) = (s + 8.32)$

For PD compensator system diagram is -





Here,

Dominant Pole: - 9.95+13.3j

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From the siso tools, we have,

dominant poles,

$$s = -9.95 + 13.3j$$

Now,

$$|K(G(s)H(s))| = 1$$

$$\Rightarrow \left| \frac{K_c(s+8.74)}{(s+15.99)(s+6.23+j)} \right| = 1$$

$$\Rightarrow K_c = \left| \frac{(s+15.99)(s+6.23+j)}{(s+8.74)} \right|_{s=-9.95+13.3j}$$

$$= 201.25$$

Here,

$$\omega_n \zeta = 9.95$$

$$\omega_n = \frac{9.95}{0.607}$$

$$= 16.39$$

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

$$= \frac{\pi}{16.39\sqrt{1-(0.607)^2}} = 0.241$$

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$$T_s = \frac{4}{\zeta \omega_n} = \frac{4}{0.607 \times 16.39}$$

$$= 0.402$$

$$\therefore K_p = \lim_{s \to 0} K_c G_c(s) H(s)$$

$$= \lim_{s \to 0} \frac{K_c (s+8.74)}{(s+15.99)(s+6.23+j)}$$

$$= \frac{201.25 \times 8.74}{15.99 \times 6.23}$$

$$= 17.65$$

$$E_{ss} = \frac{1}{1+K_p}$$

$$= \frac{1}{1+17.65}$$

$$= 0.05$$

Here, we can see that, after using PD compensator we have steady state error 0.05. Where as for un compensator system was 0.167

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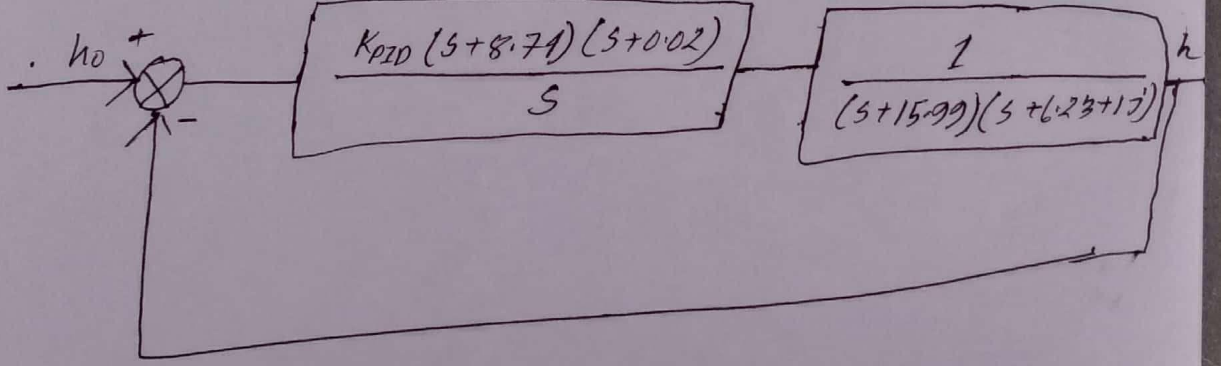
After design PD controller, now design a PID Controller that reduce the steady state error to Zero for a steap input. So we placed zero close to the origin.

