EL-MOASSER

By a group of supervisors

GUIDE ANSWERS

2nd PREP.

FIRST TERM

Maths

GPS

AL TALABA BOOKSTORE
For printing, publication & distribution
El Faggala - Cairo - Egypt
Tel.: 02/ 259 340 12 - 259 377 91
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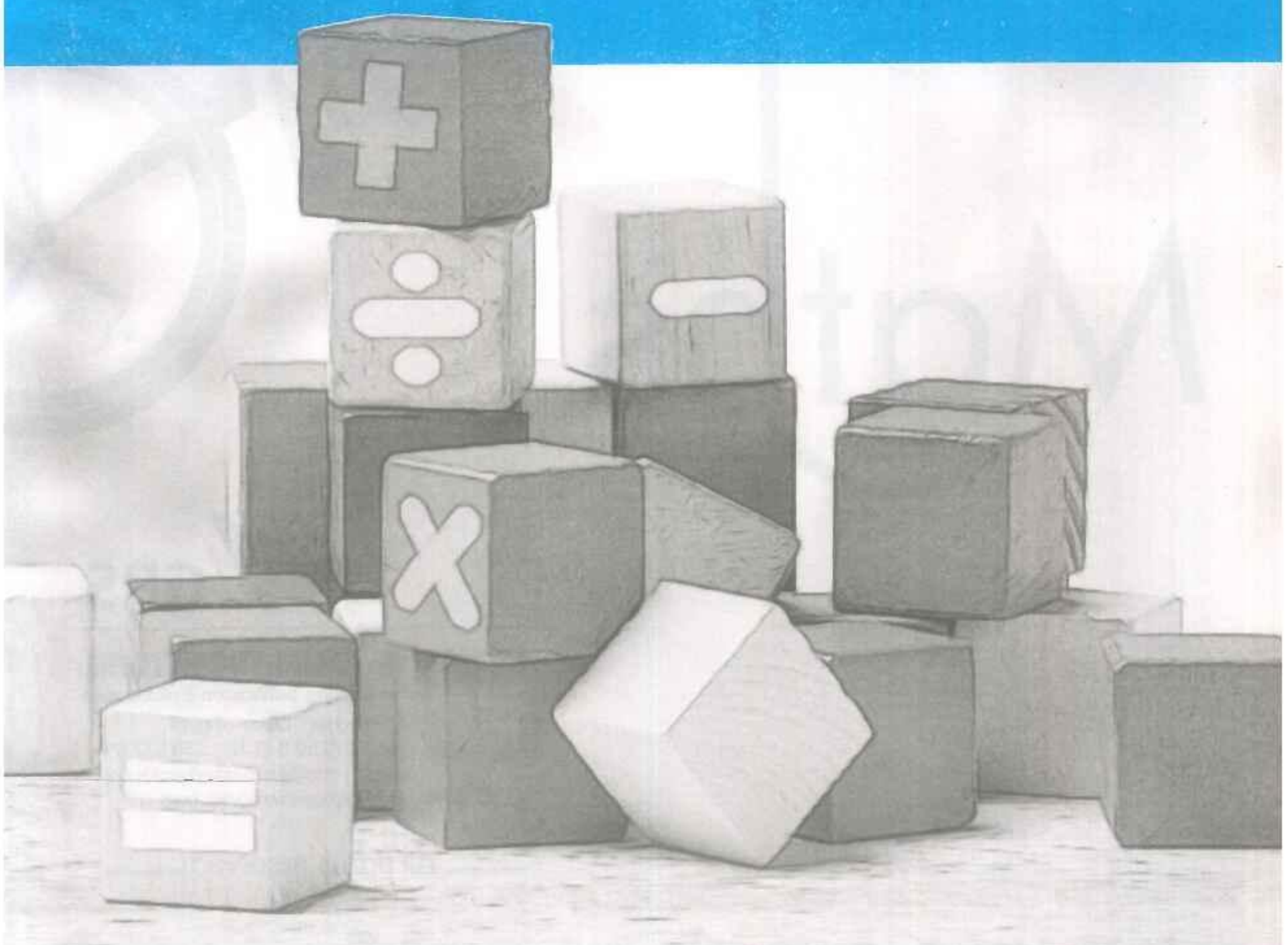
Guide Answers

Of Algebra and Statistics Exercises



Answers of unit one

Answers of Exercise 1

1

1. 2
2. -3
3. 6
4. -7
5. 0.1
6. $\frac{-2}{3}$
7. zero
8. -1
9. 6
10. 1
11. zero
12. a
13. $-3a^2$
14. 64
15. -1000
16. 64
17. 25
18. 61

2

1. c
2. a
3. d
4. d
5. a
6. d
7. c
8. c
9. d
10. b

3

1. 125
2. $-\frac{1}{64}$
3. $\because \sqrt[3]{x} = -2$ $\quad \therefore x = -8$
4. $\because \sqrt[3]{x} = -1 + 3 = 2$ $\quad \therefore x = 8$
5. -2
6. 4
7. $\because x^3 = 32 - 5 = 27$ $\quad \therefore x = 3$
8. $\because x^3 = 54 \div 2 = 27$ $\quad \therefore x = 3$
9. $\because x^3 = -200 \div \frac{1}{5} = -1000$ $\quad \therefore x = -10$

4

1. $\because x^3 = -27$ $\quad \therefore x = \sqrt[3]{-27} = -3$
   $\therefore$ The S.S. $= \{-3\}$
2. $\because 8x^3 = 8 - 7 = 1$ $\quad \therefore x^3 = \frac{1}{8}$
   $\therefore x = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$ $\quad \therefore$ The S.S. $= \{\frac{1}{2}\}$
3. $\because 2x^3 - x^3 = 3 + 5$ $\quad \therefore x^3 = 8$
   $\therefore x = \sqrt[3]{8} = 2$ $\quad \therefore$ The S.S. $= \{2\}$
4. $\because x + 3 = \sqrt[3]{343} = 7$ $\quad \therefore x = 7 - 3 = 4$
   $\therefore$ The S.S. $= \{4\}$
5. $\because (2x + 1)^3 = 20 + 7 = 27$ $\quad \therefore 2x + 1 = \sqrt[3]{27} = 3$
   $\therefore 2x = 3 - 1 = 2$ $\quad \therefore x = 2 \div 2 = 1$
   $\therefore$ The S.S. $= \{1\}$
6. $\because (5x - 2)^3 = 18 - 10 = 8$ $\quad \therefore 5x - 2 = \sqrt[3]{8} = 2$
   $\therefore 5x = 2 + 2 = 4$ $\quad \therefore x = \frac{4}{5}$
   $\therefore$ The S.S. $= \{\frac{4}{5}\}$

5

1. $-\sqrt[3]{2^9 \times 3^6} = -\sqrt[3]{(2^3 \times 3^2)^3} = -2^3 \times 3^2 = -8 \times 9 = -72$
2. $\sqrt{\sqrt[3]{729}} = \sqrt{9} = 3$
3. $\sqrt{27\sqrt[3]{27}} = \sqrt{27 \times 3} = \sqrt{81} = 9$

6

The edge length of the cube $= \sqrt[3]{27} = 3$ cm.

1. The area of one face $= 3^2 = 9$ cm^2
2. The total area $= 6 \times 3^2 = 54$ cm^2

7

The length of the inner edge $= \sqrt[3]{1000} = 10$ cm.

8

The volume of the sphere $= \frac{4}{3}\pi r^3 = \frac{1372}{81}\pi$

$\therefore r^3 = \frac{1372}{81} \times \frac{3}{4} = \frac{343}{27}$

$\therefore r = \sqrt[3]{\frac{343}{27}} = \frac{7}{3}$

$\therefore$ The diameter length of the sphere

$= 2 \times \frac{7}{3} = \frac{14}{3}$ length unit.

9

$\therefore$ The volume of the sphere $= \frac{4}{3}\pi r^3 = 113.04$

$\therefore \frac{4}{3} \times 3.14 \times r^3 = 113.04$

$\therefore r^3 = 27$ $\quad \therefore r = \sqrt[3]{27} = 3$ cm.

$\therefore$ The diameter length of the sphere $= 2 \times 3 = 6$ cm.

10

1. $\because (x^2 + 6)^3 = 1000$ $\quad \therefore x^2 + 6 = 10$
   $\therefore x^2 = 4$ $\quad \therefore x = \pm 2$
   $\therefore$ The S.S. $= \{2, -2\}$
2. $\because (x^3 - 14)^2 = 169$ $\quad \therefore x^3 - 14 = \pm 13$
   $\therefore x^3 = 14 \pm 13$ $\quad \therefore x^3 = 27$ $\quad \therefore x = 3$
   or $x^3 = 1$ $\quad \therefore x = 1$
   $\therefore$ The S.S. $= \{3, 1\}$
3. Cubing the two sides $\quad \therefore (x - 1)^2 = 25$
   $\therefore x - 1 = \pm 5$ $\quad \therefore x = 6$ or $x = -4$
   $\therefore$ The S.S. $= \{6, -4\}$

11

Cubing the two sides

$\therefore \sqrt{x} + 19 = 27$ $\quad\quad$ $\therefore \sqrt{x} = 8$

Squaring the two sides

$\therefore x = 64$ $\quad\quad$ $\therefore \sqrt[3]{x} = \sqrt[3]{64} = 4$

Answers of Exercise 2

1

The rational numbers are No.

1 , 2 , 3 , 4 , 5 , 8 , 9 ,

11 , 13 , 14 , 16 , 17 , 18 , 19

The remained numbers are irrational.

2 1 3.32 $\quad$ 2 1.9 $\quad$ 3 – 2.1

3

1 $\because \sqrt{4} < \sqrt{5} < \sqrt{9}$ $\quad\quad$ $\therefore 2 < \sqrt{5} < 3$

$\therefore$ The two numbers are 2 and 3

2 $\because \sqrt{9} < \sqrt{12} < \sqrt{16}$ $\quad\quad$ $\therefore 3 < \sqrt{12} < 4$

$\therefore$ The two numbers are 3 and 4

3 $\because \sqrt[3]{8} < \sqrt[3]{10} < \sqrt[3]{27}$ $\quad\quad$ $\therefore 2 < \sqrt[3]{10} < 3$

$\therefore$ The two numbers are 2 and 3

4 $\because \sqrt[3]{-27} < \sqrt[3]{-20} < \sqrt[3]{-8}$ $\quad\quad$ $\therefore -3 < \sqrt[3]{-20} < -2$

$\therefore$ The two numbers are –2 and –3

4

1 $\because \sqrt{1} < \sqrt{2} < \sqrt{4}$ $\quad\quad$ $\therefore 1 < \sqrt{2} < 2$ $\quad\quad$ $\therefore x = 1$

2 $\because \sqrt{64} < \sqrt{80} < \sqrt{81}$ $\quad\quad$ $\therefore 8 < \sqrt{80} < 9$ $\quad\quad$ $\therefore x = 8$

3 $\because \sqrt[3]{1} < \sqrt[3]{5} < \sqrt[3]{8}$ $\quad\quad$ $\therefore 1 < \sqrt[3]{5} < 2$ $\quad\quad$ $\therefore x = 1$

4 $\because \sqrt[3]{27} < \sqrt[3]{50} < \sqrt[3]{64}$ $\quad\quad$ $\therefore 3 < \sqrt[3]{50} < 4$ $\quad\quad$ $\therefore x = 3$

5 $\because \sqrt[3]{-125} < \sqrt[3]{-100} < \sqrt[3]{-64}$

$\therefore -5 < \sqrt[3]{-100} < -4$ $\quad\quad$ $\therefore x = -5$

6 $\because \sqrt{25} < \sqrt{35} < \sqrt{36}$ $\quad\quad$ $\therefore 5 < \sqrt{35} < 6$ $\quad\quad$ $\therefore x = 5$

5

1 $\because \sqrt{16} < \sqrt{20} < \sqrt{25}$ $\quad\quad$ $\therefore 4 < \sqrt{20} < 5$

$\because (4.1)^2 = 16.81$, $(4.2)^2 = 17.64$, $(4.3)^2 = 18.49$,

$(4.4)^2 = 19.36$, $(4.5)^2 = 20.25$

$\therefore 4.4 < \sqrt{20} < 4.5$ $\quad\quad$ $\therefore \sqrt{20} \approx 4.4$ or 4.5

Using the calculator $\sqrt{20} \approx 4.47$

2 $\because \sqrt[3]{8} < \sqrt[3]{17} < \sqrt[3]{27}$ $\quad\quad$ $\therefore 2 < \sqrt[3]{17} < 3$

$\because (2.1)^3 = 9.261$, $(2.2)^3 = 10.648$, $(2.3)^3 = 12.167$

$(2.4)^3 = 13.824$, $(2.5)^3 = 15.625$, $(2.6)^3 = 17.576$

$\therefore 2.5 < \sqrt[3]{17} < 2.6$

$\sqrt[3]{17} \approx 2.5$ or 2.6

Using the calculator $\sqrt[3]{17} \approx 2.57$

3 $\because \sqrt{4} < \sqrt{5} < \sqrt{9}$ $\quad\quad$ $\therefore 2 < \sqrt{5} < 3$

$\because (2.1)^2 = 4.41$, $(2.2)^2 = 4.84$, $(2.3)^2 = 5.29$

$\therefore 2.2 < \sqrt{5} < 2.3$ $\quad\quad$ $\therefore 3.2 < \sqrt{5} + 1 < 3.3$

$\therefore \sqrt{5} + 1 \approx 3.2$ or 3.3

Using the calculator $\sqrt{5} + 1 \approx 3.24$

4 $\because \sqrt[3]{8} < \sqrt[3]{9} < \sqrt[3]{27}$ $\quad\quad$ $\therefore 2 < \sqrt[3]{9} < 3$

$\because (2.1)^3 = 9.261$ $\quad\quad$ $\therefore 2 < \sqrt[3]{9} < 2.1$

$\therefore 1 < \sqrt[3]{9} - 1 < 1.1$ $\quad\quad$ $\therefore \sqrt[3]{9} - 1 \approx 1$ or 1.1

Using the calculator $\sqrt[3]{9} - 1 \approx 1.08$

6

1 d $\quad$ 2 b $\quad$ 3 b $\quad$ 4 c $\quad$ 5 c $\quad$ 6 b

7 b $\quad$ 8 d $\quad$ 9 c $\quad$ 10 c $\quad$ 11 d

7

1 $x^2 = \frac{10}{5} = 2$ $\quad\quad$ $\therefore x = \pm\sqrt{2}$ $\quad\quad$ $\therefore x \in \grave{\mathbb{Q}}$

2 $x^2 = \frac{9}{4}$ $\quad\quad$ $\therefore x = \pm\sqrt{\frac{9}{4}} = \pm\frac{3}{2}$ $\quad\quad$ $\therefore x \in \mathbb{Q}$

3 $x = \sqrt[3]{125}$ $\quad\quad$ $\therefore x = 5$ $\quad\quad$ $\therefore x \in \mathbb{Q}$

4 $x^3 = \frac{27}{3} = 9$ $\quad\quad$ $\therefore x = \sqrt[3]{9}$ $\quad\quad$ $\therefore x \in \grave{\mathbb{Q}}$

5 $x^2 = \frac{10}{0.1} = 100$ $\quad\quad$ $\therefore x = \pm\sqrt{100} = \pm 10$ $\quad\quad$ $\therefore x \in \mathbb{Q}$

6 $x^3 = \frac{-8}{0.001} = -8000$

$\therefore x = \sqrt[3]{-8000} = -20$ $\quad\quad$ $\therefore x \in \mathbb{Q}$

7 $x - 1 = \pm\sqrt{4} = \pm 2$ $\quad\quad$ $\therefore x = 2 + 1 = 3$

or $x = -2 + 1 = -1$ $\quad\quad$ $\therefore x \in \mathbb{Q}$

8 $x - 5 = \sqrt[3]{1} = 1$ $\quad\quad$ $\therefore x = 1 + 5 = 6$ $\quad\quad$ $\therefore x \in \mathbb{Q}$

8

1 $x^2 = \frac{25}{2} \times \frac{5}{2} = \frac{125}{4}$ $\quad \therefore x = \pm\sqrt{\frac{125}{4}}$

$\therefore$ The S.S. $= \{\sqrt{\frac{125}{4}}, -\sqrt{\frac{125}{4}}\}$

2 $x^3 = -2 \times \frac{4}{5} = -\frac{8}{5}$ $\quad \therefore x = \sqrt[3]{-\frac{8}{5}}$

$\therefore$ The S.S. $= \{\sqrt[3]{-\frac{8}{5}}\}$

3 $125\,x^3 = 27$ $\quad \therefore x^3 = \frac{27}{125}$

$\therefore x = \sqrt[3]{\frac{27}{125}} = \frac{3}{5}$

$\therefore$ The S.S. $= \varnothing$ because $\frac{3}{5} \notin \mathbb{Q}$

4 $\frac{1}{4}\,x^2 = 64$ $\quad \therefore x^2 = 64 \times 4 = 256$

$\therefore x = \pm\sqrt{256} = \pm 16$

$\therefore$ The S.S. $= \varnothing$ because $16 \notin \mathbb{Q}$, $-16 \notin \mathbb{Q}$

5 $\because (x^3 + 5)(x^2 - 3) = 0$

$\therefore x^3 + 5 = 0$ $\quad \therefore x^3 = -5$ $\quad \therefore x = -\sqrt[3]{5}$

or $x^2 - 3 = 0$ $\quad \therefore x^2 = 3$ $\quad \therefore x = \pm\sqrt{3}$

$\therefore$ The S.S. $= \{-\sqrt[3]{5}, \sqrt{3}, -\sqrt{3}\}$

6 $\because (x + \sqrt{7})(x^3 - 6) = 0$

$\therefore x + \sqrt{7} = 0$ $\quad \therefore x = -\sqrt{7}$

or $x^3 - 6 = 0$ $\quad \therefore x^3 = 6$

$\therefore x = \sqrt[3]{6}$ $\quad \therefore$ The S.S. $= \{-\sqrt{7}, \sqrt[3]{6}\}$

9

1 $\because (1.4)^2 = 1.96$, $(1.5)^2 = 2.25$, $(\sqrt{2})^2 = 2$

$\therefore \sqrt{2}$ is included between 1.4 , 1.5

2 $\because (3.31)^2 \approx 10.96$, $(3.32)^2 \approx 11.02$, $(\sqrt{11})^2 = 11$

$\therefore \sqrt{11}$ is included between 3.31 , 3.32

3 $\because (1.2)^3 = 1.728$, $(1.3)^3 = 2.197$, $(\sqrt[3]{2})^3 = 2$

$\therefore \sqrt[3]{2}$ is included between 1.2 , 1.3

4 $\because (2.4)^3 = 13.824$, $(2.5)^3 = 15.625$

$(\sqrt[3]{15})^3 = 15$

$\therefore \sqrt[3]{15}$ is included between 2.4 , 2.5

5 $\because (-2.6)^3 = -17.576$, $(-2.5)^3 = -15.625$

$(\sqrt[3]{-17})^3 = -17$

$\therefore \sqrt[3]{-17}$ is included between -2.6 , -2.5

6 $\because 2.7 - 1 = 1.7$, $(1.7)^2 = 2.89$

$2.8 - 1 = 1.8$, $(1.8)^2 = 3.24$

$\sqrt{3} + 1 - 1 = \sqrt{3}$, $(\sqrt{3})^2 = 3$

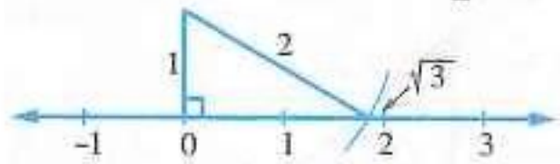
$\therefore \sqrt{3}$ is included between 1.7 , 1.8

$\therefore \sqrt{3} + 1$ is included between 2.7 , 2.8

10

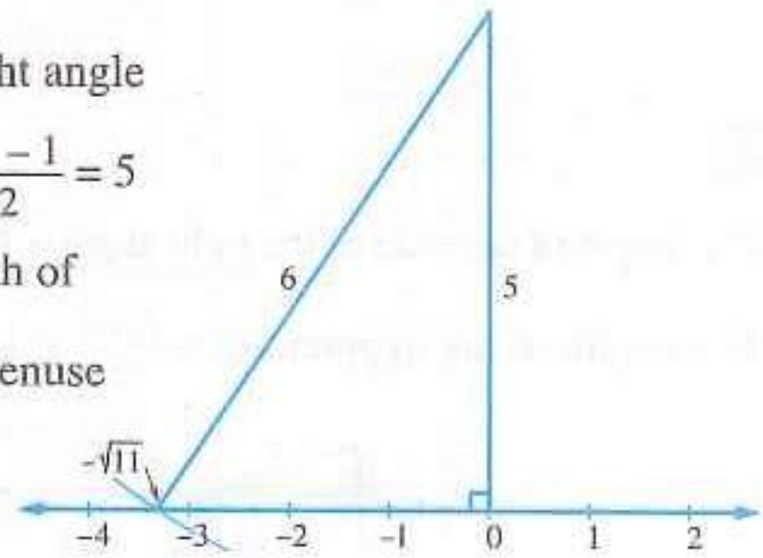
1 The length of one side of the right angle $= \frac{3-1}{2} = 1$

The length of the hypotenuse $= \frac{3+1}{2} = 2$



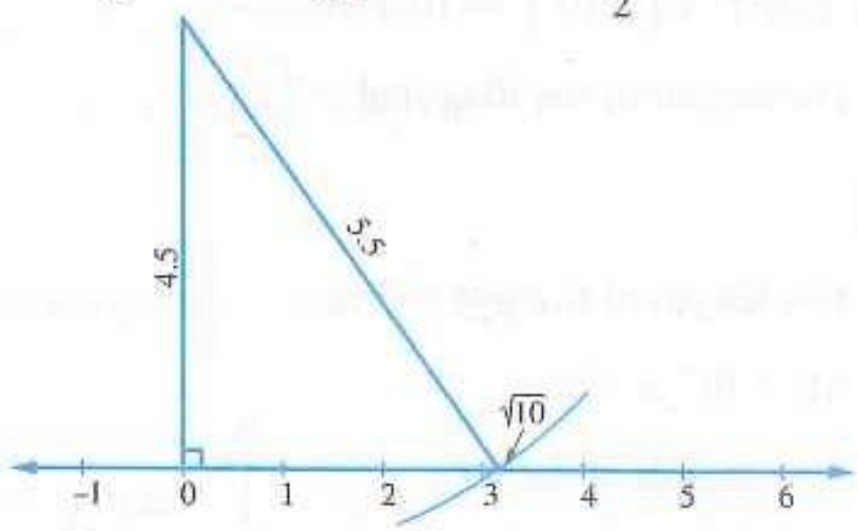
2 The length of one side of the right angle equals $\frac{11-1}{2} = 5$

The length of the hypotenuse $= \frac{11+1}{2} = 6$



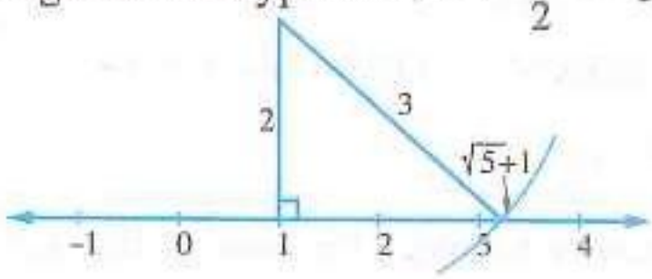
3 The length of one side of the right angle $= \frac{10-1}{2} = 4.5$

The length of the hypotenuse $= \frac{10+1}{2} = 5.5$



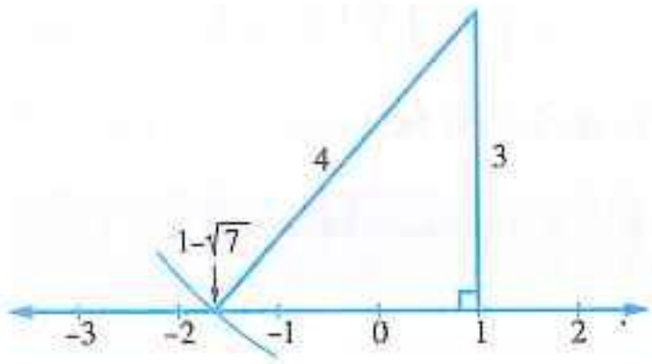
4 The length of one side of the right angle $= \frac{5-1}{2} = 2$

The length of the hypotenuse $= \frac{5+1}{2} = 3$



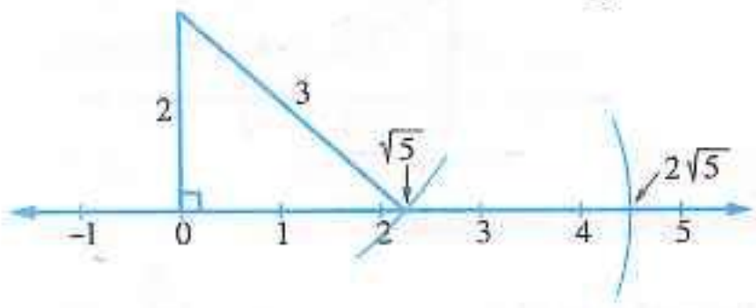
5 The length of one side of the right angle $= \frac{7-1}{2} = 3$

The length of the hypotenuse $= \frac{7+1}{2} = 4$



6 The length of one side of the right angle $= \frac{5-1}{2} = 2$

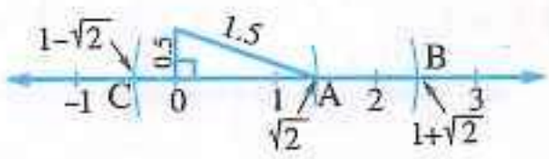
, the length of the hypotenuse $= \frac{5+1}{2} = 3$



11

The length of one side of the right angle $= \frac{2-1}{2} = 0.5$

The length of the hypotenuse $= \frac{2+1}{2} = 1.5$



12

The length of the side of the square $= \sqrt{10}$ cm.

The square of the length of the diagonal

$= (\sqrt{10})^2 + (\sqrt{10})^2 = 10 + 10 = 20$

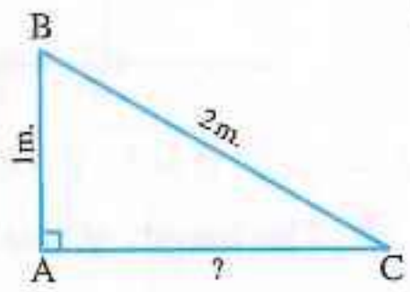
$\therefore$ The length of the diagonal $= \sqrt{20}$ cm.

13

$\because$ The length of the tree = 3 m.

$\therefore$ AB + BC = 3 m.

, $\because$ the length of the left part of the tree = 1 m.



$\therefore$ BC = 3 – 1 = 2 m.

$\because$ In Δ ABC : m ($\angle$ A) = 90°

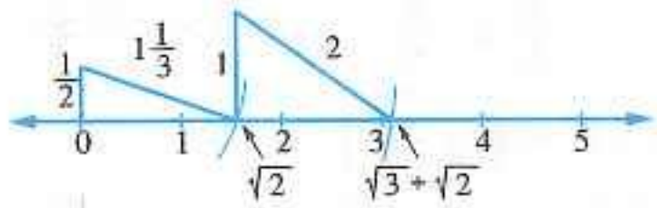
$\therefore (AC)^2 = (BC)^2 - (AB)^2 = 4 - 1 = 3$

$\therefore$ AC $= \sqrt{3}$ m.

$\therefore$ The distance between the base of the tree and the point of touching of its top with the ground $= \sqrt{3}$ m.

14

We represent on the number line the point representing the number $\sqrt{3} + \sqrt{2}$ as shown in the figure :



We find that the point representing the number $\sqrt{3} + \sqrt{2}$ lies between the point representing the number 3 and the point representing the number 4

i.e. $\sqrt{3} + \sqrt{2}$ lies between 3 and 4

Answers of Exercise 3

1

The number	Natural	Integer	Rational	Irrational	Real
-5	✗	✓	✓	✗	✓
$\sqrt{2}$	✗	✗	✗	✓	✓
$1\frac{1}{2}$	✗	✗	✓	✗	✓
$\sqrt[3]{9}$	✗	✗	✗	✓	✓
$\lvert -2 \rvert$	✓	✓	✓	✗	✓
$-\sqrt{4}$	✗	✓	✓	✗	✓
$\frac{5}{2}$	✗	✗	✓	✗	✓
0.3	✗	✗	✓	✗	✓
$\sqrt{-1}$	✗	✗	✗	✗	✗

2

1 > 2 > 3 <

4 < 5 > 6 =

7 > 8 >

3

1 a 2 d 3 b 4 a

5 d 6 c 7 a 8 b

9 b 10 a 11 d 12 c

13 d

4

1 The ascending order is :

$-\sqrt{11}$, $-\sqrt{7}$, $-\sqrt{3}$, $\sqrt{5}$, $\sqrt{8}$ and $\sqrt{15}$

2 $\because 0.6 = \sqrt{0.36}$, $\sqrt[3]{-1} = -1 = -\sqrt{1}$

$\therefore$ The ascending order is :

$-\sqrt{45}$, $-\sqrt{1}$, $\sqrt{0.36}$, $\sqrt{20}$ and $\sqrt{27}$

i.e. $-\sqrt{45}$, $\sqrt[3]{-1}$, 0.6 , $\sqrt{20}$ and $\sqrt{27}$

5

1 $\because 8 = \sqrt{64}$

$\therefore$ The descending order is :

$\sqrt{70}$, $\sqrt{64}$, $\sqrt{62}$ and $-\sqrt{50}$

i.e. $\sqrt{70}$, 8 , $\sqrt{62}$ and $-\sqrt{50}$

2 $\because 9 = \sqrt{81}$

$\therefore$ The descending order is :

$\sqrt{101}$, $\sqrt{81}$, $\sqrt{6}$, $-\sqrt{7}$, $-\sqrt{10}$ and $-\sqrt{50}$

i.e. $\sqrt{101}$, 9 , $\sqrt{6}$, $-\sqrt{7}$, $-\sqrt{10}$ and $-\sqrt{50}$

6

$\because 2^2 = 4$ $\qquad \therefore 4 > 3 > 2 > \frac{3}{2} > 0$

$\therefore 2 > \sqrt{3} > \sqrt{2} > \sqrt{\frac{3}{2}} > 0$

$\therefore$ The positive irrational numbers are

$\sqrt{3}$, $\sqrt{2}$ and $\sqrt{\frac{3}{2}}$

(There are other solutions)

7

The irrational numbers are

$-\sqrt{5}$, $-\sqrt{3}$ and $-\sqrt{2}$ (There are other solutions)

8

$\because (15)^2 = 225$, $(17)^2 = 289$

Then choosing 4 integers included

between 225 , 289

(except 256 because $\sqrt{256} = 16 \in \mathbb{Q}$)

$\therefore 225 < 235 < 245 < 255 < 265 < 289$

$\therefore 15 < \sqrt{235} < \sqrt{245} < \sqrt{255} < \sqrt{265} < 17$

$\therefore$ The four irrational numbers are

$\sqrt{235}$, $\sqrt{245}$, $\sqrt{255}$ and $\sqrt{265}$

(There are other solutions)

9

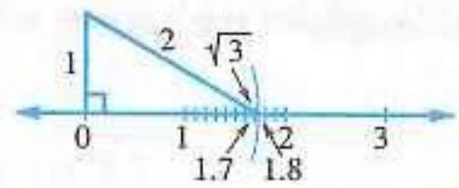
Using the calculator $\sqrt{3} \approx 1.73$

(to the nearest hundredth)

$\therefore 1.7 < \sqrt{3} < 1.8$ for representing $\sqrt{3}$

$\therefore$ The length of the hypotenuse $= \frac{3+1}{2} = 2$

, the length of one side of the right angle $= \frac{3-1}{2} = 1$



10

1 $x^2 = 6$ $\qquad \therefore x = \pm\sqrt{6} \approx \pm 2.45$

2 $x^2 = 24 \times \frac{4}{3} = 32$ $\qquad \therefore x = \pm\sqrt{32} \approx \pm 5.66$

3 $\frac{1}{2} x^2 = 5$ $\qquad \therefore x^2 = 5 \times 2 = 10$

$\therefore x = \pm\sqrt{10}$ $\qquad \therefore x \approx \pm 3.16$

4 $\frac{2}{x^3} = 16$ $\qquad \therefore x^3 = \frac{1}{8}$ $\qquad \therefore x = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$

5 $\because (x^2 - 9)(x^3 - 5) = 0$

$\therefore x^2 - 9 = 0$ $\qquad \therefore x^2 = 9$

$\therefore x = \pm\sqrt{9} = \pm 3$

or $x^3 - 5 = 0$ $\qquad \therefore x^3 = 5$

$\therefore x = \sqrt[3]{5} \approx 1.71$

6 $\because (2x^3 - 5)(x^2 + 1) = 0$

$\therefore 2x^3 - 5 = 0$ $\qquad \therefore 2x^3 = 5$

$\therefore x^3 = \frac{5}{2}$ $\qquad \therefore x = \sqrt[3]{\frac{5}{2}} \approx 1.36$

or $x^2 + 1 = 0$

$\therefore x^2 = -1$ (has no solution in $\mathbb{R}$)

11

The side length $= \sqrt{5}$ cm. , $\sqrt{5} \notin \mathbb{Q}$

12

The edge length $= \sqrt[3]{1.728} = \frac{6}{5}$ cm. , $\frac{6}{5} \in \mathbb{Q}$

13

$\because$ The total area of the cube $= 6\,\ell^2$

$\therefore 13.5 = 6\,\ell^2$ $\qquad \therefore \frac{13.5}{6} = \ell^2$

$\therefore \ell = \sqrt{\frac{13.5}{6}} = 1.5$ cm. , $1.5 \in \mathbb{Q}$

14

The diagonal length $= \sqrt{6^2 + 6^2} = \sqrt{72}$ cm.

15

The diagonal length of the rectangle $= \sqrt{(5)^2 + (7)^2} = \sqrt{74}$ cm.

$\because$ The area of the square = the area of the rectangle $= 5 \times 7 = 35$ cm.^{2}

$\therefore$ The side length of the square $= \sqrt{35}$ cm.

$\therefore$ The diagonal length of the square $= \sqrt{35 + 35}$
$= \sqrt{70}$ cm.

16

Cubing the two sides then squaring them we find that

$(\sqrt[3]{3})^3 = 3$, $3^2 = 9$, $(\sqrt{2})^3 = 2\sqrt{2}$, $(2\sqrt{2})^2 = 8$

$\because 9 > 8$ $\therefore \sqrt[3]{3} > \sqrt{2}$

17

Let the other number $= x$ $\therefore x^2 + 2^2 = 7$

$\therefore x^2 = 7 - 4 = 3$ $\therefore x = \pm\sqrt{3}$

$\therefore$ The other number is $\sqrt{3}$ or $-\sqrt{3}$

Answers of Exercise 4

1

2 $\{x : 1 \le x < 3 , x \in \mathbb{R}\}$

3 $]0 , 3]$

4 $]-2 , 3[$, $\{x : -2 < x < 3 , x \in \mathbb{R}\}$

5 $\{x : x \le 1 , x \in \mathbb{R}\}$

6 $]0 , \infty[$, $\{x : x > 0 , x \in \mathbb{R}\}$

7 $]-\infty , 4[$

8 $\{x : x \ge -2 , x \in \mathbb{R}\}$

2

1 c 2 a 3 b 4 c 5 d

3

1 $\in$ 2 $\notin$ 3 $\in$ 4 $\in$ 5 $\in$
6 $\in$ 7 $\in$ 8 $\notin$ 9 $\notin$ 10 $\notin$

4

1 $[-1 , 5[$

2 $[2 , 3[$

3 $[3 , 5[$

4 $[-1 , 2[$

5 $]-\infty , 2[\cup [5 , \infty[$

6 $]-\infty , -1[\cup [3 , \infty[$

5

1 $\mathbb{R}$

2 $[-4 , 3]$

3 $]-\infty , -4[$

4 $]3 , \infty[$

5 $]3 , \infty[$

6 $]-\infty , -4[$

6 Use the number line to get the following results :

1 $[-1 , \infty[$ 2 $[3 , 4]$ 3 $[-1 , 3[$
4 $[-1 , 4[- \{3\}$ 5 $\{3 , 4\}$
6 $]4 , \infty[$ 7 $]-\infty , -1[\cup]4 , \infty[$
8 $]-\infty , 3[$

7 Use the number line to get the following results :

1 $[2 , 4]$ 2 $[-1 , 5]$ 3 $]0 , 1[$
4 $]-2 , 3]$ 5 $[3 , 6]$ 6 $[-1 , 2[$
7 $[-3 , 2] - \{0\}$ 8 $\varnothing$
9 $\varnothing$ 10 $[-2 , 1[\cup]2 , 4]$
11 $\varnothing$ 12 $\{-1 , 5\}$

8 Use the number line to get the following results :

1 $[-3 , \infty[$ 2 $[2 , 3[$ 3 $[-4 , 3]$
4 $\mathbb{R}$ 5 $]-\infty , -1[$ 6 $]-\infty , -3[$
7 $]0 , 2]$ 8 $\mathbb{R} - [3 , 4]$

9

1 $[3 , 5]$ 2 $[3 , 5]$ 3 $\{3 , 5\}$ 4 $\varnothing$
5 $]3 , 5[$ 6 $]3 , 5[$ 7 $\varnothing$ 8 $\{3 , 5\}$
9 $[3 , 5[$ 10 $[3 , 5[$ 11 $\{3 , 4\}$ 12 $]-3 , 5]$

10

1 b 2 d 3 c
4 b 5 b 6 d

11

1 $[-3, 3]$ 2 $\mathbb{R}$ 3 $]-\infty, -1[$
4 $]-\infty, -3[$ 5 $]-2, 0]$ 6 $[0, 2]$
7 $\{1, 2\}$ 8 $\{0, 1\}$
9 $\{-1, 0, 1, 2\}$ 10 $]0, 5]$
11 $[-3, 0[$

12

1 d 2 c 3 c 4 c 5 b
6 d 7 c 8 c 9 c

13

$\because X \subset Y \quad \therefore X = X \cap Y = [4, 7]$

$, Y = X \cup Y = [3, 7]$ $, Y - X = [3, 4[$

Answers of Exercise 5

1

1 $3\sqrt{3}$ 2 zero
3 $-\sqrt[3]{7}$ 4 $6\sqrt{5}$

2

1 $3\sqrt{5}$ 2 $3\sqrt{3}-1$ 3 $8\sqrt{7}-3\sqrt{2}$
4 $7\sqrt{2}-2\sqrt[3]{2}$ 5 $\sqrt{2}$
6 $8 \times \frac{1}{2} + 2\sqrt[3]{3} - 4 - 5\sqrt[3]{3} = 4 + 2\sqrt[3]{3} - 4 - 5\sqrt[3]{3}$
$= -3\sqrt[3]{3}$

3

1 3 2 -30 3 $6\sqrt{2}$ 4 1 5 $15\sqrt{3}$
6 $2\sqrt{3} \times \frac{2\sqrt{7}}{7} \times \frac{5\sqrt{7}}{20\sqrt{3}} = 1$

4

1 $2\sqrt{2}+2\sqrt{5}$ 2 $5\sqrt{2}+2$
3 $7+2\sqrt{7}$ 4 $5\sqrt{3}+3$
5 $-6\sqrt{5}+10$ 6 $2-7+3\sqrt{7} = -5+3\sqrt{7}$
7 $-24-6\sqrt{3}+6\sqrt{3} = -24$
8 $3\sqrt{5}-5-2-2\sqrt{5} = \sqrt{5}-7$

5

1 $(\sqrt{2})^2 - (1)^2 = 2 - 1 = 1$
2 $(4)^2 - (3\sqrt{2})^2 = 16 - 18 = -2$
3 $(\sqrt{5})^2 - 2 \times 1 \times \sqrt{5} + (-1)^2 = 5 - 2\sqrt{5} + 1$
$= 6 - 2\sqrt{5}$
4 $(2\sqrt{3})^2 + 2 \times 4 \times 2\sqrt{3} + (4)^2 = 12 + 16\sqrt{3} + 16$
$= 28 + 16\sqrt{3}$
5 $3 + \sqrt{3} - 2 = 1 + \sqrt{3}$
6 $(5)^2 - 2 \times 5 \times \sqrt{3} + (-\sqrt{3})^2 - 28$
$= 25 - 10\sqrt{3} + 3 - 28 = -10\sqrt{3}$

6

1 $\frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$
2 $-\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -\frac{6\sqrt{3}}{3} = -2\sqrt{3}$
3 $\frac{2}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{6} = \frac{\sqrt{2}}{3}$
4 $\frac{6}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{6} = \sqrt{3}$
5 $\frac{\sqrt{2}+3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2+3\sqrt{2}}{2}$
6 $\frac{\sqrt{5}-15}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5-15\sqrt{5}}{10} = \frac{1-3\sqrt{5}}{2}$

7

1 b 2 c 3 a 4 d 5 c 6 b
7 d 8 b 9 c

8

1 1 , zero 2 $\sqrt{2}-1$ 3 $5\sqrt{3}$
4 1 5 2 , $\sqrt{3}$ 6 $\frac{2}{3}$
7 $4\sqrt{3}$ 8 $3+2\sqrt{2}$ 9 $\pm\sqrt{5}$
10 $8\sqrt{2}$ 11 60 cm^2
12 The additive inverse 13 $\mathbb{R}$

9

1 $\sqrt{5}-2+\sqrt{5}+2 = 2\sqrt{5}$
2 $\sqrt{5}-2-\sqrt{5}-2 = -4$
3 $(\sqrt{5}-2)(\sqrt{5}+2) = 5-4 = 1$

4 $x^2 - y^2 = (x - y)(x + y) = (-4)(2\sqrt{5}) = -8\sqrt{5}$

5 $x^2 + 2xy + y^2 = (x + y)^2 = (2\sqrt{5})^2 = 20$

6 $x^2 - 2xy + y^2 = (x - y)^2 = (-4)^2 = 16$

10

1 $x \approx 4 + 2 = 6$

and using the calculator $x \approx 5.9$ (accepted estimation)

, $y \approx 4 - 3 = 1$

and using the calculator $y \approx 1.08$ (accepted estimation)

2 $x \times y \approx 6 \times 1 = 6$

and using the calculator , the expression ≈ 6.3 (accepted estimation)

3 $x + y \approx 6 + 1 = 7$

and using the calculator , the expression ≈ 6.9 (accepted estimation)

11

The perimeter $= 2(6 + \sqrt{5} + 6 - \sqrt{5}) = 2 \times 12$
$= 24$ cm.

The area $= (6 + \sqrt{5})(6 - \sqrt{5}) = 36 - 5 = 31$ cm^2.

