Asset pricing's assignement

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Some recent developments in the theory of competition in markets with adverse selection, Martin HELLWIG (1987)[1]

Introduction

This paper studies credit market with incomplete information. In this market, one agent (entrepreneur who need to finance his project) has a private information about the quality of the project and of his future ability to repay the loan. The other agents which are the banks don't have this information. The author aim to show how this asymmetric information affects the equilibrium in the market. In this type of markets, we usually use sorting devices to incite the informed agent to reveal his private information. For example, deductibles : amount of money paid by insurer to cover partially the cost of accident, the remaining cost will be covered by the insurance company; a high risk agent chooses lower deductible. Other example is Collateral which we will use in this paper. It is known in the literature that in this kind of markets, these sorting devises correct asymmetric information in the equilibrium (when it exists). Different variants of models were used to study this issue of asymmetric information. The problem is that each variant and each game-theoretic model gives different outcome, which means that the reached results are very sensitive to small variations in the setup of models. We will see that some model specifications will lead to non existence of equilibrium and by changing a detail, the result change and we get an equilibrium or multiple equilibrium. We present the general setup of models used in the paper and then introduce 3 different versions of the baseline model and see how they lead to different outcomes.

1 General setup : Bester's (1985b)

In this paper, we consider the Bertrand paradigm to set prices; which means that we give firms market power to set prices based on their marginal cost, which leads to the same prices as in competitive models so it is like firms are not using this market power.
Entrepreneur need money to finance his project. His initial wealth is A and the investment needed is I. So I-A is the money he needs. Depending on the type of the entrepreneur (i=1 or 2), we have different returns R_i and different probabilities of success p_i. Let assume $p_1 < p_2$ and $R_1 > R_2$ and $I < p_1 R_1 < p_2 R_2$. Let denote by C the collateral and r the interest. Loan contract is thus (L,r,C). Expected payoffs are given by

$$p_i U(w + R_i - r - (I - L)) + (1 - p_i) U(w - C - (I - L)) \tag{1}$$

and bank's payoff is

$$p_i r + (1 - p_i) C - L \tag{2}$$

WLOG we restrict analysis to contracts with full debt financing ie L=I so contracts are (I,r,C)=(r,C).

Proposition 1. *(complete information) Under complete information, all loans should be made under zero collateral.*

Proof. (part of the proof) If $C > 0$ then increasing r and decreasing C will be a profitable deviation because banks are risk neutral and entrepreneurs are not. □

Remark. *Under incomplete information, C may serve as a sorting device ie a signal about i's type, because type 2 is more likely to accept an increasing in C and decreasing in r.*

Now we will see the three versions of this general model. All the following games have sequential character. So we care about sequential equilibrium. In the sense of Kreps and Wilson, we apply Nash condition (best reply) to all the decisions nodes in the tree game (regardless of the fact that the node is reached or not by the equilibrium strategic game). In the sense of Rothschild and Stiglitz, sequential equilibrium is defined in a manner that allows informed agent to optimize his choice in the second stage of the game regardless of what have been offered at stage 1.

2 First version : Rothschild and Stiglitz (1976) and Wilson (1977)

We have a two stage game. At stage 1, the uninformed agents (banks) offers contracts (r,C) and at stage 2, the informed agent (entrepreneur), choose among offered contracts.

Proposition 2. *If λ the fraction of type 1 is close to zero, the two stage game of Rothschild, Stiglitz and Wilson fails to have an equilibrium in pure strategies.*

Lemma. *(Rothschild, Stiglitz and Wilson) The only candidate for a pure strategy sequential-equilibrium outcome is a separating equilibrium (a pair of contracts) (r_1^*, C_1^*) and (r_2^*, C_2^*) such that $r_i^* = I/p_1$, $C_1^* = 0$, $r_2^* = [I - (1-p_2)C_2^*]/p_2$ and $C_2^* > 0$ high enough so that type 1 do not prefer (r_1^*, C_1^*) to (r_2^*, C_2^*).*

Proof. This can be found by solving the principal's problem given (IR) and (IC) constraints. □

Proof. (of proposition 2) using the lemma, the only candidate is the pair of contracts (r_1^*, C_1^*) and (r_2^*, C_2^*). We can show that this cannot be an equilibrium when λ is small. This is the case because the pair of contracts is Pareto dominated by a pooling contract $(\tilde{r}, \tilde{C})$. Now we should prove that there is no pooling equilibrium. Let denote (r^{**}, C^{**}) the most preferred contract to type 2 among contracts that break even on $\overline{p}$. Thus (r^{**}, C^{**}) is the only candidate for equilibrium but we can find a profitable deviation for type 2 which is $(\hat{r}, \hat{C})$ such that $\hat{r} < r^{**}$ and $\hat{C} > C^{**}$. □

3 Second version : the three-stage game

In this game, we add a third stage where the uninformed agent (banks) may accept or reject the choice of the entrepreneur in the second stage.

