STAT 111

Recitation 9

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Slides (adapted from Gemma Moran): `github.com/mohuangx/STAT111-Fall2018`

November 9, 2018

Hypothesis Tests for the Mean

- Same process as before but we need to be careful about our test-statistic.
- Let $X_1, \ldots, X_n$ be i.i.d random variables with (unknown) mean μ and (unknown) variance σ^2.
- Set $H_0 : \mu = \mu_0$ and H_1 for some number μ_0 and choose α.
- Determine test-statistic...
  - We know under the null, $\overline{X} \sim N(\mu_0, \frac{\sigma^2}{n})$ for large n.
  - But we don't know σ^2! Calculate s^2 (the sample variance).
  - Standardize:
$$T = \frac{\overline{X} - \mu_0}{s/\sqrt{n}}$$
  - It turns out T is not normally distributed so can not use z-chart! It is t-distributed with $n-1$ "degrees of freedom".

The t-distribution

- Similar to the normal distribution, but with heavier tails.
- This is to account for the uncertainty in not knowing σ^2.
- As n gets bigger, the estimate s^2 is better and the t-distribution gets closer to the normal distribution. When $n = \infty$, the t-distribution becomes the normal distribution.
- This is why we need the "degrees of freedom".

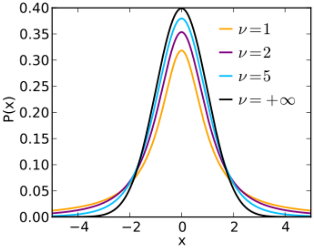


Figure: Credit: Wikipedia. ν is degrees of freedom.

Example

- A chocolate block is advertised as being 8 oz. Suppose we are concerned that the chocolate manufacturers are giving us less than 8 oz of chocolate in a block. We collect 15 blocks of chocolate and calculate:

$$\bar{x} = 7.88, \quad s^2 = 0.06.$$

Step 1 $H_0 : \mu = 8$ vs. $H_1 : \mu < 8$.

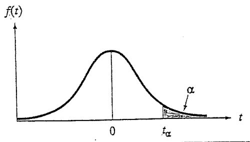
Step 2 Choose $\alpha = 0.05$.

Step 3 Test-statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{7.88 - 8}{\sqrt{0.06/15}} = -1.90$$

Step 4 Find the critical region.

Example



The numbers in the chart below give the critical values (= critical points) for a **one-sided up** t test. These values depend on the degrees of freedom (left-hand column) and the chosen value α (numerical value of the Type I Error). For example, if there are 24 degrees of freedom and α = 0.05, the critical value is 1.711.

Some manipulation is needed for **one-sided down** and **two-sided** tests.

Degrees of Freedom	α = 0.10	α = 0.050	α = 0.025	α = 0.010	α = 0.005	α = 0001	α = 0.00005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
	[illegible]	[illegible]	[illegible]	[illegible]	2.898	3.646	3.965

Here, the degrees of freedom is $n - 1 = 14$.
$P(t_{14} > 1.761) = 0.05$ so $P(t_{14} < -1.761) = 0.05$ is the critical region.

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Step 2 Choose $\alpha = 0.05$.

Step 3 Test-statistic is

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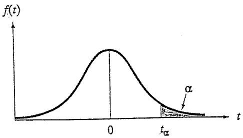
Step 4 Find the critical region $\Rightarrow t_{14} < -1.761$

Step 5 Since $t = -1.90$ is in the critical region, we reject H_0.

Example



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- For one-sided tests the critical region is: $t < -t_{n-1,\alpha}$ or $t > t_{n-1,\alpha}$ depending on the direction.
- For two-sided tests, the critical region is: $t < -t_{n-1,\alpha/2}$ and $t > t_{n-1,\alpha/2}$.