12

$(\sqrt{a} - 1) \times \frac{\sqrt{a} + 1}{4} = 1$

$\therefore \frac{a - 1}{4} = 1 \qquad \therefore a - 1 = 4 \qquad \therefore a = 5$

13

$x = \sqrt{2}$, $y = \frac{\sqrt{2}}{2}$, $z = \frac{\sqrt{2}}{4}$

$\therefore x^2 + 2y^2 + 4z^2 = (\sqrt{2})^2 + 2 \times \left(\frac{\sqrt{2}}{2}\right)^2 + 4\left(\frac{\sqrt{2}}{4}\right)^2$

$= 2 + 2 \times \frac{2}{4} + 4 \times \frac{2}{16} = 3\frac{1}{2}$

14

$\frac{1}{2}(2y) = 1 - \sqrt{2} \quad \therefore y = 1 - \sqrt{2}$

$\therefore x = -1 + \sqrt{2}$

$\therefore xy - 2\sqrt{2} = (-1 + \sqrt{2})(1 - \sqrt{2}) - 2\sqrt{2}$

$= -1 + \sqrt{2} + \sqrt{2} - 2 - 2\sqrt{2} = -3$

Answers of Exercise 6

1

1 $\sqrt{4 \times 3} = 2\sqrt{3}$ 2 $\sqrt{4 \times 7} = 2\sqrt{7}$

3 $2\sqrt{36 \times 2} = 2 \times 6\sqrt{2} = 12\sqrt{2}$

4 $\frac{2}{5}\sqrt{100 \times 10} = \frac{2}{5} \times 10\sqrt{10} = 4\sqrt{10}$

5 $\sqrt{4 \times \frac{1}{2}} = \sqrt{2}$ 6 $2\sqrt{\frac{2}{3} \times 9} = 2\sqrt{6}$

2

1 $5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$

2 $2\sqrt{5} - 3\sqrt{5} = -\sqrt{5}$

3 $3\sqrt{2} + 2\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$

4 $7\sqrt{2} - 8\sqrt{2} - 3\sqrt{2} + 4\sqrt{2} =$ zero

5 $2 \times 3\sqrt{2} + 5\sqrt{2} + \frac{1}{3} \times 9\sqrt{2}$

$= 6\sqrt{2} + 5\sqrt{2} + 3\sqrt{2} = 14\sqrt{2}$

6 $7\sqrt{2} + 5\sqrt{2} - \frac{1}{2} \times 10\sqrt{2} - \sqrt{2}$

$= 7\sqrt{2} + 5\sqrt{2} - 5\sqrt{2} - \sqrt{2} = 6\sqrt{2}$

7 $3\sqrt{3} + 5 \times 3\sqrt{2} - 10\sqrt{3} = 15\sqrt{2} - 7\sqrt{3}$

3

1 $2\sqrt{5} + 4 \times 2\sqrt{5} - \sqrt{25 \times \frac{1}{5}} = 2\sqrt{5} + 8\sqrt{5} - \sqrt{5}$

$= 9\sqrt{5}$

2 $4\sqrt{2} - 6\sqrt{2} + 3\sqrt{4 \times \frac{1}{2}} = 4\sqrt{2} - 6\sqrt{2} + 3\sqrt{2}$

$= \sqrt{2}$

3 $2\sqrt{5} + 2\sqrt{9 \times \frac{1}{3}} - 2\sqrt{3} - \sqrt{25 \times \frac{1}{5}}$

$= 2\sqrt{5} + 2\sqrt{3} - 2\sqrt{3} - \sqrt{5} = \sqrt{5}$

4 $\sqrt{3} + \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} - \sqrt{12} = \sqrt{3} + \sqrt{3} - 2\sqrt{3} =$ zero

5 $3\sqrt{2} - \sqrt{\frac{12}{6}} = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$

6 $5 + 3\sqrt{2} - \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 5 + 3\sqrt{2} - 3\sqrt{2} = 5$

4

1 $10\sqrt{6}$ 2 $6\sqrt{36} = 6 \times 6 = 36$

3 $2\sqrt{50} = 2 \times 5\sqrt{2} = 10\sqrt{2}$

4 $\sqrt{\frac{2}{7} \times \frac{7}{2}} = \sqrt{1} = 1$ 5 $3\sqrt{\frac{15}{5}} = 3\sqrt{3}$

6 $12 \times \sqrt{\frac{2}{3} \times 54} = 12\sqrt{36} = 12 \times 6 = 72$

5

1 $\sqrt{18}-\sqrt{12}=3\sqrt{2}-2\sqrt{3}$

2 $20+5\sqrt{24}=20+5\times2\sqrt{6}=20+10\sqrt{6}$

3 $(3\sqrt{5})^2-(\sqrt{7})^2=45-7=38$

4 $(\sqrt{3})^2-2\times\sqrt{3}\times\sqrt{2}+(-\sqrt{2})^2=3-2\sqrt{6}+2$

$=5-2\sqrt{6}$

5 $(\sqrt{3})^2+2\times\sqrt{3}\times\sqrt{5}+(\sqrt{5})^2-2\sqrt{15}$

$=3+2\sqrt{15}+5-2\sqrt{15}=8$

6 $3\sqrt{2}-\frac{12}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}+2\sqrt{6}-3\sqrt{2}$

$=3\sqrt{2}-2\sqrt{6}+2\sqrt{6}-3\sqrt{2}=$ zero

6

1 $\frac{\sqrt{3}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{6}}{2}$

2 $\sqrt{\frac{5}{3}}=\frac{\sqrt{5}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{15}}{3}$

3 $\frac{5\sqrt{3}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{5\sqrt{15}}{5}=\sqrt{15}$

4 $\frac{4\sqrt{3}-\sqrt{2}}{2\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{12-\sqrt{6}}{6}$

7

1 a	2 b	3 c	4 a	5 b
6 c	7 a	8 a	9 c	10 b

8

1 $\frac{1}{2}$	2 $\sqrt{2}$	3 $\sqrt{3}$	4 $\frac{3}{2}$
5 -2	6 $\sqrt{125}$	7 $\pm\frac{2\sqrt{2}}{3}$	8 20 , zero

9

1 $x+y=3+\sqrt{5}+1-\sqrt{5}=4$

$x\times y=(3+\sqrt{5})(1-\sqrt{5})=3-2\sqrt{5}-5$

$=-2-2\sqrt{5}$

2 $x+y=\sqrt{3}-\sqrt{2}+\sqrt{3}+\sqrt{2}=2\sqrt{3}$

$x\times y=(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})=3-2=1$

3 $x+y=5-3\sqrt{2}+5-3\sqrt{2}=10-6\sqrt{2}$

$x\times y=(5-3\sqrt{2})(5-3\sqrt{2})$

$=25-30\sqrt{2}+18=43-30\sqrt{2}$

10

$\because x=\frac{\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{6}}{3}$, $y=\frac{\sqrt{3}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{6}}{2}$

$\therefore 6(x+y)=6\left(\frac{\sqrt{6}}{3}+\frac{\sqrt{6}}{2}\right)=6\times\frac{\sqrt{6}}{3}+6\times\frac{\sqrt{6}}{2}$

$=2\sqrt{6}+3\sqrt{6}=5\sqrt{6}$

11 We know that $(x+y)^2=x^2+2xy+y^2$

$\therefore x^2+2xy+y^2=(2\sqrt{5}+\sqrt{2}+2\sqrt{5}-\sqrt{2})^2$

$=(4\sqrt{5})^2=16\times5=80$

12

$a^{x+y}=a^x\times a^y=a^x\div a^{-y}=6\div\sqrt{3}=\frac{6}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$=\frac{6\sqrt{3}}{3}=2\sqrt{3}$

13

1 $\frac{(\sqrt{5})^3\times(\sqrt{5})^5}{(\sqrt{2})^6\times(\sqrt{5})^6}=\frac{(\sqrt{5})^{3+5-6}}{(\sqrt{2})^6}=\frac{5}{8}$

2 $\frac{2\sqrt{2}\times(\sqrt{2})^{-3}\times(\sqrt{3})^{-3}}{(\sqrt{3})^{-3}}=2\times(\sqrt{2})^{-2}$

$=\frac{2}{(\sqrt{2})^2}=\frac{2}{2}=1$

Answers of Exercise 7

1

1 $\sqrt{5}-\sqrt{3}$ 2 $5+2\sqrt{7}$

3 $\because \sqrt{5}+\frac{2}{\sqrt{2}}=\sqrt{5}+\sqrt{2}$

$\therefore$ The conjugate number $=\sqrt{5}-\sqrt{2}$

2

1 $\frac{5}{\sqrt{7}-\sqrt{2}}\times\frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}}=\frac{5(\sqrt{7}+\sqrt{2})}{7-2}=\sqrt{7}+\sqrt{2}$

2 $\frac{\sqrt{3}}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{\sqrt{3}(2+\sqrt{3})}{4-3}=2\sqrt{3}+3$

3 $\frac{\sqrt{7}+3}{\sqrt{7}-3}\times\frac{\sqrt{7}+3}{\sqrt{7}+3}=\frac{16+6\sqrt{7}}{7-9}=-8-3\sqrt{7}$

3

$\because x = \frac{2}{\sqrt{7}-\sqrt{5}} \times \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{2(\sqrt{7}+\sqrt{5})}{7-5}$

$= \sqrt{7} + \sqrt{5}$

$\therefore (x+y)^2 = (\sqrt{7}+\sqrt{5}+\sqrt{7}-\sqrt{5})^2$

$= (2\sqrt{7})^2 = 28$

4

$x^2 y^2 = (xy)^2 = \left(\frac{4}{\sqrt{7}-\sqrt{3}} \times \frac{4}{\sqrt{7}+\sqrt{3}}\right)^2$

$= \left(\frac{16}{7-3}\right)^2 = 4^2 = 16$

5

L.H.S. $= \frac{4}{x} + 2x$

$= \frac{4}{\sqrt{5}+\sqrt{3}} + 2(\sqrt{5}+\sqrt{3})$

$= \frac{4(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} + 2(\sqrt{5}+\sqrt{3})$

$= \frac{4(\sqrt{5}-\sqrt{3})}{2} + 2(\sqrt{5}+\sqrt{3})$

$= 2(\sqrt{5}-\sqrt{3}) + 2(\sqrt{5}+\sqrt{3})$

$= 2\sqrt{5} - 2\sqrt{3} + 2\sqrt{5} + 2\sqrt{3}$

$= 4\sqrt{5} =$ R.H.S.

6

$\because b = \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{3-2}$

$= \sqrt{3} - \sqrt{2}$

We know that : $(a-b)(a+b) = a^2 - b^2$

$\therefore (\sqrt{3}+\sqrt{2}-\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2})$

$= 2\sqrt{2} \times 2\sqrt{3} = 4\sqrt{6}$

7

$\because y = \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{2(\sqrt{5}+\sqrt{3})}{5-3}$

$= \sqrt{5} + \sqrt{3}$

$\therefore x^2 + 2xy + y^2 = (x+y)^2$

$= (\sqrt{5}-\sqrt{3}+\sqrt{5}+\sqrt{3})^2 = (2\sqrt{5})^2 = 20$

8

$\because y = \frac{3}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{3(\sqrt{5}+\sqrt{2})}{5-2}$

$= \sqrt{5} + \sqrt{2}$

$, x = \sqrt{5} - \sqrt{2}$

$\therefore x$ and y are two conjugate numbers.

$\therefore x^2 - 2xy + y^2 = (x-y)^2 = (\sqrt{5}-\sqrt{2}-\sqrt{5}-\sqrt{2})^2$

$= (-2\sqrt{2})^2 = 8$

9

$\because x = 3+\sqrt{5}, y = \frac{4}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{4(3-\sqrt{5})}{9-5}$

$= 3 - \sqrt{5}$

$\therefore x$ and y are two conjugate numbers

1 $xy = (3+\sqrt{5})(3-\sqrt{5}) = 9 - 5 = 4$

2 $x^2 + y^2 = (x+y)^2 - 2xy$

$= (3+\sqrt{5}+3-\sqrt{5})^2 - 2 \times 4 = 36 - 8 = 28$

10

$\because x = \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{2(\sqrt{5}+\sqrt{3})}{5-3}$

$= \sqrt{5} + \sqrt{3}$

$, y = \frac{2}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{2(\sqrt{5}-\sqrt{3})}{5-3}$

$= \sqrt{5} - \sqrt{3}$

$\therefore x^2 - xy + y^2 = (x-y)^2 + xy$

$= (\sqrt{5}+\sqrt{3}-\sqrt{5}+\sqrt{3})^2 + (\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$

$= (2\sqrt{3})^2 + 2 = 14$

11

$\frac{x+y}{xy-1} = \frac{\sqrt{5}+\sqrt{2}+\sqrt{5}-\sqrt{2}}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})-1}$

$= \frac{2\sqrt{5}}{5-2-1} = \frac{2\sqrt{5}}{2} = \sqrt{5}$

12

$\because a = \frac{4}{\sqrt{7}-\sqrt{3}} \times \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}} = \frac{4(\sqrt{7}+\sqrt{3})}{7-3}$

$= \sqrt{7} + \sqrt{3}$

$$, b = \frac{4}{\sqrt{7}+\sqrt{3}} \times \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}} = \frac{4(\sqrt{7}-\sqrt{3})}{7-3}$$

$$= \sqrt{7}-\sqrt{3}$$

$$\therefore \frac{a-b}{ab} = \frac{\sqrt{7}+\sqrt{3}-\sqrt{7}+\sqrt{3}}{(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})} = \frac{2\sqrt{3}}{7-3} = \frac{\sqrt{3}}{2}$$

13

$$\because x = 2\sqrt{2}-\sqrt{3}\ , y = \frac{5}{2\sqrt{2}-\sqrt{3}}$$

$$\therefore y = \frac{5}{2\sqrt{2}-\sqrt{3}} \times \frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}} = \frac{5(2\sqrt{2}+\sqrt{3})}{8-3}$$

$$= 2\sqrt{2}+\sqrt{3}$$

$\therefore$ x and y are conjugate numbers.

$$\therefore \frac{x+y}{xy} = \frac{2\sqrt{2}-\sqrt{3}+2\sqrt{2}+\sqrt{3}}{(2\sqrt{2}-\sqrt{3})(2\sqrt{2}+\sqrt{3})} = \frac{4\sqrt{2}}{8-3} = \frac{4\sqrt{2}}{5}$$

14

$$\because x = \frac{5\sqrt{2}+3\sqrt{5}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{10}+15}{5}$$

$$= \sqrt{10}+3$$

$$, y = \frac{2\sqrt{5}-3\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{10}-6}{2} = \sqrt{10}-3$$

1 $x^2+y^2 = (x+y)^2 - 2xy$

$$= (\sqrt{10}+3+\sqrt{10}-3)^2 - 2(\sqrt{10}+3)(\sqrt{10}-3)$$

$$= (2\sqrt{10})^2 - 2\times(10-9) = 40-2 = 38$$

2 $xy = (\sqrt{10}+3)(\sqrt{10}-3) = 10-9 = 1$

$$\therefore x^2+y^2 = 38xy$$

15

$$\because x = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

$$, y = \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

$$\therefore x^2+y = (2-\sqrt{3})^2 + 4\sqrt{3}$$

$$= 4-4\sqrt{3}+3+4\sqrt{3} = 7$$

16

$$\because y = \sqrt{3}-\sqrt{2}$$

$$, x = \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \sqrt{3}+\sqrt{2}$$

$$\therefore (x+y)^2 = (\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2})^2$$

$$= (2\sqrt{3})^2 = 12$$

17

$$\because xy = 1$$

$$\therefore y = \frac{1}{x} = \frac{1}{\sqrt{13}+\sqrt{6}} = \frac{1}{\sqrt{13}+\sqrt{6}} \times \frac{\sqrt{13}-\sqrt{6}}{\sqrt{13}-\sqrt{6}}$$

$$= \frac{\sqrt{13}-\sqrt{6}}{7}$$

$$\therefore x^2-49y^2 = (x-7y)(x+7y)$$

$$= \left(\sqrt{13}+\sqrt{6}-7\left(\frac{\sqrt{13}-\sqrt{6}}{7}\right)\right)$$

$$\left(\sqrt{13}+\sqrt{6}+7\left(\frac{\sqrt{13}-\sqrt{6}}{7}\right)\right)$$

$$= (\sqrt{13}+\sqrt{6}-\sqrt{13}+\sqrt{6})(\sqrt{13}+\sqrt{6}+\sqrt{13}-\sqrt{6})$$

$$= 2\sqrt{6}\times 2\sqrt{13} = 4\sqrt{78}$$

18

$$\because x = \frac{4(\sqrt{7}+\sqrt{3})}{(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})} = \frac{4(\sqrt{7}+\sqrt{3})}{7-3}$$

$$= \sqrt{7}+\sqrt{3}$$

$$, y = \sqrt{7}-\sqrt{3}$$

$\therefore$ x and y are two conjugate numbers.

$$\therefore x^2y^2 = (xy)^2 = [(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})]^2$$

$$= (7-3)^2 = 4^2 = 16$$

19

$$\because y = \frac{2}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \frac{2(\sqrt{7}-\sqrt{5})}{7-5}$$

$$= \sqrt{7}-\sqrt{5}$$

$$\therefore \frac{x+y}{xy} = \frac{\sqrt{7}+\sqrt{5}+\sqrt{7}-\sqrt{5}}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})} = \frac{2\sqrt{7}}{7-5} = \sqrt{7}$$

20

$\because x = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{11+2\sqrt{30}}{6-5}$

$= 11 + 2\sqrt{30}$

$, \frac{1}{x} = \frac{1}{11+2\sqrt{30}} \times \frac{11-2\sqrt{30}}{11-2\sqrt{30}} = \frac{11-2\sqrt{30}}{121-120}$

$= 11 - 2\sqrt{30}$

$\therefore x + \frac{1}{x} = 11 + 2\sqrt{30} + 11 - 2\sqrt{30} = 22$

21

1. 4
2. $3-\sqrt{2}\ , 7$
3. $\sqrt{5}-\sqrt{3}$
4. $1-\sqrt{7}$
5. $\sqrt{3}-\sqrt{2}$
6. 20
7. 20
8. $\sqrt{5}+2$
9. $(-1\ , 2\sqrt{3})$
10. -1

22

1. $\frac{11}{2\sqrt{5}+3} = \frac{11(2\sqrt{5}-3)}{(2\sqrt{5}+3)(2\sqrt{5}-3)}$

$= \frac{11(2\sqrt{5}-3)}{20-9} = 2\sqrt{5}-3$

$\therefore a = 2\ , b = -3$

2. $\frac{3}{2\sqrt{2}-\sqrt{5}} = \frac{3(2\sqrt{2}+\sqrt{5})}{(2\sqrt{2}-\sqrt{5})(2\sqrt{2}+\sqrt{5})}$

$= \frac{3(2\sqrt{2}+\sqrt{5})}{8-5} = 2\sqrt{2}+\sqrt{5}$

$\therefore a = 2\ , b = 1$

3. $\frac{7}{\sqrt{8}+1} = \frac{7(\sqrt{8}-1)}{(\sqrt{8}+1)(\sqrt{8}-1)}$

$= \frac{7(\sqrt{8}-1)}{8-1} = \sqrt{8}-1$

$= 2\sqrt{2}-1 = a + b\sqrt{2}$

$\therefore a = -1\ , b = 2$

23

$(x+y)^2 = x^2 + 2xy + y^2$

$= \left(\sqrt{4+\sqrt{7}}\right)^2 + 2\left(\sqrt{4+\sqrt{7}}\right)\left(\sqrt{4-\sqrt{7}}\right) + \left(\sqrt{4-\sqrt{7}}\right)^2$

$= 4+\sqrt{7} + 2\sqrt{\left(4+\sqrt{7}\right)\left(4-\sqrt{7}\right)} + 4-\sqrt{7}$

$= 8 + 2\sqrt{16-7} = 8 + 2\sqrt{9} = 8+6 = 14$

24

$xy^{-1} + yx^{-1} = \frac{x}{y} + \frac{y}{x} = \frac{x^2+y^2}{xy}$

$= \frac{(\sqrt{5}+1)^2 + (\sqrt{5}-1)^2}{(\sqrt{5}+1)(\sqrt{5}-1)}$

$= \frac{6+2\sqrt{5}+6-2\sqrt{5}}{5-1} = \frac{12}{4} = 3$

25

$\because \frac{3x^2+3y^2}{xy} = \frac{3(x^2+y^2)}{xy}$

$= 3\left(\frac{x^2}{xy} + \frac{y^2}{xy}\right) = 3\left(\frac{x}{y} + \frac{y}{x}\right)$

$, \because \frac{x}{y} = \sqrt{3}-\sqrt{2}$

$\therefore \frac{y}{x} = \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \sqrt{3}+\sqrt{2}$

$\therefore$ The expression $= 3\left(\sqrt{3}-\sqrt{2}+\sqrt{3}+\sqrt{2}\right)$

$= 3\left(2\sqrt{3}\right) = 6\sqrt{3}$

26

$\because \frac{x^8y^9 - y}{(x+y)^5} = \frac{y(x^8y^8-1)}{(x+y)^5} \quad (1)$

$, x^8y^8 = (xy)^8 = \left[(\sqrt{7}+\sqrt{6})(\sqrt{7}-\sqrt{6})\right]^8$

$= 1^8 = 1$

, from (1) : $\therefore \frac{x^8y^9-y}{(x+y)^5} = \frac{(\sqrt{7}-\sqrt{6})(1-1)}{\left(\sqrt{7}+\sqrt{6}+\sqrt{7}-\sqrt{6}\right)^5}$

$= \frac{\text{zero}}{\left(2\sqrt{7}\right)^5} = \text{zero}$

Answers of Exercise 8

1

1. $\sqrt[3]{8\times 2} = 2\sqrt[3]{2}$
2. $\sqrt[3]{-27\times 2} = -3\sqrt[3]{2}$
3. $2\sqrt[3]{125\times 2} = 2\times 5\sqrt[3]{2} = 10\sqrt[3]{2}$
4. $\frac{2}{3}\sqrt[3]{-27\times 5} = \frac{2}{3}\times -3\sqrt[3]{5} = -2\sqrt[3]{5}$
5. $\sqrt[3]{\frac{1}{3}\times 27} = \sqrt[3]{9}$
6. $-2\sqrt[3]{\frac{2}{5}\times 125} = -2\sqrt[3]{50}$

2

1 $\sqrt[3]{2\times 32}=\sqrt[3]{64}=4$ 2 $\sqrt[3]{\frac{72}{9}}=\sqrt[3]{8}=2$

3 $\frac{4}{2}\sqrt[3]{\frac{-54}{-2}}=2\sqrt[3]{27}=2\times 3=6$

4 $\frac{1}{2}\times 6\sqrt[3]{10\times 100}=3\sqrt[3]{1000}=3\times 10=30$

5 $\sqrt[3]{\frac{2}{5}\times\frac{4}{25}}=\sqrt[3]{\frac{8}{125}}=\frac{2}{5}$

6 $\sqrt[3]{\frac{3}{4}\div\frac{2}{9}}=\sqrt[3]{\frac{3}{4}\times\frac{9}{2}}=\sqrt[3]{\frac{27}{8}}=\frac{3}{2}$

3

1 $2\sqrt[3]{2}-\sqrt[3]{2}=\sqrt[3]{2}$ 2 $5-2\sqrt[3]{3}$

3 $3\sqrt[3]{3}-2\sqrt[3]{3}=\sqrt[3]{3}$

4 $3\sqrt[3]{2}+2\sqrt[3]{2}-5\sqrt[3]{2}=\text{zero}$

5 $2\times 3\sqrt[3]{2}-5\sqrt[3]{2}+2\sqrt[3]{2}=3\sqrt[3]{2}$

6 $2\sqrt[3]{2}-\frac{1}{3}\times 3\sqrt[3]{2}-\sqrt[3]{2}=\text{zero}$

7 $2\sqrt[3]{2}+\sqrt[3]{250}=2\sqrt[3]{2}+5\sqrt[3]{2}=7\sqrt[3]{2}$

8 $2\sqrt[3]{3}-2\times 3\sqrt[3]{\frac{125}{9}}=2\sqrt[3]{3}-2\sqrt[3]{\frac{125}{9}\times 27}$

$=2\sqrt[3]{3}-2\times 5\sqrt[3]{3}=-8\sqrt[3]{3}$

4

1 The left hand side $=4\sqrt[3]{2}+2\sqrt[3]{2}-2\times 3\sqrt[3]{2}=\text{zero}$

= the right hand side.

2 The left hand side $=3\sqrt[3]{2}\times 2\sqrt[3]{2}\div(6\sqrt[3]{4})$

$=6\sqrt[3]{4}\div 6\sqrt[3]{4}=1=$ the right hand side.

5

1 $3\sqrt[3]{3}-2\sqrt[3]{3}-\sqrt[3]{27\times\frac{1}{9}}=3\sqrt[3]{3}-2\sqrt[3]{3}-\sqrt[3]{3}=\text{zero}$

2 $3\sqrt[3]{2}-4\sqrt[3]{\frac{1}{4}\times 8}+5\times 2\sqrt[3]{2}$

$=3\sqrt[3]{2}-4\sqrt[3]{2}+10\sqrt[3]{2}=9\sqrt[3]{2}$

3 $3\sqrt[3]{4}-2\sqrt[3]{4}-\sqrt[3]{\frac{4}{8}}=3\sqrt[3]{4}-2\sqrt[3]{4}-\frac{1}{2}\sqrt[3]{4}=\frac{1}{2}\sqrt[3]{4}$

4 $\sqrt[3]{3}-\sqrt[3]{24}+\sqrt[3]{27\times\frac{1}{9}}=\sqrt[3]{3}-2\sqrt[3]{3}+\sqrt[3]{3}=\text{zero}$

6

1 $\frac{7}{3}\times 3\sqrt{2}+3\sqrt[3]{2}-7\sqrt{2}+2\sqrt[3]{2}$

$=7\sqrt{2}+3\sqrt[3]{2}-7\sqrt{2}+2\sqrt[3]{2}=5\sqrt[3]{2}$

2 $3\sqrt{3}+\frac{1}{3}\times 3-3\sqrt{9\times\frac{1}{3}}-1$

$=3\sqrt{3}+1-3\sqrt{3}-1=\text{zero}$

3 $-2\sqrt[3]{2}+\frac{14}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}-2\sqrt{7}+3\sqrt[3]{2}$

$=-2\sqrt[3]{2}+2\sqrt{7}-2\sqrt{7}+3\sqrt[3]{2}=\sqrt[3]{2}$

4 $3\sqrt{2}+3\sqrt[3]{2}-\sqrt{\frac{216}{12}}-2\sqrt[3]{2}$

$=3\sqrt{2}+\sqrt[3]{2}-\sqrt{18}=3\sqrt{2}+\sqrt[3]{2}-3\sqrt{2}=\sqrt[3]{2}$

5 $5\sqrt{2}-\frac{1}{2}\times 10\sqrt{2}+\sqrt[3]{125}=5\sqrt{2}-5\sqrt{2}+5=5$

7

1 c 2 a 3 c 4 d 5 a

8

1 -2 2 9 3 250

4 $\frac{2}{3}\sqrt[3]{7}$ 5 $-\frac{1}{2}$ 6 1

9

1 $(\sqrt[3]{5}+1-\sqrt[3]{5}+1)^5=2^5=32$

2 $(\sqrt[3]{5}+1+\sqrt[3]{5}-1)^3=(2\sqrt[3]{5})^3=8\times 5=40$

10

$\because x-y=3+\sqrt[3]{6}-3+\sqrt[3]{6}=2\sqrt[3]{6}$

$,x+y=3+\sqrt[3]{6}+3-\sqrt[3]{6}=6$

$\therefore\left(\frac{x-y}{x+y}\right)^3=\left(\frac{2\sqrt[3]{6}}{6}\right)^3=\left(\frac{\sqrt[3]{6}}{3}\right)^3=\frac{(\sqrt[3]{6})^3}{3^3}$

$=\frac{6}{27}=\frac{2}{9}$

11

L.H.S. $=x^2+y^2=(x+y)^2-2xy$

$=(\sqrt[3]{2}+1+\sqrt[3]{2}-1)^2-2(\sqrt[3]{2}+1)(\sqrt[3]{2}-1)$

$=(2\sqrt[3]{2})^2-2((\sqrt[3]{2})^2-1)$

$=4\sqrt[3]{4}-2\sqrt[3]{4}+2=2\sqrt[3]{4}+2=$ R.H.S.

12

$$\frac{2}{\sqrt[3]{2}} = \frac{2 \times \sqrt[3]{4}}{\sqrt[3]{2} \times \sqrt[3]{4}} = \frac{2\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{2\sqrt[3]{4}}{2} = \sqrt[3]{4}$$

Another solution : $\frac{2}{\sqrt[3]{2}} = \frac{\sqrt[3]{8}}{\sqrt[3]{2}} = \sqrt[3]{\frac{8}{2}} = \sqrt[3]{4}$

Answers of Exercise 9

1

1 125 2 96 3 $4\ell^2$ 4 $6\ell^2$ 5 $8\ell^3$

2

$\because$ Area of one face $= \frac{36}{4} = 9$ cm^2.

$\therefore$ The edge length of the cube $= \sqrt{9} = 3$ cm.

1 Its total area $= 6\ell^2 = 6 \times 3^2 = 54$ cm^2.

2 Its volume $= \ell^3 = 3^3 = 27$ cm^3.

3

The edge length of the cube $= \frac{12}{4} = 3$ cm.

1 Its volume $= \ell^3 = 3^3 = 27$ cm^3.

2 Its lateral area $= 4\ell^2 = 4 \times 3^2 = 36$ cm^2.

4

The edge length of the cube $= \frac{60}{12} = 5$ cm.

1 Its volume $= \ell^3 = 5^3 = 125$ cm^3.

2 Its total area $= 6\ell^2 = 6 \times 5^2 = 150$ cm^2.

5

1 d 2 c 3 d 4 a 5 b

6 d 7 b 8 a

6

1 The volume of the cuboid $= x \times y \times z$

$= 9 \times 10 \times 5 = 450$ cm^3.

2 Its lateral area $= 2(x + y) \times z = 2(9 + 10) \times 5$

$= 190$ cm^2.

3 Its total area $= 2(xy + yz + zx)$

$= 2(9 \times 10 + 10 \times 5 + 5 \times 9)$

$= 2(90 + 50 + 45) = 370$ cm^2.

7

The volume $= x \times y \times z = \sqrt{2} \times \sqrt{3} \times \sqrt{6} = 6$ cm^3.

8

$\because$ The lateral area = the perimeter of the base × height

$\therefore$ The height $= \frac{480}{4 \times 10} = 12$ cm.

9

The area of the base $= \frac{\text{volume}}{\text{height}} = \frac{720}{5} = 144$ cm^2.

$\therefore$ The side length of the base $= \sqrt{144} = 12$ cm.

$\therefore$ The total area $= 2(xy + yz + zx)$

$= 2 \times (12 \times 12 + 12 \times 5 + 12 \times 5)$

$= 528$ cm^2.

10

$\because$ The area of the face of the cube $= \frac{294}{6} = 49$ cm^2.

$\therefore$ The edge length $= \sqrt{49} = 7$ cm.

$\therefore$ The volume of the cube $= \ell^3 = 7 \times 7 \times 7 = 343$ cm^3.

, $\because$ the volume of the cuboid $= x \times y \times z$

$= 7\sqrt{2} \times 5\sqrt{2} \times 5$

$= 350$ cm^3.

$\therefore$ The volume of the cuboid is greater than the volume of the cube

11

The volume of the cuboid $= x \times y \times z = 17 \times 7 \times 4$

$= 476$ cm^3.

, the total area $= 2(x + y) \times z + xy$

$= 2(17 + 7) \times 4 + 17 \times 7$

$= 192 + 119 = 311$ cm^2.

12

The circumference of the circle $= 2\pi r$

$= 2 \times \frac{22}{7} \times 10.5$

$= 66$ cm.

The area of the circle $= \pi r^2 = \frac{22}{7} \times (10.5)^2$

$= 346.5$ cm^2.

13

$\because$ The area of the circle $= \pi r^2$

$\therefore 154 = \frac{22}{7} r^2 \qquad \therefore r^2 = \frac{154 \times 7}{22} = 49$

$\therefore r = \sqrt{49} = 7$ cm.

The circumference $= 2\pi r = 2 \times \frac{22}{7} \times 7 = 44$ cm.

The diameter length $= 2 \times 7 = 14$ cm.

14

$\because$ The area of the circle $= \pi r^2$

$\therefore 64\pi = \pi r^2 \qquad \therefore r^2 = 64$

$\therefore r = \sqrt{64} = 8$ cm.

The circumference of the circle $= 2\pi r$

$= 2 \times 3.14 \times 8 \approx 50$ cm.

15

$\because$ The area of the circle $= 2 \times 12.32 = 24.64$ cm^2.

$\therefore \pi r^2 = 24.64 \qquad \therefore r^2 = 24.64 \times \frac{7}{22} = 7.84$

$\therefore r = \sqrt{7.84} = 2.8$ cm.

$\therefore$ The perimeter of the figure $= \pi r + 2r$

$= \frac{22}{7} \times 2.8 + 2 \times 2.8 = 14.4$ cm.

16

The area of the shaded part = the area of the great circle – the area of the small circle $= \pi r_1^2 - \pi r_2^2$

$= \pi \times 25 - \pi \times 9 = 16\pi$ cm^2.

17

Let the radius length of the circle $= x$ cm.

$\therefore$ The side length of the square $= 2x$ cm.

$\therefore$ The area of the shaded part

$= \frac{\text{the area of the square} - \text{the area of the circle}}{2}$

$= \frac{4x^2 - \pi x^2}{2} = 10\frac{5}{7} = \frac{75}{7}$

$\therefore 4x^2 - \frac{22}{7}x^2 = \frac{75}{7} \times 2$

$\therefore \frac{6}{7}x^2 = \frac{150}{7} \qquad \therefore x^2 = \frac{150}{7} \times \frac{7}{6} = 25$

$\therefore x = \sqrt{25} = 5$ cm.

$\therefore$ The perimeter of the shaded part

$= \frac{1}{2}$ the circumference of the circle

$+ \frac{1}{2}$ the perimeter of the square.

$= \frac{22}{7} \times 5 + 20 = 35\frac{5}{7}$ cm.

18

The volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (14)^2 \times 20 = 12320$ cm^3.

The total area of the cylinder $= 2\pi rh + 2\pi r^2$

$= 2 \times \frac{22}{7} \times 14 \times 20 + 2 \times \frac{22}{7} \times (14)^2 = 2992$ cm^2.

19

$\because$ The volume of the cylinder $= \pi r^2 h$

$\therefore 924 = \frac{22}{7} \times r^2 \times 6$

$\therefore r^2 = \frac{924 \times 7}{6 \times 22} = 49 \qquad \therefore r = 7$ cm.

$\therefore$ The lateral area $= 2\pi r h = 2 \times \frac{22}{7} \times 7 \times 6$

$= 264$ cm^2.

20

$\because$ The volume of the cylinder $= \pi r^2 h$

$\therefore 7536 = 3.14 \times r^2 \times 24$

$\therefore r^2 = \frac{7536}{3.14 \times 24} = 100 \qquad \therefore r = 10$ cm.

$\therefore$ The total area $= 2\pi r h + 2\pi r^2$

$= 2 \times 3.14 \times 10 \times 24 + 2 \times 3.14 \times (10)^2 = 2135.2$ cm^2.

21

The volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (7)^2 \times 10 = 1540$ cm^3.

The volume of the cube $= \ell^3 = (11)^3 = 1331$ cm^3.

$\therefore$ The volume of the cylinder is greater than the volume of the cube.

22

1 $2\pi r h$, $\pi r^2 h$ 2 2 cm. 3 20 cm.

4 r cm. 5 r cm.

23

The circumference of the base $= 2\pi r$

$\therefore 44 = 2 \times \frac{22}{7} \times r$

$\therefore r = \frac{44}{2} \times \frac{7}{22} = 7$ cm.

$\therefore$ The volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (7)^2 \times 25 = 3850$ cm^3.

24

The lateral area $= 2\pi r h$

$\therefore 52 = 2 \times \frac{22}{7} \times 4 \times h$

$\therefore h = \frac{52 \times 7}{2 \times 22 \times 4} = \frac{91}{44}$ cm.

$\therefore$ The volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 4^2 \times \frac{91}{44} = 104$ cm^3.

25

$\because$ The volume of the cylinder $= \pi r^2 h$

$\therefore 36\pi = \pi \times r^2 \times 4$

$\therefore r^2 = \frac{36}{4} = 9$ $\therefore r = 3$ cm.

$\therefore$ The edge length of the cube = 3 cm.

$\therefore$ The total area of the cube $= 6\ell^2 = 6 \times 3^2 = 54$ cm^2.

26

$\because$ The volume of the cylinder $= \pi r^2 h$

$\because h = r$

$\therefore$ The volume of the cylinder $= \pi r^3$

$\therefore 72\pi = \pi r^3$ $\therefore r^3 = 72$ $\therefore r = 2\sqrt[3]{9}$

$\therefore$ The height of the cylinder $= 2\sqrt[3]{9}$ cm.

27

The circumference of the base of the cylinder = BC

$\therefore 2\pi r = 44$ $\therefore r = \frac{44 \times 7}{2 \times 22} = 7$ cm.

The height = AB = 10 cm.

The volume $= \pi r^2 h = \frac{22}{7} \times (7)^2 \times 10 = 1540$ cm^3.

28

The volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (2.1)^3 = 38.808$ cm^3.

The surface area of the sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times (2.1)^2 = 55.44$ cm^2.

29

$\because$ The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore 4188 = \frac{4}{3} \times 3.141 \times r^3$

$\therefore r^3 = \frac{4188 \times 3}{4 \times 3.141} = 1000$ $\therefore r = 10$ cm.

30

The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore 562.5\pi = \frac{4}{3}\pi r^3$ $\therefore r^3 = \frac{562.5 \times 3}{4} = 421.875$

$\therefore r = \sqrt[3]{421.875} = 7.5$ cm.

$\therefore$ The surface area of the sphere

$= 4\pi r^2 = 4 \times \pi \times (7.5)^2 = 225\pi$ cm^2.

31

1 b	2 a	3 c	4 c
5 b	6 d	7 b	

32

The volume of the cylinder $= \pi r^2 h$

$= \pi \times (4)^2 \times 18 = 288\pi$ cm^3.

$\because$ The volume of the cylinder = the volume of the sphere.

$\therefore$ The volume of the sphere $= 288\pi$ cm^3.

$\therefore \frac{4}{3}\pi r^3 = 288\pi$

$\therefore r^3 = \frac{288 \times 3}{4} = 216$

$\therefore$ The radius length of the sphere = 6 cm.

33

$\because$ The volume of the cylinder $= \pi r^2 h$

$\therefore 7536 = 3.14 \times r^2 \times 24$

$\therefore r^2 = \frac{7536}{3.14 \times 24} = 100$

$\therefore r = \sqrt{100} = 10$ cm.

, $\because$ the radius length of the sphere

= the radius length of the cylinder base

$\therefore$ The volume of the sphere $= \frac{4}{3} \times 3.14 \times (10)^3$

$= 4186\frac{2}{3}$ cm^3.

34

The volume of the cuboid $= 77 \times 24 \times 21 = 38808$ cm^3.

$\because$ The volume of the cuboid = the volume of the sphere

$\therefore 38808 = \frac{4}{3}\pi r^3$ $\therefore r^3 = \frac{38808 \times 3 \times 7}{4 \times 22} = 9261$

$\therefore r = \sqrt[3]{9261} = 21$ cm.

35

The volume of the sphere $= \frac{4}{3}\pi (3)^3 = 36\pi$ cm^3.

$\because$ The volume of the cylinder = the volume of the sphere

$\therefore$ The volume of the cylinder $= 36\pi$ cm^3.

$\therefore \pi r^2 h = 36\pi$ $\therefore 9\pi h = 36\pi$ $\therefore h = 4$ cm.

36

$\because$ The sphere touches the six faces of the cube

$\therefore$ The edge length of the cube = 2 r

$\because$ The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore 36\pi = \frac{4}{3}\pi r^3$ $\therefore r^3 = \frac{36 \times 3}{4} = 27$

$\therefore r = 3$ cm.

$\therefore$ The edge length of the cube $= 2 \times 3 = 6$ cm.

$\therefore$ The volume of the cube $= \ell^3 = 6^3 = 216$ cm^3.

37

$\because$ The volume of the sphere
= the volume of 8 small spheres

$\therefore \frac{4}{3}\pi r_1^3 = 8 \times \frac{4}{3}\pi r_2^3$ $\quad\therefore (16.8)^3 = 8 r_2^3$

$\therefore r_2^3 = \frac{(16.8)^3}{8}$ $\quad\therefore r_2 = \frac{16.8}{2} = 8.4$ cm.

38

The volume of the sphere $= \frac{4}{3}\pi (15)^3 = 4500\pi$ cm^3.

$\because$ The volume of the cylinder

$= \frac{4}{9}$ the volume of the sphere

$\therefore \pi r^2 h = \frac{4}{9} \times 4500\pi$ $\quad\therefore r^2 \times 20 = 2000$

$\therefore r^2 = \frac{2000}{20} = 100$ $\quad\therefore r = \sqrt{100} = 10$ cm.

39

$\because$ The sum of lengths of all edges = 52 cm.

, the sum of the 4 heights $= 3 \times 4 = 12$ cm.

$\therefore$ The sum of the remained edges $= 52 - 12 = 40$ cm.

$\because$ The base is a square.

$\therefore$ The side length of the square $= \frac{40}{8} = 5$ cm.

$\therefore$ The volume $= 5 \times 5 \times 3 = 75$ cm^3.

40

The volume of the metal = the outer volume
– the inner volume $= \frac{4}{3}\pi r_1^3 - \frac{4}{3}\pi r_2^3$

$= \frac{4}{3} \times \pi\left((3.5)^3 - (2.1)^3\right) = \frac{88}{21} \times 33.614 \approx 140.859$ cm^3.

$\therefore$ The mass of the metal $= 140.859 \times 20 \approx 2817$ gm.

Answers of Exercise 10

1

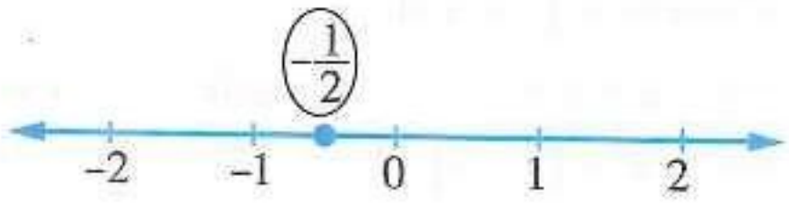
1 $\because x = -5$ $\quad\therefore$ The S.S. $= \{-5\}$

2 $\because 5x = 1 - 6 = -5$ $\quad\therefore x = \frac{-5}{5} = -1$

$\therefore$ The S.S. $= \{-1\}$

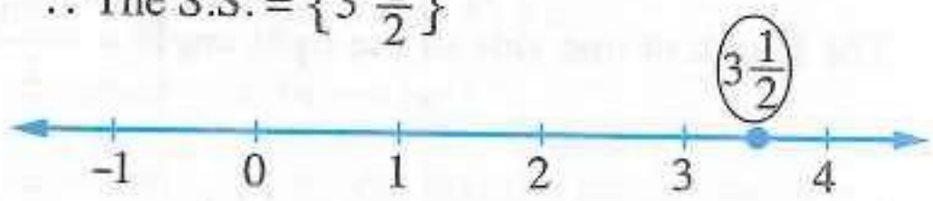
3 $\because 2x = 3 - 4 = -1$ $\quad\therefore x = -\frac{1}{2}$

$\therefore$ The S.S. $= \{-\frac{1}{2}\}$



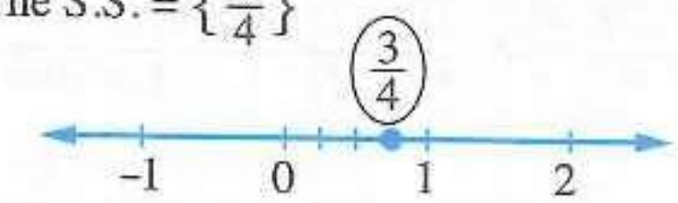
4 $\because 2x = 4 + 3 = 7$ $\quad\therefore x = \frac{7}{2} = 3\frac{1}{2}$

$\therefore$ The S.S. $= \{3\frac{1}{2}\}$



5 $\because 4x - 1 = 2$ $\quad\therefore 4x = 2 + 1 = 3$ $\quad\therefore x = \frac{3}{4}$

$\therefore$ The S.S. $= \{\frac{3}{4}\}$



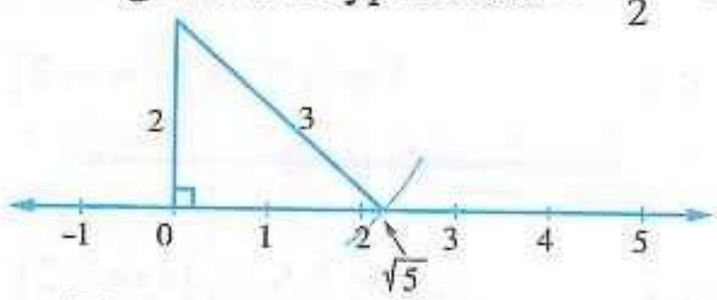
6 $\because \sqrt{5}x = 4 + 1 = 5$

$\therefore x = \frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$

$\therefore$ The S.S. $= \{\sqrt{5}\}$

The length of one side of the right angle $= \frac{5-1}{2} = 2$

$\therefore$ The length of the hypotenuse $= \frac{5+1}{2} = 3$

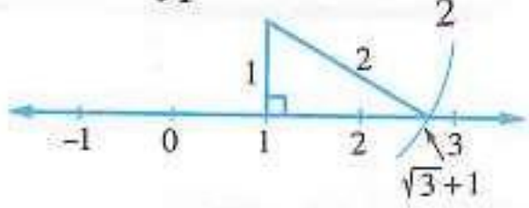


7 $\because x = \sqrt{3} + 1$ $\quad\therefore$ The S.S. $= \{\sqrt{3} + 1\}$

The length of one side of the right angle

$= \frac{3-1}{2} = 1$

The length of the hypotenuse $= \frac{3+1}{2} = 2$



8 $\because 2 - \sqrt{6}x = 8$ $\quad\therefore -\sqrt{6}x = 8 - 2 = 6$

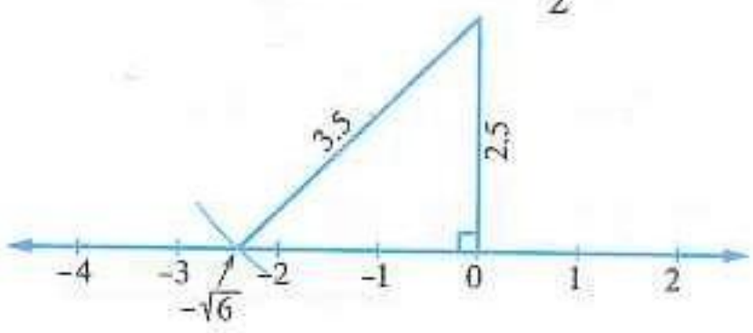
$\therefore x = -\frac{6}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{-6\sqrt{6}}{6} = -\sqrt{6}$

$\therefore$ The S.S. $= \{-\sqrt{6}\}$

The length of one side of the right angle $= \frac{6-1}{2}$

$= 2.5$

The length of the hypotenuse $= \frac{6+1}{2} = 3.5$

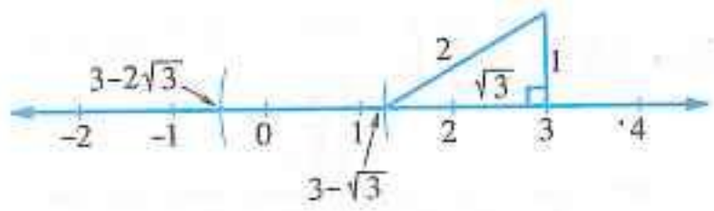


9 $\because x = 3 - 2\sqrt{3}$ $\therefore$ The S.S. $= \{3 - 2\sqrt{3}\}$

The length of one side of the right angle $= \frac{3-1}{2}$

$= 1$

The length of the hypotenuse $= \frac{3+1}{2} = 2$



2

1 a 2 c 3 c 4 d 5 a 6 d

3

1 $\because x > 3$ $\therefore$ The S.S. $=]3 , \infty[$

2 $\because x \le 2$ $\therefore$ The S.S. $=]-\infty , 2]$

3 $\because x \le 2$ $\therefore$ The S.S. $=]-\infty , 2]$

4 $\because -x > -2$ $\therefore x < 2$

$\therefore$ The S.S. $=]-\infty , 2[$

5 $\because 2x \ge -2$ $\therefore x \ge -1$

$\therefore$ The S.S. $= [-1 , \infty[$

6 $\because -5x < 5$ $\therefore x > -1$

$\therefore$ The S.S. $=]-1 , \infty[$

7 $\because \frac{1}{2}x \le 1$ $\therefore x \le 2$

$\therefore$ The S.S. $=]-\infty , 2]$

8 $\because -2x \le 4$ $\therefore x \ge -2$

$\therefore$ The S.S. $= [-2 , \infty[$

4

1 $\because 1 < x \le 4$ $\therefore$ The S.S. $=]1 , 4]$

2 $\because -8 < x < 6$ $\therefore$ The S.S. $=]-8 , 6[$

3 $\because 3 \ge x > -3$ $\therefore$ The S.S. $=]-3 , 3]$

4 $\because -4 < -x \le -2$ $\therefore 4 > x \ge 2$

$\therefore$ The S.S. $= [2 , 4[$

5 $\because -2 \le x + 1 \le 3$ $\therefore -3 \le x \le 2$

$\therefore$ The S.S. $= [-3 , 2]$

6 $\because 2 < -x \le 6$ $\therefore -2 > x \ge -6$

$\therefore$ The S.S. $= [-6 , -2[$

7 $\because -9 \le 3x \le 3$ $\therefore -3 \le x \le 1$

$\therefore$ The S.S. $= [-3 , 1]$

8 $\because 3 < 2x - 1 < 5$ $\therefore 4 < 2x < 6$

$\therefore 2 < x < 3$ $\therefore$ The S.S. $=]2 , 3[$

9 $\because -1 < \frac{1}{2}x \le 2$ $\therefore -2 < x \le 4$

$\therefore$ The S.S. $=]-2 , 4]$

10 By multiplying by 3 $\therefore 0 \le -2x + 6 < 12$

$\therefore -6 \le -2x < 6$ $\therefore 3 \ge x > -3$

$\therefore$ The S.S. $=]-3 , 3]$

5 Represent by yourself the S.S. on the number line :

1 $\because 3x - 2x < 4$ $\therefore x < 4$

$\therefore$ The S.S. $=]-\infty , 4[$

2 $\because 7x - 4x \ge 9$ $\therefore 3x \ge 9$ $\therefore x \ge 3$

$\therefore$ The S.S. $= [3 , \infty[$

3 $\because 5x - 2x < 9 + 3 \quad \therefore 3x < 12 \quad \therefore x < 4$

$\therefore$ The S.S. $=]-\infty , 4[$

4 $\because 7x - 5x \geq -8 + 12 \quad \therefore 2x \geq 4 \quad \therefore x \geq 2$

$\therefore$ The S.S. $= [2 , \infty[$

5 $\because x + x \leq 3 + 1 \quad \therefore 2x \leq 4 \quad \therefore x \leq 2$

$\therefore$ The S.S. $=]-\infty , 2]$

6 $\because -x + 2x \geq -3 - 1 \quad \therefore x \geq -4$

$\therefore$ The S.S. $= [-4 , \infty[$

6

Represent by yourself the S.S. on the number line :

1 $\because x + 3 - x \geq 2x - x \geq x - 2 - x$

$\therefore 3 \geq x \geq -2 \quad \therefore$ The S.S. $= [-2 , 3]$

2 $\because -x + x < x + x < 4 - x + x \quad \therefore 0 < 2x < 4$

$\therefore 0 < x < 2 \quad \therefore$ The S.S. $=]0 , 2[$

3 $\because 4x - 4x \leq 5x + 2 - 4x < 4x + 3 - 4x$

$\therefore 0 \leq x + 2 < 3 \quad \therefore -2 \leq x < 1$

$\therefore$ The S.S. $= [-2 , 1[$

4 $\because x - 1 - x < 3x - 1 - x \leq x + 1 - x$

$\therefore -1 < 2x - 1 \leq 1 \quad \therefore 0 < 2x \leq 2$

$\therefore 0 < x \leq 1 \quad \therefore$ The S.S. $=]0 , 1]$

5 $\because 2 + 2x - 2x \leq 3x + 3 - 2x < 5 + 2x - 2x$

$\therefore 2 \leq x + 3 < 5 \quad \therefore -1 \leq x < 2$

$\therefore$ The S.S. $= [-1 , 2[$

6 By multiplying by 6

$\therefore 3x - 4 < 6x + 6 < 3x + 9$

$\therefore -4 < 3x + 6 < 9$

$\therefore -10 < 3x < 3 \quad \therefore -\frac{10}{3} < x < 1$

$\therefore$ The S.S. $=]-\frac{10}{3} , 1[$

7

1 ≥ 3 2 < 3 3 < -3

4 $\geq -\frac{3}{2}$ 5 $\leq 2\sqrt{2}$ 6 $]2 , 4[$

7 $]-2 , 5]$ 8 $]2 , \infty[$ 9 6

8

1 a 2 b 3 c 4 c 5 c

9

$\because$ The weight of one box = 45 kg.