$$G_{PI}(s) = \frac{s+0.02}{s}$$

So the PID controller system is,



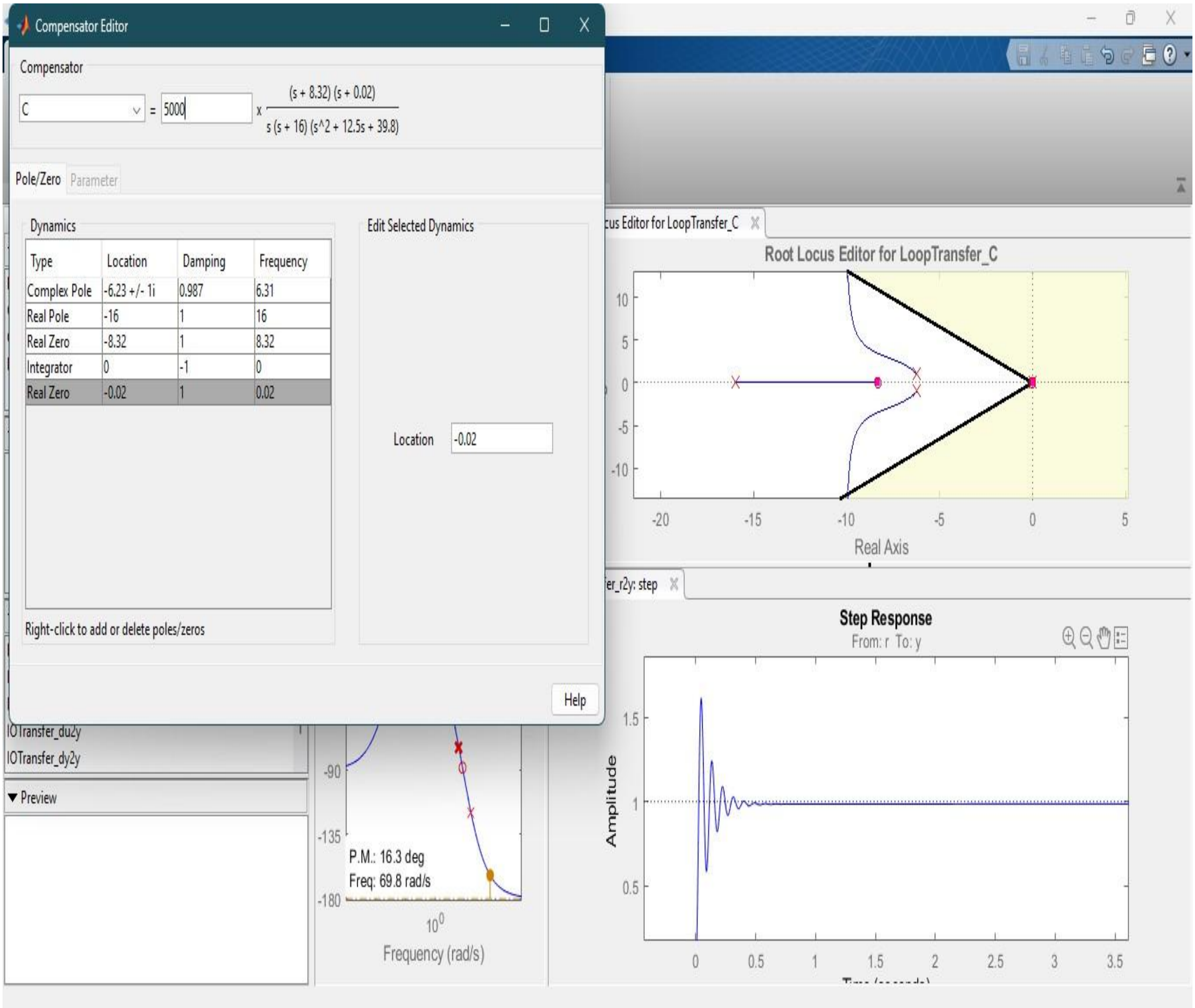
$$K_v = \lim_{s\to 0} K_{PID}$$

$$= \lim_{s\to 0} \frac{K_{PID}(s+8.74)(s+0.02)}{s(s+15.99)(s+6.23+j)}$$

$$= \alpha$$

$$\therefore E_{ss} = \frac{1}{K_V} = 0$$

So, For using PID compensator, the system become error free.



Compensator Editor
Compensator
C = 5000 x (s + 8.32) (s + 0.02) / s (s + 16) (s^2 + 12.5s + 39.8)
Pole/Zero
Parameter
Dynamics
Type | Location | Damping | Frequency
Complex Pole | -6.23 +/- 1i | 0.987 | 6.31
Real Pole | -16 | 1 | 16
Real Zero | -8.32 | 1 | 8.32
Integrator | 0 | -1 | 0
Real Zero | -0.02 | 1 | 0.02
Right-click to add or delete poles/zeros
Edit Selected Dynamics
Location -0.02
Help
Root Locus Editor for LoopTransfer_C
Real Axis
Step Response
From: r To: y
Amplitude
IOTransfer_du2y
IOTransfer_dy2y
Preview
P.M.: 16.3 deg
Freq: 69.8 rad/s
Frequency (rad/s)

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(viii)

I have chosen 'water level control system' using PID controller. The compensator is justified based on its low environmental impact. The compensator is designed to minimize water waste by ensuring that the correct amount of water is supplied. The compensator is designed to minimize the energy consumption of the system, thereby reducing system's carbon footprint.

Overall a water level control system create a positive environmental impact by reducing water waste and maintain optimal water quality.