

STAT 111

Recitation 3

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Random Variables: Questions

Q1: Let X be a random variable with the below distribution. Find the mean and the variance of X using formula (1) and then formula (2).

x	-3	-1	4	5
$P(X = x)$	0.1	0.3	0.4	0.2

Table: Probability distribution of X .

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A1: $\mu = -3 \times 0.1 - 1 \times 0.3 + 4 \times 0.4 + 5 \times 0.2 = 2.$

$$\begin{aligned}\sigma^2 &= (-3 - 2)^2(0.1) + (-1 - 2)^2(0.3) + (4 - 2)^2(0.4) + (5 - 2)^2(0.2) \\ &= 8.6.\end{aligned}$$

$$\begin{aligned}\sigma^2 &= (-3)^2(0.1) + (-1)^2(0.3) + 4^2(0.4) + 5^2(0.2) - 2^2 \\ &= 8.6.\end{aligned}$$

Random Variables: Questions

Q2: Let X be a random variable with the below distribution. Write the probability distribution of $Y = 2X$ in tableau form.

x	-3	-1	4	5
$P(X = x)$	0.1	0.3	0.4	0.2

Table: Probability distribution of X .

Random Variables: Questions

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x	-3	-1	4	5
$P(X = x)$	0.1	0.3	0.4	0.2

Table: Probability distribution of X .

A2: Probability distribution of Y :

y	-6	-2	8	10
$P(Y = y)$	0.1	0.3	0.4	0.2

Table: Probability distribution of Y .

Questions

- Q4: Suppose we have a company producing a medicine. Each day the *mean* amount of medicine produced is 500 mg and the *variance* is 900 mg^2 . Assume the amount produced each day is independent.
- (i) Let T_n be the total amount of medicine produced in a week. Find the mean and variance of T_n .

Questions

Q4: Suppose we have a company producing a medicine. Each day the *mean* amount of medicine produced is 500 mg and the *variance* is 900 mg². Assume the amount produced each day is independent.

- (i) Let T_n be the total amount of medicine produced in a week. Find the mean and variance of T_n .

$$\text{Mean}(T_n) = 5 \times 500 = 2500$$

$$\text{Var}(T_n) = 5 \times 900 = 4500$$

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- (ii) Let \bar{X} be the average amount of medicine produced in a 5-day week. Find the mean and variance of \bar{X} .

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$$\text{Mean}(T_n) = 5 \times 500 = 2500$$

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- (ii) Let \bar{X} be the average amount of medicine produced in a 5-day week. Find the mean and variance of \bar{X} .

$$\text{Mean}(\bar{X}) = 500$$

$$\text{Var}(\bar{X}) = 900/5 = 180$$

Questions

Q6: Suppose the company producing a medicine has different means and variances the amount produced on each day of the week:

Day	Mean	Variance
Monday (X_1)	450	1200
Tuesday (X_2)	550	800
Wednesday (X_3)	600	500
Thursday (X_4)	550	800
Friday (X_5)	350	1200

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Q6: Suppose the company producing a medicine has different means and variances the amount produced on each day of the week:

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Tuesday (X_2)	550	800
Wednesday (X_3)	600	500
Thursday (X_4)	550	800
Friday (X_5)	350	1200

Find mean and variance of both the sum T_n and the average \bar{X} .

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Find mean and variance of both the sum T_n and the average \bar{X} .

A6: X_1, X_2, X_3, X_4 , and X_5 are no longer *i.i.d.*!

$$\text{Mean}(T_n) = 450 + 550 + 600 + 550 + 350 = 2500$$

$$\text{Var}(T_n) = 1200 + 800 + 500 + 800 + 1200 = 4500$$

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$$\text{Mean}(T_n) = 450 + 550 + 600 + 550 + 350 = 2500$$

$$\text{Var}(T_n) = 1200 + 800 + 500 + 800 + 1200 = 4500$$

$$\text{Mean}(\bar{X}) = 1/n \times \text{Mean}(T_n) = 500$$

$$\text{Var}(\bar{X}) = 1/n^2 \times \text{Var}(T_n) = 4500/25 = 180$$

Questions

Q7: Let P_1 be the proportion of heads in 50 coin tosses, where $P(H) = 0.6$. Find $Mean(P_1)$ and $Var(P_1)$.

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A7: $Mean(P_1) = 0.6$ and $Var(P_1) = 0.6 \times 0.4/50 = 0.0048$.

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A7: $Mean(P_1) = 0.6$ and $Var(P_1) = 0.6 \times 0.4/50 = 0.0048$.

Q8: Let P_2 be the proportion of heads in 20 coin tosses, where $P(H) = 0.7$. From earlier, $Mean(P_2) = 0.7$ and $Var(P_2) = 0.0105$. Let $D = P_1 - P_2$. Find the mean and variance of D .

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Q7: Let P_1 be the proportion of heads in 50 coin tosses, where $P(H) = 0.6$. Find $Mean(P_1)$ and $Var(P_1)$.

A7: $Mean(P_1) = 0.6$ and $Var(P_1) = 0.6 \times 0.4/50 = 0.0048$.

Q8: Let P_2 be the proportion of heads in 20 coin tosses, where $P(H) = 0.7$. From earlier, $Mean(P_2) = 0.7$ and $Var(P_2) = 0.0105$. Let $D = P_1 - P_2$. Find the mean and variance of D .

A8: $Mean(D) = 0.6 - 0.7 = -0.1$
 $Var(D) = 0.0048 + 0.0105 = 0.0153$