Let the number of boxes be x

, $\because$ the maximum weight that the lift can carry is 2200 kg.

$\therefore$ The weight of boxes $\leq$ the maximum weight that the lift can carry.

$\therefore 45x \leq 2200 \quad \therefore x \leq 48\frac{8}{9}$

$\therefore$ The maximum number of boxes can the lift carry in one time is 48 boxes.

10

$\because -4 < -2x < 2 \quad \therefore 2 > x > -1 \quad \therefore$ The S.S. $=]-1 , 2[$

$\because \sqrt{3} \approx 1.7 \quad \therefore \sqrt{3} \in]-1 , 2[$

11

$\because a + 3 \leq x \leq b + 3 \quad \therefore$ The S.S. $= [a + 3 , b + 3]$

$\therefore [4 , 7] = [a + 3 , b + 3]$

$\therefore a + 3 = 4 \quad \therefore a = 1$

$, b + 3 = 7 \quad \therefore b = 4$

12

$\because \frac{1}{5} \leq \frac{2x + 1}{5} \leq 1 \quad \therefore 1 \leq 2x + 1 \leq 5$

$\therefore 0 \leq 2x \leq 4 \quad \therefore 0 \leq x \leq 2$

$\therefore$ The S.S. $= [0 , 2] \quad \therefore m = 0 , m + n = 2 \quad \therefore n = 2$

13

$\because 5 \leq \frac{2x}{3} + 1 \leq 7 \quad \therefore 4 \leq \frac{2x}{3} \leq 6$

$\therefore 12 \leq 2x \leq 18 \quad \therefore 6 \leq x \leq 9$

$\therefore 4 \leq x - 2 \leq 7 \quad \therefore$ The smallest value of $x - 2$ is 4

14

Multiply both sides by $(\sqrt{3} - \sqrt{5})$

$\therefore x \leq (\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{5})$

"Note that the sign changed because $(\sqrt{3} - \sqrt{5})$ is a negative number because $\sqrt{3} < \sqrt{5}$"

$\therefore x \leq -2 \quad \therefore$ The S.S. $=]-\infty , -2]$

Answers of unit two

Answers of Exercise 11

1 (5 , 14) , (2 , 5) , (0 , – 1) , (– 3 , – 10)

2 The ordered pair (– 1 , 3) satisfies the relation.

3
1. (1 , – 3) , (2 , – 1) , (3 , 1) , (5 , 5)
2. (0 , 5) , (2 , 6) , (4 , 7) , (6 , 8)
3. (0 , 2) , (3 , 2) , (5 , 2) , (– 4 , 2)
4. (2.5 , 7) , (2.5 , 3) , (2.5 , – 7) , (2.5 , 4)

There are other solutions.

4

1.

x	0	1	2	3
y	1	5	9	13

2.

x	– 4	– 3	– 2
y	– 5	0	5

3.

a	1	4	3
b	– 3	0	– 1

4.

a	2	5	– 1
b	– 1	0	– 2

5 1. 7 2. – 9 3. zero 4. – 1

6 ∵ (3 , 6) satisfies the relation $y = k\,x$

∴ $6 = 3\,k$ ∴ $k = 2$

7 ∵ (3 , 1) satisfies the relation $y - 3\,x = a$

∴ $1 - 3 \times 3 = a$ ∴ $a = -8$

8 ∵ (– 3 , 2) satisfies the relation $3\,x + b\,y = 1$

∴ $3 \times (-3) + b \times 2 = 1$ ∴ $2\,b = 9 + 1$

∴ $2\,b = 10$ ∴ $b = 5$

9 ∵ (3 , a) satisfies the relation $y - 2\,x = 4$

∴ $a - 2 \times 3 = 4$ ∴ $a = 10$

10 ∵ (k , 2 k) satisfies the relation $x + y = 15$

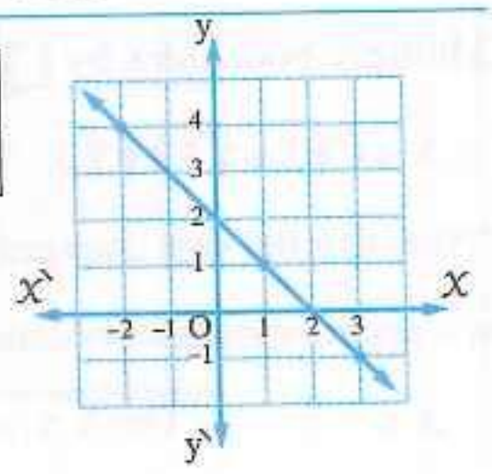
∴ $k + 2\,k = 15$ ∴ $3\,k = 15$ ∴ $k = 5$

11 1. $x = 3$ 2. y = zero

12

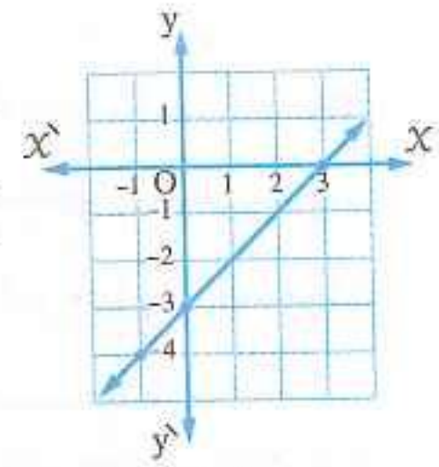
1.

x	1	3	– 2
y	1	– 1	4



2.

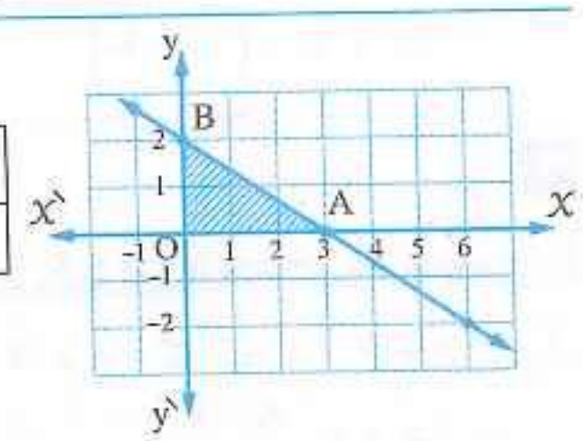
x	0	3	– 1
y	– 3	0	– 4



From 3 to 8 represent the relations graphically by yourself.

13 $y = -\frac{2}{3}x + 2$

x	0	3	6
y	2	0	– 2



From the graph :

The area of Δ OAB $= \frac{1}{2} \times 3 \times 2 = 3$ square units.

14 ∵ The straight line intersects x-axis at (3 , b)

∴ $b = 0$

∴ (3 , 0) satisfies the relation $2\,x - y = a$

∴ $2 \times 3 - 0 = a$ ∴ $a = 6$

15

1. b	2. c	3. b	4. b
5. d	6. a	7. b	8. a
9. d	10. c	11. c	12. c
13. c	14. c		

16 Let the first number be x and the second be y

∴ $2\,x + y = 12$ ∴ $y = 12 - 2\,x$

∵ The two numbers are even natural numbers.

∴ x has the values 0 , 2 , 4 , 6 then we can register the different possibilities to the two numbers in the following table :

x	0	2	4	6
y	12	8	4	0

17 Let the length of the rectangle = x cm. and the width = y cm. ∴ $x > y$

, ∵ the perimeter of the rectangle = 14 cm.

∴ $2\,(x + y) = 14$ ∴ $x + y = 7$

We can record the different possibilities of the length and the width of the rectangle in the opposite table :

x	6	5	4
y	1	2	3

18 Let the number of bills of L.E. 5 be x , then its value = $5x$

and let the number of bills of L.E. 20 be y , then its value = 20 y

$\therefore 5x + 20y = 65$ where x and y are natural numbers.

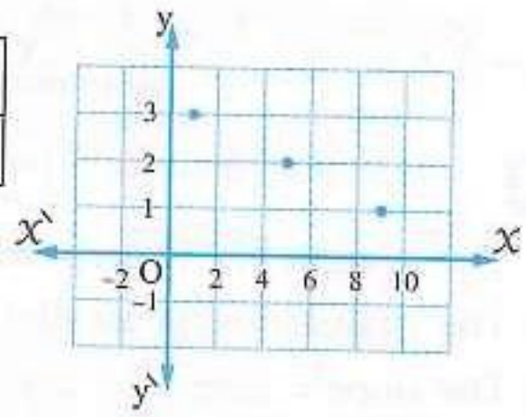
$\therefore x + 4y = 13$ $\qquad \therefore y = \frac{13 - x}{4}$

$\because x \leq 10$, $(13 - x)$ is divisible by 4

i.e. x has the values 9 , 5 and 1

, then we can write the different possibilities in the following table :

x	1	5	9
y	3	2	1



19 Let the store sold in this week x computer's table and y chairs.

$\therefore 100x + 50y = 500$

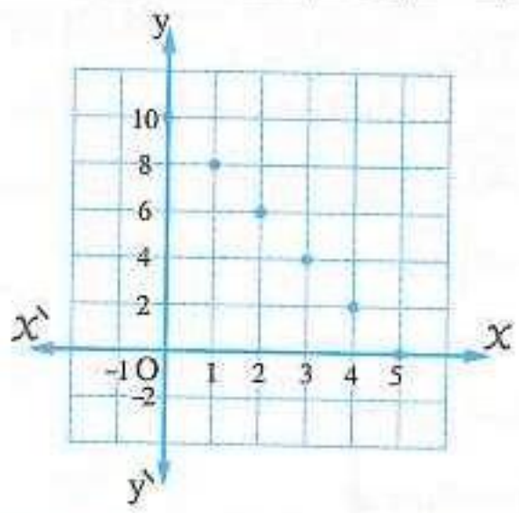
where x and y are two natural numbers.

$\therefore 2x + y = 10$ $\qquad \therefore y = 10 - 2x$

$\therefore x$ is not more than 5

$\therefore$ We can write the expectations which represent the number of computer's tables and the number of chairs in the following table :

x	0	1	2	3	4	5
y	10	8	6	4	2	0



20 Let the length of any of the two congruent sides in the triangle be x cm. and the length of the third side be y cm.

$\because$ the perimeter of the triangle = 19

$\therefore 2x + y = 19$ $\qquad \therefore y = 19 - 2x$

$\therefore x$ and y are positive integers then x is not more than 9 and from the inequality of the triangle then x has the values 5 , 6 , 7 , 8 and 9

, then we can write all the possibilities in the following table :

x	5	6	7	8	9
y	9	7	5	3	1

Answers of Exercise 12

1

Figure (1) the slope is positive.

Figure (2) the slope is negative.

Figure (3) the slope is undefined.

Figure (4) the slope equals zero.

2

1 negative 2 zero 3 undefined 4 positive

3

1 zero 2 undefined 3 x-axis 4 $\overleftrightarrow{BC}$ or $\overleftrightarrow{AC}$

4

1 The slope of $\overleftrightarrow{AB} = \frac{4-3}{3-1} = \frac{1}{2}$

2 The slope of $\overleftrightarrow{AB} = \frac{0-2}{5-1} = -\frac{1}{2}$

3 The slope of $\overleftrightarrow{AB} = \frac{5-2}{6-3} = 1$

4 The slope of $\overleftrightarrow{AB} = \frac{-1+1}{4-2} = 0$

5 The slope of $\overleftrightarrow{AB} = \frac{3-3}{2-1} = 0$

6 The slope of $\overleftrightarrow{AB} = \frac{4-2}{5-5} = \frac{2}{\text{zero}}$ undefined.

7 The slope of $\overleftrightarrow{AB} = \frac{2+1}{3-3} = \frac{3}{\text{zero}}$ undefined.

8 The slope of $\overleftrightarrow{AB} = \frac{1+2}{4-3} = \frac{3}{1} = 3$

9 The slope of $\overleftrightarrow{AB} = \frac{1-3}{2+1} = \frac{-2}{3}$

10 The slope of $\overleftrightarrow{NK} = \frac{-7+2}{-1-4} = \frac{-5}{-5} = 1$

11 The slope of $\overleftrightarrow{EO} = \frac{0+1}{0+3} = \frac{1}{3}$

12 The slope of $\overleftrightarrow{AB} = \frac{-1+9}{-1+6} = \frac{8}{5}$

5

1 Taking the two points (0 , 0) , (1 , 2) which lie on the straight line we find that :

the slope $= \frac{2-0}{1-0} = 2$

2 Taking the two points (0 , – 1) , (– 2 , 3) which lie on the straight line we find that :

the slope $= \frac{3-(-1)}{-2-0} = \frac{4}{-2} = -2$

6

$\because m(\angle M) = 45°$ $\therefore \Delta MNL$ is an isosceles triangle.

$\therefore ML = LN$

$\because$ The length of $\overline{ML} = 4$ units.

$\therefore$ The length of $\overline{LN} = 4$ units.

$\therefore N = (3 , 6)$

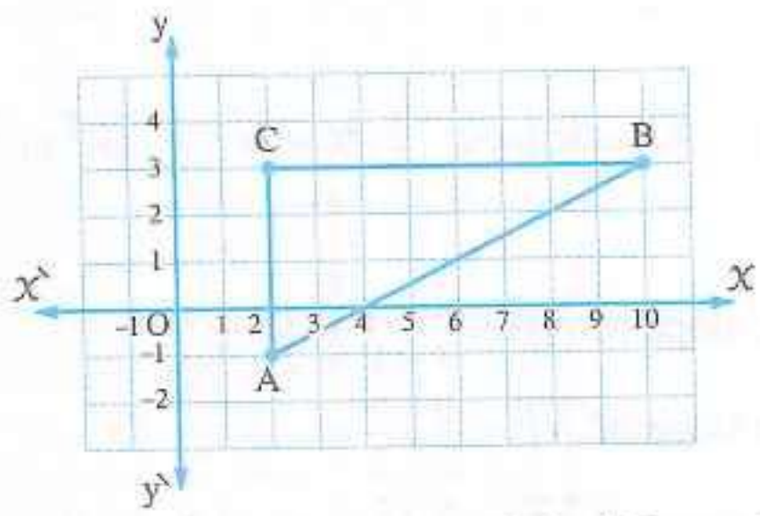
, the slope of $\overleftrightarrow{MN} = \frac{6-2}{3-7} = \frac{4}{-4} = -1$

7

the slope of $\overleftrightarrow{AB} = \frac{3+1}{10-2} = \frac{1}{2}$

the slope of $\overleftrightarrow{BC} = \frac{3-3}{2-10} =$ zero

the slope of $\overleftrightarrow{AC} = \frac{3+1}{2-2} = \frac{4}{0}$ (undefined)



from the graph we find that ΔABC is right-angled.

8

$\because$ The slope of the straight line which passes through the two points $(1 , 3)$ and $(3 , k)$ equals 3

$\therefore \frac{k-3}{3-1} = 3$ $\therefore \frac{k-3}{2} = 3$

$\therefore k - 3 = 6$ $\therefore k = 9$

9

$\because$ The slope of the straight line which passes through the two points $(3 , c)$ and $(5 , -2)$ equals -3

$\therefore \frac{-2-c}{5-3} = -3$ $\therefore \frac{-2-c}{2} = -3$

$\therefore -2 - c = -6$ $\therefore c = 4$

10

$-2 = \frac{2-4}{x-(-1)}$ $\therefore -2 = \frac{-2}{x+1}$

$\therefore x + 1 = 1$ $\therefore x = 0$

11

$\because \frac{-1-y}{3-(-2)} = -0.6$ $\therefore \frac{-1-y}{5} = -0.6$

$\therefore -1 - y = -3$ $\therefore y = 2$

12

$\because$ The straight line is parallel to x-axis

$\therefore$ The slope = zero $\therefore \frac{k-4}{2-3} =$ zero

$\therefore k - 4 =$ zero. $\therefore k = 4$

13

$\because$ The straight line is parallel to y-axis

$\therefore$ The slope is undefined

$\therefore x_2 - x_1 =$ zero $\therefore 6 - 2x = 0$

$\therefore -2x = -6$ $\therefore x = 3$

14

$\because$ The straight line is perpendicular to y-axis

$\therefore$ The straight line is parallel to x-axis

$\therefore$ The slope = zero $\therefore y_2 - y_1 =$ zero

$\therefore 3y - 6 = 0$ $\therefore 3y = 6$ $\therefore y = 2$

15

$\because$ The slope of the straight line passing through the two points $(-5 , 11)$ and $(0 , 8) = \frac{8-11}{0-(-5)} = \frac{-3}{5}$ (1)

$\because$ The slope of the straight line passing through the two points $(0 , 8)$ and $(5 , 5) = \frac{5-8}{5-0} = \frac{-3}{5}$ (2)

from (1) and (2) we find that the three points are collinear.

(lying on the straight line whose slope $= -\frac{3}{5}$)

16

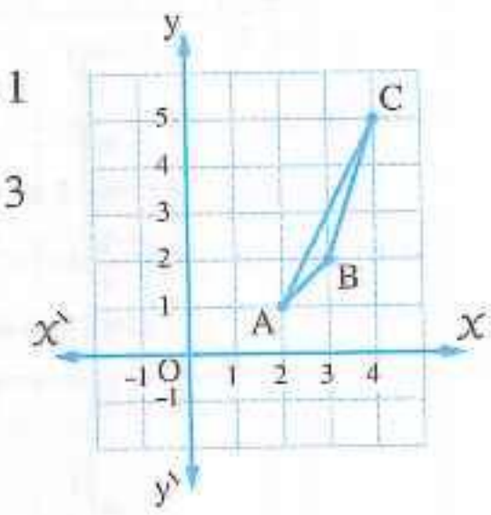
The slope of $\overleftrightarrow{AB} = \frac{2-1}{3-2} = 1$

, the slope of $\overleftrightarrow{BC} = \frac{5-2}{4-3} = 3$

and the slope of $\overleftrightarrow{AC}$

$= \frac{5-1}{4-2} = 2$

We observe that the three points are not collinear.



17

1 $\because$ The slope of $\overleftrightarrow{AB} = \frac{2-1}{2-1} = 1$

$\because$ The slope of $\overleftrightarrow{BC} = \frac{-3-2}{-3-2} = 1$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ and the point B is a common point.

$\therefore$ The points A , B and C are collinear.

2 ∵ The slope of $\overleftrightarrow{AB} = \frac{7-(-3)}{-6-4} = \frac{10}{-10} = -1$

∵ The slope of $\overleftrightarrow{BC} = \frac{-4-7}{5-(-6)} = \frac{-11}{11} = -1$

∴ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ and the point B is a common point.

∴ The points A , B and C are collinear.

3 ∵ The slope of $\overleftrightarrow{AB} = \frac{4-12}{2-(-2)} = \frac{-8}{4} = -2$

∵ The slope of $\overleftrightarrow{BC} = \frac{-4-4}{6-2} = \frac{-8}{4} = -2$

∴ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ and the point B is a common point.

∴ The points A , B and C are collinear.

18

1 ∵ The slope of $\overleftrightarrow{AB} = \frac{0-1}{3-2} = \frac{-1}{1} = -1$

∵ The slope of $\overleftrightarrow{BC} = \frac{-1-0}{5-3} = -\frac{1}{2}$

∴ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

∴ The points A , B and C are not collinear.

2 ∵ The slope of $\overleftrightarrow{AB} = \frac{1-2}{3-(-1)} = -\frac{1}{4}$

∵ The slope of $\overleftrightarrow{BC} = \frac{2-1}{7-3} = \frac{1}{4}$

∴ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

∴ The points A , B and C are not collinear.

3 ∵ The slope of $\overleftrightarrow{AB} = \frac{2-(-3)}{2-0} = \frac{5}{2}$

∵ The slope of $\overleftrightarrow{BC} = \frac{-3-2}{-3-2} = 1$

∴ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

∴ The points A , B and C are not collinear.

19

∵ The slope of $\overleftrightarrow{AB} = \frac{5-3}{2+1} = \frac{2}{3}$

∵ The slope of $\overleftrightarrow{BC} = \frac{5-1}{2-8} = -\frac{2}{3}$

∴ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

∴ $C \notin \overleftrightarrow{AB}$

20

∵ The slope of the straight line which passes through the two points (4 , 1) and (– 2 , 7)

$= \frac{7-1}{-2-4} = \frac{6}{-6} = -1$

∵ The slope of the straight line which passes through the two points (– 2 , 7) and (3 , y)

$= \frac{y-7}{3-(-2)} = \frac{y-7}{5}$

∵ The three points are collinear.

∴ $\frac{y-7}{5} = -1$ ∴ $y - 7 = -5$

∴ $y = -5 + 7$ ∴ $y = 2$

21

∵ The slope of the straight line which passes through the two points (3 , – 1) and (x , 1) equals $\frac{2}{3}$

∴ $\frac{1-(-1)}{x-3} = \frac{2}{3}$ ∴ $\frac{2}{x-3} = \frac{2}{3}$

∴ $x - 3 = 3$ ∴ $x = 6$

∵ The slope of the straight line which passes through the two points (3 , – 1) and (9 , y) equals $\frac{2}{3}$

∴ $\frac{y-(-1)}{9-3} = \frac{2}{3}$ ∴ $\frac{y+1}{6} = \frac{2}{3}$ ∴ $3y + 3 = 12$

∴ $3y = 9$ ∴ $y = 3$

Answers of Exercise 13

1

∵ The uniform velocity $= \frac{\text{the covered distance}}{\text{the taken time}}$

$= \frac{180}{3} = 60$ km./hr.

∴ The covered distance = the taken time × the uniform velocity = 60 × 5 = 300 km.

2

∵ The rate of consumption of fuel

$= \frac{\text{the amount of consumed fuel}}{\text{time}}$

$= \frac{2.47}{3} = \frac{247}{300}$ litre/hr.

∴ The consumed amount

= the rate of consumption × time

$= \frac{247}{300} \times 10 = 8\frac{7}{30}$ litre.

3

1 At the moment of starting the motion, the body is at a distance of 2 metres from the fixed point.

At t = 6 , the body is at a distance of 8 metres.

Taking the two points (0 , 2) and (6 , 8) on the straight line which represents the relation between t and d

$\therefore$ The slope $= \frac{8-2}{6-0} = \frac{6}{6} = 1$

It represents the velocity of the body within a going trip.

2 At the moment of starting the motion , the body is at t distance of 12 metres from the fixed point.

At t = 6 the body is at a distance of 2 metres. Taking the two points (0 , 12) and (6 , 2) on the straight line representing the relation between t and d

$\therefore$ The slope $= \frac{2-12}{6-0} = \frac{-10}{6} = -\frac{5}{3}$

It represents the velocity of the body within the returning back.

3 On starting the motion , the body is at a distance of 8 metres from the fixed point.

At t = 6 the body is at a distance of 8 metres.

$\because$ The straight line representing the relation is horizontal. $\therefore$ The slope = zero

It means that the body is rest.

4

Taking two points on the straight line representing the relation between t and d say (0 , 50) and (4 , 150)

$\therefore$ The uniform velocity = the slope of the straight line

$= \frac{150-50}{4-0} = \frac{100}{4} = 25$ km./hr.

5

1 Taking two points on the straight line representing the relation between t and d say (0 , 50) and (2 , 200)

$\therefore$ The unifrom velocity = the slope of the straight line $= \frac{200-50}{2-0} = 75$ km./hr.

2 from the graph :

The car is at a distance = 275 km. from the point 0 after passing 3 hours from the moment of beginning the motion.

6

1 The velocity within the first 3 hours = the slope of the striaght line $\overleftrightarrow{OB} = \frac{125}{3} = 41\frac{2}{3}$ km./hr.

The velocity within the next 2 hours = the slope of the straight line $\overleftrightarrow{BC} = \frac{125}{2} = 62\frac{1}{2}$ km./hr.

2 The average velocity within the all trip

$= \frac{\text{total distance}}{\text{total time}} = \frac{250}{5} = 50$ km./hr.

7

1 The velocity within the first 3 hours = the slope of the striaght line $= \frac{60-20}{3-0} = \frac{40}{3} = 13\frac{1}{3}$ km./hr.

2 The velocity within the next 4 hours = the slope of the straight line $= \frac{0-60}{7-3} = \frac{-60}{4} = -15$ km./hr.

The negative sign means that the bicycle returns back with velocity 15 km./hr.

The total distance = 40 + 60 = 100 km.

8

1 The slope of the straight line $\overleftrightarrow{AB}$

$= \frac{60-20}{4-0} = \frac{40}{4} = 10$

It means the increasing of the capital within the first 4 years with rate equals 10 thousands pounds/year.

The slope of $\overleftrightarrow{BC} = \frac{60-60}{6-4}$ = zero

It means constancy of the capital within the fifth and sixth years.

The slope of $\overleftrightarrow{CD} = \frac{50-60}{8-6} = \frac{-10}{2} = -5$

It means decreasing of the capital within the 7th and 8th years with rate = 5 thousands/year.

2 The starting capital of the company = 20 thousand pounds.

9

1 The slope of $\overleftrightarrow{AB} = \frac{125-50}{8-0} = \frac{75}{8} = 9\frac{3}{8}$

It means that the increase in height goes with respect to the increase in age.

The slope of $\overleftrightarrow{BC} = \frac{175-125}{18-8} = \frac{50}{10} = 5$

It means that the increase in height goes with respect to the increase in age but with a rate less than the rate within the first 8 years.

The slope of $\overleftrightarrow{CD} = \frac{175-175}{22-18} = 0$

It denotes the constancy in height inspite of the increase in age after 18 years.

2 $\because$ The height of the person at age 30 years = 175 cm. and the height of the person at age 8 years = 125 cm.

$\therefore$ The difference = 175 – 125 = 50 cm.

10

1 The greatest capacity of the tank = 70 litre.

2 The tank will be empty after 30 hours.

3 The remained fuel = 35 litre.

4 Taking the two points (0 , 70) , (30 , 0) on the straight line representing the relation.

∴ The rate of consumption of the fuel =

The slope of the straight line $= \frac{70-0}{0-30} = -2\frac{1}{3}$ litre/hr.

The negative sign means the rate of consumption.

i.e. The amount of fuel is consumpted with rate $2\frac{1}{3}$ litre/hr.

11

1 100 pages.

2 Taking the two points (0 , 100) and (3 , 40) on the straight line representing the relation.

we find that the rate of decreasing the number of pages = the slope of the straight line

$= \frac{40-100}{3-0} = \frac{-60}{3} = -20$ pages/hr.

The negative sign expresses the decreasing in the number of remained pages with rate 20 page/h.

3 ∵ The remained pages decreases with rate 20 page/h.

∵ There are 100 pages.

∴ The person finishes reading the book after $\frac{100}{20} = 5$ hours.

12

Let the covered distance be d km

The amount of the remained fuel in the tank be y litre.

At the beginning the covered distance = zero

The amount of remained fuel = 40 litre

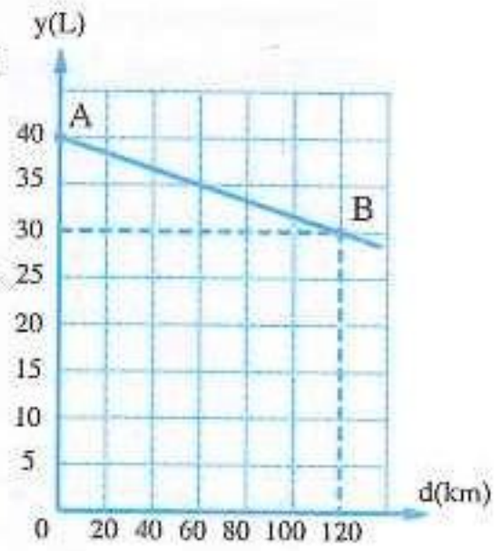
We express this by the point A (0 , 40)

∵ After covering distance.

The amount of remained fuel

$= \frac{3}{4} \times 40 = 30$ litre.

We expresses this by the point B (120 , 30)



∴ The rate of decreasing the amount of fuel

= the slope of $\overleftrightarrow{AB}$

$= \frac{30-40}{120-0} = \frac{-10}{120} = -\frac{1}{12}$

∴ The amount of fuel decreases with the rate one litre for every 12 km.

∴ The distance covered by the car when the tank becomes empty = 12 × 40 = 480 km.

13

1 100 km.

2 The train A took 2 hours

the train B took $2\frac{1}{2}$ hours.

3 The average speed $= \frac{\text{total distance}}{\text{total time}}$

with respect to the train A

The average speed $= \frac{100}{2} = 50$ km./hr.

with respect to the train B

the average speed $= \frac{100}{2.5} = 40$ km./hr.

4 It means that the train A was at rest from half past ten till half past eleven.

14

1 The velocity of the bicycle = the slope of the straight line $= \frac{200-125}{8-2} = 12.5$ km./h.

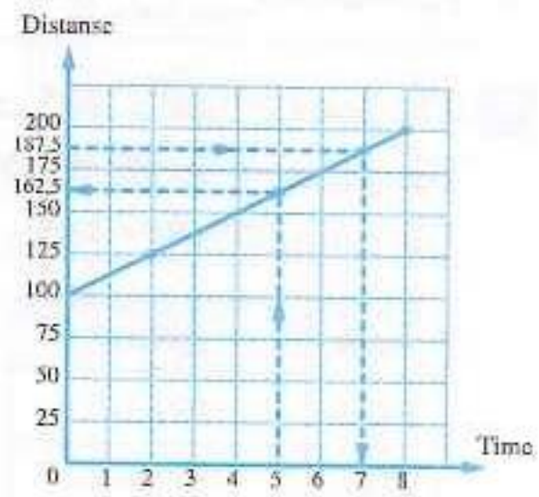
2 300 minutes = 5 hours ,

then the bicycle is at distance = 162.5 km.

3 7 hours.

4 From the graph.

the starting point is far from the fixed point = 100 km.



Answers of unit three

Answers of Exercise 14

1

Sets	Tallies	Freq.	Sets	Freq.
30 –	////	4	30 –	4
40 –	~~////~~	5	40 –	5
50 –	~~////~~ //	7	50 –	7
60 –	~~////~~ ///	8	60 –	8
70 –	~~////~~ /	6	70 –	6
80 –	////	4	80 –	4
90 –	~~////~~ /	6	90 –	6
Total		40	Total	40

The set which has the highest frequency is 60 –
The sets which have the lowest frequency are 80 – , 30 –

2

1

Sets	Tallies	Freq.	Sets	Freq.
20 –	///	3	20 –	3
24 –	//	2	24 –	2
28 –	~~////~~ /	6	28 –	6
32 –	~~////~~ //	7	32 –	7
36 –	~~////~~ ~~////~~ //	12	36 –	12
Total		30	Total	30

2 12 students.

3 The least height = 165 cm.
The greatest height = 200 cm.
The range = 200 – 165 = 35 cm.

Sets	Tallies	Freq.	Sets	Freq.
165 –	~~////~~ ///	8	165 –	8
170 –	~~////~~ ~~////~~	10	170 –	10
175 –	~~////~~ ~~////~~ ~~////~~	15	175 –	15
180 –	~~////~~ /	6	180 –	6
185 –	~~////~~ ~~////~~	10	185 –	10
190 –	////	4	190 –	4
195 –	/	1	195 –	1
200 –	/	1	200 –	1
Total		55	Total	55

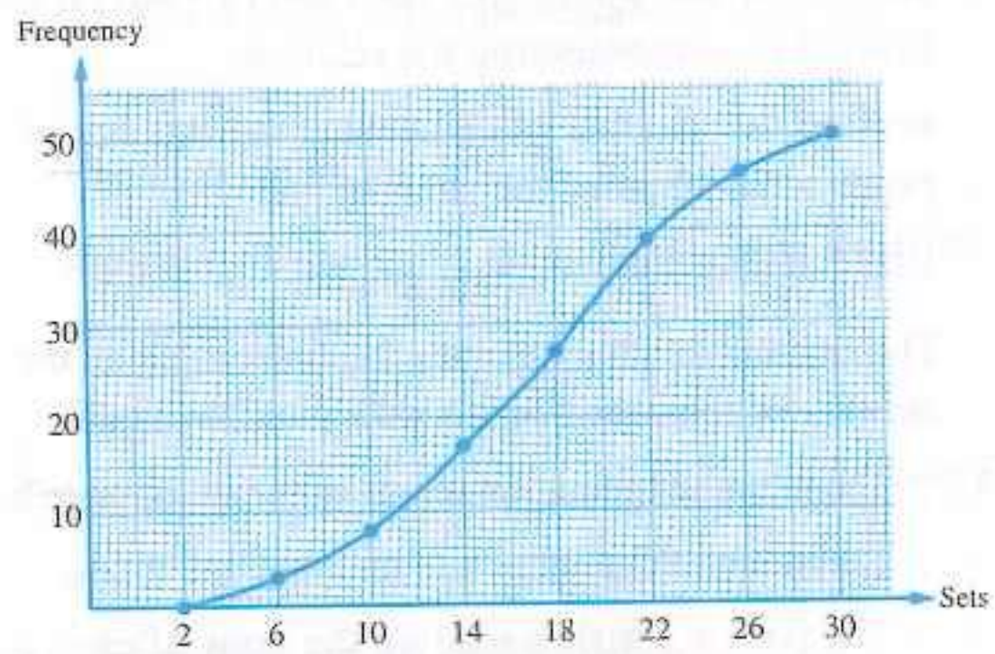
1 39 soldiers.
2 $\frac{22}{55} \times 100\% = 40\%$

Answers of Exercise 15

First : Problems on the ascending cumulative frequency curve.

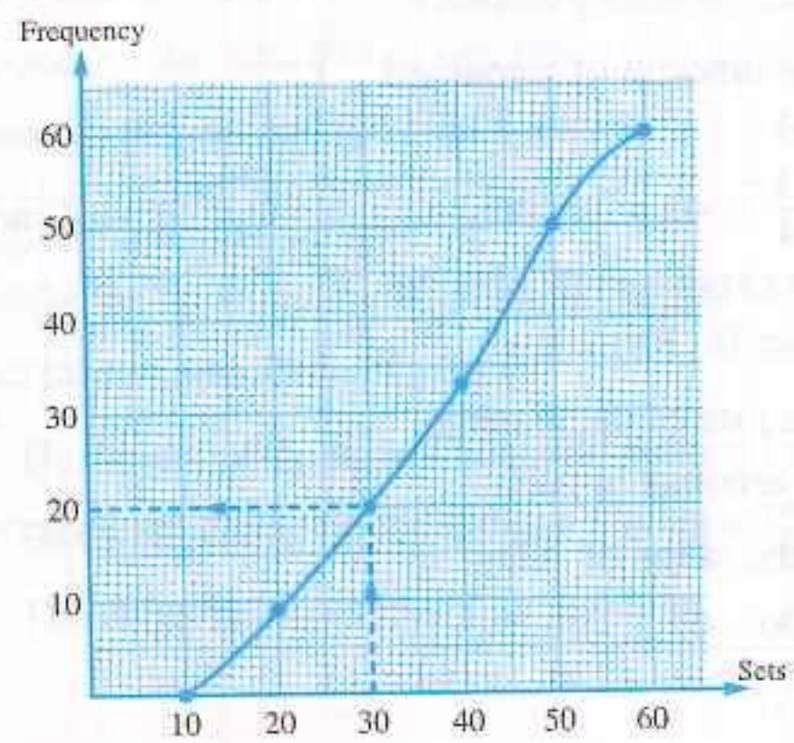
1

The upper boundaries of sets	Ascending cumulative frequency
less than 2	0
less than 6	3
less than 10	8
less than 14	17
less than 18	27
less than 22	39
less than 26	46
less than 30	50



2

The upper boundaries of sets	Ascending cumulative frequency
less than 10	0
less than 20	9
less than 30	20
less than 40	33
less than 50	50
less than 60	60



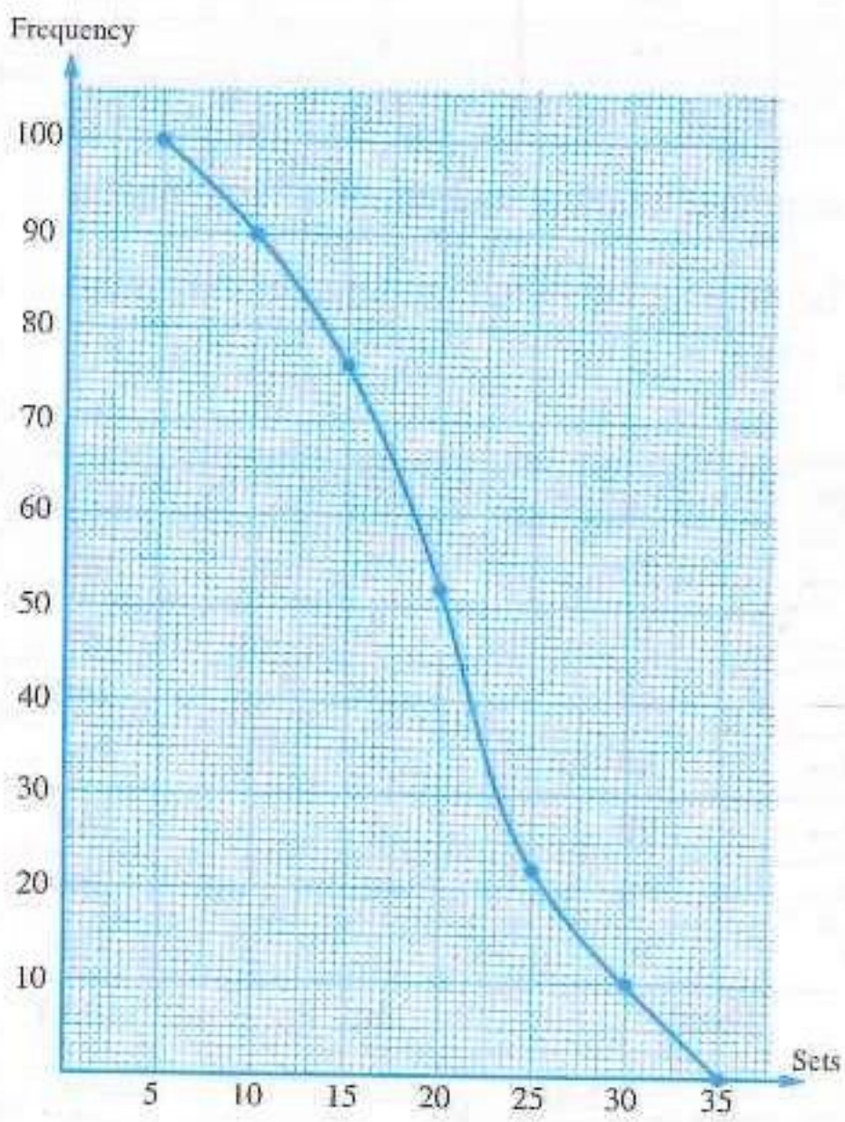
From the graph :

The number of failed pupils = 20 pupils.

Second : Problems on the descending cumulative frequency curve.

3

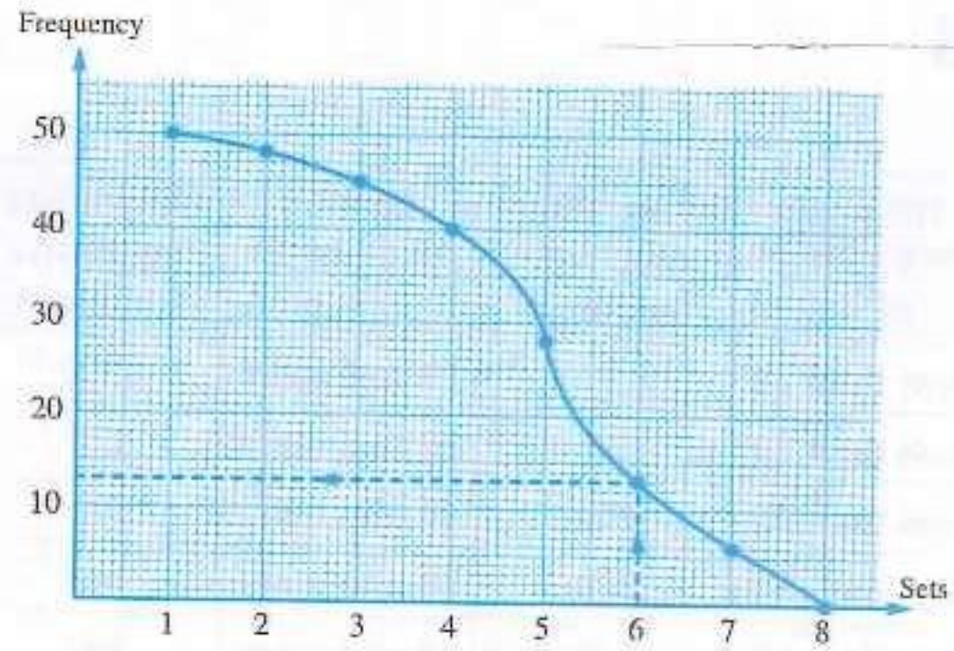
The lower limits of sets	Descending cumulative frequency
5 and more	100
10 and more	90
15 and more	76
20 and more	52
25 and more	22
30 and more	10
35 and more	0



4

1

The lower boundaries of sets	Descending cumulative frequency
1 and more	50
2 and more	48
3 and more	45
4 and more	40
5 and more	28
6 and more	13
7 and more	6
8 and more	0



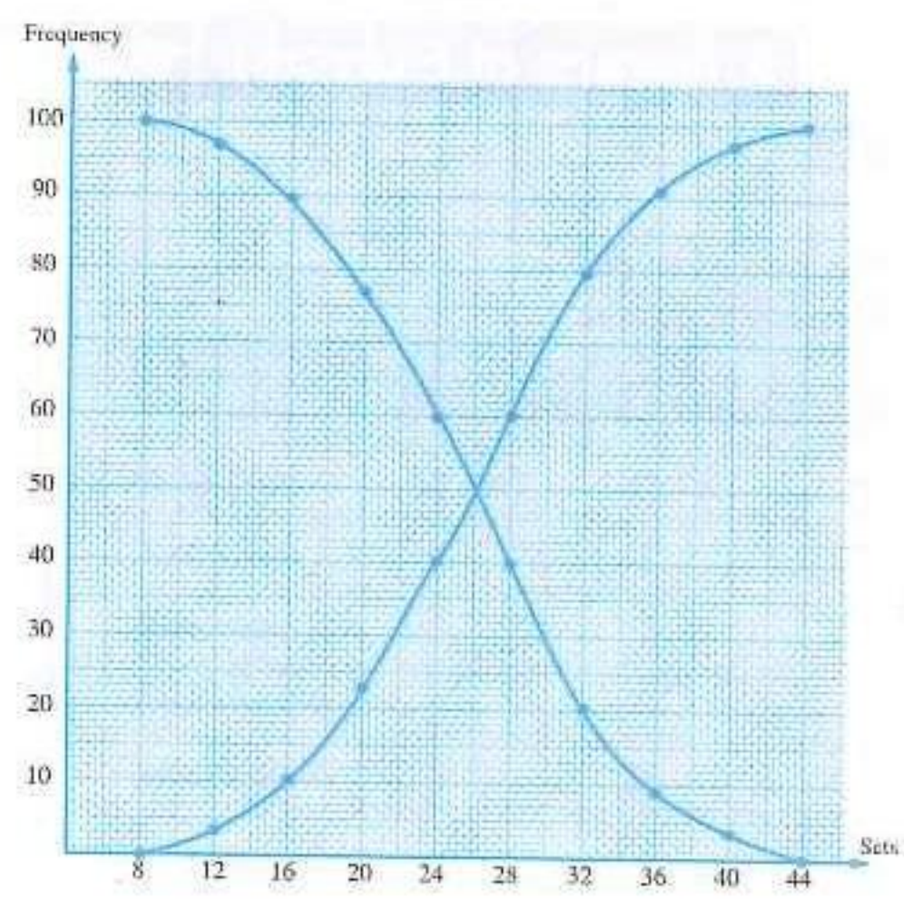
2 **From the graph :** The number of pupils who study 6 hours and more daily = 13 pupils.

3 The percentage of the number of pupils who study 6 hours and more daily = $\frac{13}{50} \times 100\ \% = 26\ \%$

Third : Problems on the two curves together.

5

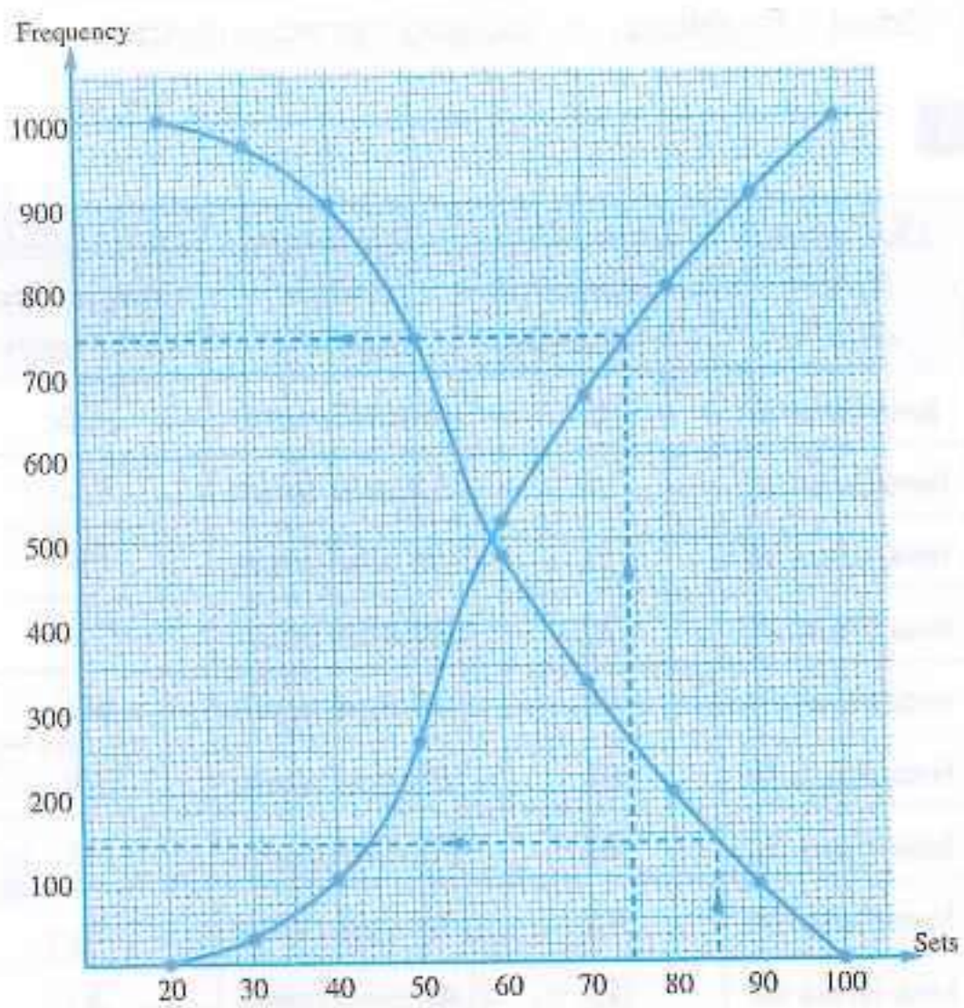
The upper limits of sets	Ascending cumulative frequency	The lower limits of sets	Descending cumulative frequency
less than 8	0	8 and more	100
less than 12	4	12 and more	96
less than 16	11	16 and more	89
less than 20	23	20 and more	77
less than 24	41	24 and more	59
less than 28	61	28 and more	39
less than 32	80	32 and more	20
less than 36	91	36 and more	9
less than 40	97	40 and more	3
less than 44	100	44 and more	0



6

1

The upper boundaries of sets	Ascending cumulative frequency	The lower boundaries of sets	Descending cumulative frequency
less than 20	0	20 and more	1000
less than 30	30	30 and more	970
less than 40	100	40 and more	900
less than 50	260	50 and more	740
less than 60	520	60 and more	480
less than 70	670	70 and more	330
less than 80	800	80 and more	200
less than 90	910	90 and more	90
less than 100	1000	100 and more	0



2 740 students. 3 140 students.

