Governing equations (1-D steady radial conduction)

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

Integrating once

$$r\frac{dT}{dr} = C_1 \Rightarrow \frac{dT}{dr} = \frac{C_1}{r}$$

Integrate again

$$T(r) = C_1 \ln r + C_2$$

Applying Boundary Conditions

At $r = a$, $T = T_i$; At $r = b$; $T = T_o$

$$\begin{cases} T_i = C_1 \ln a + C_2 \\ T_o = C_1 \ln b + C_2 \end{cases}$$

$$\Rightarrow C_1 = \frac{T_o - T_i}{\ln(b/a)}$$

then $C_2 = T_i - C_1 \ln a$

Final expression for temp dist

$$T(r) = T_i + \frac{T_o - T_i}{\ln(b/a)} \ln\left(\frac{r}{a}\right) \quad \text{— ①}$$

Heat flux and heat rate per unit length

Radial heat (Fourier)

$$q_r(r) = -k\frac{dT}{dr} = -k\frac{C_1}{r} = -k\frac{T_o - T_i}{\ln(b/a)}\frac{1}{r} \quad \text{— ②}$$

Given values

$T_i = -1°C$ also $r = \frac{a+b}{2} = 0.175$

$T_o = 0°C$

$a = 0.15 m$

$b = 0.20 m$

$K = 3 W/m °C$

Subst in ①

we get $T(0.175) = \underline{\underline{-0.464°C}}$

Subst in ②

we get $q_r(0.175) = -59.6 W/m^2$

heat transfer per unit length

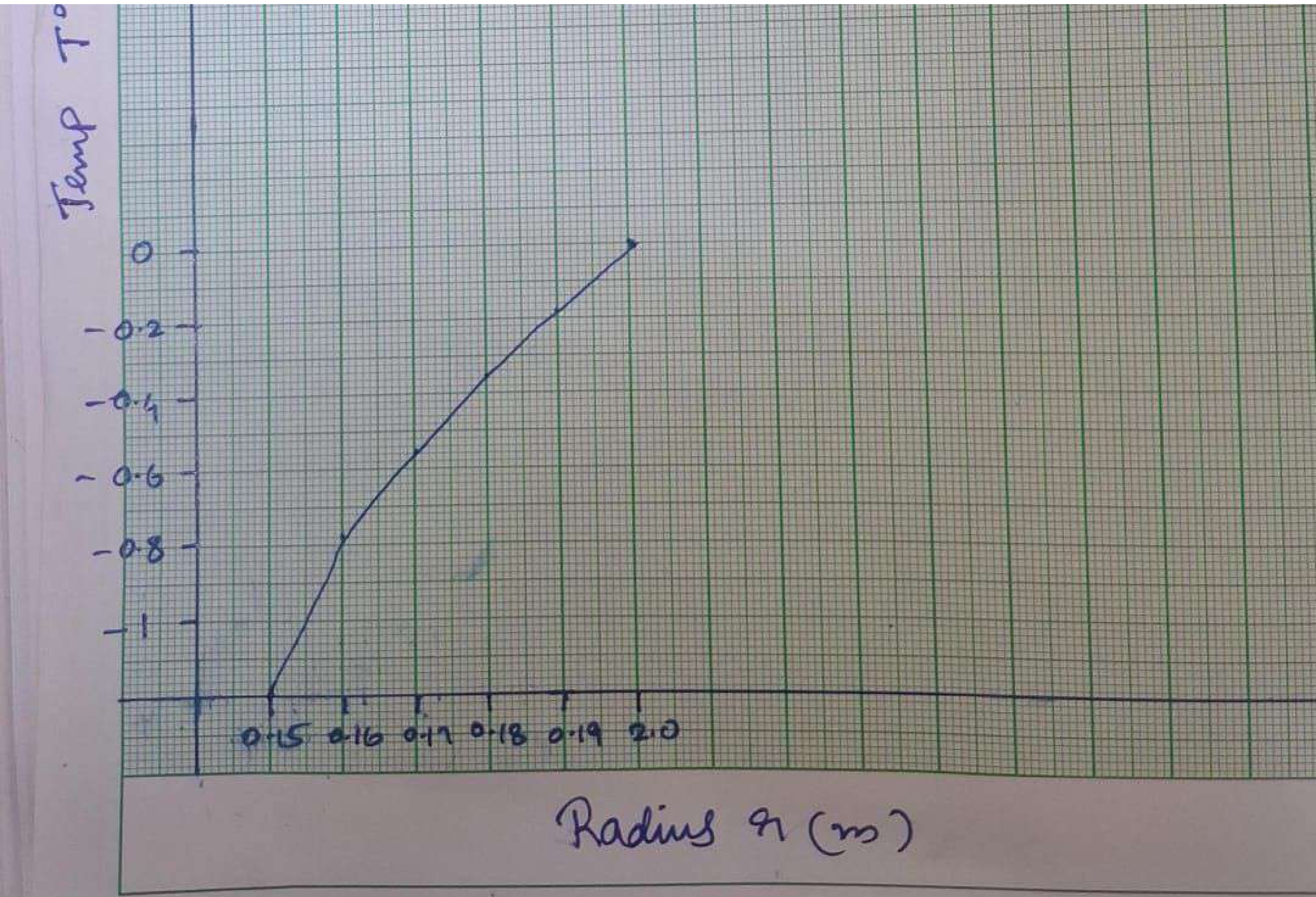
$$Q = \frac{2\pi k (T_o - T_i)}{\ln(b/a)} = \underline{\underline{65.52\ W/m}}$$

We know inner radius is 0.15m and outer is 0.20m.

Temp dist throughout the thickness

r	r/0.15	ln(r/0.15)	T(r) °C
0.15	1.00	0	−1
0.16	1.0667	0.0645	−0.78
0.17	1.1333	0.1252	−0.56
0.18	1.2000	0.1823	−0.37
0.19	1.2667	0.2364	−0.18
2.0	1.333	0.2877	0

Plotting r vs T(r)



Temp T°
0
-0.2
-0.4
-0.6
-0.8
-1
0.15
0.16
0.17
0.18
0.19
2.0
Radius r (m)