Proposition 3. *The optimal contract for type 2 (r^{**}, C^{**}) is a sequential equilibrium of the three stage game.*

Proof. To show this, we will prove that the deviation $(\hat{r}, \hat{C})$ is not possible because it will be rejected by the bank at the third stage. To see this, let

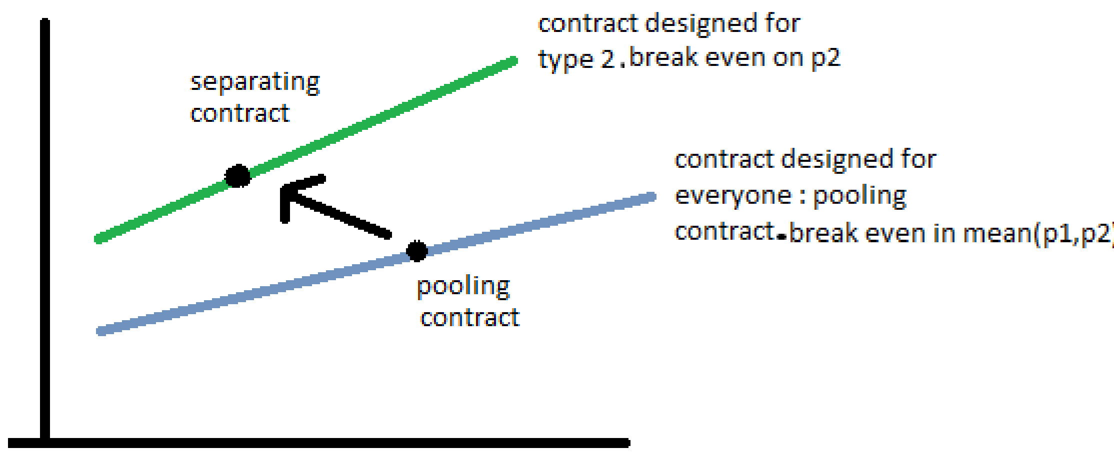

Figure 1: Illustration of the proof

proceed by absurd : suppose that $(\hat{r}, \hat{C})$ is accepted at third stage and find a contradiction. I will illustrate this in the graph. Let start from the pooling contract (r^{**}, C^{**}). The separating contract is a profitable deviation for type 2 so he will move to it;the pooling equilibrium break even in $\hat{p}$ but now we only have type 1 in the pooling contract which means profit is negative for the bank, thus it will be rejected at the third stage. Therefore, everyone will apply for $(\hat{r}, \hat{C})$, but again this contract is designed to be profitable for only type 2 and not both types, so profit is negative which means that this contract will be rejected by the bank at the third stage : contradiction. Conclusion : (r^{**}, C^{**}) is the sequential equilibrium of this the three stage game. □

Remark. *(r^{**}, C^{**}) is a Wilson equilibrium because the only profitable deviation $(\hat{r}, \hat{C})$ becomes unprofitable after removing (r^{**}, C^{**}).*

Remark. *The separating contract pair (r_1^*, C_1^*) and (r_2^*, C_2^*) and all pooling contract that Pareto dominates the separating contract are sequential equilibrium, by using the stability criterion (Kohlberg and Mertens(1986)) we can show that the pooling contract is the most robust equilibrium.*

4 Third version : Cho and Kreps (1986)

It is a three stage game. In the first step, the informed agent move first by announcing a signal which is the collateral C. In the second step, the uninformed agents (Banks) offer contracts given the signal. At the last step, the informed choose one bank.

Proposition 4. *The separating pair* (r_1^*, C_1^*) *and* (r_2^*, C_2^*) *is a sequential equilibrium.*

Proof. If type i deviates to C^{**}, the bank will know that it should be type 1 deviation because type 1 is the high risk type and he would like to be treated by a pooling contract. But then the bank will set an interest different then r^{**} to make the deviation unprofitable for type 1. This means that separating contract will remain as an equilibrium. □

Remark. *Again, we can show that any pooling contract that Pareto dominates the separating pair* (r_1^*, C_1^*) *and* (r_2^*, C_2^*) *is also sequential equilibrium. using the Kohlberg-Mertens criterion singles out the separating equilibrium because the pooling lack robustness.*

Conclusion

Different model specifications lead to very different outcomes. In the particular case of having large proportion of low risk type, there is no sequential equilibrium in pure strategies of the two stage game. But the addition of another stage led to big change : the game has always a sequential equilibrium. If the informed agent moves first, then the separating contract will be the equilibrium and if the uninformed agent moves first, a pooling contract will emerge. Working on the regulation in markets with adverse selection (credit or insurance markets) presents a big challenge because it is not easy to know which is the appropriate model since we cannot determine for instance who make the first move in the market.

References

[1] Martin HELLWIG. *Some recent developments in the theory of competition in markets with adverse selection.* European Economic Review, 1987.