Answers of Exercise 16

1

1 $\frac{\text{The sum of values}}{\text{Number of values}}$

2 Its lower limit, its upper limit.

3 10 4 11 5 14 6 3940

2

1 c 2 d 3 c 4 b 5 a

3

Sets	Centre of Sets "$\mathcal{X}$"	Frequency "f"	Center of sets × frequency "$\mathcal{X} \times f$"
5 –	10	6	60
15 –	20	8	160
25 –	30	4	120
35 –	40	2	80
Total		20	420

$\therefore$ The mean $= \frac{420}{20} = 21$

4 1

Sets	"$\mathcal{X}$"	"f"	"$\mathcal{X} \times f$"
10 –	15	1	15
20 –	25	2	50
30 –	35	4	140
40 –	45	2	90
50 –	55	1	55
Total		10	350

$\therefore$ The mean of marks of students $= \frac{350}{10} = 35$ marks.

2 The number of failed students = 3 students.

5

Sets	"$\mathcal{X}$"	"f"	"$\mathcal{X} \times f$"
15 –	20	2	40
25 –	30	3	90
35 –	40	5	200
45 –	50	8	400
55 –	60	6	360
65 –	70	4	280
75 –	80	2	160
Total		30	1530

$\therefore$ The mean $= \frac{1530}{30} = 51$

6

Sets	"$\mathcal{X}$"	"f"	"$\mathcal{X} \times f$"
140 –	142	12	1704
144 –	146	20	2920
148 –	150	38	5700
152 –	154	22	3388
156 –	158	17	2686
160 –	162	11	1782
Total		120	18180

$\therefore$ The mean $= \frac{18180}{120} = 151.5$ cm.

7

1 25 – , 10

2

Sets	"x"	"f"	"$x \times f$"
5 –	10	3	30
15 –	20	10	200
25 –	30	12	360
35 –	40	10	400
45 –	50	5	250
Total		40	1240

$\therefore$ The mean $= \frac{1240}{40} = 31$ marks.

3 The number of students whose marks are not less than 35 = 15 students.

8 The missed number is 5

Sets	"x"	"f"	"$x \times f$"
6 –	8	2	16
10 –	12	3	36
14 –	16	5	80
18 –	20	8	160
22 –	24	6	144
26 –	28	4	112
30 –	32	2	64
Total		30	612

$\therefore$ The mean $= \frac{612}{30} = 20.4$ kg.

9

1 $3k + 4k = 50 - (7 + 10 + 8 + 4)$

$\therefore 7k = 21 \qquad \therefore k = \frac{21}{7} = 3$

2

Sets	"x"	"f"	"$x \times f$"
30 –	32.5	7	227.5
35 –	37.5	9	337.5
40 –	42.5	12	510
45 –	47.5	10	475
50 –	52.5	8	420
55 –	57.5	4	230
Total		50	2200

$\therefore$ The mean $= \frac{2200}{50} = 44$ kg.

10

1 $k - 2 = 50 - (4 + 5 + 8 + 7 + 5 + 1)$

$\therefore k - 2 = 20 \qquad \therefore k = 22$

2

Sets	"x"	"f"	"$x \times f$"
2 –	4	4	16
6 –	8	5	40
10 –	12	8	96
14 –	16	20	320
18 –	20	7	140
22 –	24	5	120
26 –	28	1	28
Total		50	760

$\therefore$ The mean $= \frac{760}{50} = 15.2$ days.

11 $\because$ The total of marks of the student in 5 months

$= 5 \times 23.8 = 119$ marks.

, let the required mark of the sixth month be x

$\therefore \frac{119 + x}{6} = 24 \qquad \therefore 119 + x = 144$

$\therefore x = 144 - 119 = 25$ marks.

$\therefore$ The mark of the student in the 6th month is 25

12 $\because$ The total of marks of Magdi in 4 exams

$= 4 \times 16 = 64$ marks.

, let the required mark be $x \quad \therefore \frac{64 + x}{5} = 18$

$\therefore 64 + x = 90 \quad \therefore x = 90 - 64 \quad \therefore x = 26$ marks.

$\therefore$ The mark of Magdi in the 5th exam should be 26 marks.

Answers of Exercise 17

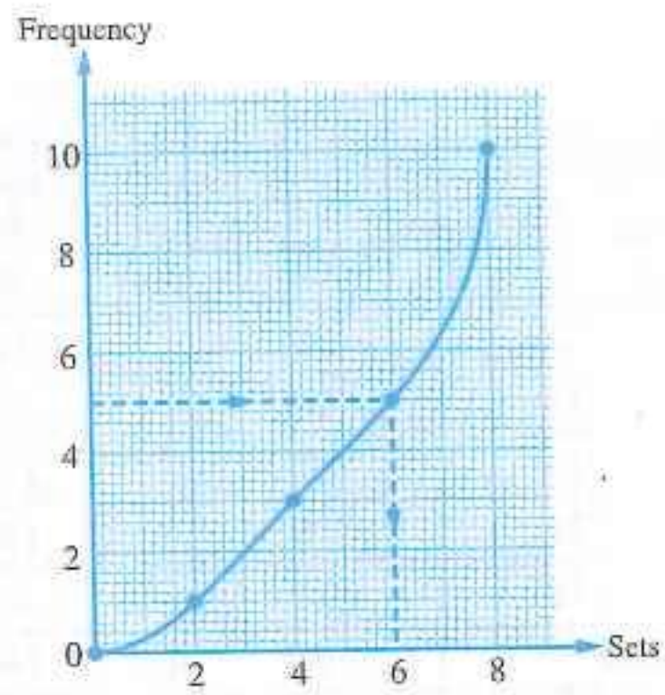
1

1 b 2 c 3 a 4 d

5 b 6 d 7 d

2

The upper limits of sets	Ascending cumulative frequency
less than 0	0
less than 2	1
less than 4	3
less than 6	5
less than 8	10

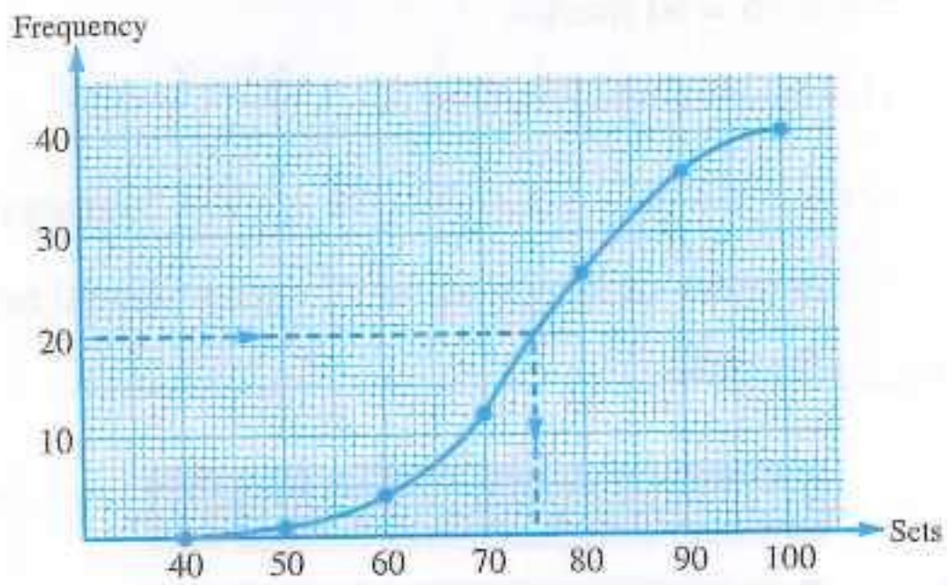


$\because$ The order of the median $= \frac{10}{2} = 5$

$\therefore$ The median $= 6$

3

The upper boundaries of sets	Ascending cumulative frequency
less than 40	0
less than 50	1
less than 60	4
less than 70	12
less than 80	26
less than 90	36
less than 100	40

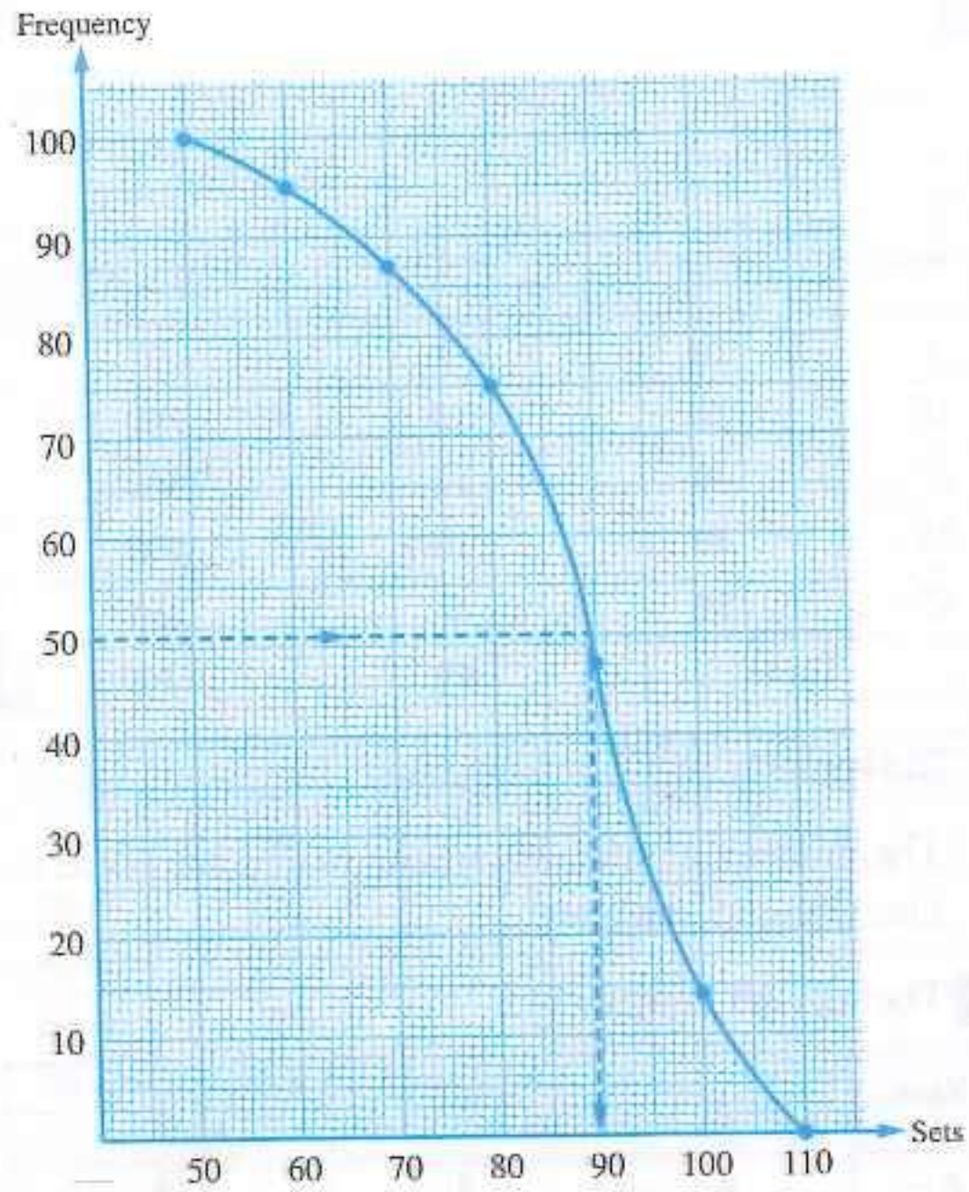


$\because$ The order of the median $= \frac{40}{2} = 20$

$\therefore$ The percentage of intelligence $\approx 75\%$

4

The lower boundaries of sets	Descending cumulative frequency
50 and more	100
60 and more	95
70 and more	87
80 and more	75
90 and more	47
100 and more	14
110 and more	0

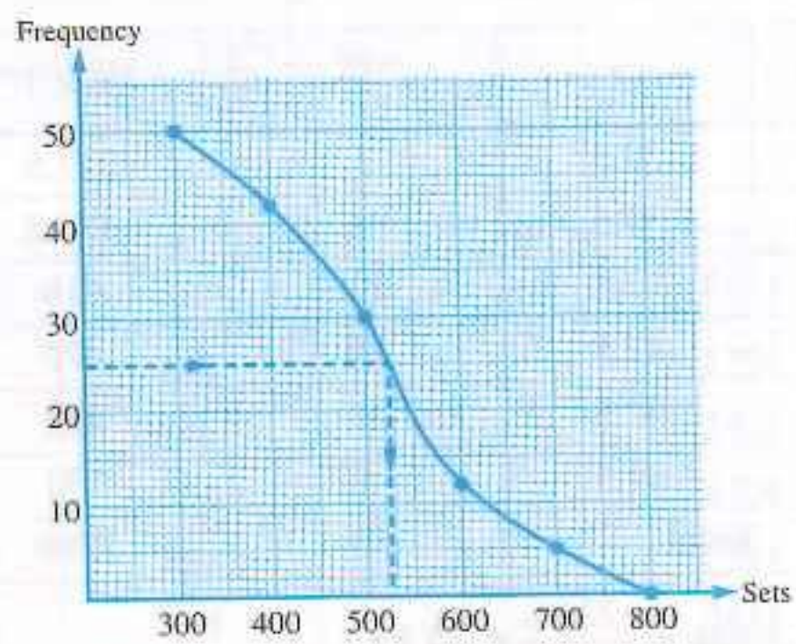


$\because$ The order of the median $= \frac{100}{2} = 50$

$\therefore$ The median of working hours ≈ 89.5 hours.

5

The lower boundaries of sets	Descending cumulative frequency
300 and more	50
400 and more	42
500 and more	30
600 and more	12
700 and more	5
800 and more	0

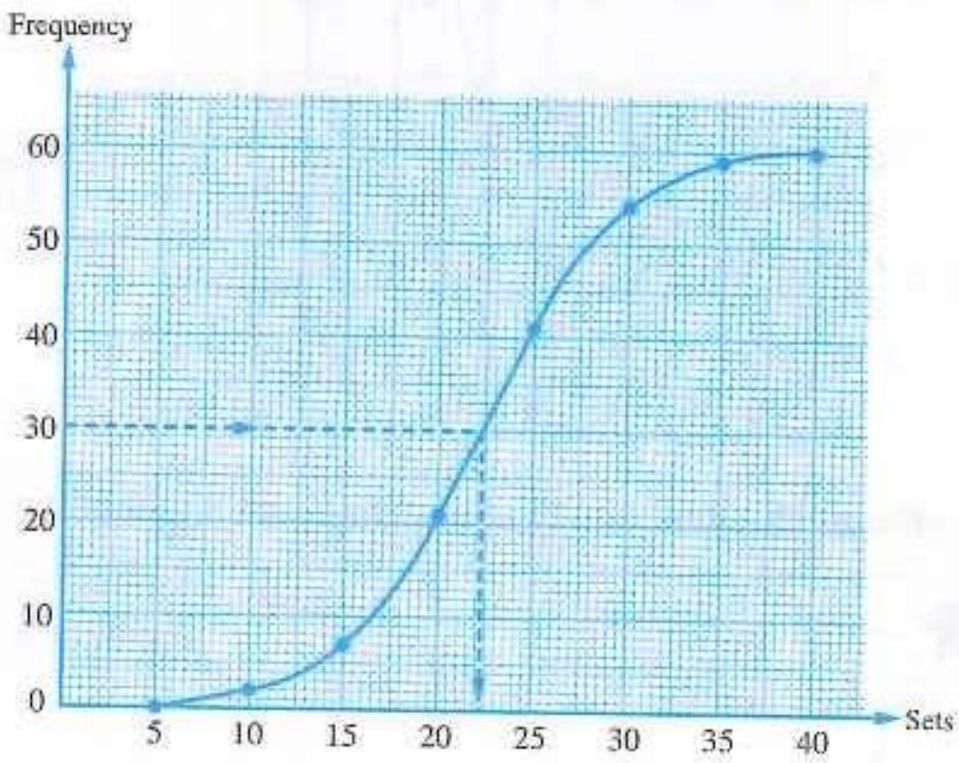


$\because$ The order of the median $= \frac{50}{2} = 25$

$\therefore$ The median wage $= 520$ pounds.

6

The upper limits of sets	Ascending cumulative frequency
less than 5	0
less than 10	2
less than 15	7
less than 20	21
less than 25	41
less than 30	54
less than 35	59
less than 40	60



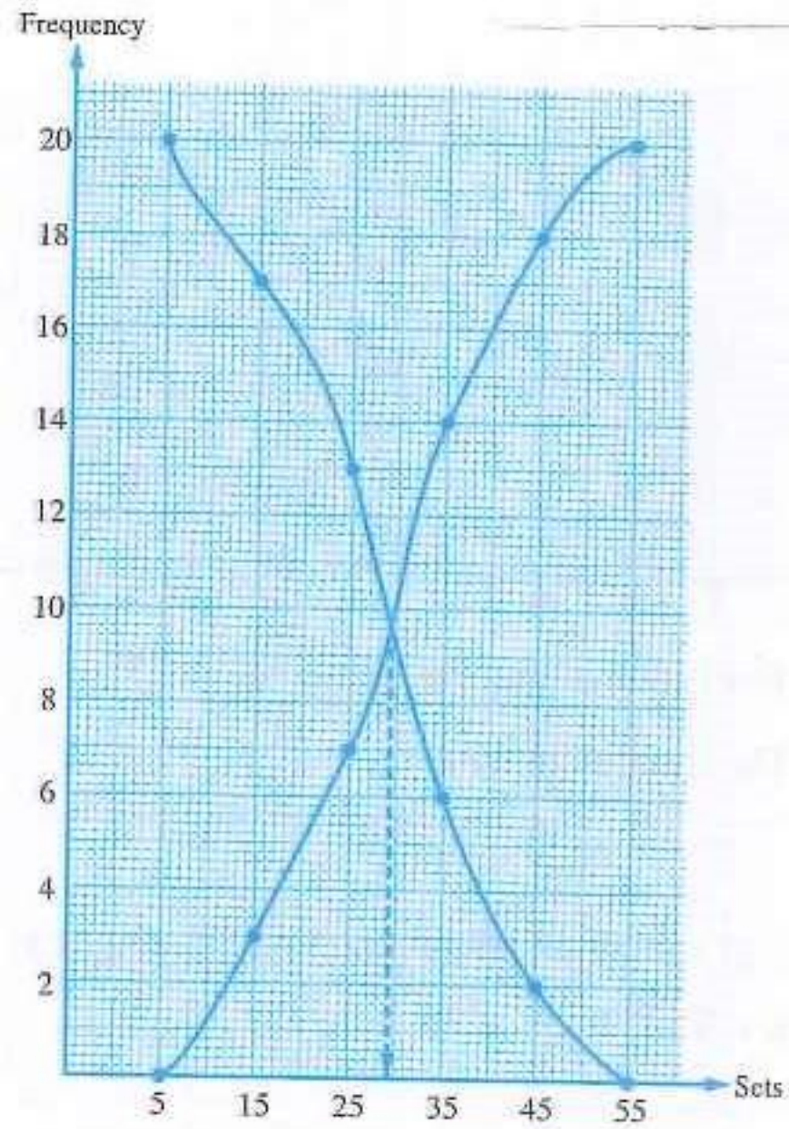
$\because$ The order of the median $= \frac{60}{2} = 30$

$\therefore$ The median mark $\approx$ 22 marks.

7

The upper limits of sets	Ascending cumulative frequency
less than 5	0
less than 15	3
less than 25	7
less than 35	14
less than 45	18
less than 55	20

The lower limits of sets	Descending cumulative frequency
5 and more	20
15 and more	17
25 and more	13
35 and more	6
45 and more	2
55 and more	0



From the graph we find that the median weight $\approx$ 29 kg.

8

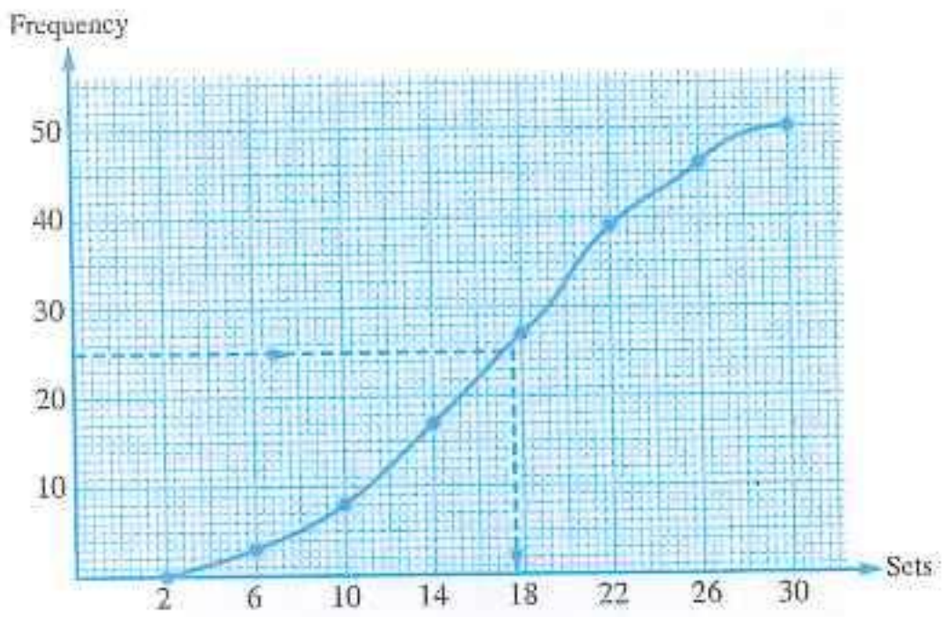
1

Sets	"x"	"f"	"$x \times f$"
2 –	4	3	12
6 –	8	5	40
10 –	12	9	108
14 –	16	10	160
18 –	20	12	240
22 –	24	7	168
26 –	28	4	112
Total		50	840

$\therefore$ The mean $= \frac{840}{50} = 16.8$

2 We form the ascending cumulative frequency table.

The upper limits of sets	Ascending cumulative frequency
less than 2	0
less than 6	3
less than 10	8
less than 14	17
less than 18	27
less than 22	39
less than 26	46
less than 30	50



$\because$ The order of the median $= \frac{50}{2} = 25$

$\therefore$ The median ≈ 17.6

9

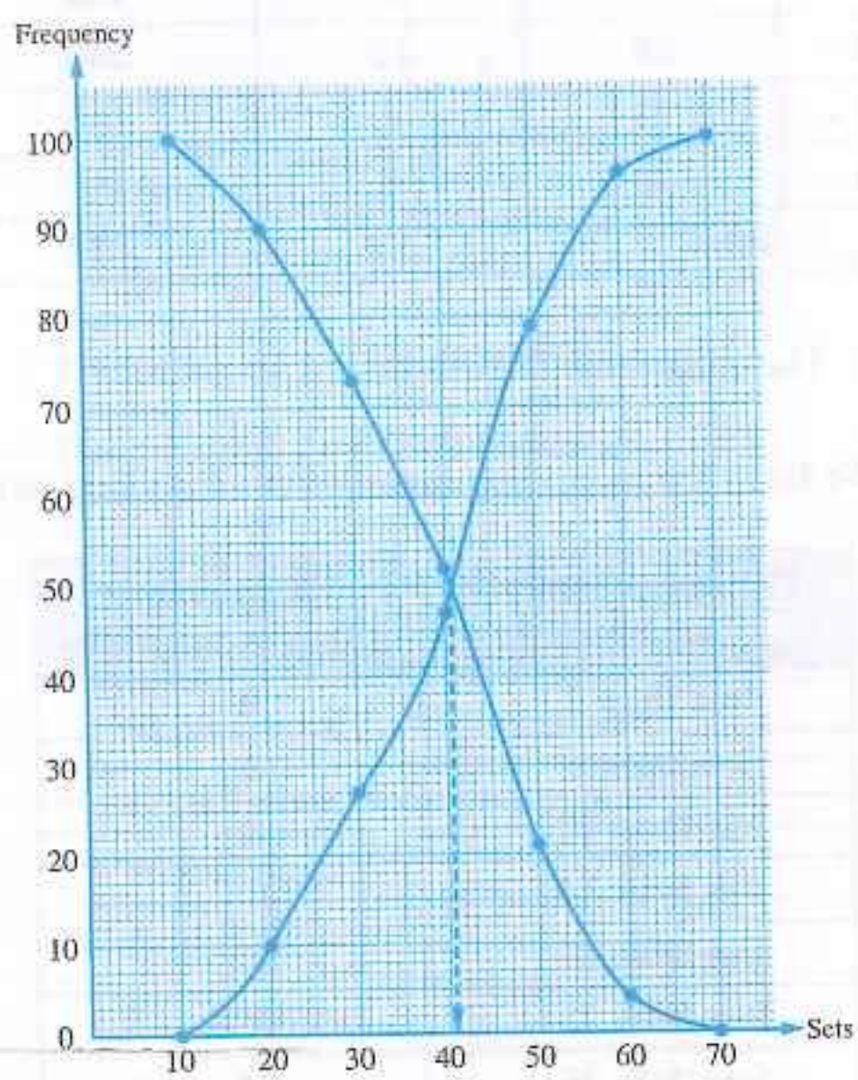
1 $x = 30$, $k + 2 = 100 - (10 + 17 + 20 + 32 + 4)$

$\therefore k + 2 = 17 \qquad \therefore k = 15$

2

The upper limits of sets	Ascending cumulative frequency
less than 10	0
less than 20	10
less than 30	27
less than 40	47
less than 50	79
less than 60	96
less than 70	100

The lower limits of sets	Descending cumulative frequency
10 and more	100
20 and more	90
30 and more	73
40 and more	53
50 and more	21
60 and more	4
70 and more	0



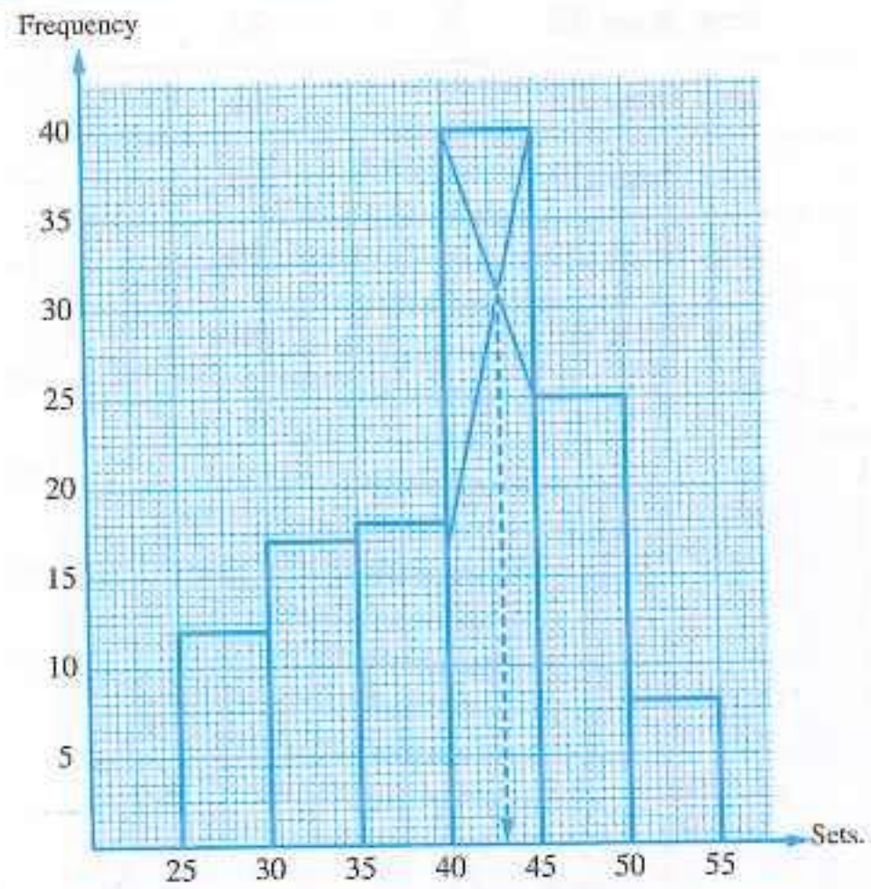
The median ≈ 41

Answers of Exercise 18

1 1 b 2 b 3 a 4 c

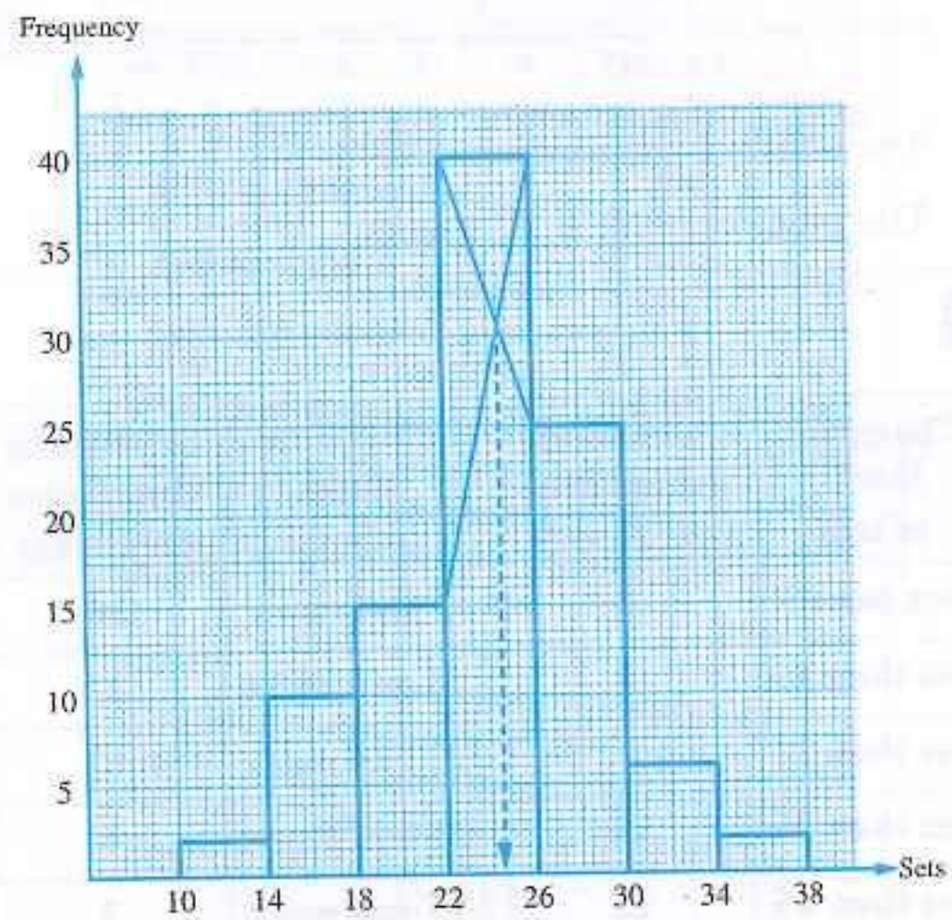
5 d 6 b 7 d

2



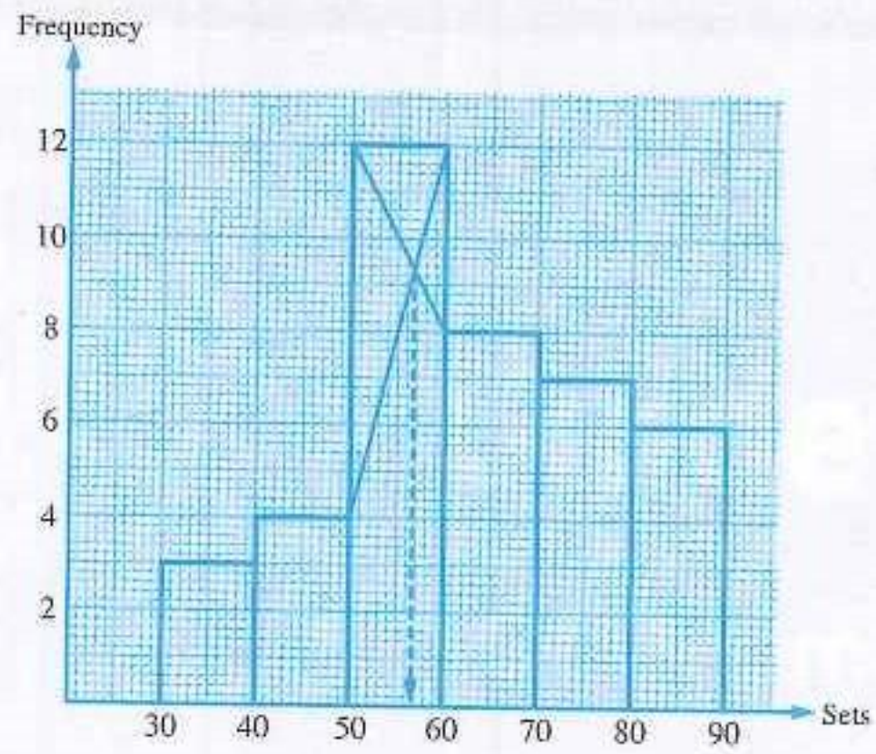
From the graph : The mode age = 43 years.

3



From the graph : The mode mark ≈ 24.5 marks.

4



From the graph : The mode = 57 marks.

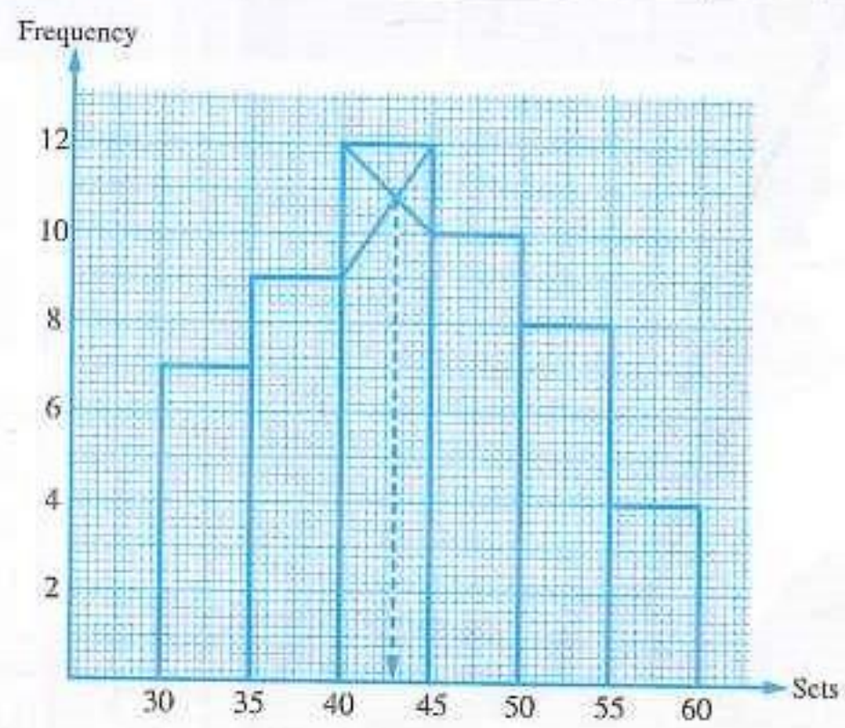
5

1 $\because k + 4 + 3k + 4k + 3k + 1 + 3k - 1 + k + 1 = 50$

$\therefore 15k + 5 = 50 \quad \therefore 15k = 45 \quad \therefore k = 3$

2

Weight in kg.	30 –	35 –	40 –	45 –	50 –	55 –	Total
number of students	7	9	12	10	8	4	50



From the graph : The mode weight ≈ 43 kg.

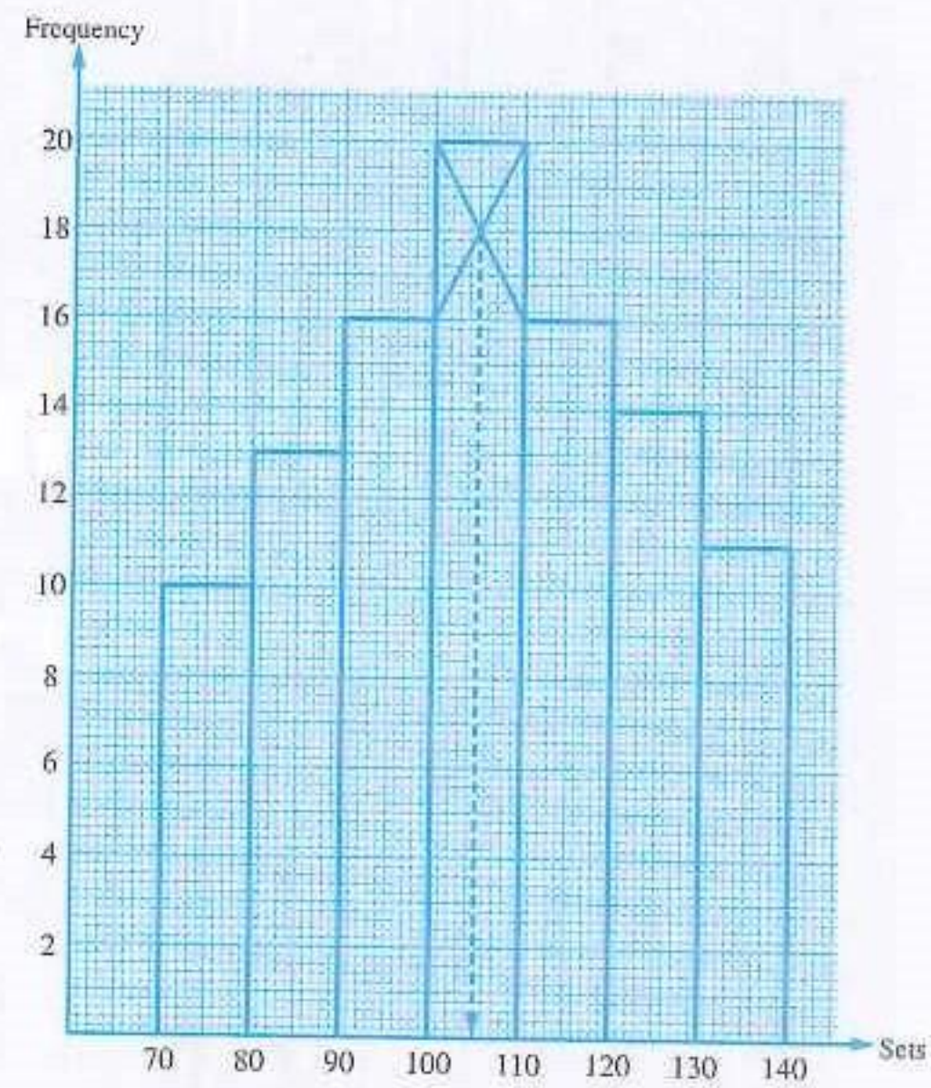
6

1 $x = 110$

$\therefore k - 4 = 100 - (10 + 13 + 20 + 16 + 14 + 11)$

$\therefore k - 4 = 16 \quad \therefore k = 20$

2



From the graph : The mode wage = L.E. 105

7

1 $\because 3k + 4k = 50 - (7 + 10 + 8 + 4)$

$\therefore 7k = 21 \quad \therefore k = \frac{21}{7} = 3$

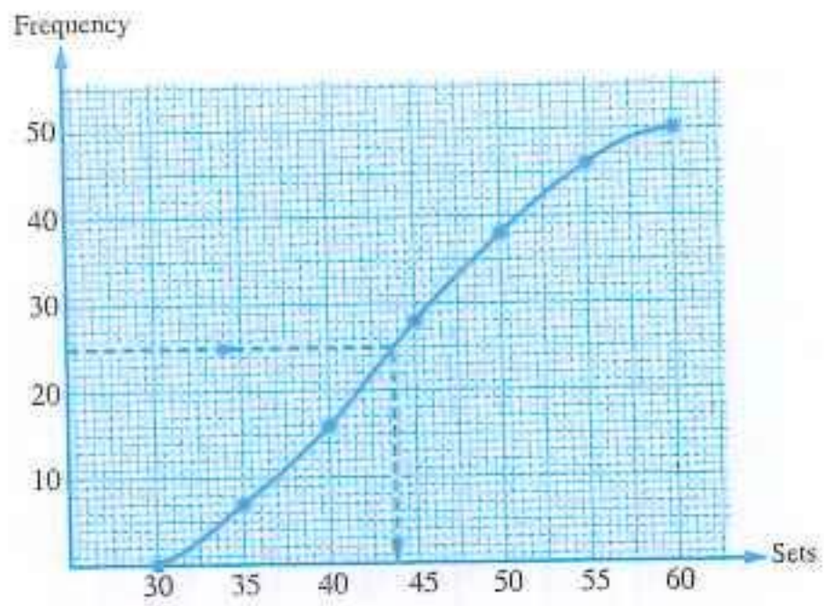
2

Sets	"x"	"f"	"$x \times f$"
30 –	32.5	7	227.5
35 –	37.5	9	337.5
40 –	42.5	12	510
45 –	47.5	10	475
50 –	52.5	8	420
55 –	57.5	4	230
Total		50	2200

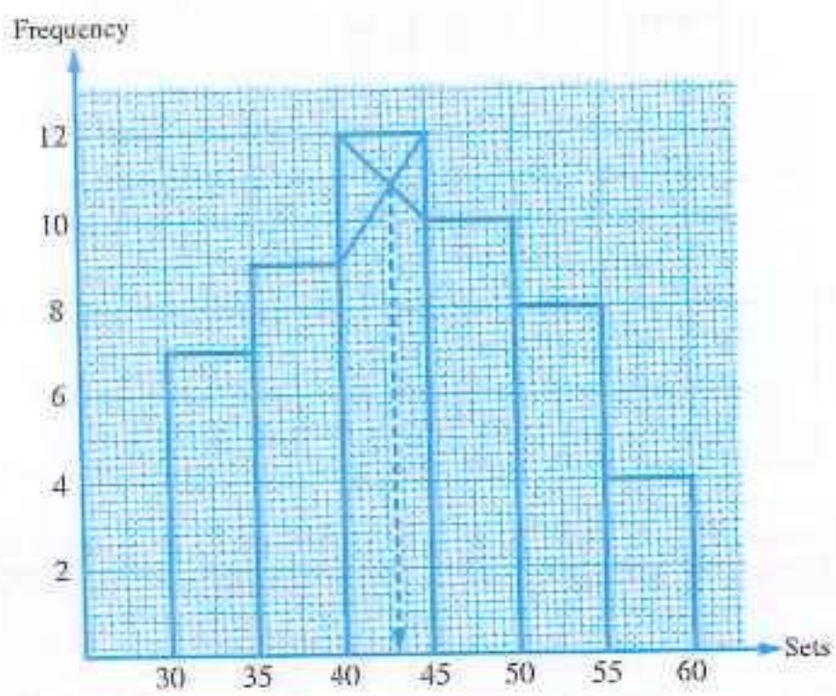
$\therefore$ The mean $= \frac{2200}{50} = 44$ kg.

3

The upper limits of sets	Ascending cumulative frequency
less than 30	0
less than 35	7
less than 40	16
less than 45	28
less than 50	38
less than 55	46
less than 60	50



4



From the graph : The mode weight = 43 kg.

5 The order of the median = $\frac{50}{2} = 25$

$\therefore$ The median $\approx$ 43.5 kg.

Answers of accumulative basic skills

1

1 6 2 0 3 15 4 154 5 21

6 $\frac{2}{3}$ 7 7500 8 4 9 12 10 9

11 27 12 6 , 8 , 2

2

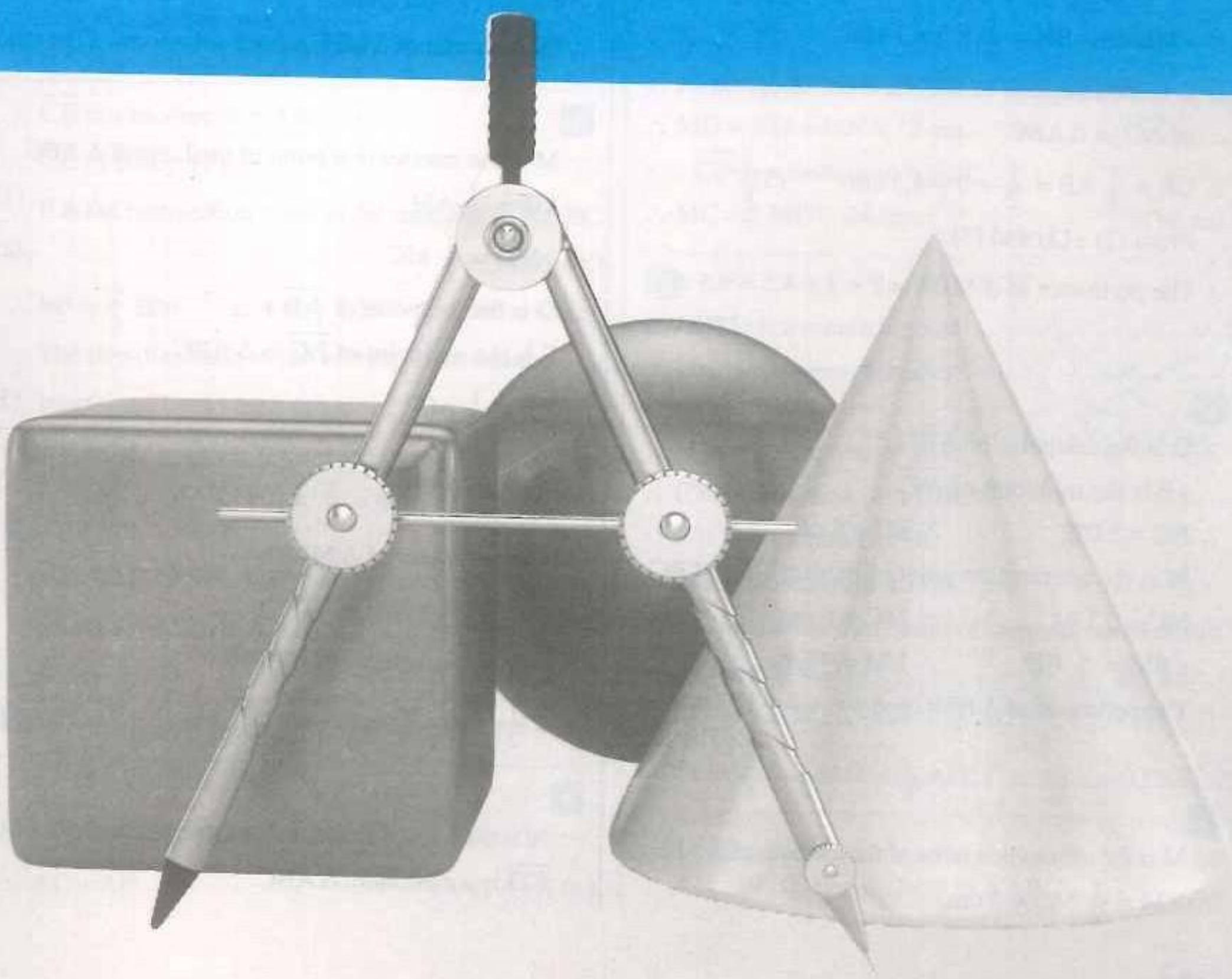
1 c 2 c 3 a 4 a 5 d

6 c 7 d 8 b 9 d 10 d

11 c 12 a

Guide Answers

Of Geometry Exercises



Answers of unit four

Answers of Exercise 1

1

1 a median 2 3 3 one point 4 1 : 2
5 2 : 1 6 4 7 16

2

1 d 2 a 3 d 4 d
5 a 6 d 7 c

3

1 8 cm. , 5 cm. 2 6 cm. , 4 cm. , $\frac{1}{3}$, $\frac{2}{3}$
3 6 cm. , 3 cm. , 4 cm. 4 5 cm. , 12 cm. , 27 cm.

