

# STAT 111

## Recitation 1

Mo Huang

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Office Hours: Wednesdays 3:00 - 4:00 pm, JMHH F96

Slides: [github.com/mohuangx/STAT111-Spring2019](https://github.com/mohuangx/STAT111-Spring2019)

September 7, 2018

# Logistics

- ▶ Section 207: 1:00 pm - 1:50 pm
- ▶ Section 208: 2:00 pm - 2:50 pm
  
- ▶ Every Friday, I will:
  1. Collect the homework for that week;
  2. Give you your graded homework; and
  3. Give you the homework for the week after (also posted on Canvas).
  
- ▶ If you do not collect your homework, it will be available in the STAT 111 box in the Statistics Department (4th Floor, JMHH).
  
- ▶ If you know in advance that you will not be able to attend a recitation, give your homework to Professor Ewens on Thursdays or put it in my mailbox (at entrance to the Statistics Department)
  
- ▶ Questions about course materials should be asked during office hours or in an email to Professor Ewens.

# Statistics and Probability

- ▶ **Probability theory** is *deductive* (or “top-down logic”). We start with a theory and then consider the implications resulting from that theory.
- ▶ **Statistics** is *inductive* (or “bottom-up logic”). We start with finite observations and then attempt to make objective statements about the world from a sample.
- ▶ Statistical methods use probability theory to draw conclusions from observed data.

## Statistics and Probability: Example

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- ▶ A new medicine is proposed (“new medicine”) which cures 1,643 people out of 2,000.

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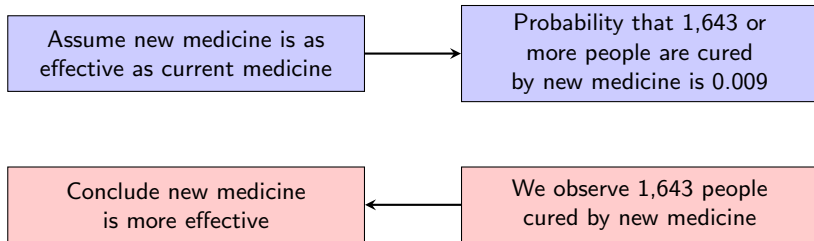
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Probability theory



Statistics

## Probability: Questions

Q1: A smoke alarm consists of two parts,  $A$  and  $B$ . If there is smoke, part  $A$  will detect it with probability 0.96 and part  $B$  will detect it with probability 0.98. The probability that they both detect it is 0.95. What is the probability that smoke will not be detected?

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A1: Let  $A$  be the event that part  $A$  detects smoke and let  $B$  be the event that part  $B$  detects smoke. What is the event that smoke is not detected?



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$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.96 + 0.98 - 0.95 \\ &= 0.99 \end{aligned}$$

$$\begin{aligned} P([A \cup B]^C) &= 1 - P(A \cup B) \\ &= 0.01 \end{aligned}$$

## Probability: Questions

Q2: A six-sided die is biased such that an odd number is twice as likely to occur as an even number. That is, the probability of an odd number is  $2/9$  and an even number is  $1/9$ . Let  $A$  be the event that an even number occurs. Let  $B$  be the event that a number greater than or equal to 4 occurs. Find  $P(A)$ ,  $P(B)$ ,  $P(A \cup B)$  and  $P(A \cap B)$ .

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$$\begin{aligned}P(A) &= P(2, 4 \text{ or } 6) \\&= 1/9 + 1/9 + 1/9 \\&= 3/9\end{aligned}$$

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$$\begin{aligned}P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\&= 3/9 + 4/9 - 2/9 \\&= 5/9\end{aligned}$$



## Probability: Questions

A2:  $P(A) = 3/9$ ,  $P(B) = 4/9$ ,  $P(A \cap B) = 2/9$ ,  $P(A \cup B) = 5/9$

Q3: Following Q2, find  $P(A|B)$ .

## Probability: Questions

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Q3: Following Q2, find  $P(A|B)$ .

A3:

$$\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ &= \frac{2/9}{4/9} \\ &= 1/2 \end{aligned}$$

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Q4: Are  $A$  and  $B$  independent?

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Q4: Are  $A$  and  $B$  independent?

A4: No, because  $P(A|B) \neq P(A)$ . (Recall  $P(A) = 1/3$ ).

## Probability: Questions

Q5: An unfair coin ( $P(H) = 0.4$ ) was flipped twice. Given that on at least one flip a head occurred, what is the probability that a head occurred on both flips?

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$$\begin{aligned} P(\text{at least one } H) &= P(HH) + P(HT) + P(TH) \\ &= (0.4)^2 + (0.4)(0.6) + (0.6)(0.4) \\ &= 0.64. \end{aligned}$$

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$$\begin{aligned}P(HH|\text{at least one } H) &= \frac{P(HH \cap \text{at least one } H)}{P(\text{at least one } H)} \\&= \frac{P(HH)}{P(\text{at least one } H)} \\&= \frac{(0.4)^2}{0.64} \\&= 0.25\end{aligned}$$