STAT 111

Recitation 10

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Slides (adapted from Gemma Moran): `github.com/mohuangx/STAT111-Fall2018`

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Two-sample t-test

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- Answer: Let $X_{11}, \ldots, X_{1m}$ represent the heights of the m students in 201 with mean μ_1 and let $X_{21}, \ldots, X_{2n}$ represent the heights of the n students in 202 with mean μ_2. We can test $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$.

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- This is an example of a two-sample t-test!
- Two-sample t-test: Let $X_{11}, \ldots, X_{1m}$ be i.i.d random variables with (unknown) mean μ_1 and (unknown) variance σ^2. Let $X_{21}, \ldots, X_{2n}$ be i.i.d random variables with (unknown) mean μ_2 and (unknown) variance σ^2. We want to test whether or not $\mu_1 = \mu_2$.

Example

- Suppose we want to test whether there is a difference in height between class 201 ($X_{11}, \ldots, X_{1m}$) and 202 ($X_{21}, \ldots, X_{2n}$). We observe that $\bar{x}_1 = 66.7$, $s_1^2 = 10.5$, $m = 28$, and $\bar{x}_2 = 65.6$, $s_2^2 = 12.3$, $n = 34$.

Step 1 $H_0 : \mu_1 = \mu_2$ vs. $H_1 : \mu_1 \neq \mu_2$. Two-sided test.

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$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{m} + \frac{1}{n}}}, \text{ where } s = \sqrt{\frac{(m-1)s_1^2 + (n-1)s_2^2}{m+n-2}}$$

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$$t = \frac{66.7 - 65.6}{3.390\sqrt{\frac{1}{28} + \frac{1}{34}}} = 1.254$$

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So we need to look at t_{60}. What is the critical region?

$t \geq t_{60,0.025} = 2.000$ and $t \leq -t_{60,0.025} = -2.000$.

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Step 3 Test-statistic is $t = 1.254$.

Step 4 Find the critical region: $t \geq 2.000$ and $t \leq -2.000$

Step 5 Do we reject H_0?

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Step 3 Test-statistic is $t = 1.254$.

Step 4 Find the critical region: $t \geq 2.000$ and $t \leq -2.000$

Step 5 Do we reject H_0? No, $t = 1.254$ is not in the critical region.