4

$\because$ $\overline{AD}$, $\overline{BE}$ are two medians in Δ ABC ,
$\overline{AD} \cap \overline{BE} = \{M\}$
$\therefore$ M is the point of concurrence of the medians of Δ ABC
$\therefore MD = \frac{1}{3} AD = \frac{1}{3} \times 6 = 2$ cm. (1)
, $ME = \frac{1}{3} BE = \frac{1}{3} \times 9 = 3$ cm. (2)
$\because$ D is the midpoint of $\overline{BC}$, E is the midpoint of $\overline{AC}$ in Δ ABC
$\therefore DE = \frac{1}{2} AB = \frac{1}{2} \times 9 = 4.5$ cm. (3)
From (1) , (2) and (3) :
$\therefore$ The perimeter of Δ MDE = 2 + 3 + 4.5 = 9.5 cm.
(The req.)

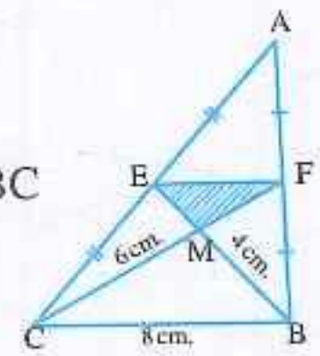
5

$\because$ D is the midpoint of $\overline{AB}$
, E is the midpoint of $\overline{AC}$
$\therefore BC = 2\ DE$ $\therefore BC = 8$ cm.
$\because$ M is the intersection point of medians of Δ ABC
$\therefore MC = 2\ DM$ $\therefore MC = 6$ cm.
, $BM = \frac{2}{3} BE$ $\therefore BM = 4$ cm.
$\therefore$ The perimeter of Δ BMC = 8 + 6 + 4 = 18 cm.
(The req.)

6

$\because$ M is the intersection point of the medians of Δ ABC
$\therefore XM = \frac{1}{2} MC = 4$ cm.
$\therefore$ The perimeter of Δ MXY = 4 + 5 + 3 = 12 cm.
(First req.)
, AM = 2 MY = 6 cm.
$\because$ X is the midpoint of $\overline{AB}$,
Y is the midpoint of $\overline{BC}$
$\therefore$ AC = 2 XY = 10 cm.
$\therefore$ The perimeter of Δ MAC = 6 + 8 + 10 = 24 cm.
(Second req.)

7



$\because$ F is the midpoint of $\overline{AB}$
, E is the midpoint of $\overline{AC}$
$\therefore$ $\overline{BE}$, $\overline{CF}$ are two medians in Δ ABC
$\therefore$ M is the intersection point of the medians of Δ ABC
$\therefore ME = \frac{1}{2} MB = 2$ cm. (1)
, $MF = \frac{1}{2} MC = 3$ cm. (2)
$\because$ F is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$
$\therefore FE = \frac{1}{2} BC = 4$ cm. (3)
From (1) , (2) and (3) :
$\therefore$ The perimeter of Δ MFE = 2 + 3 + 4 = 9 cm. (The req.)

8

$\because$ M is the intersection point of medians of Δ ABC
$\therefore MF = \frac{1}{2} AM$ (1)
, $MD = \frac{1}{2} MC$ (2)
$\because$ D is the midpoint of $\overline{AB}$,
F is the midpoint of $\overline{BC}$ in Δ ABC
$\therefore DF = \frac{1}{2} AC$ (3)
By adding (1) , (2) and (3) :
$\therefore MF + MD + DF = \frac{1}{2} AM + \frac{1}{2} MC + \frac{1}{2} AC$
$\therefore$ The perimeter of Δ MFD
$= \frac{1}{2} (AM + MC + AC)$
$= \frac{1}{2}$ the perimeter of Δ AMC
$= \frac{1}{2} \times 36 = 18$ cm. (The req.)

9

$\because$ M is the point of concurrence of the medians of Δ ABC
$\therefore$ $\overline{CD}$ is a median in Δ ABC

$\therefore DM = \frac{1}{2} MC = 3$ cm.

, $\because \Delta AMD$ is a right-angled triangle at M

$\therefore (AM)^2 = (AD)^2 - (DM)^2 = 25 - 9 = 16$

$\therefore AM = 4$ cm.

$\therefore ME = \frac{1}{2} AM = 2$ cm. (The req.)

10

$\because$ ABCD is a parallelogram

$\therefore$ The two diagonals bisect each other.

$\therefore$ M is the midpoint of $\overline{AC}$

$\therefore \overline{DM}$ is a median in ΔADC

$\because DE = 2\ EM$

$\therefore$ E is the intersection point of the medians of ΔADC

$\because E \in \overline{FC}$

$\therefore \overline{CF}$ is a median in ΔACD $\therefore AF = FD$ (Q.E.D.)

11

$\because$ The two diagonals of the rectangle bisect each other.

$\therefore$ M is the midpoint of $\overline{AC}$

$\therefore \overline{BM}$ is a median in ΔABC

$\because$ E is the midpoint of $\overline{AB}$

$\therefore \overline{CE}$ is a median in ΔABC

$\because \overline{CE} \cap \overline{BM} = \{F\}$

$\therefore$ F is the intersection point of the medians of ΔABC

(First req.)

$\therefore BF = \frac{2}{3} BM$ $\therefore 4 = \frac{2}{3} BM$ $\therefore BM = 6$ cm.

$\because$ The two diagonals of the rectangle are equal in length and bisect each other.

$\therefore AM = BM = 6$ cm. (Second req.)

12

$\because$ D is the midpoint of $\overline{BC}$

$\therefore \overline{AD}$ is a median in ΔABC

$\because AM = \frac{2}{3} AD$

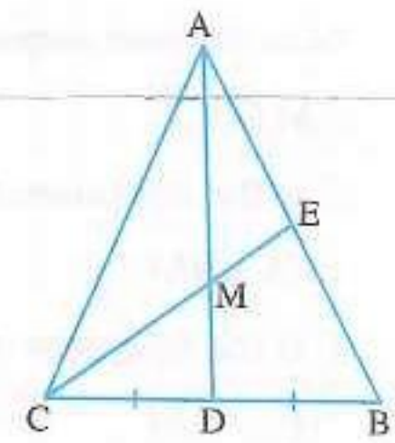
$\therefore$ M is the intersection point of the medians of ΔABC

$\therefore \overline{CF}$ is a median in ΔABC

$\therefore$ F is the midpoint of $\overline{AB}$ $\therefore BF = \frac{1}{2} AB$

$\because AC = AB$ $\therefore BF = \frac{1}{2} AC$ (Q.E.D.)

13



$\because$ D is the midpoint of $\overline{BC}$

$\therefore \overline{AD}$ is a median in ΔABC

$\because AM = 2\ MD$

$\therefore$ M is the intersection point of the medians of ΔABC

$\because M \in \overline{CE}$

$\therefore \overline{CE}$ is a median in ΔABC

$\therefore EM = \frac{1}{3} EC = \frac{1}{3} \times 12 = 4$ cm. (The req.)

14

$\because$ M is the point of concurrence of the medians of ΔABC

$\therefore \overline{CD}$ is a median in ΔABC

$\therefore$ D is the midpoint of $\overline{AB}$

In ΔAMB :

$\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BM}$

$\therefore \overline{MD}$, $\overline{AE}$ are two medians in ΔAMB

$\therefore$ N is the point of concurrence of the medians of ΔAMB

$\therefore MN = 2\ ND$ $\therefore x + 3 = 2(x - 1)$

$\therefore x + 3 = 2x - 2$ $\therefore 3 + 2 = 2x - x$

$\therefore x = 5$

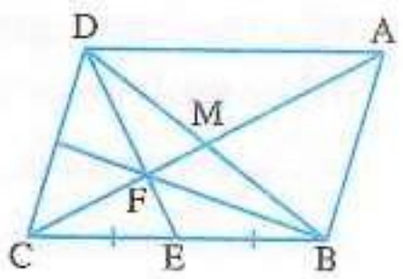
$\therefore ND = 5 - 1 = 4$ cm. , $MN = 5 + 3 = 8$ cm.

$\therefore MD = ND + MN = 12$ cm.

, $\because \overline{CD}$ is a median in ΔABC

$\therefore MC = 2\ MD = 24$ cm. (The req.)

15



$\because$ ABCD is a parallelogram

$\therefore$ The two diagonals bisect each other.

$\therefore$ M is the midpoint of $\overline{BD}$

$\therefore \overline{CM}$ is a median in ΔDBC

$\because$ E is the midpoint of $\overline{BC}$

$\therefore \overline{DE}$ is a median in ΔDBC

$\therefore$ F is the intersection point of the medians of ΔDBC

$\therefore \overrightarrow{BF}$ bisects $\overline{CD}$ (Q.E.D. 1)

$\therefore CF = \frac{2}{3} CM$, $\because CM = \frac{1}{2} AC$

$\therefore CF = \frac{2}{3} \times \frac{1}{2} AC = \frac{1}{3} AC$ (Q.E.D. 2)

16

$\because \overline{AD}$ and $\overline{BE}$ are medians in ΔABC

$\therefore$ M is the intersection point of the medians of Δ ABC

, $\because$ M $\in \overline{CF}$ $\therefore$ $\overline{CF}$ is a median in Δ ABC

$\therefore$ F is the midpoint of $\overline{AB}$

In Δ ABM :

$\because$ F is the midpoint of $\overline{AB}$, N is the midpoint of $\overline{BM}$

$\therefore$ $\overline{NF}$ // $\overline{AM}$ $\therefore$ $\overline{NF}$ // $\overline{MD}$ (1)

In Δ BMC :

$\because$ D is the midpoint of $\overline{BC}$, N is the midpoint of $\overline{BM}$

$\therefore$ $\overline{ND}$ // $\overline{CM}$ $\therefore$ $\overline{ND}$ // $\overline{MF}$ (2)

From (1) and (2) :

$\therefore$ The figure FNDM is a parallelogram. (Q.E.D.)

17

$\because$ D is the midpoint of $\overline{BC}$

$\therefore$ $\overline{AD}$ is a median in Δ ABC

, $\because$ AM = 2 MD , M $\in \overline{AD}$

$\therefore$ M is the intersection point of the medians of Δ ABC

, $\because$ M $\in \overline{BE}$

$\therefore$ $\overline{BE}$ is a median in Δ ABC $\therefore$ BM = 2 ME

$\therefore$ BM = 4 cm. $\therefore$ BE = 2 + 4 = 6 cm.

, $\because$ Δ BCE in which :

D is the midpoint of $\overline{BC}$, $\overline{DF}$ // $\overline{BE}$

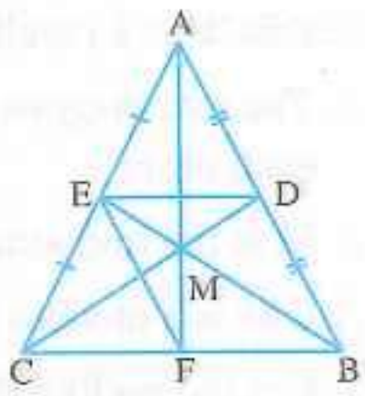
$\therefore$ F is the midpoint of $\overline{EC}$

$\therefore$ DF = $\frac{1}{2}$ BE = 3 cm. (The req.)

18

$\because$ $\overline{CD}$ and $\overline{BE}$ are two medians in Δ ABC

$\therefore$ M is the intersection point of the medians of Δ ABC



$\therefore$ $\overline{AF}$ is a median in Δ ABC

$\therefore$ F is the midpoint of $\overline{BC}$

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore$ $\overline{FE}$ // $\overline{AB}$, FE = $\frac{1}{2}$ AB

$\therefore$ $\overline{FE}$ // $\overline{BD}$, FE = BD

$\therefore$ DBFE is a parallelogram (Q.E.D.)

Answers of Exercise 2

1

1 3 2 half the length of the hypotenuse

3 right 4 half the length of the hypotenuse

5 twice 6 twice

2

1 3 2 10 3 8 4 18 , 9 , $\frac{1}{3}$, 3

5 5 , 5 , 15 6 9 , 8 , 10 , 27

3

1 b 2 b 3 c

4 a 5 b 6 a

4

In Δ ADC :

$\because$ m ($\angle$ D) = 90° , E is the midpoint of $\overline{AC}$

$\therefore$ DE = $\frac{1}{2}$ AC (1)

In Δ ABC :

$\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

$\therefore$ AB = $\frac{1}{2}$ AC (2)

From (1) and (2) :

$\therefore$ AB = DE (Q.E.D.)

5

In Δ LXZ :

$\because$ D is the midpoint of $\overline{LX}$, E is the midpoint of $\overline{LZ}$

$\therefore$ DE = $\frac{1}{2}$ XZ (1)

From Δ XYZ :

$\because$ m ($\angle$ Y) = 90° , M is the midpoint of $\overline{XZ}$

$\therefore$ YM = $\frac{1}{2}$ XZ (2)

From (1) and (2) : $\therefore$ DE = YM (Q.E.D.)

6

In Δ ACD :

$\because$ E is the midpoint of $\overline{AD}$, F is the midpoint of $\overline{CD}$

$\therefore$ EF = $\frac{1}{2}$ AC $\therefore$ AC = 8 cm.

In Δ ABC :

$\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

$\therefore$ AB = $\frac{1}{2}$ AC = 4 cm. (The req.)

7

In Δ ABC :

$\because$ m ($\angle$ BAC) = 90° , D is the midpoint of $\overline{BC}$

$\therefore$ BC = 2 AD = 2 × 3 = 6 cm.

In Δ CBE : $\because$ m ($\angle$ CBE) = 90° , m ($\angle$ E) = 30°

$\therefore$ EC = 2 BC = 2 × 6 = 12 cm.

, $\because$ F is the midpoint of $\overline{CE}$

$\therefore$ BF = $\frac{1}{2}$ EC = $\frac{1}{2}$ × 12 = 6 cm. (The req.)

8

In Δ ABC :

$\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 60°

$\therefore$ m ($\angle$ CAB) = 30° $\therefore$ BC = $\frac{1}{2}$ AC

$\because$ DE = BC $\therefore$ DE = $\frac{1}{2}$ AC

$\because$ $\overline{DE}$ is a median in Δ ACD

$\therefore$ m ($\angle$ ADC) = 90° (Q.E.D.)

9

In Δ ABC :

$\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

$\therefore$ AB = $\frac{1}{2}$ AC

, $\because$ AB = DE = 5 cm. $\therefore$ DE = $\frac{1}{2}$ AC

$\because$ $\overline{DE}$ is a median in Δ ACD

$\therefore$ m ($\angle$ ADC) = 90° (Q.E.D.)

10

In Δ DBC :

$\because$ E is the midpoint of $\overline{BC}$, $\overline{EF}$ // $\overline{BD}$

$\therefore$ EF = $\frac{1}{2}$ BD

, $\because$ AM = EF $\therefore$ AM = $\frac{1}{2}$ BD

$\because$ $\overline{AM}$ is a median in Δ ABD

$\therefore$ m ($\angle$ BAD) = 90° (Q.E.D.)

11

$\because$ $\angle$ ADC is an exterior angle of Δ ABD

$\therefore$ m ($\angle$ ADC) = 33° + 27° = 60°

$\therefore$ In Δ ADC : m ($\angle$ DAC) = 180° − (60° + 90°) = 30°

$\therefore$ DC = $\frac{1}{2}$ AD

$\therefore$ AD = 8 cm. (The req.)

12

In Δ ADB :

$\because$ m ($\angle$ ADB) = 90° , AE = EB

$\therefore$ DE = $\frac{1}{2}$ AB

Similarly in Δ ACB :

$\because$ m ($\angle$ ACB) = 90° , AE = EB

$\therefore$ CE = $\frac{1}{2}$ AB $\therefore$ DE = CE

$\therefore$ Δ CED is an isosceles triangle. (Q.E.D.)

13

In Δ LYE :

$\because$ m ($\angle$ YLE) = 90° , m ($\angle$ E) = 30°

$\therefore$ LY = $\frac{1}{2}$ YE = 5 cm.

In Δ ZYX :

$\because$ m ($\angle$ ZYX) = 90° , L is the midpoint of $\overline{ZX}$

$\therefore$ YL = $\frac{1}{2}$ ZX $\therefore$ ZX = 10 cm. (The req.)

14

In Δ ABC :

$\because$ m ($\angle$ ABC) = 90° , m ($\angle$ C) = 30°

$\therefore$ AC = 2 AB = 14 cm.

, $\because$ D is the midpoint of $\overline{AC}$

$\therefore$ BD = $\frac{1}{2}$ AC = 7 cm.

, in Δ DEC :

$\because$ m ($\angle$ DEC) = 90° , m ($\angle$ C) = 30°

$\therefore$ DE = $\frac{1}{2}$ DC , $\because$ DC = $\frac{1}{2}$ AC = 7 cm.

$\therefore$ DE = 3.5 cm. (The req.)

15

In Δ ABC :

$\because$ m ($\angle$ ABC) = 90° , m ($\angle$ C) = 30°

$\therefore$ AB = $\frac{1}{2}$ AC = 4 cm.

$\because$ X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{BC}$

$\therefore$ XY = $\frac{1}{2}$ AC = 4 cm.

In Δ XBY :

$\because$ m ($\angle$ XBY) = 90°

, Z is the midpoint of $\overline{XY}$

$\therefore$ BZ = $\frac{1}{2}$ XY = 2 cm. (The req.)

16

In Δ MED :

$\because$ m ($\angle$ MED) = 90° $\therefore$ $(MD)^2 = 3^2 + 4^2 = 25$

$\therefore$ MD = $\sqrt{25}$ = 5 cm.

, $\because$ M is the point of concurrence of the medians of Δ ABC

$\therefore$ AD = 3 MD = 15 cm.

, $\because$ m ($\angle$ BAC) = 90°

, $\overline{AD}$ is a median in Δ ABC

$\therefore$ BC = 2 AD = 30 cm. (The req.)

17

$\because$ ABCD is a parallelogram

$\therefore$ m ($\angle$ C) = m ($\angle$ A) = 60°

$\therefore$ In Δ DEC :

m ($\angle$ EDC) = 180° – (60° + 90°) = 30°

$\therefore$ CE = $\frac{1}{2}$ DC $\therefore$ DC = 8 cm.

$\therefore$ The perimeter of the parallelogram ABCD

= (12 + 8) × 2 = 40 cm. (The req.)

18

$\because$ m ($\angle$ BAD) = 90° , m ($\angle$ BAE) = 30°

$\therefore$ m ($\angle$ DAF) = 60°

From Δ AFD :

$\because$ m ($\angle$ AFD) = 90° , m ($\angle$ DAF) = 60°

$\therefore$ m ($\angle$ ADF) = 30° $\therefore$ AD = 2 AF = 8 cm.

$\therefore$ The area of the square = 64 cm^2. (The req.)

19

In Δ BCE : $\because$ m ($\angle$ EBC) = 30°

$\therefore$ CE = $\frac{1}{2}$ BE (1)

, $\because$ m ($\angle$ EBC) = 30° $\therefore$ m ($\angle$ ABE) = 60°

$\therefore$ In Δ ABE :

$\because$ m ($\angle$ EAB) = 30°

, $\because$ m ($\angle$ AEB) = 90°

$\therefore$ BE = $\frac{1}{2}$ AB (2)

From (1) and (2) :

$\therefore$ CE = $\frac{1}{2} \times \frac{1}{2}$ AB = $\frac{1}{4}$ AB (Q.E.D.)

20

In Δ ABC :

$\because$ m ($\angle$ ABC) = 90° , m ($\angle$ A) = 30°

$\therefore$ AC = 2 BC = 16 cm.

, m ($\angle$ C) = 180° – (90° + 30°) = 60°

In Δ BCD :

$\because$ m ($\angle$ BDC) = 90° , m ($\angle$ C) = 60°

$\therefore$ m ($\angle$ CBD) = 180° – (90° + 60°) = 30°

$\therefore$ CD = $\frac{1}{2}$ BC = 4 cm.

$\therefore$ AD = AC – CD = 16 – 4 = 12 cm. (The req.)

21

In Δ ABC : $\because$ m ($\angle$ B) = 30° , m ($\angle$ C) = 90°

$\therefore$ AC = $\frac{1}{2}$ AB

, $\because$ E is the midpoint of $\overline{BC}$

, O is the midpoint of $\overline{AC}$

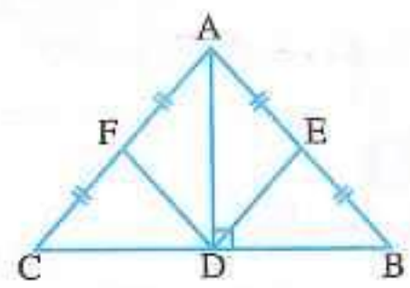
$\therefore$ EO = $\frac{1}{2}$ AB $\therefore$ EO = AC

In Δ DEO : $\because$ X is the midpoint of $\overline{DE}$

, Y is the midpoint of $\overline{DO}$

$\therefore$ XY = $\frac{1}{2}$ EO $\therefore$ XY = $\frac{1}{2}$ AC (Q.E.D)

22



In Δ ADB

$\because$ m ($\angle$ ADB) = 90°

, E is the midpoint of $\overline{AB}$

$\therefore$ DE = $\frac{1}{2}$ AB

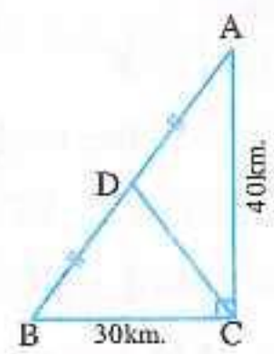
In Δ ADC : $\because$ m ($\angle$ ADC) = 90°

, F is the midpoint of $\overline{AC}$ $\therefore$ DF = $\frac{1}{2}$ AC

$\therefore$ DE + DF = $\frac{1}{2}$ AB + $\frac{1}{2}$ AC but AB = AC (Given)

$\therefore$ DE + DF = $\frac{1}{2}$ AB + $\frac{1}{2}$ AB = AB (Q.E.D.)

23



Let the service station lie at the point D which is the midpoint of $\overline{AB}$

$\therefore$ The road length = the length of $\overline{CD}$

In Δ ACB :

$\because$ m ($\angle$ ACB) = 90°

$\therefore$ $(AB)^2 = (AC)^2 + (BC)^2 = 1600 + 900 = 2500$

$\therefore$ AB = 50 km.

, $\because$ D is the midpoint of $\overline{AB}$

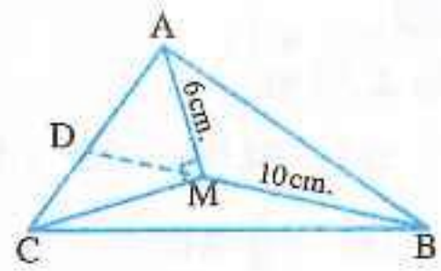
$\therefore$ CD = $\frac{1}{2}$ AB = $\frac{1}{2}$ × 50 = 25 km.

$\therefore$ The length of the road 25 km. (The req.)

24

Constr : Draw $\overrightarrow{BM}$ to intersect $\overline{AC}$ at D

Proof : $\because$ M is the point of concurrence of the medians of Δ ABC



$\therefore$ MD = $\frac{1}{2}$ BM = 5 cm.

In Δ AMC : $\because$ m ($\angle$ AMC) = 90°

, $\overline{MD}$ is a median

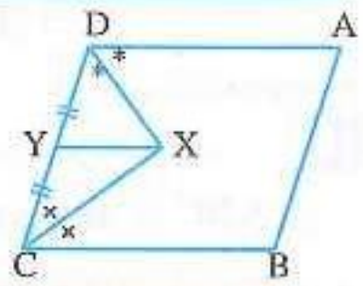
$\therefore MD = \frac{1}{2} AC$ $\therefore AC = 10$ cm. (First req.)

In ΔAMC : $\because m(\angle AMC) = 90°$

$\therefore (MC)^2 = (10)^2 - (6)^2 = 64$

$\therefore MC = \sqrt{64} = 8$ cm. (Second req.)

25



$\because \overline{DA} // \overline{CB}$

, $\overleftrightarrow{DC}$ is a transversal.

$\therefore m(\angle ADC) + m(\angle DCB) = 180°$

$\therefore \frac{1}{2} m(\angle ADC) + \frac{1}{2} m(\angle DCB) = 90°$

$\therefore m(\angle XDC) + m(\angle DCX) = 90°$

but the sum of the measures of the interior angles of a triangle XDC = 180°

$\therefore m(\angle DXC) = 90°$, $\because DY = YC$

$\therefore XY = \frac{1}{2} DC$ $\therefore XY = YC$ (Q.E.D.)

Answers of Exercise 3

1

1. $x = 50°$
2. $x = 56°$
3. $y = 63°$
4. $\ell = 65°$, $z = 50°$
5. $x = 54°$, $y = 117°$
6. $x = 69°$, $y = 111°$
7. $x = 120°$
8. $x = 68°$, $y = 127°$

2

1. congruent
2. 60°
3. F
4. 50°
5. 70°
6. 20°

3

1. b
2. c
3. c
4. b
5. a
6. b
7. d
8. a
9. b
10. b

4

In ΔABC :

$\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

$= \frac{180° - 40°}{2} = 70°$ (First req.)

$\because m(\angle ABC) = m(\angle ACB)$,

$\angle ABD$ supplements $\angle ABC$

, $\angle ACE$ supplements $\angle ACB$

$\because$ The supplementaries of the congruent angles are congruent

$\therefore \angle ABD \equiv \angle ACE$ (Second req.)

5

From ΔABC :

$\because AB = AC$ $\therefore m(\angle B) = m(\angle ACB) = 70°$

$\therefore m(\angle BAC) = 180° - (2 \times 70°) = 40°$

In ΔACD :

$\because AC = CD$ $\therefore m(\angle CAD) = m(\angle D)$

$\because \angle ACB$ is an exterior angle of ΔACD

$\therefore m(\angle ACB) = m(\angle CAD) + m(\angle D)$

$\therefore m(\angle CAD) = \frac{70°}{2} = 35°$

$\therefore m(\angle BAD) = m(\angle BAC) + m(\angle CAD)$

$= 40° + 35° = 75°$ (The req.)

6

$\because \angle ACD$ is an exterior angle of ΔABC

$\therefore m(\angle ACD) = 30° + 40° = 70°$

From ΔACD : $\because AC = AD$

$\therefore m(\angle D) = m(\angle ACD) = 70°$ (First req.)

$\therefore m(\angle CAD) = 180° - (70° + 70°) = 40°$ (Second req.)

7

$\because \Delta ACD$ is an equilateral triangle

$\therefore m(\angle CAD) = 60°$ (1)

From ΔABC : $\because AB = BC$

$\therefore m(\angle BAC) = m(\angle BCA) = \frac{180° - 40°}{2} = 70°$ (2)

From (1) and (2) :

$\therefore m(\angle BAD) = 60° + 70° = 130°$ (The req.)

8

In ΔABD : $\because AB = AD$

$\therefore m(\angle ADB) = m(\angle ABD) = \frac{180° - 120°}{2} = 30°$

(First req.)

$\because \overline{AD} // \overline{BC}$, $\overleftrightarrow{DC}$ is a transversal to them

$\therefore m(\angle C) + m(\angle ADC) = 180°$

$\therefore m(\angle C) = 180° - (65° + 30°) = 85°$ (Second req.)

9

$\because \overrightarrow{AD} // \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them.

$\therefore m(\angle C) = m(\angle DAC) = 30°$ (alternate angles)

In ΔABC : $\because AC = BC$

$\therefore m(\angle CAB) = m(\angle B) = \frac{180° - 30°}{2} = 75°$ (The req.)

10

$\because$ Δ DEC is an equilateral triangle.

$\therefore$ m ($\angle$ ECD) = 60° (1)

From Δ ABC :

$\because$ AB = AC $\therefore$ m ($\angle$ B) = m ($\angle$ ACB)

, $\because$ m ($\angle$ B) + m ($\angle$ ACB) = 180° – 80° = 100°

$\therefore$ m ($\angle$ B) = m ($\angle$ ACB) = $\frac{100°}{2}$ = 50° (2)

From (1) and (2) :

$\therefore$ m ($\angle$ BCD) = 50° + 60° = 110° (The req.)

11

In Δ ABC :

$\because$ BA = BC , m ($\angle$ B) = 80°

$\therefore$ m ($\angle$ BAC) = m ($\angle$ BCA) = $\frac{180° - 80°}{2}$ = 50°

$\therefore$ m ($\angle$ DAC) = 114° – 50° = 64°

In Δ ADC :

$\because$ DA = DC , m ($\angle$ DAC) = 64°

$\therefore$ m ($\angle$ ADC) = 180° – (64° × 2) = 52° (The req.)

12

From Δ ABC :

$\because$ AB = AC

$\therefore$ m ($\angle$ B) = m ($\angle$ BCA)

$\because$ m ($\angle$ B) + m ($\angle$ BCA) = 180° – 48° = 132°

$\therefore$ m ($\angle$ B) = m ($\angle$ BCA) = $\frac{132°}{2}$ = 66° (First req.)

$\because$ $\overline{CD}$ bisects $\angle$ ACB

$\therefore$ m ($\angle$ BCD) = $\frac{66°}{2}$ = 33° (Second req.)

13

$\because$ Δ ABC is an equilateral triangle

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) = 60°

$\therefore$ $\frac{1}{2}$ m ($\angle$ ABC) = $\frac{1}{2}$ m ($\angle$ ACB) = 30°

$\because$ $\overrightarrow{BD}$ bisects $\angle$ ABC , $\overrightarrow{CD}$ bisects $\angle$ ACB

$\therefore$ m ($\angle$ DBC) = m ($\angle$ DCB) = 30°

$\therefore$ From Δ DBC :

m ($\angle$ D) = 180° – (2 × 30°) = 120° (The req.)

14

$\because$ Δ ABC is an equilateral triangle.

$\therefore$ m ($\angle$ ABC) = 60° (1)

From Δ DBC :

$\because$ DB = DC , m ($\angle$ D) = 100°

$\therefore$ m ($\angle$ DBC) = m ($\angle$ DCB) = $\frac{180° - 100°}{2}$ = 40° (2)

From (1) and (2) :

$\therefore$ m ($\angle$ ABD) = m ($\angle$ ABC) – m ($\angle$ DBC)

= 60° – 40° = 20° (The req.)

15

$\because$ Δ ABC is an equilateral triangle.

$\therefore$ m ($\angle$ ACB) = m ($\angle$ B) = m ($\angle$ BAC) = 60°

$\therefore$ m ($\angle$ ACD) = 120°

In Δ ACD :

$\because$ AC = CD $\therefore$ m ($\angle$ CAD) = m ($\angle$ D)

$\because$ m ($\angle$ CAD) + m ($\angle$ D) = 180° – 120° = 60°

$\therefore$ m ($\angle$ CAD) = $\frac{60°}{2}$ = 30°

$\therefore$ m ($\angle$ BAD) = 60° + 30° = 90°

$\therefore$ $\overline{BA} \perp \overline{AD}$ (Q.E.D.)

Another solution :

$\because$ C is the midpoint of $\overline{BD}$

$\therefore$ $\overline{AC}$ is a median in Δ ABD

, $\because$ AC = $\frac{1}{2}$ BD

$\therefore$ $\angle$ BAD is right

$\therefore$ $\overline{BA} \perp \overline{AD}$ (Q.E.D.)

16

From Δ ABC :

$\because$ AB = AC $\therefore$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ $\Delta\Delta$ ABD , ACE in them :

$$\begin{cases} AB = AC \\ m(\angle B) = m(\angle C) \\ BD = EC \end{cases}$$

$\therefore$ Δ ABD $\equiv$ Δ ACE then we deduce that AD = AE

$\therefore$ Δ ADE is an isosceles triangle. (Q.E.D. 1)

$\therefore$ m ($\angle$ ADE) = m ($\angle$ AED)

$\therefore$ $\angle$ ADE $\equiv$ $\angle$ AED (Q.E.D. 2)

17

$\because$ $\Delta\Delta$ ADE , BCE in them :

$$\begin{cases} AD = CB \\ AE = EB \\ m(\angle A) = m(\angle B) \end{cases}$$

$\therefore \Delta ADE \equiv \Delta BCE$, then we deduce that $DE = CE$

In ΔDEC :

$\because DE = CE$ $\therefore m(\angle EDC) = m(\angle ECD)$

$\because m(\angle DEC) = 40°$

$\therefore m(\angle EDC) + m(\angle ECD) = 180° - 40° = 140°$

$\therefore m(\angle EDC) = \frac{140°}{2} = 70°$ (The req.)

18

$\because \angle LZX$ is an exterior angle of ΔXYZ

$\therefore m(\angle X) + m(\angle Y) = 130°$

$\because ZX = ZY$ $\therefore m(\angle X) = m(\angle Y)$

$\therefore m(\angle Y) = \frac{130°}{2} = 65°$

$\because \overrightarrow{LM} // \overline{XY}$, $\overleftrightarrow{LY}$ is a transversal to them

$\therefore m(\angle MLY) = m(\angle Y) = 65°$ (The req.)

19

$\because \overrightarrow{AE} // \overline{BC}$ and $\overleftrightarrow{BD}$ is a transversal to them.

$\therefore m(\angle B) = m(\angle DAE)$ (corresponding angles)

, $\because \overrightarrow{AE} // \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them.

$\therefore m(\angle C) = m(\angle EAC)$ (alternate angles)

but $m(\angle B) = m(\angle C)$ because $AB = AC$

$\therefore m(\angle DAE) = m(\angle EAC)$

i.e. $\overrightarrow{AE}$ bisects $\angle DAC$ (Q.E.D.)

20

$\because B \in \overline{AD}$

$\therefore m(\angle ABC) + m(\angle CBE) + m(\angle EBD) = 180°$ (1)

$\because$ The sum of measures of the angles of the triangle = 180°

$\therefore m(\angle ABC) + m(\angle A) + m(\angle C) = 180°$ (2)

From (1) and (2) :

$\therefore m(\angle CBE) + m(\angle EBD) = m(\angle A) + m(\angle C)$

, $\because m(\angle CBE) = m(\angle EBD)$ (Given)

, $m(\angle A) = m(\angle C)$ (because $BA = BC$)

$\therefore m(\angle CBE) = m(\angle C)$ and they are alternate angles.

$\therefore \overrightarrow{BE} // \overline{AC}$ (Q.E.D.)

21

In ΔDEC :

$\because DE = DC$

$\therefore m(\angle DEC) = m(\angle C) = \frac{180° - 40°}{2} = 70°$

$\because \overline{AD} // \overline{EC}$, $\overleftrightarrow{DE}$ is a transversal to them

$\therefore m(\angle ADE) = m(\angle DEC) = 70°$ (alternate angles)

$\because AD = AE$

$\therefore m(\angle AED) = m(\angle ADE) = 70°$ (First req.)

In ΔAED :

$\because m(\angle EAD) = 180° - (70° + 70°) = 40°$

$\because m(\angle BAD) = m(\angle C) = 70°$

(from properties of the parallelogram)

$\therefore m(\angle BAE) = 70° - 40° = 30°$ (Second req.)

22

From ΔABC :

$\because AB = AC$ $\therefore m(\angle B) = m(\angle C)$

$\therefore 2x + 13 = 3x - 17$ $\therefore x = 30°$

$\therefore m(\angle B) = m(\angle C) = 2 \times 30 + 13 = 73°$

$\therefore m(\angle A) = 180° - (73° + 73°) = 34°$ (The req.)

23

1 $x = 60°$, $y = 121°$ 2 $x = 45°$, $y = 105°$

3 $x = 44°$

4 $x = 75°$, $y = 15°$ 5 $x = 25°$, $y = 92°$

6 $y = 110°$, $\ell = 40°$, $z = 70°$

7 $x = 30°$, $y = 40°$ 8 $x = 70°$, $y = 50°$

9 $x = 120°$

24

1 3 cm. 2 5° 3 5°

4 66.5° 5 7° 6 22°

25

In $\Delta\Delta$ EBD and CBD :

$\overline{BD}$ is a common side

$m(\angle EBD) = m(\angle CBD)$

$m(\angle EDB) = m(\angle CDB)$

$\therefore \Delta EBD \equiv \Delta CBD$, then we deduce that :

$BE = BC$, $m(\angle BED) = m(\angle C)$

, $\because BA = BC$ $\therefore BA = BE$

$\therefore m(\angle A) = m(\angle BEA)$

, $\because m(\angle BEA) + m(\angle BED) = 180°$

$\therefore m(\angle A) + m(\angle C) = 180°$ (Q.E.D.)

26

$\Delta\Delta$ XYM , MZL in them :

$\begin{cases} XY = MZ \\ YM = LZ \\ m(\angle Y) = m(\angle Z) = 90° \end{cases}$

$\therefore \Delta XYM \equiv \Delta MZL$, then we deduce that :

$XM = ML$, $m(\angle XMY) = m(\angle MLZ)$

$\because \angle MLZ$ complements $\angle LMZ$

$\therefore \angle XMY$ complements $\angle LMZ$

$\therefore m(\angle XML) = 90°$

$\therefore$ From ΔXLM :

$\because MX = ML$, $m(\angle XML) = 90°$

$\therefore m(\angle MXL) = m(\angle MLX) = \frac{180° - 90°}{2} = 45°$

(The req.)

27

From ΔBDC :

$\because BD = CD$

$\therefore m(\angle DBC) = m(\angle BCD)$ (1)

$\because \angle ADB$ is an exterior angle of ΔCBD

$\therefore m(\angle ADB) = m(\angle DBC) + m(\angle BCD)$

from (1) : $m(\angle ADB) = 2\, m(\angle BCD)$ (2)

In ΔABD :

$\because AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB)$

from (2) :

$\therefore m(\angle ABD) = m(\angle ADB) = 2\, m(\angle BCD)$

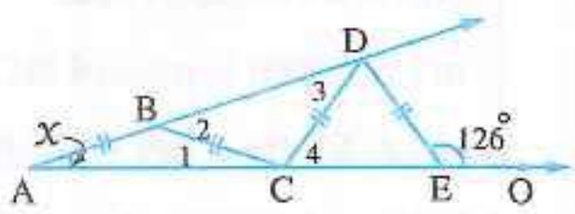
$\because \angle BAE$ is an exterior angle of ΔABD

$\therefore m(\angle BAE) = m(\angle ABD) + m(\angle ADB)$

$= 2\, m(\angle BCD) + 2\, m(\angle BCD)$

$= 4\, m(\angle BCD)$ (Q.E.D.)

28



In ΔABC :

$\because BC = BA$

$\therefore m(\angle A) = m(\angle 1) = x$

, $\because \angle 2$ is an exterior angle of ΔABC

$\therefore m(\angle 2) = m(\angle A) + m(\angle 1) = x + x = 2x$

In ΔDBC : $\because CB = CD$

$\therefore m(\angle 3) = m(\angle 2) = 2x$

, $\because \angle 4$ is an exterior angle of ΔACD

$\therefore m(\angle 4) = m(\angle A) + m(\angle 3) = x + 2x = 3x$ (1)

, $\because m(\angle DEC) = 180° - 126° = 54°$

In ΔCDE : $\because DC = DE$

$\therefore m(\angle 4) = m(\angle DEC) = 54°$ (2)

From (1) and (2) : $\therefore 3x = 54°$

$\therefore x = \frac{54°}{3} = 18°$ (The req.)

Answers of Exercise 4

1

1 AB = AC	2 YX = YZ	3 XY = XZ
4 AB = AC = BC	5 ML = MN	6 BA = BC
7 ZX = ZY	8 CB = CA	9 AC = AB

2

1 congruent , isosceles	2 equilateral	
3 isosceles	4 isosceles	
5 equilateral	6 6	7 60°

3

$\because B \in \overline{DC}$ $\therefore m(\angle ABC) = 180° - 125° = 55°$

In ΔABC : $m(\angle C) = 180° - (55° + 70°) = 55°$

$\therefore m(\angle ABC) = m(\angle C)$

$\therefore AB = AC$

$\therefore \Delta ABC$ is an isosceles triangle. (Q.E.D.)

4

$\because Y \in \overline{ZL}$ $\therefore m(\angle XYZ) = 180° - 120° = 60°$

, $\because XY = XZ$

$\therefore \Delta XYZ$ is an equilateral triangle. (Q.E.D.)

5

$\because B \in \overrightarrow{AD}$ $\therefore m(\angle ABC) = 180° - 120° = 60°$

Similarly : $m(\angle ACB) = 60°$

$\therefore m(\angle A) = 180° - (60° + 60°) = 60°$

$\therefore m(\angle A) = m(\angle ABC) = m(\angle ACB)$

$\therefore \Delta ABC$ is an equilateral triangle. (Q.E.D.)

6

$\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{DB}$ is a transversal to them.

$\therefore m(\angle DBC) = m(\angle ADB) = 40°$ (alternate angles)

In ΔDBC : $m(\angle C) = 180° - (100° + 40°) = 40°$

$\therefore m(\angle DBC) = m(\angle C)$ $\therefore DB = DC$

$\therefore \Delta DBC$ is an isosceles triangle. (Q.E.D.)

7

$\because \overline{XY} \parallel \overline{AC}$, $\overleftrightarrow{AB}$ is a transversal to them.

$\therefore m(\angle A) = m(\angle ABX) = 62°$ (alternate angles)

$\therefore m(\angle ABC) = 180° - (62° + 56°) = 62°$

$\therefore m(\angle ABC) = m(\angle A)$ $\therefore CA = CB$ (Q.E.D.)

8

$\because AB = AC$ $\therefore m(\angle B) = m(\angle C)$ (1)

, $\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore m(\angle AXY) = m(\angle B)$ (corresponding angles) (2)

Similarly $m(\angle AYX) = m(\angle C)$ (3)

From (1) , (2) and (3) :

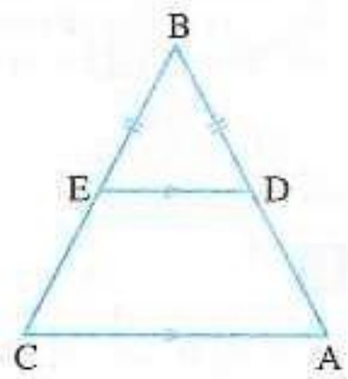
$\therefore m(\angle AXY) = m(\angle AYX)$ $\therefore AX = AY$

$\therefore \Delta AXY$ is an isosceles triangle. (Q.E.D. 1)

$\because AB = AC$, $AX = AY$ subtracting

$\therefore XB = YC$ (Q.E.D. 2)

9



From ΔEBD : $\because DB = EB$

$\therefore m(\angle BDE) = m(\angle BED)$ (1)

$\because \overline{DE} \parallel \overline{AC}$, $\overleftrightarrow{AD}$ is a transversal to them

$\therefore m(\angle A) = m(\angle BDE)$

(corresponding angles) (2)

Similarly $m(\angle C) = m(\angle BED)$ (3)

From (1) , (2) and (3) : $\therefore m(\angle A) = m(\angle C)$

$\therefore AB = BC$ (Q.E.D.)

10

$\because MB = MC$ $\therefore m(\angle B) = m(\angle C)$ (1)

$\because \overline{AD} \parallel \overline{BC}$ and $\overleftrightarrow{AC}$ is a transversal to them.

$\therefore m(\angle A) = m(\angle C)$ (alternate angles) (2)

similarly $m(\angle D) = m(\angle B)$ (3)

from (1) , (2) and (3) $\therefore m(\angle A) = m(\angle D)$

$\therefore MA = MD$ (Q.E.D.)

11

$\because AB = AC$ $\therefore m(\angle B) = m(\angle C)$ (1)

$\because \overline{AB} \parallel \overline{DE}$, $\overleftrightarrow{BE}$ is a transversal to them.

$\therefore m(\angle B) = m(\angle DEF)$ (corresponding angles) (2)

similarly $m(\angle C) = m(\angle DFE)$ (3)

from (1) , (2) and (3)

$\therefore m(\angle DEF) = m(\angle DFE)$

$\therefore DE = DF$ (Q.E.D. 1)

In $\Delta\Delta$ ABC , DEF :

$\because m(\angle B) = m(\angle DEF)$, $m(\angle C) = m(\angle DFE)$

$\therefore m(\angle BAC) = m(\angle EDF)$ (Q.E.D. 2)

12

$\because \overline{ED} \parallel \overline{BC}$, $\overleftrightarrow{DB}$ is a transversal to them.

$\therefore m(\angle EDB) = m(\angle DBC)$ (alternate angles)

but $m(\angle EBD) = m(\angle DBC)$

$\therefore m(\angle EDB) = m(\angle EBD)$ $\therefore EB = ED$

$\therefore \Delta EBD$ is an isosceles triangle. (Q.E.D.)

13

$\because \overrightarrow{AE} \parallel \overline{BC}$ and $\overleftrightarrow{DB}$ is a transversal to them.

$\therefore m(\angle DAE) = m(\angle B)$ (corresponding angles)

$\because \overrightarrow{AE} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them.

$\therefore m(\angle EAC) = m(\angle C)$ (alternate angles)

but $m(\angle DAE) = m(\angle EAC)$

$\therefore m(\angle B) = m(\angle C)$ $\therefore AB = AC$ (Q.E.D.)

14

$\because m(\angle ABC) = m(\angle ACB)$ $\therefore AB = AC$

$\therefore \Delta\Delta$ ADB , AEC in them :

$$\begin{cases} AB = AC \\ DB = EC \\ m(\angle D) = m(\angle E) = 90° \end{cases}$$

$\therefore \Delta ADB \equiv \Delta AEC$

$\therefore m(\angle DAB) = m(\angle CAE)$ (Q.E.D.)

15

In ΔYZX :

$\because YZ = YX$ $\therefore m(\angle Z) = \frac{180° - 50°}{2} = 65°$

$\because m(\angle ZMX) = 50° + 15° = 65°$

$\therefore$ In ΔMZX : $m(\angle Z) = m(\angle ZMX)$ $\therefore MX = ZX$

$\therefore \Delta MZX$ is an isosceles triangle. (Q.E.D.)

16

In ΔABC : $\because AB = AC$

$\therefore m(\angle ACB) = m(\angle ABC) = \frac{180° - 70°}{2} = 55°$

$\because m(\angle MCA) = 25°$ $\therefore m(\angle MCB) = 55° - 25° = 30°$

$\therefore m(\angle MBC) = m(\angle MCB)$ $\therefore MB = MC$

$\therefore \Delta MBC$ is an isosceles triangle. (Q.E.D.)

17

$\because \angle ADC$ is an exterior angle of ΔADB

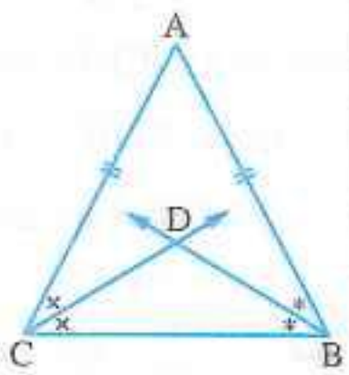
$\therefore m(\angle ADC) = 40° + 30° = 70°$

$\because AD = AC \qquad \therefore m(\angle C) = m(\angle ADC) = 70°$

$\therefore$ In ΔABC : $m(\angle BAC) = 180° - (40° + 70°) = 70°$

$\therefore m(\angle BAC) = m(\angle C) \qquad \therefore AB = BC$ (Q.E.D.)

