STAT 111

Recitation 2

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The Binomial Distribution: Questions

Q1: Let X be the number of heads if I toss an unbiased coin 3 times ($n = 3, \theta = 0.5$). Find the probability distribution of X in "tableau" form.

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$$P(X = 0) = \binom{3}{0}(0.5)^0(0.5)^3 = 0.125$$

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x	0	1	2	3
$P(X = x)$	0.125	0.375	0.375	0.125

Table: Probability distribution of X using the tableau method.

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n	i	*0.05*	*0.10*	*0.15*	*0.20*	*0.25*
18	0	0.3972	0.1501	0.0536	0.0180	0.0056
	1	0.3763	0.3002	0.1704	0.0811	0.0338
	2	0.1683	0.2835	0.2556	0.1723	0.0958
	3	0.0473	0.1680	0.2406	0.2297	0.1704
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A3:

$$\begin{aligned} P(X \leq 4) &= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) \\ &= 0.0536 + 0.1704 + 0.2556 + 0.2406 + 0.1592 \\ &= 0.8794 \end{aligned}$$

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A2: Finding 3 successes with $\theta = 0.8$ is the same as finding 9 failures with θ of failure 0.2. Hence, $P(X = 3) = 0.0001$.

θ

n	i	0.05	0.10	0.15	0.20	0.25	0.30	0.35
12	0	0.5404	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057
	1	0.3413	0.3766	0.3012	0.2062	0.1267	0.0712	0.0368
	2	0.0988	0.2301	0.2924	0.2835	0.2323	0.1678	0.1088
	3	0.0173	0.0852	0.1720	0.2362	0.2581	0.2397	0.1954
	4	0.0021	0.0213	0.0683	0.1329	0.1936	0.2311	0.2367
	5	0.0002	0.0038	0.0193	0.0532	0.1032	0.1585	0.2039
	6	0.0000	0.0005	0.0040	0.0155	0.0401	0.0792	0.1281
	7	0.0000	0.0000	0.0006	0.0033	0.0115	0.0291	0.0591
	8	0.0000	0.0000	0.0001	0.0005	0.0024	0.0078	0.0199
	9	0.0000	0.0000	0.0000	0.0001	0.0004	0.0015	0.0048
	10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0008
	11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
	12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Random Variables: Questions

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Q3: Let X be a binomial random variable with $n = 2$ and $\theta = 0.8$. Find the mean of X using the binomial table.

A3:

$$\begin{aligned}\mu &= 0 \times 0.04 + 1 \times 0.32 + 2 \times 0.64 \\ &= 1.6\end{aligned}$$