STAT 111

Recitation 7

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Slides: `github.com/mohuangx/STAT111-Spring2019`

March 22, 2019

Estimation Summary

- Binomial parameter θ:
  Estimate: p
  95% confidence interval: $p \pm \sqrt{1/n}$
- Mean μ:
  Estimate: $\bar{x}$
  95% confidence interval: $\bar{x} \pm 2\frac{s}{\sqrt{n}}$
- Difference between proportions $\theta_1 - \theta_2$:
  Estimate: $p_1 - p_2$
  95% confidence interval: $p_1 - p_2 \pm \sqrt{\frac{1}{n} + \frac{1}{m}}$
- Difference between means $\mu_1 - \mu_2$:
  Estimate: $\bar{x}_1 - \bar{x}_2$
  95% confidence interval: $\bar{x}_1 - \bar{x}_2 \pm 2\sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{m}}$

Note:

$$s = \sqrt{\frac{x_1^2 + x_2^2 + \cdots + x_n^2 - n(\bar{x}^2)}{n-1}}.$$

Linear Regression

- How does the growth height of a plant in a greenhouse depend on the amount of water that we give it?
- Let x be the amount of water we plan to give the plant (fixed).
- Let Y be the (future) growth height of a plant (random variable).
- Linear Regression Model: for the ith observation,
  - Mean of $Y_i = \alpha + \beta x_i$, Variance of $Y_i = \sigma^2$.

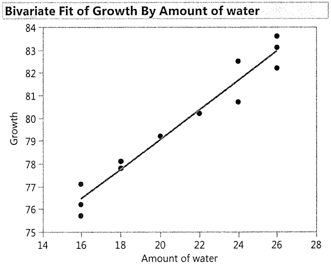

Figure: Data: $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$

Linear Regression

- Mean of $Y_i = \alpha + \beta x_i$, Variance of $Y_i = \sigma^2$. How do we estimate α, β?

$$\text{Calculate:} \quad s_{xx} = x_1^2 + x_2^2 + \cdots + x_n^2 - n(\overline{x}^2) = \sum_{i=1}^{n} x_i^2 - n(\overline{x}^2)$$

$$s_{yy} = y_1^2 + y_2^2 + \cdots + y_n^2 - n(\overline{y}^2) = \sum_{i=1}^{n} y_i^2 - n(\overline{y}^2)$$

$$s_{xy} = x_1 y_1 + x_2 y_2 + \cdots + x_n y_n - n(\overline{xy}) = \sum_{i=1}^{n} x_i y_i - n(\overline{xy}).$$

- Estimate β by b and α by a:

$$b = s_{xy}/s_{xx}$$
$$a = \overline{y} - b\overline{x}$$

- Estimate σ^2 by s_r^2:

$$s_r^2 = (s_{yy} - b^2 s_{xx})/(n-2).$$

- How accurate is the estimate b of β?

$$\text{Standard deviation of } b : \frac{s_r}{\sqrt{s_{xx}}} \quad \Rightarrow \quad 95\% \text{ C.I. } b \pm 2\frac{s_r}{\sqrt{s_{xx}}}$$

Example

Plant number	1	2	3	4	5	6	7	8	9	10	11	12
Amount of water	16	16	16	18	18	20	22	24	24	26	26	26
Growth height	76.2	77.1	75.7	78.1	77.8	79.2	80.2	82.5	80.7	83.1	82.2	83.6

- $\overline{x} = 21, \overline{y} = 79.7, s_{xx} = 188, s_{yy} = 83.54, s_{xy} = 122.4$

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- Estimate $\alpha,\ \beta$

$$b = s_{xy}/s_{xx} = 0.6511$$

$$a = \overline{y} - \beta\overline{x} = 79.7 - (0.6511)(21) = 66.03$$

- Estimate σ^2

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$$s_r^2 = \frac{83.54 - (0.6511)^2(188)}{10} = 0.3850$$

- Find a 95% C.I. for β.

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- Find a 95% C.I. for β.

$$95\% \text{ C.I. for } b : 0.6511 \pm 2\frac{\sqrt{0.3850}}{\sqrt{188}} \Rightarrow (0.56, 0.74)$$