18



$\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

$\therefore \frac{1}{2} m(\angle ABC) = \frac{1}{2} m(\angle ACB)$

$\because m(\angle DBC) = \frac{1}{2} m(\angle ABC)$

$m(\angle DCB) = \frac{1}{2} m(\angle ACB)$

$\therefore m(\angle DBC) = m(\angle DCB) \qquad \therefore DB = DC$

$\therefore \Delta DBC$ is an isosceles triangle. (Q.E.D.)

19

$\because \Delta ABC$ is an equilateral triangle

$\therefore m(\angle ACB) = 60°$

$\because \angle ACB$ is an exterior angle of ΔDCF

$\therefore m(\angle D) = 60° - 30° = 30°$

$\therefore m(\angle D) = m(\angle F) \qquad \therefore CD = CF$

$\therefore \Delta DCF$ is an isosceles triangle. (Q.E.D.)

20

$\because DA = DC \qquad \therefore m(\angle C) = m(\angle DAC) = 30°$

$\because \angle ADB$ is an exterior angle of ΔADC

$\therefore m(\angle ADB) = 30° + 30° = 60°$

$\because DA = DB$

$\therefore \Delta ABD$ is an equilateral triangle. (Q.E.D. 1)

$\because m(\angle BAD) = 60°$, $m(\angle DAC) = 30°$

$\therefore m(\angle BAC) = 90°$

$\therefore \Delta ABC$ is a right-angled triangle. (Q.E.D. 2)

21

$\because \overline{ED} \parallel \overline{AC}$, $\overleftrightarrow{EC}$ is a transversal to them.

$\therefore m(\angle DEC) = m(\angle ACE)$ (alternate angles)

$\because m(\angle DEC) = m(\angle AEC)$

$\therefore m(\angle ACE) = m(\angle AEC)$

$\therefore AE = AC$ (1)

, $\because \overline{DE} \parallel \overline{AC}$, $\overleftrightarrow{AB}$ is a transversal to them.

$\therefore m(\angle A) = m(\angle BED) = 60°$

(corresponding angles) (2)

from (1) and (2) :

$\therefore \Delta AEC$ is an equilateral triangle. (Q.E.D.)

22

In ΔABC : $m(\angle ACB) = 180° - (60° + 90°) = 30°$

In ΔECD : $m(\angle ECD) = 180° - (30° + 90°) = 60°$

$\because C \in \overline{BD}$

$\therefore m(\angle ACE) = 180° - (30° + 60°) = 90°$

In ΔACE : $m(\angle CAE) = 180° - (90° + 45°) = 45°$

$\therefore m(\angle CAE) = m(\angle CEA) = 45° \qquad \therefore CA = CE$

In ΔECD : $\because m(\angle D) = 90°$, $m(\angle CED) = 30°$

$\therefore CE = 2\ CD = 6$ cm. but $AC = CE$

$\therefore AC = 6$ cm. (The req.)

23

In ΔADE : $\because \angle ADE \equiv \angle AED \qquad \therefore AD = AE$

$D \in \overline{BC}$, $E \in \overline{BC}$

$\because \angle ADB$ supplements $\angle ADE$,

$\angle AEC$ supplements $\angle AED$

but $m(\angle ADE) = m(\angle AED)$

$\therefore m(\angle ADB) = m(\angle AEC)$

(supplementaries of the congruent angles are congruent)

$\therefore \Delta\Delta ADB$, AEC in them :

$$\begin{cases} m(\angle ADB) = m(\angle AEC) \\ AD = AE \\ BD = CE \end{cases}$$

$\therefore \Delta ADB \equiv \Delta AEC$

We deduce that $AB = AC$

$\therefore \Delta ABC$ is an isosceles triangle. (Q.E.D.)

24

In ΔBMC : $\because m(\angle MBC) = m(\angle MCB)$

$\therefore MB = MC$

, $\because$ m ($\angle$ ABM) = m ($\angle$ MCD)

(complementaries of equal angles in measure are equal in measure)

$\therefore$ $\Delta\Delta$ ABM , DCM in them :

AB = DC (two sides in a square)

BM = CM (proved)

m ($\angle$ ABM) = m ($\angle$ DCM) (proved)

$\therefore$ Δ ABM $\equiv$ Δ DCM we deduce that AM = DM

$\therefore$ Δ AMD is an isosceles triangle. (Q.E.D.)

25

In $\Delta\Delta$ ABF , AME : m ($\angle$ B) = m ($\angle$ AME) = 90°

m ($\angle$ BAF) = m ($\angle$ MAE) ($\overrightarrow{AE}$ bisects $\angle$ BAC)

$\therefore$ m ($\angle$ AFB) = m ($\angle$ E) (1)

, $\because$ $\overline{AD}$ // $\overline{BF}$, $\overleftrightarrow{AF}$ is a transversal to them.

$\therefore$ m ($\angle$ DAE) = m ($\angle$ AFB) (alternate angles) (2)

from (1) and (2) : $\therefore$ m ($\angle$ E) = m ($\angle$ DAE)

$\therefore$ DA = DE (Q.E.D.)

26

$\because$ m ($\angle$ B) = m ($\angle$ C) $\therefore$ AB = AC

$\therefore 2x - 1 = x + 3$ $\therefore 2x - x = 3 + 1$ $\therefore x = 4$

$\therefore$ AB = AC = 2 × 4 − 1 = 7 cm. , BC = 9 − 4 = 5 cm.

$\therefore$ The perimeter of Δ ABC = 7 + 7 + 5 = 19 cm. (The req.)

27

1 $\because 3x + x + 50° + 30° = 180°$

$\therefore 4x + 80° = 180°$

$\therefore 4x = 180° - 80° = 100°$ $\therefore x = \frac{100°}{4} = 25°$

$\therefore$ m ($\angle$ A) = 3 × 25° = 75° ,

m ($\angle$ B) = 25° + 50° = 75°

$\therefore$ m ($\angle$ A) = m ($\angle$ B) $\therefore$ CB = CA

2 $\because 2z + 3z - 10° + z + 40° = 180°$

$\therefore 6z + 30° = 180°$

$\therefore 6z = 180° - 30° = 150°$ $\therefore z = \frac{150°}{6} = 25°$

$\therefore$ m ($\angle$ B) = 3 × 25° − 10° = 65°

, m ($\angle$ C) = 25° + 40° = 65°

$\therefore$ m ($\angle$ B) = m ($\angle$ C) $\therefore$ AB = AC

3 $\because$ $\angle$ DBC is an exterior angle of Δ ABC

$\therefore 3x = x - 20° + x + 70°$

$\therefore 3x = 2x + 50°$ $\therefore x = 50°$

$\therefore$ m ($\angle$ A) = 50° − 20° = 30° ,

m ($\angle$ C) = 50° + 70° = 120°

$\therefore$ m ($\angle$ ABC) = 180° − (30° + 120°) = 30°

$\therefore$ m ($\angle$ A) = m ($\angle$ ABC) $\therefore$ CB = CA

28

1 c 2 b

Answers of Exercise 5

1

1 An axis of symmetry. 2 3 3 1 4 zero

5 Bisects it and it is perpendicular to the base.

6 Bisects the base and is perpendicular to it.

7 Bisects each of the base and the vertex angle.

8 The straight line perpendicular to it at its middle.

9 at equal distances 10 3 11 1 12 3

2

1 35° 2 70° 3 55° 4 2 5 $\overrightarrow{AD}$

3

1 a 2 c 3 b 4 b 5 a

6 a 7 b 8 a

4

$\because$ BA = BC , $\overline{BD} \perp \overline{AC}$

$\therefore$ $\overrightarrow{BD}$ bisects each of $\angle$ ABC , $\overline{AC}$

$\therefore$ AC = 2 AD = 40 cm.

, m ($\angle$ DBC) = $\frac{1}{2}$ m ($\angle$ ABC) = 45° (1) (First req.)

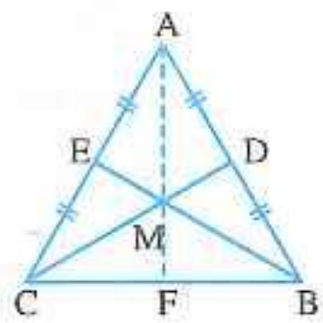
$\because$ ΔABC in which m ($\angle$B) = 90° , BA = BC

$\therefore$ m ($\angle$ C) = 45° (2)

From (1) and (2) : $\therefore$ DB = DC

$\therefore$ ΔDBC is an isosceles triangle. (Second req.)

5



In ΔABC : $AB = AC$,

M is the point of

intersection of its medians

$\therefore$ $\overline{AF}$ is a median of ΔABC

$\therefore$ $\overline{AM} \perp \overline{BC}$ (Q.E.D. 1)

, $\overrightarrow{AM}$ bisects $\angle BAC$ (Q.E.D. 2)

6

In ΔABC : $\because AB = AC$, $\overline{AD} \perp \overline{BC}$

$\therefore BC = 2 \times 5 = 10$ cm. (First req.)

In the right-angled triangle ADB at D

$AD = \sqrt{(13)^2 - (5)^2} = 12$ cm.

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times 10 \times 12 = 60$ cm^2.

(Second req.)

7

$\because AB = AC$, $\overline{AD} \perp \overline{BC}$ $\therefore BD = \frac{1}{2} BC = 5$ cm.

, $m(\angle BAC) = 2 \times 30^\circ = 60^\circ$

$\therefore$ ΔABC is an equilateral triangle.

$\because AB = 10$ cm.

$\because$ In ΔADB which is right-angled at D

$\therefore AD = \sqrt{(10)^2 - (5)^2} = 5\sqrt{3}$ cm. (First req.)

The number of axes of symmetry of $\Delta ABC = 3$

(Second req.)

The area of $\Delta ABC = \frac{1}{2} \times 10 \times 5\sqrt{3} = 25\sqrt{3}$ cm^2.

(Third req.)

8

In ΔABC : $\because AB = AC$, $\overrightarrow{AE}$ bisects $\angle BAC$

$\therefore BE = \frac{1}{2} BC$ (Q.E.D.1)

, $\overline{AE} \perp \overline{BC}$

$\therefore$ $\overleftrightarrow{AE}$ is the axis of symmetry of $\overline{BC}$, $D \in \overleftrightarrow{AE}$

$\therefore BD = CD$ (Q.E.D.2)

9

$\because C \in \overline{BD}$, $m(\angle ACD) = 130^\circ$

$\therefore m(\angle ACB) = 180^\circ - 130^\circ = 50^\circ$

From ΔABC : $m(\angle B) = 180^\circ - (80^\circ + 50^\circ) = 50^\circ$

$\therefore m(\angle B) = m(\angle ACB)$

$\therefore$ ΔABC is an isosceles triangle.

, $\overrightarrow{AE}$ bisects $\angle BAC$

$\therefore$ $\overline{AE} \perp \overline{BC}$, E is the midpoint of $\overline{BC}$ (Q.E.D.)

10

$\because m(\angle ABX) = m(\angle ACY)$

$\therefore m(\angle ABC) = m(\angle ACB)$

(The supplementaries of congruent angles are congruent)

$\therefore AB = AC$

$\because$ $\overline{AD}$ is a median of ΔABC which is isosceles.

$\therefore$ $\overline{AD} \perp \overline{BC}$ (Q.E.D.)

11

$\because$ $\overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{DB}$ is a transversal to them

$\therefore m(\angle ADB) = m(\angle DBC)$ (alternate angles)

but $m(\angle ABD) = m(\angle DBC)$

$\therefore m(\angle ADB) = m(\angle ABD)$

$\therefore$ In ΔABD : $AB = AD$ (Q.E.D.1)

, $\because$ $\overrightarrow{AE}$ bisects $\angle BAD$ $\therefore$ $\overline{AE} \perp \overline{BD}$ (Q.E.D.2)

, $BE = ED$ (Q.E.D.3)

12

In ΔACD :

$\because$ E is the midpoint of $\overline{AD}$

, $\overline{CE} \perp \overline{AD}$ $\therefore DC = AC$

$\therefore$ ΔACD is an isosceles triangle.

, $\because$ $\angle ADC$ is an exterior angle of ΔADB

$\therefore m(\angle ADC) = 20^\circ + 30^\circ = 50^\circ$

From ΔCDE :

$m(\angle DCE) = 180^\circ - (90^\circ + 50^\circ) = 40^\circ$

, $\because$ $\overrightarrow{CE}$ bisects $\angle ACD$

$\therefore m(\angle ACE) = m(\angle DCE) = 40^\circ$ (The req.)

13

In ΔADC :

$\because$ E is the midpoint of $\overline{DC}$

, $\overline{AE} \perp \overline{DC}$ $\therefore AD = AC$

$\therefore \Delta ADC$ is an isosceles triangle.

$\therefore m(\angle ADC) = m(\angle C) = 70°$

, $\because \angle ADC$ is an exterior angle of ΔABD

$\therefore m(\angle ADC) = m(\angle B) + m(\angle BAD)$

, $\because BD = AC$, $AD = AC$

$\therefore m(\angle B) = m(\angle BAD) = \frac{70°}{2} = 35°$ (The req.)

14

In ΔXYL : $\because XL = XY$, M is the midpoint of $\overline{LY}$

$\therefore \overleftrightarrow{XM}$ is the axis of $\overline{LY}$

similarly in ΔZYL , $\overleftrightarrow{ZM}$ is the axis of $\overline{LY}$

$\therefore$ X , M and Z are on the same straight line. (Q.E.D.)

15

$\because AB = AC$ $\therefore A \in$ the axis of $\overline{BC}$ (1)

, $m(\angle ABC) = m(\angle ACB)$,

$\because m(\angle ABD) = m(\angle ACD)$

by subtracting :

$\therefore m(\angle DBC) = m(\angle DCB)$

$\therefore DB = DC$ $\therefore D \in$ the axis of $\overline{BC}$ (2)

From (1) and (2) :

$\therefore \overleftrightarrow{AD}$ is the axis of $\overline{BC}$ (Q.E.D.)

16

$\because \overline{AD}$ bisects the base of ΔABC which is an isosceles triangle.

$\therefore \overline{AD} \perp \overline{BC}$ $\therefore m(\angle ADB) = 90°$

$\because \overleftrightarrow{XY} // \overline{BC}$, $\overleftrightarrow{AD}$ is a transversal to them.

$\therefore m(\angle YAD) = m(\angle ADB) = 90°$ (alternate angles)

$\therefore \overline{AD} \perp \overleftrightarrow{XY}$ (Q.E.D.)

17

$\because AB = AC$, $EB = EC$ $\therefore \overleftrightarrow{AE}$ is the axis of $\overline{BC}$

$\therefore BD = DC$ (First req.)

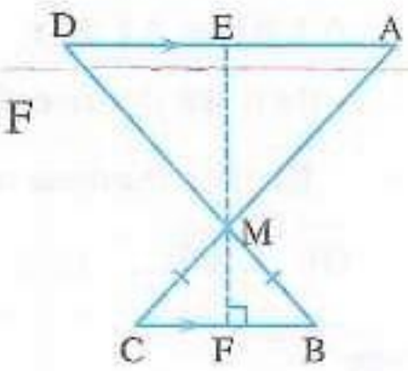
$\therefore DC = 3$ cm.

In ΔADC which is right-angled at D

$\therefore AD = \sqrt{(10)^2 - (3)^2} = \sqrt{100 - 9} = \sqrt{91}$ cm.

(Second req.)

18 **Constr. :**



Draw $\overleftrightarrow{MF} \perp \overline{BC}$ to meet $\overline{BC}$ at F and $\overline{AD}$ at E

Proof : $\because \overline{AD} // \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them.

$\therefore m(\angle A) = m(\angle C)$ similarly $m(\angle B) = m(\angle D)$

$\because MB = MC$ $\therefore m(\angle B) = m(\angle C)$

$\therefore m(\angle A) = m(\angle D)$ $\therefore AM = DM$

$\therefore \Delta AMD$ is an isosceles triangle. (Q.E.D.1)

In ΔMBC : $\because MB = MC$, $\overline{MF} \perp \overline{BC}$

$\therefore \overleftrightarrow{MF}$ is the axis of symmetry of ΔMBC

$\because \overline{AD} // \overline{BC}$, $\overleftrightarrow{FE}$ is a transversal to them.

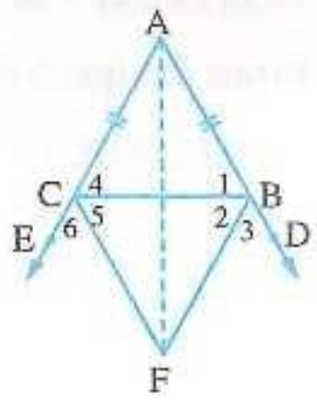
$\therefore m(\angle AEM) = m(\angle BFM) = 90°$

$\therefore \overline{ME} \perp \overline{AD}$

, $\because MA = MD$ $\therefore \overleftrightarrow{ME}$ is the axis of ΔAMD

$\therefore \overleftrightarrow{EF}$ is the axis of symmetry of each of ΔAMD , ΔBMC (Q.E.D.2)

19



$\because AB = AC$

$\therefore m(\angle 1) = m(\angle 4)$ (1)

$\because m(\angle DBC) = 180° - m(\angle 1)$ (2)

, $m(\angle BCE) = 180° - m(\angle 4)$ (3)

From (1) , (2) and (3) :

$\therefore m(\angle DBC) = m(\angle BCE)$

$\therefore \frac{1}{2} m(\angle DBC) = \frac{1}{2} m(\angle BCE)$

$\therefore m(\angle 2) = m(\angle 5)$ $\therefore FB = FC$

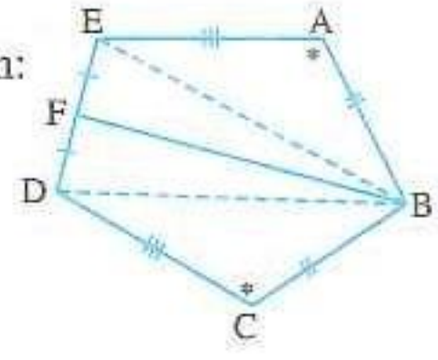
$\therefore \Delta BFC$ is an isosceles triangle. (Q.E.D.1)

$\because AB = AC$, $FB = FC$

$\therefore \overleftrightarrow{AF}$ is the axis of symmetry of $\overline{BC}$ (Q.E.D.2)

20

Constr. : Draw $\overline{BD}$, $\overline{BE}$



Proof: $\Delta\Delta ABE$, CBD in them:

$$\begin{cases} m(\angle A) = m(\angle C) \\ AB = CB \ , \\ AE = CD \end{cases}$$

$\therefore \Delta ABE \equiv \Delta CBD$

, then we deduce that: $BE = BD$

, $\because \overline{BF}$ is a median of ΔBED which is isosceles

$\therefore \overline{BF} \perp \overline{DE}$ (Q.E.D.)

21

1 c 2 b 3 c 4 b 5 a 6 b

22

In ΔABD :

$\because$ E is the midpoint of $\overline{AB}$

, $\overline{DE} \perp \overline{AB}$ $\therefore DA = DB$

$\therefore m(\angle A) = m(\angle ABD)$ (1)

, in ΔDBC :

$\because$ O is the midpoint of $\overline{BC}$

, $\overline{DO} \perp \overline{BC}$ $\therefore DB = DC$

$\therefore m(\angle DBC) = m(\angle C)$ (2)

, $\because m(\angle ABD) + m(\angle DBC) = 130°$ (3)

From (1) , (2) and (3) :

$\therefore m(\angle A) + m(\angle C) = 130°$

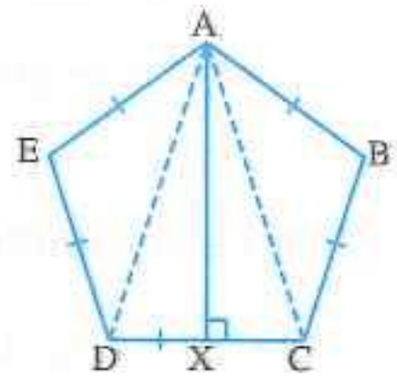
From the quadrilateral ABCD

$\therefore m(\angle ADC) = 360° - (130° + 130°) = 100°$ (The req.)

23

Constr. :

Draw $\overline{AD}$, $\overline{AC}$



Proof :

$\because$ ABCDE is a regular pentagon

$\therefore$ The measure of each interior angle = 108°

, $AB = BC = CD = DE = EA$

In ΔABC : $\because AB = BC$, $m(\angle ABC) = 108°$

$\therefore m(\angle BAC) = \frac{180° - 108°}{2} = 36°$

$\therefore \Delta ABC$, ΔAED in them :

$$\begin{cases} AB = AE \\ BC = ED \\ m(\angle ABC) = m(\angle AED) = 108° \end{cases}$$

$\therefore \Delta ABC \equiv \Delta AED$

Then we deduce that : $m(\angle EAD) = m(\angle BAC) = 36°$

, $AC = AD$ $\therefore \Delta ADC$ is an isosceles triangle.

, $\because \overline{AX} \perp \overline{CD}$

$\therefore m(\angle CAX) = m(\angle DAX)$

$= \frac{108° - (36° + 36°)}{2} = 18°$ (The req.)

Answers of unit five

Answers of Exercise 6

1

1 > 2 > 3 < 4 <

5 < 6 > 7 > , <

2

1 $m(\angle 1) < m(\angle 3)$ 2 $m(\angle 4) < m(\angle 2)$

3 $m(\angle 3) < m(\angle 5)$ 4 $m(\angle 6) < m(\angle 2)$

5 $m(\angle 1) < m(\angle 3) < m(\angle 5)$

6 $m(\angle 3) < m(\angle 5) < m(\angle 7)$

7 $m(\angle 1) < m(\angle 3) < m(\angle 5) < m(\angle 7)$

3

$\because \overleftrightarrow{AD} \perp \overline{BC}$ from its midpoint.

$\therefore \overleftrightarrow{AD}$ is the axis of symmetry of $\overline{BC}$

$\therefore AB = AC$

$\because E \in \overline{AB}$ $\therefore AB > AE$

$\therefore AC > AE$ (Q.E.D.)

4

$\because \overline{AB} \parallel \overline{CD}$, $\overleftrightarrow{BC}$ is a transversal.

$\therefore m(\angle BCD) = m(\angle ABC)$ (alternate angles)

$\therefore m(\angle BCD) + m(\angle ACB) > m(\angle ABC)$

$\therefore m(\angle ACD) > m(\angle ABC)$ (1) (Q.E.D.1)

$\because E \in \overrightarrow{CD}$ $\therefore \angle ADE$ is an exterior angle of ΔACD

$\therefore m(\angle ADE) > m(\angle ACD)$ (2)

From (1) and (2) :

$\therefore m(\angle ADE) > m(\angle ABC)$ (Q.E.D.2)

5

$\because E \in \overrightarrow{CB}$ $\therefore \angle ABE$ is an exterior angle of ΔABC

$\therefore m(\angle ABE) > m(\angle A)$ (1)

$\Delta\Delta ABM$, CDM in them :

$$\begin{cases} AM = MC \\ MB = MD \\ m(\angle AMB) = m(\angle DMC) \quad \text{(V.O.A)} \end{cases}$$

$\therefore \Delta ABM \equiv \Delta CDM$, then we deduce that :

$m(\angle A) = m(\angle ACD)$ and from (1) :

$\therefore m(\angle ABE) > m(\angle ACD)$ (Q.E.D.)

6

$\because$ The figure is a parallelogram.

$\therefore AD = BC$, $AB = CD$

$\because DX < BY$ $\therefore AX > CY$

$\therefore AX + AB > CY + CD$ (Q.E.D.)

7

$\because D \in \overline{AB}$ $\therefore \angle ADC$ is an exterior angle of ΔDBC

$\therefore m(\angle ADC) > m(\angle B)$

But $m(\angle ADC) = m(\angle ACD)$

because ΔADC in which $AD = AC$

$\therefore m(\angle ACD) > m(\angle B)$

$\therefore m(\angle ACD) + m(\angle DCB) > m(\angle B)$

$\therefore m(\angle ACB) > m(\angle B)$ (Q.E.D.)

8

In ΔAXY : $\because m(\angle AXY) = m(\angle AYX)$

$\therefore AX = AY$ (1)

$\because AC > AB$ $\therefore AY + YC > AX + XB$ (2)

From (1) and (2) :

$\therefore YC > XB$ (Q.E.D.)

9

$\because \angle ADC$ is an exterior angle of ΔDBC

$\therefore m(\angle ADC) > m(\angle B)$

But $m(\angle B) = m(\angle ACB)$

(because $AB = AC$ in ΔABC)

$\therefore m(\angle ADC) > m(\angle ACB)$ (Q.E.D.)

10

$\because m(\angle ACB) > m(\angle ABC)$

$\therefore$ The supplement

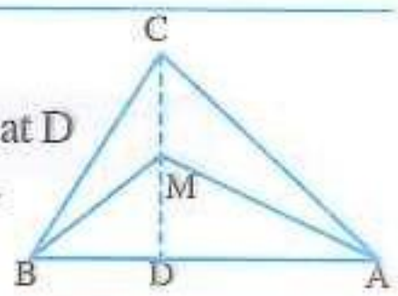
of $\angle ABC >$ the supplement of $\angle ACB$

$\therefore m(\angle ABD) > m(\angle ACE)$

$\therefore \frac{1}{2} m(\angle ABD) > \frac{1}{2} m(\angle ACE)$

i.e. $m(\angle ABX) > m(\angle ACY)$ (Q.E.D.)

11



Const : Draw $\overline{CM}$ to intersect $\overline{BA}$ at D

Proof : $\because \angle AMD$ is an exterior angle of ΔAMC

$\therefore m(\angle AMD) > m(\angle ACM)$ (1)

$\because \angle BMD$ is an exterior angle of ΔCMB

$\therefore m(\angle BMD) > m(\angle BCM)$ (2)

Adding (1) and (2)

$\therefore$ m ($\angle$ AMD) + m ($\angle$ BMD) > m ($\angle$ ACM) + m ($\angle$ BCM)

$\therefore$ m ($\angle$ AMB) > m ($\angle$ C) (Q.E.D.)

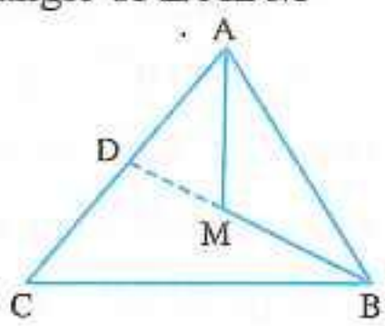
Another solution :

Const : Draw $\overrightarrow{BM}$ to intersect $\overline{AC}$ at D

Proof : $\because$ $\angle$ AMB is an exterior angle of Δ ADM

$\therefore$ m ($\angle$ AMB) > m ($\angle$ ADM)

, $\because$ $\angle$ ADM is an exterior angle of Δ BCD



$\therefore$ m ($\angle$ ADM) > m ($\angle$ C)

$\therefore$ m ($\angle$ AMB) > m ($\angle$ C) (Q.E.D.)

12

$\because$ m ($\angle$ B) > m ($\angle$ C)

$\therefore$ m ($\angle$B) + $\frac{1}{2}$m ($\angle$ BAC) > m ($\angle$ C) + $\frac{1}{2}$m ($\angle$ BAC)

$\therefore$ m ($\angle$ B) + m ($\angle$ BAD) > m ($\angle$C) + m ($\angle$ CAD)

but m ($\angle$ B) + m ($\angle$ BAD) = m ($\angle$ CDA)

(an exterior angle of ΔABD),

m ($\angle$ C) + m ($\angle$ CAD) = m ($\angle$ BDA)

(an exterior angle of ΔACD)

$\therefore$ m ($\angle$ ADC) > m ($\angle$ ADB)

$\because$ Their sum = 180° $\therefore$ m ($\angle$ ADC) > $\frac{180^\circ}{2}$

i.e. m ($\angle$ ADC) > 90°

i.e. $\angle$ ADC is an obtuse angle. (Q.E.D.)

13

$\because$ AC = AD $\therefore$ m ($\angle$ D) = m ($\angle$ ACD)

$\because$ m ($\angle$ ACB) > m ($\angle$ ABC)

$\therefore$ m ($\angle$ ACB) + m ($\angle$ ACD) > m ($\angle$ ABC) + m ($\angle$ D)

$\therefore$ m ($\angle$ BCD) > m ($\angle$ B) + m ($\angle$ D)

but the sum of measures of the interior angles of ΔBCD = 180°

$\therefore$ m ($\angle$ BCD) > $\frac{180^\circ}{2}$ i.e. m ($\angle$ BCD) > 90°

i.e. $\angle$ BCD is an obtuse angle. (Q.E.D.)

Answers of Exercise 7

1

1 The angle of the greater measure

2 $\angle$ A 3 m ($\angle$ D)

4 m ($\angle$ A) < m ($\angle$ B) < m ($\angle$ C)

2

1 > , > , < 2 < , < , > 3 > , > , >

3

1 $\because$ $\overline{BC}$ is the longest side

$\therefore$ $\angle$ A is the greatest angle in measure

$\because$ $\overline{AC}$ is the shortest side

$\therefore$ $\angle$ B is the smallest angle in measure

$\therefore$ The ascending order of measures of the angles is :

m ($\angle$ B) , m ($\angle$ C) and m ($\angle$ A)

2 $\because$ $\overline{BC}$ is the longest side.

$\therefore$ $\angle$ A is the greatest angle in measure.

$\because$ $\overline{AB}$ is the shortest side.

$\therefore$ $\angle$ C is the smallest angle in measure.

$\therefore$ The ascending order of the measures of the angles is : m ($\angle$ C) , m ($\angle$ B) and m ($\angle$ A)

4

In ΔABC : $\because$ AC > AB

$\therefore$ m ($\angle$ ABC) > m ($\angle$ ACB) (1)

In ΔBDC : $\because$ DB = DC

$\therefore$ m ($\angle$ DBC) = m ($\angle$ DCB) (2)

Adding (1) and (2) :

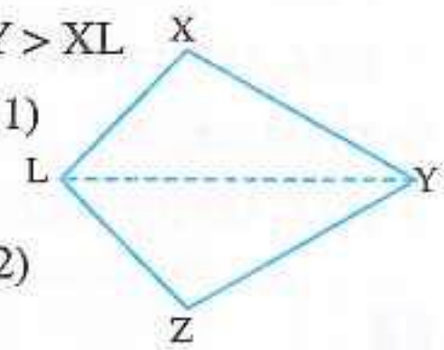
$\therefore$ m ($\angle$ ABC) + m ($\angle$ DBC) > m ($\angle$ ACB) + m ($\angle$ DCB)

$\therefore$ m ($\angle$ ABD) > m ($\angle$ ACD) (Q.E.D.)

5

Construction : Draw $\overline{YL}$

Proof : In ΔXYL $\because$ XY > XL



$\therefore$ m ($\angle$ XLY) > m ($\angle$ XYL) (1)

In ΔZYL : $\because$ YZ > ZL

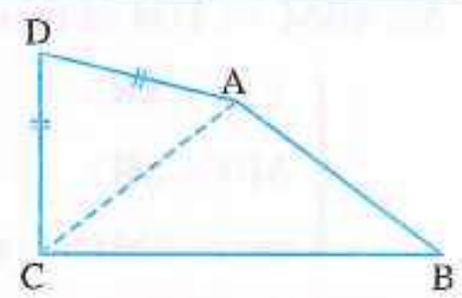
$\therefore$ m ($\angle$ ZLY) > m ($\angle$ ZYL) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ XLY) + m ($\angle$ ZLY) > m ($\angle$ XYL) + m ($\angle$ ZYL)

$\therefore$ m ($\angle$ XLZ) > m ($\angle$ XYZ) (Q.E.D.)

6



Construction : Draw $\overline{AC}$

Proof : In ΔABC

$\because$ BC > AB

$\therefore$ m ($\angle$ BAC) > m ($\angle$ ACB) (1)

In ΔDAC : $\because$ DA = DC

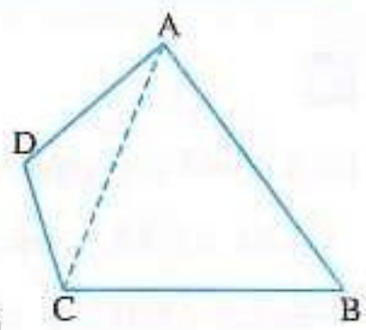
$\therefore$ m ($\angle$ DAC) = m ($\angle$ DCA) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ BAC) + m ($\angle$ DAC) > m ($\angle$ ACB) + m ($\angle$ DCA)

$\therefore$ m ($\angle$ BAD) > m ($\angle$ BCD) (Q.E.D.)

7



Construction : Draw $\overline{AC}$

Proof : In ΔABC

$\because$ AB > BC

$\therefore$ m ($\angle$ ACB) > m ($\angle$ BAC) (1)

In ΔADC : $\because$ AD > DC

$\therefore$ m ($\angle$ ACD) > m ($\angle$ CAD) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ BCD) > m ($\angle$ BAD) (Q.E.D.)

8

In ΔMBC : $\because$ MC > MB

$\therefore$ m ($\angle$ MBC) > m ($\angle$ MCB)

$\because$ m ($\angle$ MBC) = $\frac{1}{2}$ m ($\angle$ ABC)

, m ($\angle$ MCB) = $\frac{1}{2}$ m ($\angle$ ACB)

$\therefore$ $\frac{1}{2}$ m ($\angle$ ABC) > $\frac{1}{2}$ m ($\angle$ ACB)

$\therefore$ m ($\angle$ ABC) > m ($\angle$ ACB) (Q.E.D.)

9

In ΔDBC : $\because$ DB > DC

$\therefore$ m ($\angle$ DCB) > m ($\angle$ DBC)

In ΔABC : $\because$ AB = AC

$\therefore$ m ($\angle$ ACB) = m ($\angle$ ABC)

$\therefore$ m ($\angle$ ACB) – m ($\angle$ DCB) < m ($\angle$ ABC) – m ($\angle$ DBC)

$\therefore$ m ($\angle$ ACD) < m ($\angle$ ABD)

i.e. m ($\angle$ ABD) > m ($\angle$ ACD) (Q.E.D.)

10

In ΔABC : $\because$ AB > AC $\therefore$ m ($\angle$ C) > m ($\angle$ B) (1)

$\because$ $\overline{XY}$ // $\overline{BC}$ and $\overleftrightarrow{AC}$ is a transversal.

$\therefore$ m ($\angle$ AYX) = m ($\angle$ C) (corresponding angles) (2)

Similarly : $\because$ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal.

$\therefore$ m ($\angle$ AXY) = m ($\angle$ B) (3)

From (1) , (2) and (3) :

$\therefore$ m ($\angle$ AYX) > m ($\angle$ AXY) (Q.E.D.)

11

$\because$ AB > AC $\therefore$ m ($\angle$ C) > m ($\angle$ B)

But m ($\angle$ C) = m ($\angle$ AED) (corresponding angles)

, m ($\angle$ B) = m ($\angle$ ADE) (corresponding angles)

$\therefore$ m ($\angle$ AED) > m ($\angle$ ADE)

In ΔADE :

$\because$ m ($\angle$ A) = 90°

$\therefore$ m ($\angle$ AED) + m ($\angle$ ADE) = 90°

, $\because$ m ($\angle$ AED) > m ($\angle$ ADE)

$\therefore$ m ($\angle$ AED) > $\frac{90°}{2}$

$\therefore$ m ($\angle$ AED) > 45° (Q.E.D.)

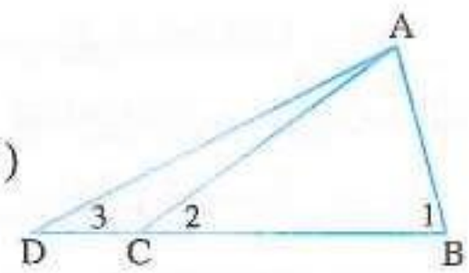
12

$\because$ AB > AC , BD = CE Subtracting $\therefore$ AD > AE

$\therefore$ In ΔADE : $\because$ AD > AE

$\therefore$ m ($\angle$ AED) > m ($\angle$ ADE) (Q.E.D.)

13



In ΔABC : $\because$ AC > AB

$\because$ m ($\angle$ 1) > m ($\angle$ 2) (1)

But $\angle$ 2 is an exterior angle of ΔACD

$\because$ m ($\angle$ 2) > m ($\angle$ 3) (2)

From (1) and (2) : $\therefore$ m ($\angle$ 1) > m ($\angle$ 3)

$\therefore$ m ($\angle$ ABD) > m ($\angle$ D) (Q.E.D.)

14

$\because$ ΔABC is an equilateral triangle.

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) = 60°

$\because$ m ($\angle$ EBC) < m ($\angle$ ECB) Subtracting

$\therefore$ m ($\angle$ ABC) – m ($\angle$ EBC) > m ($\angle$ ACB) – m ($\angle$ ECB)

$\therefore$ m ($\angle$ ABE) > m ($\angle$ ACE) (1) (Q.E.D.1)

$\because$ m ($\angle$ A) = m ($\angle$ B)

$\therefore$ m ($\angle$ A) = m ($\angle$ ABE) + m ($\angle$ EBC)

$\therefore$ m ($\angle$ A) > m ($\angle$ ABE) and from (1) :

$\therefore$ m ($\angle$ A) > m ($\angle$ ABE) > m ($\angle$ ACE) (Q.E.D.2)

15

In ΔXBC : $\because$ XC > XB

$\therefore$ m ($\angle$ XBC) > m ($\angle$ XCB)

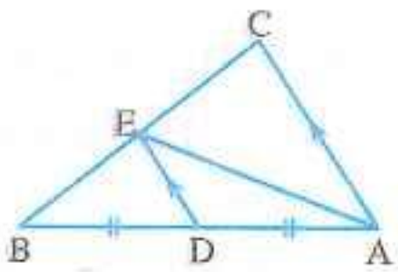
$\because$ ABCD is a rectangle.

i.e. m ($\angle$ ABC) = m ($\angle$ DCB) = 90°

$\therefore$ 90° – m ($\angle$ XBC) < 90° – m ($\angle$ XCB)

$\therefore$ m ($\angle$ ABX) < m ($\angle$ XCD) (Q.E.D.)

16



In ΔABC : $\because$ D is the midpoint of $\overline{AB}$, $\overline{DE} \parallel \overline{AC}$

$\therefore DE = \frac{1}{2} AC$

, $\because AD = \frac{1}{2} AB$, $\because AB > AC$

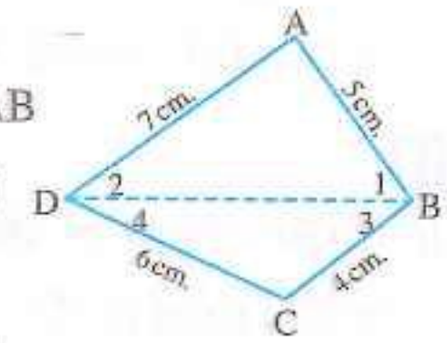
$\therefore \frac{1}{2} AB > \frac{1}{2} AC$

$\therefore AD > DE$ $\therefore m(\angle AED) > m(\angle DAE)$

But $m(\angle AED) = m(\angle CAE)$ (alternate angles)

$\therefore m(\angle CAE) > m(\angle DAE)$ (Q.E.D.)

17



1 **Construction :** Draw $\overline{BD}$

Proof : In ΔABD : $\because AD > AB$

$\therefore m(\angle 1) > m(\angle 2)$ (1)

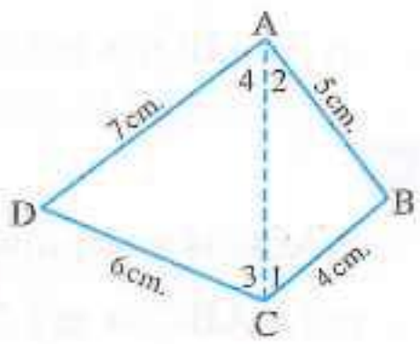
In ΔCBD : $\because CD > CB$

$\therefore m(\angle 3) > m(\angle 4)$ (2)

Adding (1) and (2) :

$\therefore m(\angle 1) + m(\angle 3) > m(\angle 2) + m(\angle 4)$

$\therefore m(\angle ABC) > m(\angle ADC)$ (Q.E.D.1)



2 **Construction :** Draw $\overline{AC}$

Proof : In ΔABC :

$\because BA > BC$

$\therefore m(\angle 1) > m(\angle 2)$ (3)

In ΔADC : $\because AD > DC$

$\therefore m(\angle 3) > m(\angle 4)$ (4)

Adding (3) and (4) :

$\therefore m(\angle 1) + m(\angle 3) > m(\angle 2) + m(\angle 4)$

$\because m(\angle BCD) > m(\angle BAD)$ (Q.E.D.2)

3 $\because$ The sum of measure of the interior angles of the quadrilateral = 360°

and from the two preceding requirements.

$\therefore m(\angle B) + m(\angle C) > \frac{360°}{2}$

$\therefore m(\angle B) + m(\angle C) > 180°$ (Q.E.D.3)

18

$\because \overline{AE}$ is a median in ΔABD , $m(\angle A) = 90°$

$\therefore AE = \frac{1}{2} BD$

, $\because$ E is the midpoint of $\overline{BD}$, $\overline{EX} \parallel \overline{AC}$

$\therefore EX = \frac{1}{2} DC$, $\because AE > EX$

$\therefore \frac{1}{2} BD > \frac{1}{2} DC$ $\therefore BD > DC$

$\therefore m(\angle C) > m(\angle DBC)$ (Q.E.D.)

19

In ΔABM : $\because AM > BM$ $\therefore m(\angle ABM) > m(\angle A)$ (1)

$\because AM = CM$ $\therefore$ In ΔCBM : $MC > MB$

$\therefore m(\angle MBC) > m(\angle C)$ (2)

Adding (1) and (2) :

$\therefore m(\angle ABM) + m(\angle MBC) > m(\angle A) + m(\angle C)$

$\therefore m(\angle ABC) > m(\angle A) + m(\angle C)$

$\therefore \angle ABC$ is an obtuse angle. (Q.E.D)

20

In ΔABD : $\because m(\angle B) = 90° - m(\angle BAD)$ (1)

From ΔACD: $m(\angle C) = 90° - m(\angle CAD)$ (2)

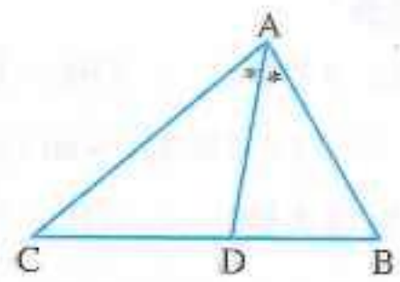
From ΔABC : $AC > AB$ $\therefore m(\angle B) > m(\angle C)$ (3)

From (1) , (2) and (3) :

$\therefore 90° - m(\angle BAD) > 90° - m(\angle CAD)$

$\therefore m(\angle BAD) < m(\angle CAD)$ (Q.E.D.)

21



In ΔABC : $\because AC > AB$

$\therefore m(\angle B) > m(\angle C)$

$\because m(\angle BAD) = m(\angle DAC)$

($\overrightarrow{AD}$ bisects $\angle A$)

$\therefore m(\angle B) + m(\angle BAD) > m(\angle C) + m(\angle DAC)$

$\because \angle ADC$ is an exterior angle of ΔABD

$\therefore m(\angle ADC) = m(\angle B) + m(\angle BAD)$

$\because \angle ADB$ is an exterior angle of ΔADC

$\therefore m(\angle ADB) = m(\angle C) + m(\angle DAC)$

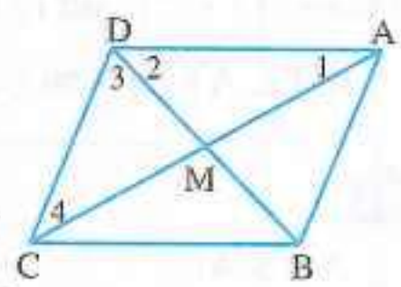
$\therefore m(\angle ADC) > m(\angle ADB)$

$\because m(\angle ADC) + m(\angle ADB) = 180°$

$\therefore m(\angle ADC) > \frac{180°}{2}$ *i.e.* $m(\angle ADC) > 90°$

i.e. $\angle ADC$ is an obtuse angle. (Q.E.D.)

22



Let $\overline{CA} \cap \overline{DB} = \{M\}$

$\because$ The two diagonals in the parallelogram bisects each other

, $\because AC > BD$ $\therefore MA > MD$,

$MC > MD$,

From ΔAMD : $\because AM > MD$

$\therefore m(\angle 2) > m(\angle 1)$ (1)

From ΔDMC : $\because MC > MD$

$\therefore m(\angle 3) > m(\angle 4)$ (2)

Adding (1) and (2) :

$\therefore m(\angle 2) + m(\angle 3) > m(\angle 1) + m(\angle 4)$

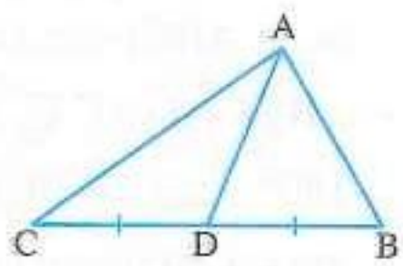
$\therefore m(\angle D) > m(\angle 1) + m(\angle 4)$

$\therefore$ In ΔADC :

$\because m(\angle D) > m(\angle CAD) + m(\angle ACD)$

$\therefore \angle D$ is an obtuse angle. (Q.E.D.)

23



$\because$ The perimeter of ΔACD

$= CD + DA + AC$

The perimeter of ΔABD

$= BD + DA + AB$

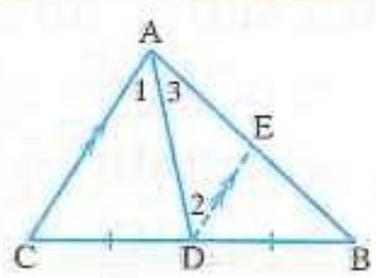
$\because$ The perimeter of $\Delta ACD >$ the perimeter of ΔABD

$\therefore CD + DA + AC > BD + DA + AB$

But $CD = BD$

$\therefore AC > AB \therefore m(\angle B) > m(\angle C)$ (Q.E.D.)

24



Construction : Draw $\overrightarrow{DE} // \overline{AC}$ to intersect $\overline{AB}$ at E

Proof : In ΔABC : $\because \overline{DE} // \overline{AC}$,

D is the midpoint of $\overline{BC}$

$\therefore$ E is the midpoint of $\overline{BA}$

$\therefore AE = \frac{1}{2}AB$, $DE = \frac{1}{2}AC$

, $\because AB > AC$ $\therefore AE > DE$

$\because m(\angle 2) > m(\angle 3)$ (1)

$\because \overline{DE} // \overline{AC}$, $\overrightarrow{AD}$ is a transversal to them

$\therefore m(\angle 1) = m(\angle 2)$ (Alternate angles)

From (1) : $\therefore m(\angle 1) > m(\angle 3)$

$\therefore m(\angle BAD) < m(\angle CAD)$ (Q.E.D.)

Answers of Exercise 8

1

1. A side greater in length than that opposite to the other angle , greater in measure than the measure of the angle opposite to the other side.
2. The shortest side.
3. The hypotenuse.
4. The length of the line segment drawn from the given point perpendicular to the given straight line.
5. $\overline{AB}$ 6. $\overline{AC}$ 7. $\overline{BC}$

2

1. c 2. a 3. d 4. a

3

1. > , > , < 2. > , > , >
3. > , > , > , > 4. > , < , < , >

4 $YZ < XY < XZ$

5 $AC > AB > BC$

6

$\because \overline{AE} // \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal.

$\therefore m(\angle C) = m(\angle EAC) = 30°$ (alternate angles) (1)

, $\because \overline{AE} // \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal.

$\therefore m(\angle B) = m(\angle DAE) = 70°$

(corresponding angles) (2)

From (1) and (2) : $\therefore m(\angle B) > m(\angle C)$

$\therefore AC > AB$ (Q.E.D.)

7

$\because C \in \overrightarrow{AE}$ $\therefore m(\angle ACB) = 180° - 120° = 60°$

$\because B \in \overrightarrow{CD}$ $\therefore m(\angle ABC) = 180° - 110° = 70°$

$\therefore m(\angle A) = 180° - (60° + 70°) = 50°$

$\therefore m(\angle ACB) > m(\angle A)$ $\therefore AB > BC$ (Q.E.D.)

8

In ΔABC : $\because AB = AC$

$\therefore m(\angle ACB) = m(\angle B) = 65°$

$\therefore m(\angle DCB) = 65° + 20° = 85°$

In ΔDBC : $\therefore m(\angle D) = 180° - (65° + 85°) = 30°$

$\therefore$ In ΔDAC : $m(\angle D) > m(\angle ACD)$

$\therefore AC > AD$ but $AB = AC$

$\therefore AB > AD$ (Q.E.D.)

9

In ΔDBC : $\because DB = DC$

$\therefore m(\angle B) = m(\angle DCB) = \frac{180° - 100°}{2} = 40°$

$\because \overrightarrow{CD}$ bisects $\angle ACB$ $\therefore m(\angle ACD) = 40°$

$\because D \in \overline{AB}$

$\therefore$ m ($\angle$ ADC) = 180° – 100° = 80°

$\therefore$ In Δ ADC : m ($\angle$ A) = 180° – (40° + 80°)= 60°

$\therefore$ m ($\angle$ ADC) > m ($\angle$ A)

$\therefore$ AC > DC but DC = DB

$\therefore$ AC > DB (Q.E.D.)

10

$\because \overline{AD} // \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal.

$\therefore$ m ($\angle$ ACB) = m ($\angle$ DAC) = 30° (alternate angles)

In Δ ABC : $\because$ m ($\angle$ BAC) > m ($\angle$ ACB)

$\therefore$ BC > AB (Q.E.D.)

11

In Δ ACM : $\because$ m ($\angle$ C) = 90° $\therefore$ AM > CM (1)

In Δ BDM : $\because$ m ($\angle$ D) = 90° $\therefore$ BM > DM (2)

Adding (1) and (2) : $\therefore$ AM + MB > CM + MD

$\therefore$ AB > CD (Q.E.D.)

12

In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB)

$\because$ m ($\angle$ ABM) < m ($\angle$ ACM)

$\therefore$ m ($\angle$ ABC) – m ($\angle$ ABM) > m ($\angle$ ACB) – m ($\angle$ ACM)

$\therefore$ m ($\angle$ MBC) > m ($\angle$ MCB)

From Δ MBC : $\therefore$ MC > MB (Q.E.D.)

13

$\because \overline{DE} // \overline{BC}$, $\overleftrightarrow{DB}$ is a transversal.

$\therefore$ m ($\angle$ ADE) = m ($\angle$ B) (corresponding angles)

, $\because$ $\angle$ B is an obtuse angle

$\therefore$ $\angle$ ADE is an obtuse angle

$\therefore$ In Δ ADE :

m ($\angle$ ADE) > m ($\angle$ AED) (obtuse and acute angles)

$\therefore$ AE > AD (Q.E.D.)

14

In Δ ABC : $\because$ AB > AC $\therefore$ m ($\angle$ C) > m ($\angle$ B) (1)

$\because \overline{DE} // \overline{BC}$ and $\overleftrightarrow{DC}$ is a transversal.

$\therefore$ m ($\angle$ D) = m ($\angle$ C) (alternate angles) (2)

$\therefore \overline{DE} // \overline{BC}$, $\overleftrightarrow{BE}$ is a transversal.

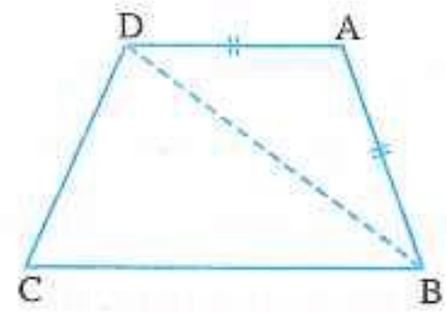
$\therefore$ m ($\angle$ E) = m ($\angle$ B) (alternate angles) (3)

From (1) , (2) and (3) :

$\therefore$ m ($\angle$ D) > m ($\angle$ E) and from Δ ADE

$\therefore$ AE > AD (Q.E.D.)

15



Const. : Draw $\overline{BD}$

Proof : In Δ ADB

$\because$ AD = AB

$\therefore$ m ($\angle$ ADB) = m ($\angle$ ABD)

, $\because$ m ($\angle$ ADC) > m ($\angle$ ABC)

$\therefore$ m ($\angle$ ADC) – m ($\angle$ ADB) > m ($\angle$ ABC) – m ($\angle$ ABD)

$\therefore$ m ($\angle$ BDC) > m ($\angle$ DBC)

$\therefore$ In Δ BDC : BC > CD (Q.E.D.)

16

In Δ ABC : $\because$ AB > AC

$\therefore$ m ($\angle$ ABC) < m ($\angle$ ACB)

, $\because$ B $\in \overline{AD}$, C $\in \overline{AE}$

$\therefore$ 180° – m ($\angle$ ABC) > 180° – m ($\angle$ ACB)

$\therefore$ m ($\angle$ CBD) > m ($\angle$ BCE)

$\because \overrightarrow{BF}$ bisects $\angle$ DBC , $\overrightarrow{CF}$ bisects $\angle$ BCE

$\therefore$ m ($\angle$ FBC) > m ($\angle$ BCF) (Q.E.D.1)

$\therefore$ CF > BF (Q.E.D.2)

17

In Δ ABD : $\because$ BD = AD

$\therefore$ m ($\angle$ BAD) = m ($\angle$ B)

$\therefore$ m ($\angle$ BAD) + m ($\angle$ DAC) > m ($\angle$ B)

$\therefore$ m ($\angle$ BAC) > m ($\angle$ B) $\therefore$ BC > AC (Q.E.D.)

18

In Δ DBC : $\because$ m ($\angle$ B) > m ($\angle$ DCB)

$\therefore$ DC > DB but DB = AD $\therefore$ DC > AD

$\therefore$ In Δ ADC : m ($\angle$ A) > m ($\angle$ ACD) (Q.E.D.1)

$\because$ m ($\angle$ BDC) = 180° – (70° + 50°) = 60°

$\because$ $\angle$ BCD is an exterior angle of Δ ADC

$\therefore$ m ($\angle$ BDC) = m ($\angle$ A) + m ($\angle$ ACD) = 60°

$\because$ m ($\angle$ A) > m ($\angle$ ACD) $\therefore$ m ($\angle$ ACD) < 30°

$\therefore$ m ($\angle$ ACD) + m ($\angle$ DCB) < 30° + 50°

$\therefore$ m ($\angle$ ACB) < 80°

$\therefore$ $\angle$ ACB is an acute angle. (Q.E.D.2)

19

In Δ AFB : $\because$ FA = FB

$\therefore$ m ($\angle$ FBA) = m ($\angle$ FAB) = 50° (1)

$\because$ $\angle$ AFD is an exterior angle of Δ AFB

$\therefore$ m ($\angle$ AFD) = 50° + 50° = 100° (2)

$\therefore$ In Δ AFD :

$\because$ FA = FD $\quad \therefore m(\angle FDA) = \frac{180° - 100°}{2} = 40°$

From (1) and (2) : $\therefore$ In Δ ABD

$m(\angle ABD) > m(\angle ADB)$

$\therefore AD > AB$ (Q.E.D.1)

In Δ ABD : $\because \overline{AF}$ is a median , $AF = \frac{1}{2}BD$

$\therefore m(\angle DAB) = 90°$

$\therefore \overline{BC}$ is a hypotenuse of Δ BAC

$\therefore BC > AC$ (Q.E.D.2)

20

$\because \angle ADB$ is an exterior angle of Δ ADC

$\therefore m(\angle ADB) > m(\angle C)$

, $\because m(\angle C) = m(\angle B)$, (AB = AC in Δ ABC)

$\therefore m(\angle ADB) > m(\angle B)$

And from Δ ABD : AB > AD (Q.E.D.)

21

$\Delta\Delta$ ABD , AED in them

$m(\angle B) = m(\angle AED)$, $m(\angle BAD) = m(\angle DAE)$

$\therefore m(\angle ADB) = m(\angle ADE)$

$\therefore \Delta\Delta$ ABD , AED

In them $\begin{cases} m(\angle BAD) = m(\angle EAD) \\ m(\angle ADB) = m(\angle ADE) \\ \overline{AD} \text{ is a common side} \end{cases}$

$\therefore \Delta ABD \equiv \Delta AED$, then we deduce that :

BD = DE (Q.E.D.1)

$\therefore$ In Δ DEC

$\because m(\angle DEC) = 90° \quad \therefore DC > DE$

$\because DE = DB \quad \therefore DC > DB$ (Q.E.D.2)

22

$\because m(\angle ADC) = 180° - 110° = 70°$

$\therefore \Delta$ ACD in which $m(\angle ADC) > m(\angle C)$

$\therefore AC > AD$ (1)

$\because \Delta$ ADB is an obtuse-angled at D

$\therefore AB > AD$ (2)

By adding (1) and (2) : $\therefore AB + AC > 2\,AD$ (Q.E.D.)

23

In Δ ABC : $\because m(\angle B) = 90°$

$\therefore$ The hypotenuse $\overline{AC}$ is the longest side.

$\therefore AB < AC$, $BC < AC$

By adding : $\therefore AB + BC < 2\,AC$ (Q.E.D.)

24

$\because \overline{AD} \parallel \overline{CE}$, $\overleftrightarrow{AC}$ is a transversal.

$\therefore m(\angle DAC) = m(\angle ACE)$

$\therefore m(\angle BCE) > m(\angle DAC)$

$\therefore m(\angle BCE) > m(\angle BAD)$, ($\overrightarrow{AD}$ bisects $\angle$ BAC) (1)

$\because \overline{AD} \parallel \overline{CE}$ and $\overleftrightarrow{BE}$ is a transversal.

$\therefore m(\angle BAD) = m(\angle E)$ (corresponding angles) (2)

From (1) and (2) :

$\therefore m(\angle BCE) > m(\angle E)$ and from Δ BCE

$\therefore BE > BC$ (Q.E.D.)

25

$\because \Delta$ XYM is right-angled at Y

$\therefore \angle$ XMY is an acute angle.

$\therefore \angle$ XMZ is an obtuse angle.

$\because \Delta$ XMZ is an obtuse-angled at M

$\therefore XZ > XM$ (Q.E.D.)

26

In Δ ABC : $\because AB > BC$

$\therefore m(\angle C) > m(\angle A)$ (1)

, $\because m(\angle ABC) = 90°$

$\therefore \angle$ A complements $\angle$ C (2)

, in Δ ABD : $\because m(\angle ADB) = 90°$

$\therefore \angle$ A complements $\angle$ ABD (3)

From (2) and (3) :

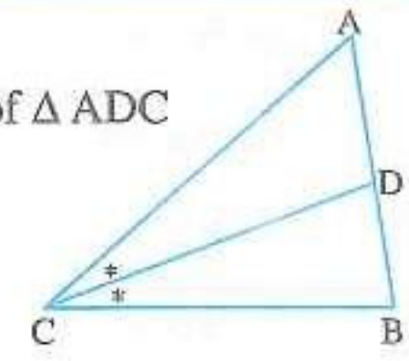
$\therefore m(\angle C) = m(\angle ABD)$

, from (1) :

$\therefore m(\angle ABD) > m(\angle A)$

$\therefore$ In Δ ABD : AD > BD (Q.E.D.)

27



$\because \angle$ BDC is an exterior angle of Δ ADC

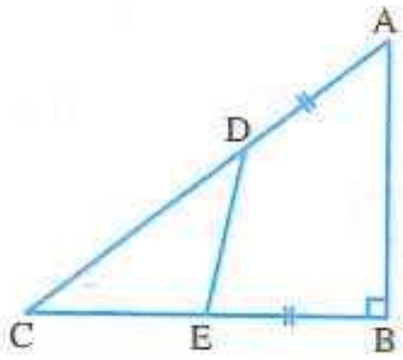
$\therefore m(\angle BDC) > m(\angle ACD)$

$\because m(\angle BCD) = m(\angle ACD)$

$\therefore m(\angle BDC) > m(\angle BCD)$

In Δ DBC : $\therefore BC > BD$ (Q.E.D.)

28



In Δ ABC : $\because$ m ($\angle$ B) = 90°

$\therefore$ AC > BC

$\because$ AD = BE

$\therefore$ AC – AD > BC – BE

$\therefore$ DC > EC

$\therefore$ In Δ DEC : $\because$ DC > EC

$\therefore$ m ($\angle$ CED) > m ($\angle$ CDE) (Q.E.D.)

29

$\because$ m ($\angle$ A) + m ($\angle$ B) + m ($\angle$ C) = 180°

$\therefore 5x + 2° + 6x - 10° + x + 20° = 180°$

$\therefore 12x + 12° = 180°$ $\therefore 12x = 180° - 12° = 168°$

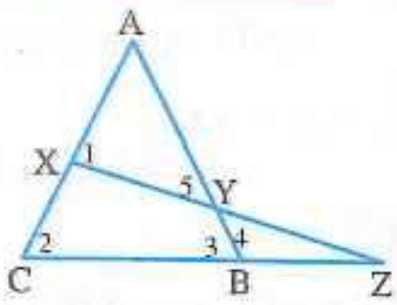
$\therefore x = \frac{168°}{12°} = 14°$ $\therefore$ m ($\angle$ A) = 5 × 14° + 2° = 72°

, m ($\angle$ B) = 6 × 14° – 10° = 74°

m ($\angle$ C) = 14° + 20° = 34°

$\therefore$ AB < BC < AC (The req.)

30



$\because$ $\angle$ 1 is an exterior angle of Δ XZC

$\therefore$ m ($\angle$ 1) > m ($\angle$ 2)

But m ($\angle$ 3) = m ($\angle$ 2) , (AB = AC in Δ ABC)

$\therefore$ m ($\angle$ 1) > m ($\angle$ 3)

But $\angle$ 3 is an exterior angle of Δ YZB

$\therefore$ m ($\angle$ 3) > m ($\angle$ 4) $\therefore$ m ($\angle$ 1) > m ($\angle$ 4)

, $\because$ m ($\angle$ 4) = m ($\angle$ 5) (V.O.A.)

$\therefore$ m ($\angle$ 1) > m ($\angle$ 5)

and from Δ AYX : $\therefore$ AY > AX (Q.E.D.)

Answers of Exercise 9

1

1. $\because 3 + 4 < 9$ $\therefore$ lengths are not suitable
2. $\because 5 + 7 > 8$ $\therefore$ lengths are suitable
3. $\because 4 + 6 = 10$ $\therefore$ lengths are not suitable
4. $\because 6 + 8 > 13$ $\therefore$ lengths are suitable
5. $\because 3 + 4 > 5$ $\therefore$ lengths are suitable
6. $\because 9 + 9 < 19$ $\therefore$ lengths are not suitable

2

Let the length of the third side be ℓ

1. $\because 9 - 6 < \ell < 9 + 6$ $\therefore 3 < \ell < 15$
   $\therefore \ell \in]3 , 15[$
2. $\because 3 - 3 < \ell < 3 + 3$ $\therefore 0 < \ell < 6$
   $\therefore \ell \in]0 , 6[$
3. $\because 3.2 - 2.9 < \ell < 3.2 + 2.9$
   $\therefore 0.3 < \ell < 6.1$ $\therefore \ell \in]0.3 , 6.1[$
4. $\because 7.3 - 5.7 < \ell < 7.3 + 5.7$
   $\therefore 1.6 < \ell < 13$ $\therefore \ell \in]1.6 , 13[$

3

1 b	2 b	3 c	4 d	5 a
6 b	7 d	8 b	9 a	10 a

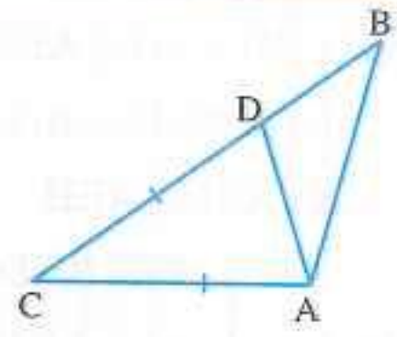
4

In Δ XLY : XL + LY > XY (The triangle inequality)

But XL = LZ $\therefore$ LZ + LY > XY

$\therefore$ YZ > XY (Q.E.D.)

5



In Δ ABC :

$\because$ CA + AB > BC

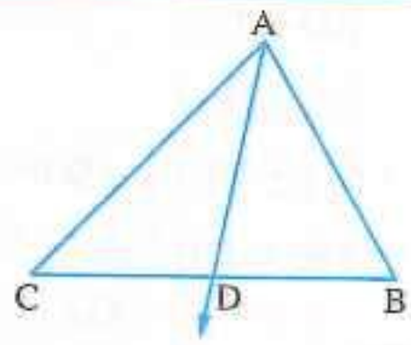
(triangle inequality)

$\therefore$ CA + AB > BD + DC

But CA = DC

$\therefore$ AB > BD (Q.E.D.)

6



From Δ ABD :

AD + DB > AB

(triangle inequality) (1)

From Δ ADC : AD + DC > AC

(Triangle inequality) (2)

Adding (1) and (2) :

$\therefore$ BD + DC + 2 AD > AB + AC (Q.E.D.)

7

From Δ ABM : MA + MB > AB

(Triangle inequality) (1)

From Δ BMC : MB + MC > BC

(Triangle inequality) (2)

From Δ AMC : MA + MC > AC (Triangle inequality) (3)

Adding (1) , (2) and (3) :

$\therefore$ 2 MA + 2 MB + 2 MC > AB + BC + AC

$\therefore$ MA + MB + MC > $\frac{1}{2}$ the perimeter of Δ ABC

(Q.E.D.)

8

From Δ AEZ : AE + AZ > EZ (Triangle inequality) (1)

From Δ EBF :

EB + BF > EF (Triangle inequality) (2)

From Δ ZFC : ZC + CF > ZF (Triangle inequality) (3)

Adding (1) , (2) and (3) :

$\therefore$ AB + AC + BC > EZ + EF + ZF

$\therefore$ The perimeter of Δ ABC > the perimeter of Δ EFZ

(Q.E.D.)

9

In Δ DAC : DA + DC > AC (1)

In Δ DBC : DB + DC > BC (2)

In Δ DBA : DB + DA > AB (3)

Adding (1) , (2) and (3) :

$\therefore$ 2 (DA + DB + DC) > AC + BC + AB

$\therefore$ AC + BC + AB < 2 (DA + DB + DC)

$\therefore$ The perimeter of Δ ABC < 2 (DA + DB + DC)

(Q.E.D.)

10

$\because$ 7 – 3 < AC < 7 + 3 $\therefore$ 4 < AC < 10

$\therefore$ AC $\in$]4 , 10[$\therefore$ AB < AC

$\therefore$ m ($\angle$ C) < m ($\angle$ B) (The req.)

11

Assuming that ABC is a triangle

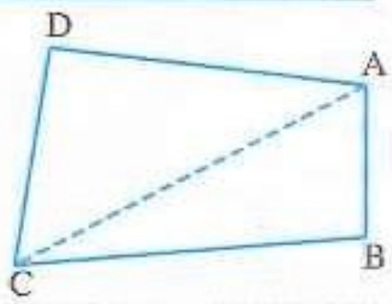
$\because$ AB < AC + BC adding AB to both sides

$\therefore$ 2 AB < AC + BC + AB

$\therefore$ AB < $\frac{1}{2}$ the perimeter of Δ ABC

$\therefore$ The length of any side in the triangle is less than the half of the perimeter of the triangle (Q.E.D.)

12



Construction : Draw $\overline{AC}$

Proof : From Δ ABC

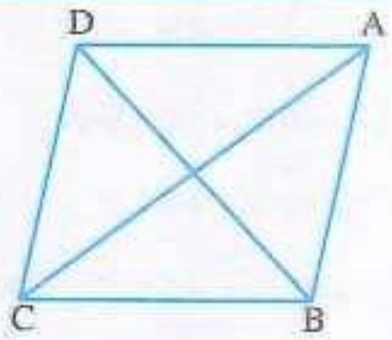
AB + BC > AC (Triangle inequality) (1)

From Δ ADC :

AC + CD > AD (Triangle inequality) (2)

From (1) and (2) : $\therefore$ AB + BC + CD > AD (Q.E.D.)

13



Let ABCD be a quadrilateral

In Δ ABC : AB + BC > AC (1)

In Δ BCD : BC + CD > BD (2)

In Δ ACD : AD + CD > AC (3)

In Δ ABD : AB + AD > BD (4)

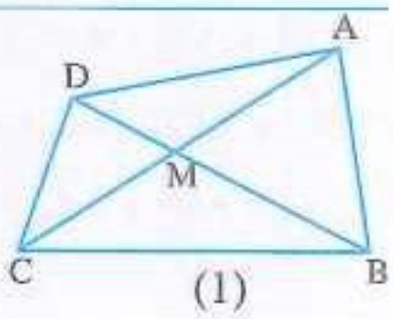
Adding (1) , (2) , (3) , and (4) :

$\because$ 2 AB + 2 BC + 2 CD + 2 AD > 2 AC + 2 BD

$\therefore$ AB + BC + CD + AD > AC + BD

$\therefore$ The sum of lengths of the two diagonals in any convex quadrilateral is less than the perimeter of the quadrilateral. (Q.E.D.)

14



Let ABCD be a quadrilateral ,

$\overline{AC} \cap \overline{BD} = \{M\}$

From Δ ABM : AB < MA + MB (1)

From Δ BMC : BC < MB + MC (2)

From Δ CMD : CD < MC + MD (3)

From Δ AMD : AD < MA + MD (4)

Adding (1) , (2) , (3) and (4)

$\therefore$ AB + BC + CD + AD

< 2 MA + 2 MC + 2 MB + 2 MD

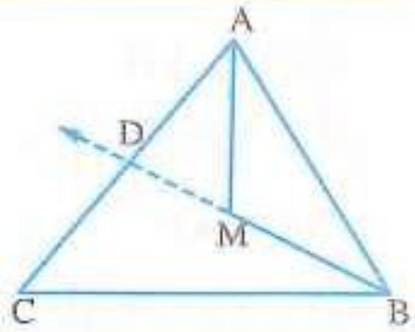
$\therefore$ AB + BC + CD + DA

< 2 (MA + MC) + 2 (MB + MD)

$\therefore$ AB + BC + CD + DA < 2 (AC + BD)

$\therefore$ The perimeter of the quadrilateral ABCD < twice the sum of lengths of the two diagonals. (Q.E.D.)

15



Construction :

Draw $\overrightarrow{BM}$ to cut $\overline{AC}$ at D

Proof :

In Δ BDC :

BC + DC > BD (Triangle inequality)

$\therefore$ BC + DC > BM + MD (1)

$\because$ In Δ AMD : AD + MD > AM (Triangle inequality)

$\therefore$ AD > AM – MD (2)

Adding (1) , (2) :

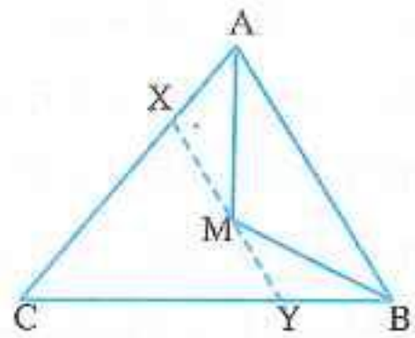
$\therefore BC + AD + DC > BM + MD + AM - MD$

$\therefore BC + AC > BM + AM$

$\therefore AM + MB < BC + AC$ (Q.E.D.)

Another solution :

Construction :



Draw $\overline{XY}$ Passing through the point M where $X \in \overline{AC}$, $Y \in \overline{BC}$

Proof : In Δ CXY

$\because CY + CX > XM + MY$ adding BY and AX to both sides.

$\therefore CY + BY + CX + AX > XM + AX + MY + BY$

$\therefore BC + AC > XM + AX + MY + BY$

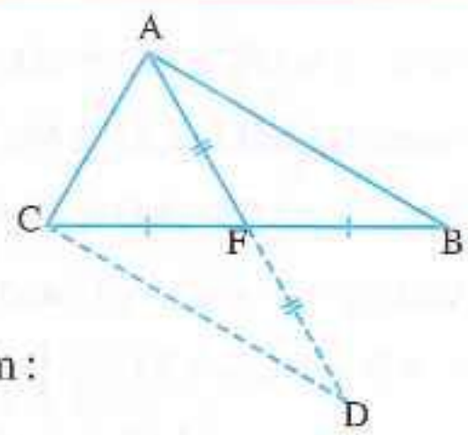
$\because XM + AX > AM$, $MY + BY > MB$

$\therefore BC + AC > AM + MB$

$\therefore AM + MB < AC + BC$ (Q.E.D.)

16

Construction :



Extend $\overline{AF}$ as its length to D then draw $\overline{CD}$

Proof : $\Delta\Delta$ AFB , DFC in them :

$$\begin{cases} AF = DF \text{ const.} \\ BF = FC \text{ (given)} \\ m(\angle AFB) = m(\angle DFC) \quad \text{(V.O.A.)} \end{cases}$$

$\therefore$ The two triangles are congruent then we deduce that AB = DC but in Δ ACD we find that :

$AC + CD > AD$ (triangle inequality)

$\therefore AC + AB > AD$

$\because AD = 2\,AF$

$\therefore AC + AB > 2\,AF$ (1) (Q.E.D.1)

From Δ ABC : $\because AB + AC > BC$

i.e. $AB + AC > 2\,BF$ (2)

Adding (1) and (2) : $2\,AB + 2\,AC > 2\,AF + 2\,BF$

Dividing by 2 : $\therefore AB + AC > AF + BF$ (Q.E.D.2)

Answers of accumulative basic skills

1

1 $2\sqrt{10}$	2 2 : 3	3 5	4 150°
5 18	6 54	7 $\frac{1}{2}$ P – y	8 $5\sqrt{3}$
9 108°	10 60	11 9	12 19

2

1 (b)	2 (c)	3 (b)	4 (d)
5 (a)	6 (c)	7 (a)	8 (c)
9 (d)	10 (a)	11 (c)	12 (b)
13 (d)	14 (c)		

Guide Answers

Of The Notebook (Algebra and Statistics)

Answers of the accumulative tests on Algebra and Statistics

Accumulative test 1

1 1 c 2 c 3 c 4 d

2 1 -8 2 zero 3 5 4 2

3 1 $\{-1\}$ 2 $\{\frac{1}{2}\}$

Accumulative test 2

1 1 c 2 a 3 c 4 b

2 1 4 2 2 3 25 4 zero

3 [a] Prove by yourself.

[b] Prove by yourself.

Accumulative test 3

1 1 c 2 c 3 a 4 c 5 d

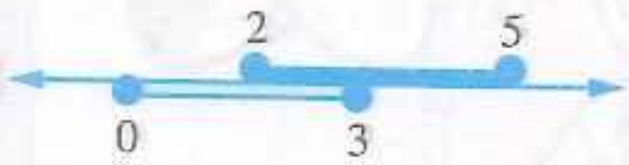
2 1 $\varnothing$ 2 $\varnothing$ 3 4 4 $\varnothing$ 5 $\{2\}$

Accumulative test 4

1 1 c 2 b 3 c 4 c

2 1 $\{3 , 5\}$ 2 $\mathbb{R} - \{1\}$ 3 4 4 $\mathbb{R}$

3 1 $X = \{x : x \in \mathbb{R} , 2 \le x \le 5\}$

2 

3 $X - Y =]3 , 5]$ $\because \sqrt{29} \simeq 5.39$

$\therefore \sqrt{29} \notin X - Y$

4

$X \cup Y = [-1 , \infty[$, $X \cap Y = [-1 , 3[$

, $Y - X =]4 , \infty[$

Accumulative test 5

1 1 c 2 c 3 c 4 b

2 1 $]-1 , 5[$ 2 96 3 $5\sqrt{3}$ 4 24

3 $4 + 2\sqrt{3}$

4 1 22 2 56

Accumulative test 6

1 1 b 2 d 3 c 4 b

2 1 $[2 , 3]$ 2 $\varnothing$ 3 $\sqrt{5} - 3$ 4 $-\sqrt{2}$

3

1 $A \cap B = [-2 , 3[$ 2 $B - A = [3 , 5]$

4 1 $2\sqrt{6}$ 2 zero

Accumulative test 7

1 1 c 2 b 3 c 4 b

2 1 zero 2 $\sqrt{3}$ 3 $\{0\}$ 4 2

3 [a] 7

[b] $10 - 4\sqrt{5}$

4 Prove by yourself , $\frac{4}{5}$

Accumulative test 8

1 1 a 2 b 3 b 4 d

2 1 $]-\infty , 0[$ 2 $3\sqrt{2}$ 3 $3\sqrt[3]{2}$ 4 6

3 1 $6\sqrt[3]{2}$ 2 -1

4 [a]

1 $X \cap Y = [-2 , 1]$

2 $X - Y =]1 , 3]$

[b] 1

Accumulative test 9

1 1 c 2 c 3 a 4 d

2 1 -4 2 3 cm. 3 $\{0 , 1\}$ 4 $5\sqrt{5}$

3 [a] 4 cm.

[b] Prove by yourself , 4

4 [a] 36π cm.^{2}

[b] $7\sqrt{5} - 4\sqrt[3]{2}$

Accumulative test 10

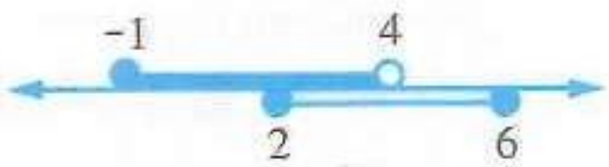
1 1 b 2 b 3 d 4 c

2 1 $[-2 , \infty[$ 2 $]1 , \infty[$ 3 $\{-1\}$ 4 15

3 [a] 15 cm.

[b] $[-2 , 3[$, represent by yourself.

4 [a]



1 $X \cup Y = [-1 , 6]$

2 $X \cap Y = [2 , 4[$

[b] $[-1 , 2[$, represent by yourself.

Accumulative test 11

1 1 b 2 c 3 c 4 a

2 1 6 2 12 3 y 4 3

3 [a] Find by yourself.

[b] $3\sqrt{3}$

4 [a] $[3 , \infty[$, represent by yourself.

[b] Represent by yourself , 2 square units.

Accumulative test 12

1 1 a 2 c 3 d 4 d

2 1 36 2 zero 3 5 4 y

3 [a] Represent by yourself , -1

[b] $[0 , 2]$, represent by yourself.

4 [a] Prove by yourself.

[b] zero.

Accumulative test 13

1 1 a 2 c

2 1 zero 2 $(2 , 3)$ 3 -3

3 1 $\frac{75}{8}$, 5 , zero

2 50 cm.

4 [a] 440 cm.^{2} , 1540 cm.^{3}

[b] Represent by yourself , $\left(\frac{-1}{2} , 0\right)$, $(0 , 1)$

5 1 $]0 , 3]$ 2 $[-2 , \infty[$

3 $]-\infty , 0]$

Accumulative test 14

1 1 c 2 b

2 1 4 2 2

3 Form by yourself.

4 [a] $]1 , 5[$

[b] Find and represent by yourself.

5 [a] zero

[b] $y = \sqrt{5} - 2$, prove by yourself.

Accumulative test 15

1 1 b 2 b

2 1 1 2 5

3 1 20 workers.

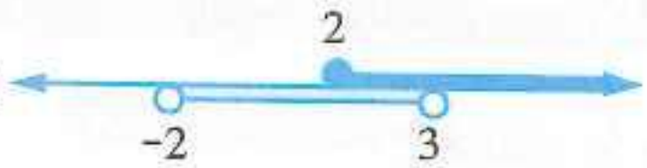
2 Graph by yourself.

4 1 $\{\sqrt{2}\}$

2 $[-1 , 5[$, represent by yourself.

5 [a] $\frac{2}{3}$, the point $C \notin \overleftrightarrow{AB}$

[b]



1 $X \cap Y = [2 , 3[$

2 $X \cup Y =]-2 , \infty[$

3 $X - Y = [3 , \infty[$

Accumulative test 16

1 1 a 2 b 3 c 4 b

2 1 zero 2 20 3 $\sqrt{2} - 1$ 4 $2\sqrt{5}$

3 1 $\mathcal{X} = 60$, $k = 14$

2 The arithmetic mean = 50.6

4 [a] 14 cm.

[b] $\mathcal{X} = \sqrt{7} + \sqrt{3}$, $y = \sqrt{7} - \sqrt{3}$, $\mathcal{X}^2 y^2 = 16$

Accumulative test 17

1 1 a 2 b 3 c 4 c

2 1 5 2 6 3 7 4 24

3 [a] Graph by yourself.

[b] 1 $k = 15$

2 The median = 52 approximately.

4 [a] $[-2 , 1]$

[b]

1 $X \cap Y = [5 , 7]$

2 $X \cup Y =]3 , \infty[$

3 $X - Y =]3 , 5[$

Accumulative test 18

1 1 c 2 d 3 c 4 d

2 1 6 2 3 3 3 4 3

3 [a] $\{\sqrt{7}\}$

[b] 7

4 [a] 4

[b] 1 $\mathcal{X} = 110$, $k = 20$

2 The mode of wages ≃ L.E. 105

Answers of October tests on Algebra and Statistics

Answers of Test 1

1 1 d 2 c 3 a

2 1 $\mathbb{R}^+$ or $\mathbb{R} - \{0\}$
2 $\{\sqrt{5}, -\sqrt{3}\}$ 3 $]1, 4[$

3 $\because (\sqrt{2})^2 = \sqrt{2} \times \sqrt{2} = 2$, $(1.4)^2 = 1.96$
, $(1.5)^2 = 2.25$
$\therefore 1.96 < 2 < 2.25$ $\therefore \sqrt{1.96} < \sqrt{2} < \sqrt{2.25}$
$\therefore 1.4 < \sqrt{2} < 1.5$
$\therefore \sqrt{2}$ lies between 1.4 and 1.5

4 27 litres $\times$ 1000 = 27000 cm^3.
$\because$ Volume of the cube $= \ell^3$
$\therefore \ell^3 = 27000$ $\therefore \ell = \sqrt[3]{27000}$
$\therefore \ell = 30$ cm.

Answers of Test 2

1 1 d 2 c 3 c

2 1 3 2 $\mathbb{R}$ 3 $\varnothing$

3 $\because 2 + x^3 = 1$ $\therefore x^3 = 1 - 2 = -1$
$\therefore x = \sqrt[3]{-1} = -1$ $\therefore$ The S.S. $= \{-1\}$

4 $X \cup Y = [-1, \infty[$

$Y - X =]4, \infty[$

Answers of November tests on Algebra and Statistics

Answers of Test 1

1 1 c 2 b 3 c

2 1 96 2 $3\sqrt{2}$ 3 $\pm \frac{2\sqrt{2}}{3}$

3 $2\sqrt{9 \times 2} + \sqrt{25 \times 2} + \frac{1}{3} \times \sqrt{81 \times 2}$
$= 2 \times 3\sqrt{2} + 5\sqrt{2} + \frac{1}{3} \times 9\sqrt{2}$
$= 6\sqrt{2} + 5\sqrt{2} + 3\sqrt{2} = 14\sqrt{2}$

4 $\because -1 < 3x + 5 < 11$
$\therefore -6 < 3x < 6$
$\therefore -2 < x < 2$
$\therefore$ The S.S. $=]-2, 2[$

Answers of Test 2

1 1 c 2 d 3 b

2 1 1 , zero 2 $\sqrt{125}$ 3 $2\sqrt[3]{2}$

3 $\because$ The volume of the cylinder $= \pi r^2 h$
$\therefore 924 = \frac{22}{7} \times r^2 \times 6$
$\therefore r^2 = \frac{924 \times 7}{22 \times 6} = 49$ $\therefore r = 7$ cm.
$\therefore$ The lateral area $= 2\pi r h = 2 \times \frac{22}{7} \times 7 \times 6$
$= 264$ cm^2.

4 $\because b = \frac{1}{\sqrt{3} + \sqrt{2}} \times \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}} = \frac{\sqrt{3} - \sqrt{2}}{3 - 2}$
$= \sqrt{3} - \sqrt{2}$
$\therefore a^2 - b^2 = (\sqrt{3} + \sqrt{2})^2 - (\sqrt{3} - \sqrt{2})^2$
$= 3 + 2\sqrt{6} + 2 - 3 + 2\sqrt{6} - 2 = 4\sqrt{6}$

Answers of important questions on Algebra and Statistics

Unit one

First Answers of multiple choice questions

1 (a)	2 (c)	3 (a)	4 (d)	5 (b)
6 (c)	7 (c)	8 (a)	9 (b)	10 (a)
11 (c)	12 (b)	13 (b)	14 (c)	15 (c)
16 (d)	17 (c)	18 (c)	19 (b)	20 (d)
21 (a)	22 (a)	23 (c)	24 (b)	25 (d)
26 (c)				

Second Answers of complete questions

1 $\mathbb{R}$	2 $\varnothing$	3 12	4 $\{0\}$
5 $]3, 4]$	6 $\sqrt{3}-\sqrt{2}$		7 $\sqrt{5}+2$
8 $]0, \infty[$	9 12	10 3 , 4	11 $\sqrt{50}$
12 64 cm^3	13 $\frac{4}{3}$	14 3	15 12 cm.
16 20π	17 2	18 2	19 $(-1, 2\sqrt{3})$
20 5	21 $]-2, 3]$		22 $\{-1, 0, 1\}$
23 $[-3, 0[$		24 2	

Third Answers of essay questions

1

1. $X \cup Y = [-3, 5]$
2. $X \cap Y = [-1, 2[$
3. $X - Y = [-3, -1[$

2

- $B - A = [3, 5]$
- $A \cap B = [-2, 3[$
- $A \cup B =]-\infty, 5]$
- $\grave{A} = [3, \infty[$

3 $\because y = \frac{2}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{2(\sqrt{3}-1)}{3-1} = \sqrt{3}-1$

$\therefore \frac{xy}{x-y} = \frac{(\sqrt{3}+1)(\sqrt{3}-1)}{\sqrt{3}+1-\sqrt{3}+1} = \frac{3-1}{2} = 1$

4 $\because x = \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{\sqrt{5}-2}{5-4} = \sqrt{5}-2$

, $y = \sqrt{5}+2$ $\quad\therefore x$, y are conjugate.

$\therefore x^2 y^2 = (xy)^2 = [(\sqrt{5}-2)(\sqrt{5}+2)]^2$

$= [5-4]^2 = 1$

5 $\because y = \frac{2}{\sqrt{7}-\sqrt{5}} \times \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{2(\sqrt{7}+\sqrt{5})}{7-5}$

$= \sqrt{7}+\sqrt{5}$

$\therefore x^2 + 2xy + y^2 = (x+y)^2$

$= (\sqrt{7}-\sqrt{5}+\sqrt{7}+\sqrt{5})^2$

$= (2\sqrt{7})^2 = 28$

6 $\because y = \frac{2}{x}$

$= \frac{2}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \frac{2(\sqrt{7}-\sqrt{5})}{7-5}$

$= \sqrt{7}-\sqrt{5}$

, $x = \sqrt{7}+\sqrt{5}$

$\therefore x$, y are two conjugate numbers

$\therefore x^2 + xy + y^2 = (x+y)^2 - xy$

$= (\sqrt{7}+\sqrt{5}+\sqrt{7}-\sqrt{5})^2$

$- (\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})$

$= (2\sqrt{7})^2 - 2 = 26$

7 $(x+y)^2 = x^2 + 2xy + y^2 = (\sqrt{4+\sqrt{7}})^2$

$+ 2(\sqrt{4+\sqrt{7}})(\sqrt{4-\sqrt{7}})$

$+ (\sqrt{4-\sqrt{7}})^2$

$= 4+\sqrt{7}+2\sqrt{(4+\sqrt{7})(4-\sqrt{7})}$

$+ 4-\sqrt{7}$

$= 8 + 2\sqrt{16-7} = 8 + 2\sqrt{9} = 8+6 = 14$

8 $\because x = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{11+2\sqrt{30}}{6-5}$

$= 11 + 2\sqrt{30}$

, $\frac{1}{x} = \frac{1}{11+2\sqrt{30}} \times \frac{11-2\sqrt{30}}{11-2\sqrt{30}}$

$= \frac{11-2\sqrt{30}}{121-120} = 11 - 2\sqrt{30}$

$\therefore x + \frac{1}{x} = 11 + 2\sqrt{30} + 11 - 2\sqrt{30} = 22$

9 $\because 5x - 3 < 2x + 9$ $\therefore 5x - 2x < 9 + 3$

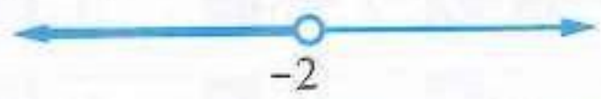
$\therefore 3x < 12$ $\therefore x < 4$

$\therefore$ The S.S. $=]-\infty , 4[$

10 $\because 5 - 3x > 11$ $\therefore -3x > 6$

$\therefore x < -2$

$\therefore$ The S.S. $=]-\infty , -2[$



11 $\because 1 < 2x + 3 \leq 9$ $\therefore -2 < 2x \leq 6$

$\therefore -1 < x \leq 3$

$\therefore$ The S.S. $=]-1 , 3]$

12 $\because \frac{x}{3} + 2 > 3$ $\therefore \frac{x}{3} > 1$

$\therefore x > 3$

$\therefore$ The S.S. $=]3 , \infty[$

13 $\because 4x + 3 > 5x + 2 > 4x$ $\therefore 3 > x + 2 > 0$

$\therefore 1 > x > -2$

$\therefore$ The S.S. $=]-2 , 1[$

14 The expression $= \sqrt[3]{2 \times 4} + 2\sqrt[3]{2} - 2\sqrt[3]{2} - 2$

$= 2 - 2 =$ zero

15 The expression $= \sqrt{25 \times 3} - 2\sqrt{9 \times 3} + \sqrt{9 \times \frac{1}{3}}$

$= 5\sqrt{3} - 6\sqrt{3} + \sqrt{3} =$ zero

16 The expression $= \sqrt[3]{27 \times 3} + \sqrt[3]{8 \times 3} - \sqrt[3]{27 \times \frac{1}{9}}$

$= 3\sqrt[3]{3} + 2\sqrt[3]{3} - \sqrt[3]{3} = 4\sqrt[3]{3}$

17 The expression $= \sqrt{4 \times 3} + \sqrt[3]{27 \times 2} - 2\sqrt{3}$

$- \sqrt[3]{8 \times 2}$

$= 2\sqrt{3} + 3\sqrt[3]{2} - 2\sqrt{3} - 2\sqrt[3]{2}$

$= \sqrt[3]{2}$

18 The expression $= \sqrt{25 \times 2} + \sqrt[3]{27 \times 2} - 5\sqrt{4 \times \frac{1}{2}}$

$- \sqrt[3]{8 \times 2}$

$= 5\sqrt{2} + 3\sqrt[3]{2} - 5\sqrt{2} - 2\sqrt[3]{2}$

$= \sqrt[3]{2}$

19 $\because (x - 2)^3 = 125$ $\therefore x - 2 = 5$

$\therefore x = 7$

$\therefore$ The S.S. $= \{7\}$

20 [1] $\because$ The volume of the sphere $= \frac{4}{3} \pi r^3$

$\therefore 36 \pi = \frac{4}{3} \times \pi \times r^3$ $\therefore r^3 = \frac{36 \times 3}{4} = 27$

$\therefore r = 3$ cm.

[2] The area of the sphere $= 4 \pi r^2$

$= 4 \times 3^2 \times \pi = 36 \pi$ cm^2.

21 $\because$ The volume of the cylinder $= \pi r^2 h$

, $\because h = r$

$\therefore$ The volume of the cylinder $= \pi h^3$

$\therefore 27 \pi = \pi h^3$ $\therefore h^3 = 27$ $\therefore h = 3$ cm.

22 The volume of the cylinder

$= \pi r^2 h = \pi \left(4\sqrt{2}\right)^2 \times 9 = 288 \pi$ cm^3.

23 The volume of the cylinder

$= \pi r^2 h = \frac{22}{7} \times 7^2 \times 10 = 1540$ cm^3.

, The lateral area $= 2 \pi r h$

$= 2 \times \frac{22}{7} \times 7 \times 10 = 440$ cm^2.

24 $\because$ The volume of the sphere

$= \frac{4}{3} \pi r^3 = \frac{4}{3} \times 3^3 \times \pi = 36 \pi$ cm^3.

, $\because$ The volume of the cylinder

$=$ The volume of the sphere

$\therefore$ The volume of the cylinder $= 36 \pi$ cm^3.

$\therefore \pi r^2 h = 36 \pi$ $\therefore 9 \pi h = 36 \pi$

$\therefore h = \frac{36}{9} = 4$ cm.

25 $\because$ The volume of the cylinder

$= \pi r^2 h = 3^2 \times 4 \times \pi = 36 \pi$ cm^3.

, ∵ the volume of the sphere

= The volume of the cylinder

∴ The volume of the sphere = 36π cm^3.

∴ $\frac{4}{3}\pi r^3 = 36\pi$

∴ $r^3 = \frac{36 \times 3}{4} = 27$ ∴ r = 3 cm.

26 ∵ The lateral area = $2\pi r h$

∴ $440 = 2 \times \frac{22}{7} \times 10 \times r$

∴ $r = \frac{440 \times 7}{2 \times 22 \times 10} = 7$ cm.

∴ The volume of the cylinder

$= \pi r^2 h = \frac{22}{7} \times 7^2 \times 10 = 1540$ cm^3.

Unit two

First Answers of multiple choice questions

1 (b)	2 (c)	3 (b)	4 (a)	5 (c)
6 (b)	7 (c)	8 (c)	9 (a)	10 (b)

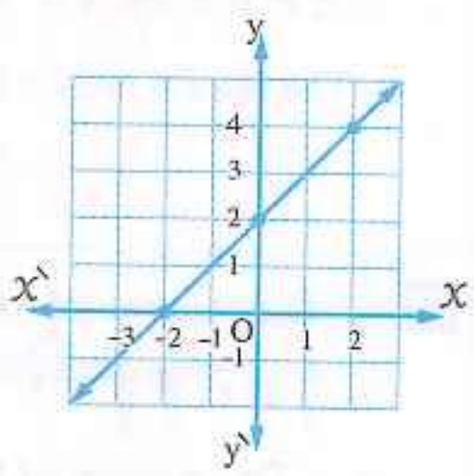
Second Answers of complete questions

1 1	2 undefined	3 zero
4 4	5 x-axis , zero	6 – 11
7 $\overleftrightarrow{BC}$ or $\overleftrightarrow{AC}$	8 – 1 , zero	9 3
10 (0 , 8)		

Third Answers of essay questions

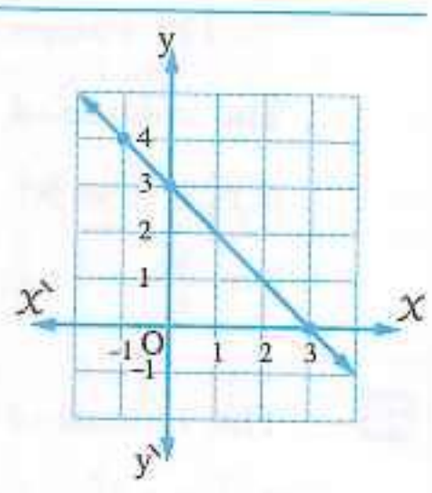
1 $y = x + 2$

x	– 2	0	2
y	0	2	4



2 $y = 3 - x$

x	– 1	0	3
y	4	3	0

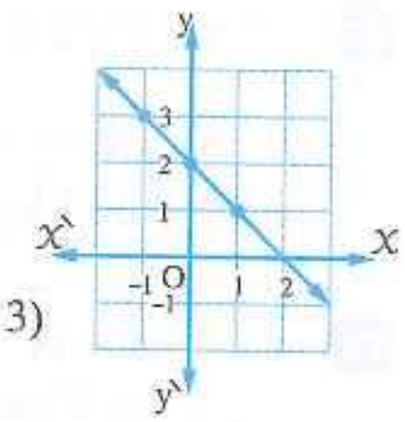


From the graph :

The point of intersection with x-axis is : (3 , 0)

3 $y = 2 - x$

x	– 1	0	1
y	3	2	1



Taking the two points (– 1 , 3) and (0 , 2) which lie on the straight line we find that :

the slope = $\frac{2-3}{0+1} = -1$

4 At $x = 0$ ∴ $2 \times 0 + 3y = 12$

∴ $3y = 12$ ∴ $y = 4$

∴ The straight line intersects y-axis at (0 , 4)

At $y = 0$ ∴ $2x + 3 \times 0 = 12$

∴ $2x = 12$ ∴ $x = 6$

∴ The straight line intersects x-axis at (6 , 0)

5 ∵ The slope of $\overleftrightarrow{AB} = \frac{4-3}{2+1} = \frac{1}{3}$

, the slope of $\overleftrightarrow{BC} = \frac{5-4}{5-2} = \frac{1}{3}$

∴ The slope of $\overleftrightarrow{AB}$ = The slope of $\overleftrightarrow{BC}$

and the point B is a common point

∴ The points A , B and C are collinear.

6 ∵ The slope of $\overleftrightarrow{AB} = \frac{3+1}{-1-2} = \frac{-4}{3}$

, the slope of $\overleftrightarrow{BC} = \frac{3-3}{2+1}$ = zero

∴ The slope of $\overleftrightarrow{AB} \neq$ The slope of $\overleftrightarrow{BC}$

∴ The points A , B and C are not collinear.

7 ∵ $\frac{k-17}{6-4} = 4$ ∴ $\frac{k-17}{2} = 4$

∴ $k - 17 = 8$ ∴ $k = 25$

8 ∵ (k , 2 k) satisfies the relation : $3x + y = 30$

∴ $3k + 2k = 30$ ∴ $5k = 30$ ∴ $k = 6$

9 The slope of $\overleftrightarrow{AB} = \frac{5-3}{2+1} = \frac{2}{3}$

, ∵ the slope of $\overleftrightarrow{BC} = \frac{1-5}{8-2} = \frac{-4}{6} = \frac{-2}{3}$

∴ The slope of $\overleftrightarrow{AB} \neq$ The slope of $\overleftrightarrow{BC}$

∴ $C \notin \overleftrightarrow{AB}$

10 $\because$ The slope of $\overleftrightarrow{AB} = \frac{5-3}{3-3} = \frac{2}{\text{zero}}$

$\therefore$ The slope of $\overleftrightarrow{AB}$ is undefined

$\therefore$ $\overleftrightarrow{AB}$ // y-axis

Unit three

First Answers of multiple choice questions

1 (b) **2** (d) **3** (c) **4** (a) **5** (d)

6 (c) **7** (a) **8** (b) **9** (c) **10** (c)

11 (c) **12** (c) **13** (b) **14** (a) **15** (c)

Second Answers of complete questions

1 5 **2** 3 **3** 7 **4** the third

5 central tendency **6** 4 **7** 40 **8** 3

9 10 **10** median

Third Answers of essay questions

1

Sets	x	f	$x \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		**20**	**540**

$\therefore$ The mean $= \frac{540}{20} = 27$

2 1 $n = 60$

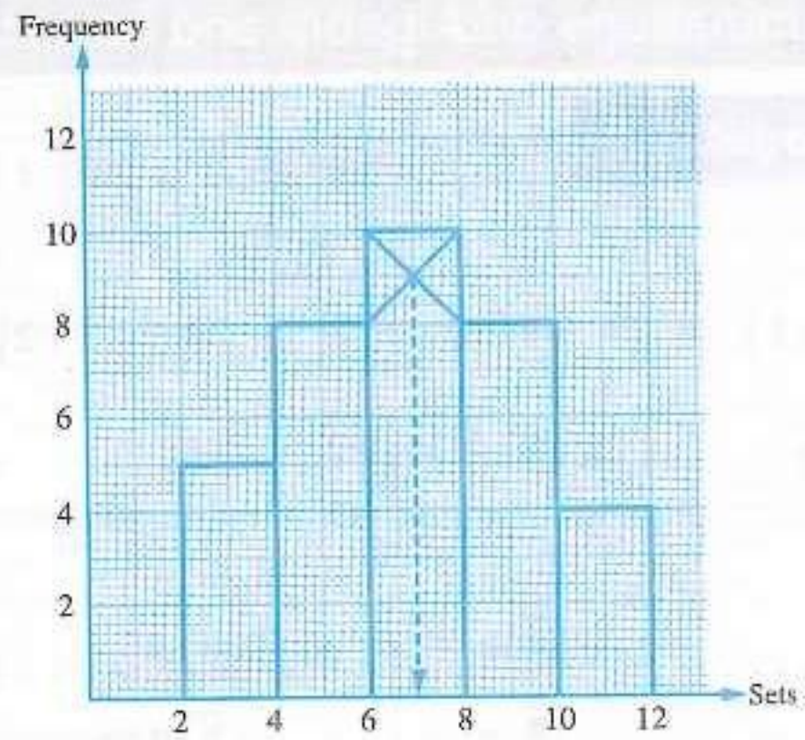
$k = 100 - (10 + 22 + 25 + 20 + 8) = 15$

2

Sets	x	f	$x \times f$
20 –	25	10	250
30 –	35	15	525
40 –	45	22	990
50 –	55	25	1375
60 –	65	20	1300
70 –	75	8	600
Total		**100**	**5040**

$\therefore$ The mean $= \frac{5040}{100} = 50.4$

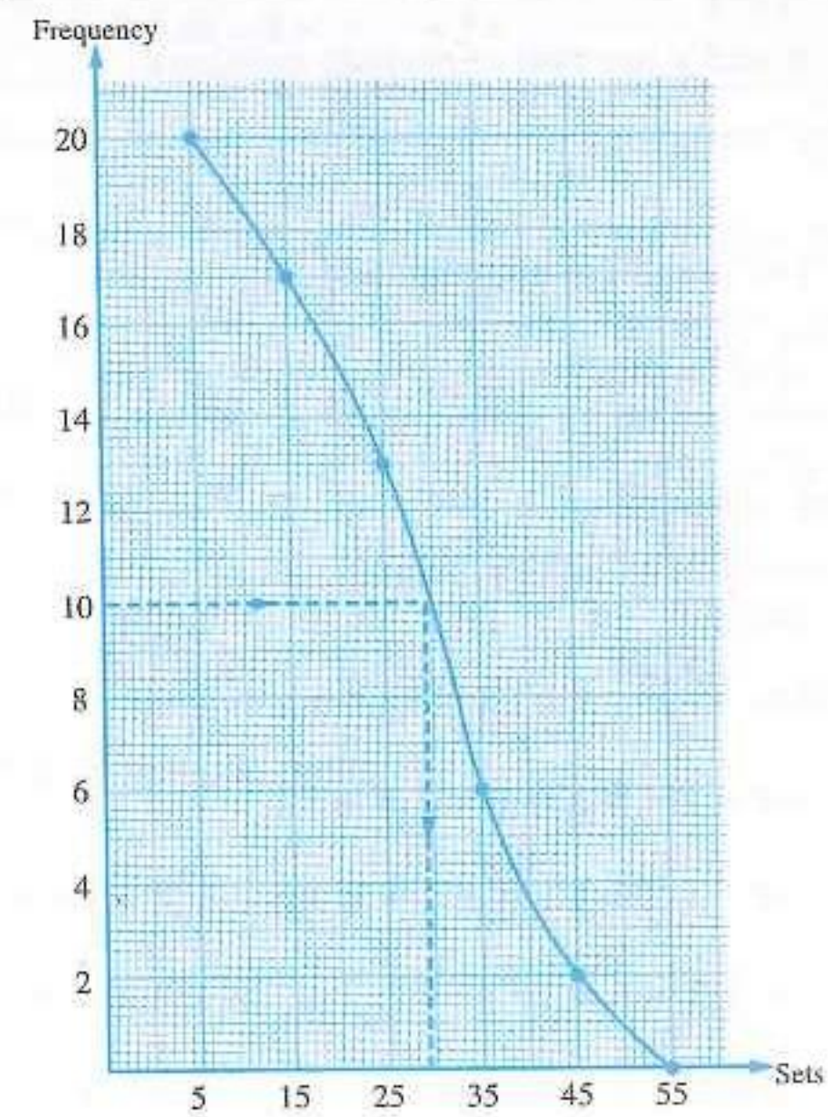
3



From the graph : The mode mark = 7 marks

4

The lower boundaries of sets	Descending cumulative frequency
5 and more	20
15 and more	17
25 and more	13
35 and more	6
45 and more	2
55 and more	0



$\because$ The order of the median $= \frac{20}{2} = 10$

$\therefore$ The median $\simeq 29$

Answers of the school book model examinations on Algebra and Statistics

Model 1

1

1 $\{-1\}$ 2 20 3 $[-2, 2]$

4 24 5 $\sqrt{3}-\sqrt{2}$

2

1 d 2 c 3 c

4 c 5 a 6 b

3

[a] $\sqrt{2\times 9}+\sqrt[3]{2\times 27}-3\sqrt{2}-\frac{1}{2}\sqrt[3]{2\times 8}$

$=3\sqrt{2}+3\sqrt[3]{2}-3\sqrt{2}-\frac{1}{2}\times 2\sqrt[3]{2}$

$=2\sqrt[3]{2}$

[b] $\because x=\frac{3}{\sqrt{5}-\sqrt{2}}\times\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}=\frac{3\left(\sqrt{5}+\sqrt{2}\right)}{5-2}$

$=\sqrt{5}+\sqrt{2}$

$\therefore$ x and y are two conjugate numbers.

4

[a] $\because$ The area of the square $=\frac{1}{2}d^2$

$\therefore \frac{1}{2}d^2=1089$

$\therefore d^2=2178$

$\therefore d=\sqrt{2178}=33\sqrt{2}$ cm.

[b] $\because 6\times\frac{3x+1}{6}<6(x+1)<6\times\frac{x+4}{2}$

$\therefore 3x+1<6x+6<3x+12$

$\therefore 3x-3x+1<6x-3x+6<3x-3x+12$

$\therefore 1<3x+6<12$ $\therefore 1-6<3x<12-6$

$\therefore -5<3x<6$ $\therefore -\frac{5}{3}<x<2$

$\therefore$ The S.S. $=\left]-\frac{5}{3}, 2\right[$

5

[a] The volume of the cylinder $=\pi r^2 h$

$=\left(4\sqrt{2}\right)^2\times 9\times\pi$

$=288\pi$ cm^3

$\because$ the volume of the cylinder

= the volume of the sphere

$\therefore$ The volume of the sphere $=288\pi$ cm^3

$\therefore \frac{4}{3}\pi r^3=288\pi$

$\therefore r^3=288\times\frac{3}{4}=216$

$\therefore r=\sqrt[3]{216}=6$ cm.

[b]

Sets	x	f	$x\times f$
5 –	10	7	70
15 –	20	10	200
25 –	30	12	360
35 –	40	13	520
45 –	50	8	400
Total		**50**	**1550**

$\therefore$ The mean $=\frac{1550}{50}=31$

Model 2

1

1 $\sqrt{3}+\sqrt{5}$ 2 6 3 $3+\sqrt{10}$

4 3 5 $]3, 4]$

2

1 b 2 a 3 b

4 c 5 c 6 d

3

[a] $\frac{\sqrt{3}\left(\sqrt{5}+\sqrt{3}\right)+\sqrt{5}\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}$

$=\frac{\sqrt{15}+3+5-\sqrt{15}}{5-3}=\frac{8}{2}=4$

[b] The left hand side

$=\sqrt[3]{2\times 64}+\sqrt[3]{2\times 8}-2\sqrt[3]{2\times 27}$

$=4\sqrt[3]{2}+2\sqrt[3]{2}-2\times 3\sqrt[3]{2}=6\sqrt[3]{2}-6\sqrt[3]{2}=0$

= the right hand side

4

[a] $\because -2-7 < 3x+7-7 \le 10-7$

$\therefore -9 < 3x \le 3$

$\therefore -3 < x \le 1$

$\therefore$ The S.S. $=]-3 , 1]$

[b] $x^4 - 2x^2 + 1 = (x^2-1)^2$

$= \left(\left(\sqrt{2}+\sqrt{3}\right)^2 - 1\right)^2 = \left(2+2\sqrt{6}+3-1\right)^2$

$= \left(4+2\sqrt{6}\right)^2$

$= 16 + 16\sqrt{6} + 24$

$= 40 + 16\sqrt{6}$

5

[a] 20

[b]

Sets	x	f	$x \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		20	540

$\therefore$ The mean $= \frac{540}{20} = 27$

Model for the merge students

1

1 $\sqrt{3}-\sqrt{2}$ 2 $3\sqrt{6}$ 3 3

4 5 5 $\varnothing$

2

1 a 2 b 3 a 4 a 5 a

3

1 $\{5 , -5\}$ 2 $[0 , 2]$ 3 7

4 irrational

5

4

1 ✔ 2 ✔ 3 ✔ 4 ✘ 5 ✔

5

[a] The centre $= \frac{8+4}{2} = 6$

[b]

Sets	The centre of the set « x »	Frequency « f »	$x \times f$
5 –	10	7	$10 \times 7 = 70$
15 –	20	10	$20 \times 10 = 200$
25 –	30	12	$30 \times 12 = 360$
35 –	40	13	$40 \times 13 = 520$
45 –	50	8	$50 \times 8 = 400$
Total		50	1550

$\therefore$ The arithmetic mean $= \frac{\sum (x \times f)}{\sum (f)}$

$= \frac{1550}{50} = 31$

Answers of the schools examinations on Algebra and Statistics

1 Cairo

1 1 (d) 2 (a) 3 (a) 4 (d) 5 (c)

2 1 $3\sqrt{2}$ 2 $\varnothing$ 3 3 4 $-\frac{6}{5}$

3

[a] $\sqrt{9\times2}+\sqrt{25\times2}-\sqrt{9\times6}$

$=3\sqrt{2}+5\sqrt{2}-3\sqrt{6}$

$=8\sqrt{2}-3\sqrt{6}$

[b] $\because x=\frac{4}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}=\frac{4\left(3+\sqrt{5}\right)}{9-5}=3+\sqrt{5}$

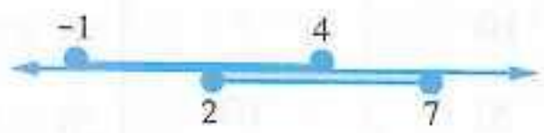
, $y=3-\sqrt{5}$

$\therefore$ x and y are two conjugate numbers.

, $(x+y)^2=\left(3+\sqrt{5}+3-\sqrt{5}\right)^2=(6)^2=36$

4

[a]



1 $X\cup Y=[-1\ ,7]$

2 $X\cap Y=[2\ ,4]$

[b] $\because$ (3 m , 2 m) satisfies the relation

$\therefore 2m=2\times3m-8$ $\therefore 2m=6m-8$

$\therefore 2m-6m=-8$ $\therefore -4m=-8$

$\therefore m=2$

5

[a] 1 $\because 8x^3-20=7$ $\therefore 8x^3=27$

$\therefore x^3=\frac{27}{8}$ $\therefore x=\frac{3}{2}$

$\therefore$ The S.S. $=\left\{\frac{3}{2}\right\}$

2 $\because 3x+7\le10$

$\therefore 3x\le3$ $\therefore x\le1$

$\therefore$ The S.S. $=]-\infty\ ,1]$

[b]

Sets	x	f	$x\times f$
0 –	2	2	4
4 –	6	10	60
8 –	10	8	80
12 –	14	7	98
16 –	18	3	54
Total		**30**	**296**

$\therefore$ The arithmetic mean $=\frac{296}{30}=9\ \frac{13}{15}$

2 Cairo

1 1 (a) 2 (b) 3 (a) 4 (c) 5 (a)

2 1 $4\sqrt{3}$ 2 -5 3 5 4 $\mathbb{R}$

3

[a] $\because -2<3x+7<10$

$\therefore -9<3x<3$ $\therefore -3<x<1$

$\therefore$ The S.S. $=]-3\ ,1[$

[b] $\because y=\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$=\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}=\sqrt{5}+\sqrt{3}$

, $x=\sqrt{5}-\sqrt{3}$

$\therefore$ x and y are two conjugate numbers.

, $x^2+y^2=\left(\sqrt{5}-\sqrt{3}\right)^2+\left(\sqrt{5}+\sqrt{3}\right)^2$

$=5-2\sqrt{15}+3+5+2\sqrt{15}+3=16$

4

[a]

1 $X\cap Y=[2\ ,4]$ 2 $X\cup Y=]-1\ ,\infty[$

3 $X-Y=]-1\ ,2[$

[b] $\because$ The volume of the sphere $=\frac{4}{3}\pi r^3$

$\therefore 36\pi=\frac{4}{3}\pi r^3$ $\therefore r^3=27$

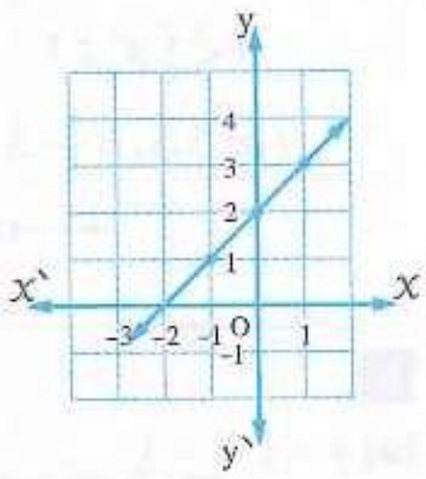
$\therefore r=3$ cm.

$\therefore$ The area $=4\pi r^2=4\pi\times9=36\pi$ cm^2.

5

[a] $y - x = 2$

x	-1	0	1
y	1	2	3



[b]

Sets	x	f	$x \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		20	540

$\therefore$ The mean $= \frac{540}{20} = 27$

3 Giza

1 1 (c) 2 (c) 3 (a) 4 (a) 5 (b)

2 1 1 2 8 3 $\varnothing$ 4 2 , 3

3

[a] $\sqrt[3]{64 \times 2} + \sqrt[3]{8 \times 2} - 2\sqrt[3]{27 \times 2}$

$= 4\sqrt[3]{2} + 2\sqrt[3]{2} - 6\sqrt[3]{2} = 0$

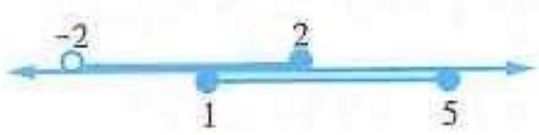
[b] $x\,y = (\sqrt{5} - 3)(\sqrt{5} + 3) = 5 - 9 = -4$

[c] $\because$ The mode = 6

$\therefore 2x = 6$ $\therefore x = 3$

4

[a]



$]-2 , 2] \cap [1 , 5] = [1 , 2]$

[b] The volume $= 2 \times 2\sqrt{5} \times 3\sqrt{5} = 60$ cm^3.

[c] $\because$ 4 , 5 , 11 , 13 , 20

$\therefore$ The median = 11

5

[a] $\because -3 \le 2x + 1 < 7$

$\therefore -4 \le 2x < 6$ $\therefore -2 \le x < 3$

$\therefore$ The S.S. $= [-2 , 3[$

[b] $\because x\,y = 1$ $\therefore y = \frac{1}{x}$

$\therefore y = \frac{1}{\sqrt{5}+2} = \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{\sqrt{5}-2}{5-4} = \sqrt{5} - 2$

$\because x = \sqrt{5} + 2$

$\therefore x$ and y are two conjugate numbers.

[c]

Sets	x	f	$x \times f$
5 –	10	20	200
15 –	20	30	600
25 –	30	15	450
35 –	40	25	1000
45 –	50	10	500
Total		100	2750

$\therefore$ The mean $= \frac{2750}{100} = 27.5$

4 Giza

1 1 (c) 2 (b) 3 (c) 4 (d) 5 (b)

2 1 64 2 $288\,\pi$ 3 undefined 4 3

3

[a] $\sqrt[3]{8 \times 3} + \sqrt{4 \times 3} - 2\sqrt[3]{3} - 2\sqrt{3}$

$= 2\sqrt[3]{3} + 2\sqrt{3} - 2\sqrt[3]{3} - 2\sqrt{3} = 0$

[b] $\because y = \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{3-2} = \sqrt{3} - \sqrt{2}$

$\therefore x^2 - y^2 = (\sqrt{3}+\sqrt{2})^2 - (\sqrt{3}-\sqrt{2})^2$

$= 3 + 2\sqrt{6} + 2 - 3 + 2\sqrt{6} - 2 = 4\sqrt{6}$

4

[a] The volume $= 3 \times 4 \times 5 = 60$ cm^3.

, the total area $= 2\,(3 \times 4 + 4 \times 5 + 5 \times 3)$

$= 2\,(12 + 20 + 15) = 94$ cm^2.

[b] $\because 3x - 1 \geq 8$

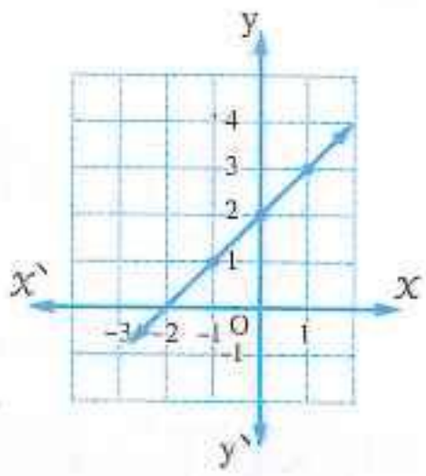
$\therefore 3x \geq 9$ $\therefore x \geq 3$

$\therefore$ The S.S. $= [3 , \infty[$

5

[a] $y = x + 2$

x	-1	0	1
y	1	2	3



[b]

Sets	x	f	$x \times f$
4 –	6	2	12
8 –	10	4	40
12 –	14	8	112
16 –	18	6	108
20 –	22	4	88
Total		**24**	**360**

$\therefore$ The mean $= \frac{360}{24} = 15$

5 Alexandria

1 1 (a) 2 (c) 3 (c) 4 (b) 5 (c)

2 1 zero 2 11 3 3 4 undefined

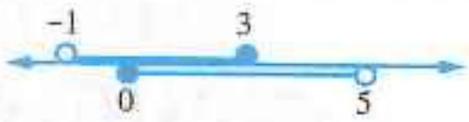
3

[a] $\sqrt[3]{27 \times 2} + 2\sqrt[3]{8 \times \frac{1}{4}} - \sqrt[3]{2}$

$= 3\sqrt[3]{2} + 2\sqrt[3]{2} - \sqrt[3]{2} = 4\sqrt[3]{2}$

[b] 1 $A \cap B = [0 , 3]$

2 $B - A =]3 , 5[$



4

[a] $\because$ The volume of the sphere $= \frac{4}{3} \pi r^3$

$\therefore 288 \pi = \frac{4}{3} \pi r^3$

$\therefore r^3 = 216$ $\therefore r = 6$ cm.

$\therefore$ The area $= 4 \pi r^2 = 4 \pi \times 36 = 144 \pi$ cm^2.

[b] $\because -2 < 3x + 7 \leq 10$

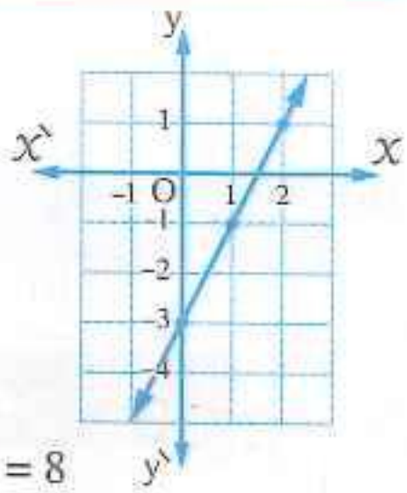
$\therefore -9 < 3x \leq 3$ $\therefore -3 < x \leq 1$

$\therefore$ The S.S. $=]-3 , 1]$

5

[a] $y = 2x - 3$

x	0	1	2
y	-3	-1	1



[b] 1 $k = 40 - (7 + 9 + 12 + 4) = 8$

2

Sets	x	f	$x \times f$
5 –	10	7	70
15 –	20	9	180
25 –	30	12	360
35 –	40	8	320
45 –	50	4	200
Total		**40**	**1130**

$\therefore$ The arithmetic mean $= \frac{1130}{40} = 28.25$

6 El-Kalyoubia

1 1 (b) 2 (b) 3 (b) 4 (d) 5 (d)

2 1 $]1 , 5[$ 2 2 3 3^5 4 $(4 , 0)$

3

[a] 1 $X \cap Y = [1 , 2]$

2 $X - Y = [-1 , 1[$

[b] $\sqrt{25 \times 3} - 2\sqrt{9 \times 3} - 2\sqrt{9 \times \frac{1}{3}}$

$= 5\sqrt{3} - 6\sqrt{3} - 2\sqrt{3} = -3\sqrt{3}$

4

[a] $\because$ The slope $= \frac{1 - k - 3}{3 - 2k} = 1$

$\therefore 1 - k - 3 = 3 - 2k$ $\therefore -k + 2k = 3 + 3 - 1$

$\therefore k = 5$

[b] $\because 4x \le 5x + 2 \le 4x + 3$

$\therefore 0 \le x + 2 \le 3$ $\quad\quad \therefore -2 \le x \le 1$

$\therefore$ The S.S. $= [-2 , 1]$

5

[a] The volume $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 27 = 36\pi$ cm^3.

, the area $= 4\pi r^2 = 4\pi \times 9 = 36\pi$ cm^2.

[b]

Sets	x	f	$x \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		**20**	**540**

$\therefore$ The mean $= \frac{540}{20} = 27$

7 El-Monofia

1 1 (d) 2 (b) 3 (b) 4 (c) 5 (a)

2 1 $\sqrt{5} - \sqrt{2}$ 2 3 3 $]-\infty , -3[$ 4 $]3 , 7]$

3

[a] $\because$ The slope of $\overleftrightarrow{AB} = \frac{4+1}{-3-2} = -1$

, the slope of $\overleftrightarrow{BC} = \frac{5-4}{-4+3} = -1$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ and the point B is a common point.

$\therefore$ The points A , B and C are collinear.

[b] $\because$ The volume of the cylinder $= \pi r^2 \times h$

$\therefore \pi r^2 \times 4 = 64\pi$ $\quad\quad \therefore r^2 = \frac{64}{4} = 16$

$\therefore r = 4$ cm.

4

[a]

1 $X \cap Y = [2 , 4[$ 2 $X \cup Y = [-1 , \infty[$

3 $X - Y = [-1 , 2[$

[b] $\because -2 < 5x + 3 < 13$

$\therefore -5 < 5x < 10$ $\quad\quad \therefore -1 < x < 2$

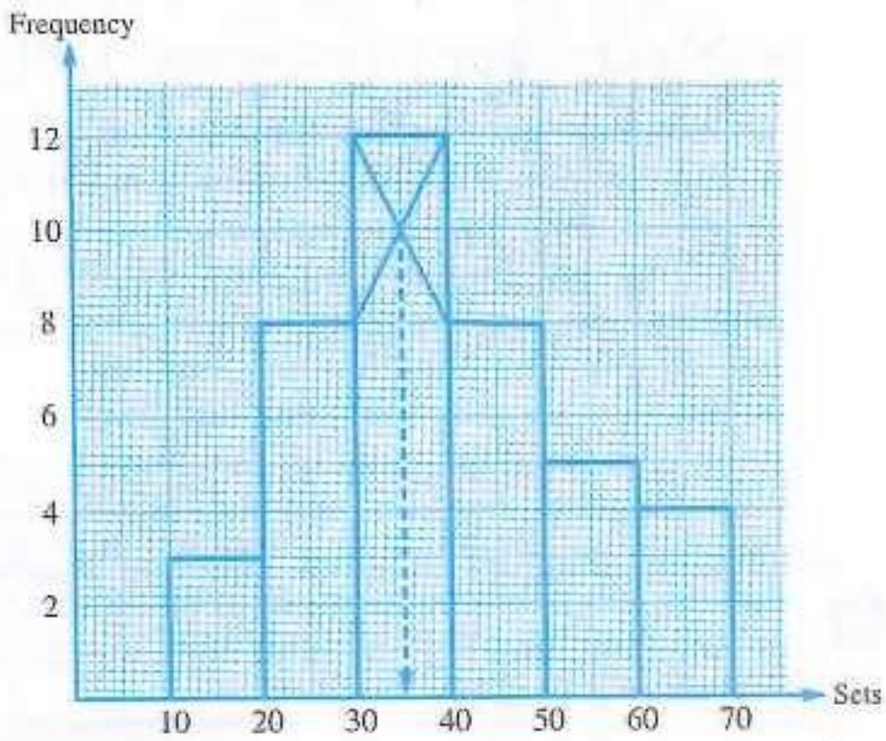
$\therefore$ The S.S. $=]-1 , 2[$

5

[a] $3\sqrt[3]{8 \times 2} + \sqrt{25 \times 2} - 2\sqrt[3]{27 \times 2} - 2\sqrt{4 \times 2}$

$= 6\sqrt[3]{2} + 5\sqrt{2} - 6\sqrt[3]{2} - 4\sqrt{2} = \sqrt{2}$

[b]



From the graph : The mode = 35

8 El-Gharbia

1 1 (d) 2 (b) 3 (c) 4 (c) 5 (b)

2 1 9 2 24 3 – 3 4 2

3

[a] $\sqrt{25 \times 5} + 2\sqrt{16 \times 5} - \sqrt{4 \times 5} - \sqrt{9 \times 5}$

$= 5\sqrt{5} + 8\sqrt{5} - 2\sqrt{5} - 3\sqrt{5} = 8\sqrt{5}$

[b] 1 $\because (x-3)^3 = 2^3$ $\quad\quad \therefore x - 3 = 2$

$\therefore x = 5$ $\quad\quad \therefore$ The S.S. $= \{5\}$

2 $\because 8 + 3x < 14$

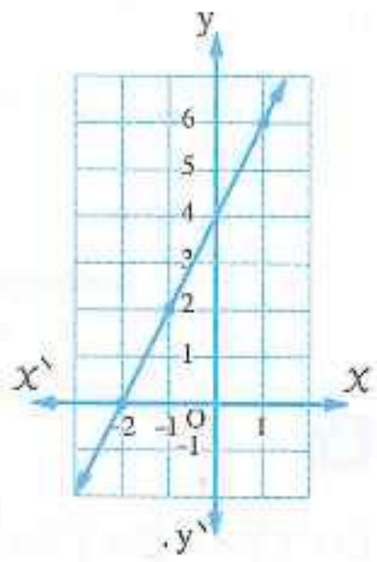
$\therefore 3x < 6$ $\quad\quad \therefore x < 2$

$\therefore$ The S.S. $=]-\infty , 2[$

4

[a] $y - 2x = 4$

x	-2	-1	1
y	0	2	6



Taking the two points

$(-2, 0), (1, 6)$

$\therefore$ The slope $= \frac{6-0}{1+2} = \frac{6}{3} = 2$

[b] $\because x = \frac{3}{\sqrt{6}-\sqrt{3}} \times \frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}}$

$= \frac{3(\sqrt{6}+\sqrt{3})}{6-3} = \sqrt{6}+\sqrt{3}$

$\therefore \frac{xy}{x-y} = \frac{(\sqrt{6}+\sqrt{3})(\sqrt{6}-\sqrt{3})}{(\sqrt{6}+\sqrt{3})-(\sqrt{6}-\sqrt{3})}$

$= \frac{6-3}{2\sqrt{3}} = \frac{3}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{2}$

5

[a] 1 $X \cap Y =]1, 5]$

2 $Y - X =]5, 7]$

[b] 1 $\because k - 1 = 20 - (4 + 6 + 3 + 2) = 5$

$\therefore k = 6$

2

Sets	x	f	$x \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		**20**	**540**

$\therefore$ The arithmetic mean $= \frac{540}{20} = 27$

9 Ismailia

1 1 (c) 2 (d) 3 (b) 4 (a) 5 (c)

2 1 6 2 0 , 25 3 5 4 $\overleftrightarrow{AB}$ or $\overleftrightarrow{AC}$

3

[a] The slope of $\overleftrightarrow{AB} = \frac{5-3}{2+1} = \frac{2}{3}$

[b] $\sqrt[3]{64 \times 2} + \sqrt[3]{8 \times 2} - 2\sqrt[3]{27 \times 2}$

$= 4\sqrt[3]{2} + 2\sqrt[3]{2} - 6\sqrt[3]{2} = 0$

4

[a] $\because (x^2 + 4)(x^2 - 9) = 0$

$\therefore x^2 + 4 = 0$

$\therefore x^2 = -4$ (has no solution in $\mathbb{R}$)

or $x^2 - 9 = 0$ $\therefore x^2 = 9$

$\therefore x = \pm 3$ $\therefore$ The S.S. $= \{-3, 3\}$

[b] The volume of the cylinder $= \pi r^2 h$

$\therefore \frac{22}{7} \times 7^2 \times 10 = 1540$ cm^3.

5

[a] $\because y = \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \frac{\sqrt{3}+\sqrt{2}}{3-2} = \sqrt{3}+\sqrt{2}$

$\therefore x + y = \sqrt{3} - \sqrt{2} + \sqrt{3} + \sqrt{2} = 2\sqrt{3}$

[b]

Sets	x	f	$x \times f$
5 –	10	2	20
15 –	20	1	20
25 –	30	3	90
35 –	40	3	120
45 –	50	1	50
Total		**10**	**300**

$\therefore$ The arithmetic mean $= \frac{300}{10} = 30$

10 Kafr El-Sheikh

1 1 (d) 2 (a) 3 (d) 4 (d) 5 (b)

2 1 $\frac{3}{2}$ 2 $3\sqrt{2}$ 3 4 4 8

3

[a] $\because -1 < 3x - 7 \le 5$ $\therefore 6 < 3x \le 12$

$\therefore 2 < x \le 4$ $\therefore$ The S.S. $=]2, 4]$

[b] $\therefore y = \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3} = \sqrt{5}+\sqrt{3}$

$, x = \sqrt{5}-\sqrt{3}$

$\therefore$ x and y are two conjugate numbers.

$, \frac{x+y}{xy} = \frac{\sqrt{5}-\sqrt{3}+\sqrt{5}+\sqrt{3}}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)} = \frac{2\sqrt{5}}{5-3} = \sqrt{5}$

4

[a] 1 $A \cap B = [1 , 3]$

2 $A \cup B =]-3 , 6[$

[b] $\sqrt{25 \times 2} - \sqrt{9 \times 2} - 2\sqrt{2} = 5\sqrt{2} - 3\sqrt{2} - 2\sqrt{2} = 0$

5

[a] (0 , 5) , (1 , 4) , (2 , 3)

[b]

Sets	x	f	$x \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		20	540

$\therefore$ The arithmetic mean $= \frac{540}{20} = 27$ marks

11 El-Fayoum

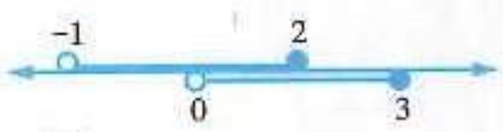
1 1 (b) 2 (a) 3 (b) 4 (a) 5 (d)

2 1 $5\ mn^2$ 2 7 3 $\frac{1}{4}$ 4 36

3

[a] 1 $A \cup B =]-1 , 3]$

2 $A \cap B =]0 , 2]$



[b] $\because x = \frac{5}{\sqrt{7}-\sqrt{2}} \times \frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}}$

$= \frac{5\left(\sqrt{7}+\sqrt{2}\right)}{7-2} = \sqrt{7}+\sqrt{2}$

$, y = \sqrt{7}-\sqrt{2}$

$\therefore$ x and y are two conjugate numbers.

$, (x+y)^2 = \left(\sqrt{7}+\sqrt{2}+\sqrt{7}-\sqrt{2}\right)^2$

$= \left(2\sqrt{7}\right)^2 = 28$

4

[a] 1 $\because$ The lateral area of the cube $= 4\ \ell^2$

$\therefore 4\ \ell^2 = 64$ $\therefore \ell^2 = 16$

$\therefore$ The edge length of the cube (ℓ) = 4 cm.

2 The total area of the cube

$= 6\ \ell^2 = 6 \times 4^2 = 96\ cm^2$.

3 The volume of the cube $= \ell^3 = 4^3 = 64\ cm^3$.

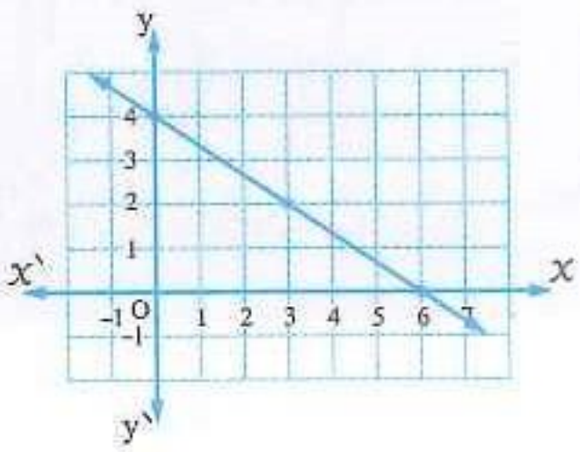
[b] $2\sqrt{9 \times 3} + 3\sqrt{4 \times 3} - 4\sqrt{3} + \frac{1}{3}\sqrt{9 \times 3}$

$= 6\sqrt{3} + 6\sqrt{3} - 4\sqrt{3} + \sqrt{3} = 9\sqrt{3}$

5

[a] $2x + 3y = 12$

x	0	3	6
y	4	2	0



[b] A = 12

$, B = 40 - (5 + 10 + 14 + 6) = 5$

Sets	x	f	$x \times f$
0 –	2	5	10
4 –	6	10	60
8 –	10	14	140
12 –	14	6	84
16 –	18	5	90
Total		40	384

$\therefore$ The arithmetic mean $= \frac{384}{40} = 9.6$

12 Souhag

1 1 (b) 2 (c) 3 (a) 4 (c) 5 (d) 6 (b)

2 1 3 2 12 3 $6\sqrt{6}$ 4 $\sqrt{5}+2$ 5 2 6 $\{-3 , 3\}$

3

[a] $\because -2 < 3x + 4 \leq 16$

$\therefore -6 < 3x \leq 12 \qquad \therefore -2 < x \leq 4$

$\therefore$ The S.S. $=]-2, 4]$

[b] $\sqrt{25 \times 3} - 2\sqrt{9 \times 3} + \sqrt{9 \times \frac{1}{3}}$

$= 5\sqrt{3} - 6\sqrt{3} + \sqrt{3} = 0$

4

[a] $\because r = h$

$\therefore$ The volume of the cylinder $= \pi r^2 h$

$= \pi r^3 = 27\pi$

$\therefore r^3 = 27 \qquad \therefore r = 3$ cm.

[b] $\because y = \dfrac{2}{\sqrt{7} - \sqrt{5}} \times \dfrac{\sqrt{7} + \sqrt{5}}{\sqrt{7} + \sqrt{5}}$

$= \dfrac{2(\sqrt{7} + \sqrt{5})}{7 - 5} = \sqrt{7} + \sqrt{5}$

$\therefore (x + y)^2 = (\sqrt{7} - \sqrt{5} + \sqrt{7} + \sqrt{5})^2 = (2\sqrt{7})^2 = 28$

5

[a]

1 $X \cup Y = [-3, 6]$

2 $X - Y =]4, 6]$

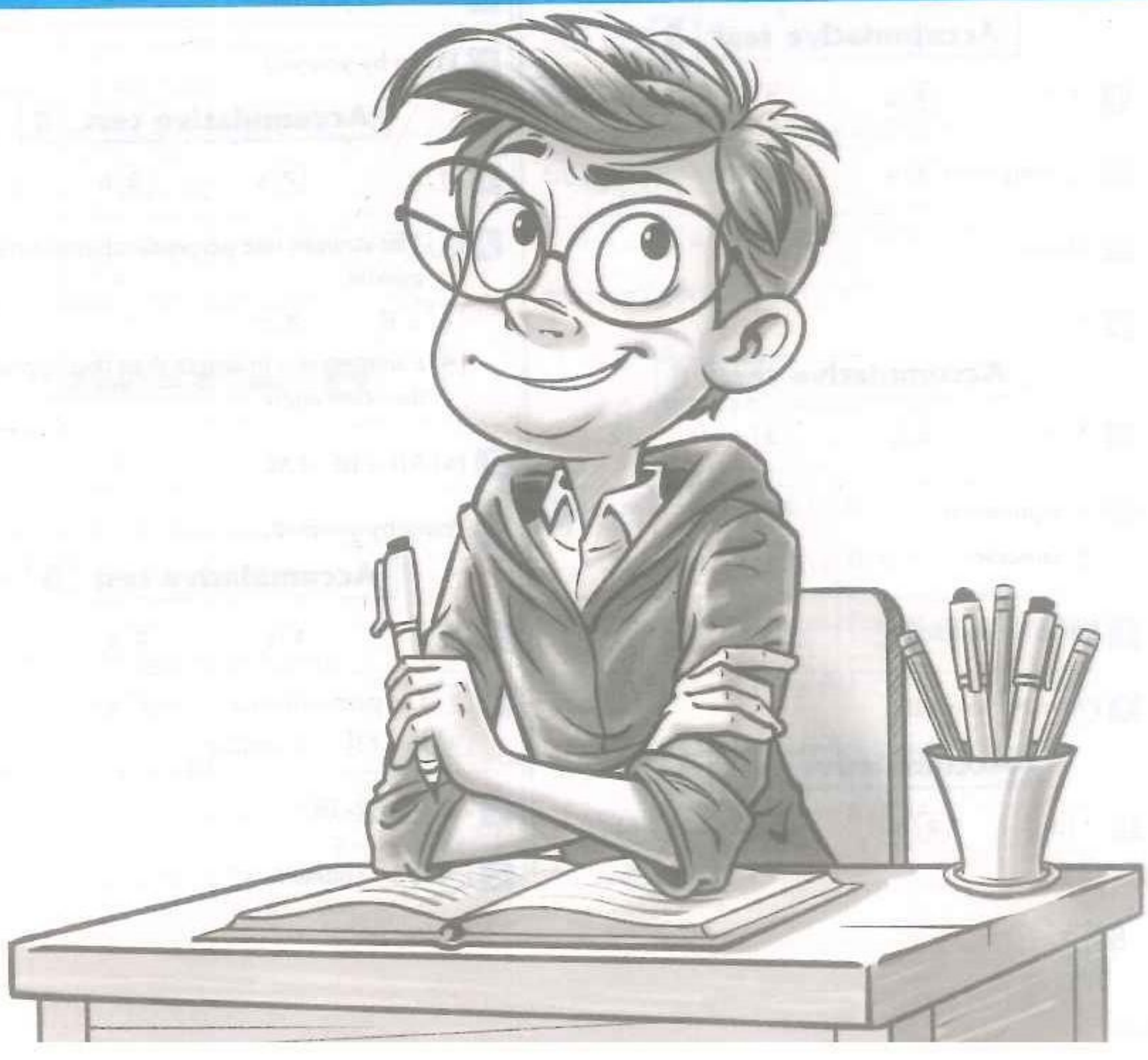
[b]

Sets	x	f	$x \times f$
5 –	10	7	70
15 –	20	10	200
25 –	30	12	360
35 –	40	13	520
45 –	50	8	400
Total		**50**	**1550**

$\therefore$ The arithmetic mean $= \dfrac{1550}{50} = 31$

Guide Answers

Of The Notebook (Geometry)



Answers of the accumulative tests on Geometry

Accumulative test 1

1 [1] c [2] b [3] a [4] c

2 [1] 2 : 1 [2] 4
[3] 3 [4] one point

3 6 cm.

4 15 cm.

Accumulative test 2

1 [1] a [2] a [3] c [4] c

2 [1] $\frac{1}{2}$ the length of the hypotenuse
[2] 4 [3] 4 [4] 2.5 cm.

3 Prove by yourself.

4 [1] 12 cm. [2] 4 cm. [3] 26 cm.

Accumulative test 3

1 [1] c [2] a [3] a [4] b

2 [1] congruent [2] 4 [3] 50° [4] 120°

3 24 cm.

4 70°

Accumulative test 4

1 [1] a [2] a [3] d [4] a

2 [1] equilateral [2] 1 : 2
[3] isosceles [4] 3

3 Prove by yourself.

4 Prove by yourself.

Accumulative test 5

1 [1] d [2] b [3] b [4] a

2 [1] the angle at this vertex is right
[2] 30° [3] 12° [4] the vertex angle

3 Prove by yourself.

4 6 cm.

Accumulative test 6

1 [1] b [2] a [3] d [4] a

2 [1] 50° [2] >
[3] bisects the base and is perpendicular to it
[4] one axis of symmetry

3 Prove by yourself.

4 [1] 25° [2] 2 cm.

Accumulative test 7

1 [1] c [2] c [3] c [4] b

2 [1] the angle of the greater measure
[2] ≡ [3] $\overline{BC}$ [4] 105°

3 Prove by yourself.

4 Prove by yourself.

Accumulative test 8

1 [1] b [2] a [3] b [4] b

2 [1] The straight line perpendicular to it from its middle
[2] ∠ B [3] >
[4] a side greater in length than that opposite to the other angle

3 [a] AB < BC < AC

4 Prove by yourself.

Accumulative test 9

1 [1] b [2] c [3] a [4] b

2 [1] The perpendicular [2] <
[3]]5 , 13[[4] obtuse

3 AC > AB > BC

4 Prove by yourself.

Answers of October tests on Geometry

Answers of Test 1

1 1 d 2 a 3 d

2 1 twice 2 120° 3 1

3 In Δ ABC :

$\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$ (given)

$\therefore$ BC = 2DE = 2 × 4 = 8 cm.

, $\because$ $\overline{BE}$ and $\overline{CD}$ are two medians

, $\overline{BE} \cap \overline{CD} = \{M\}$

$\therefore$ M is the point of intersection of the medians

$\therefore$ MC = 2DM = 2 × 3 = 6 cm.

, $BM = \frac{2}{3}\ BE = \frac{2}{3} \times 6 = 4$ cm.

$\therefore$ The perimeter of Δ BMC = 8 + 6 + 4 = 18 cm.

(The req.)

4 In Δ ABC : $\because$ AB = AC

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$

In Δ BCD : $\because$ BD = CD

$\therefore m(\angle DBC) = m(\angle DCB) = \frac{180^\circ - 120^\circ}{2} = 30^\circ$

$\therefore m(\angle ABD) = 70^\circ - 30^\circ = 40^\circ$ (The req.)

Answers of Test 2

1 1 b 2 a 3 c

2 1 one point 2 the angle at this vertex is right

3 50°

3 $\because$ Δ ACD is an equilateral triangle

$\therefore m(\angle CAD) = 60^\circ$

In Δ ABC : $\because$ AB = BC

$\therefore m(\angle BAC) = m(\angle BCA) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$

$\therefore m(\angle BAD) = 60^\circ + 70^\circ = 130^\circ$ (The req.)

4 In Δ ABC :

$\because m(\angle BAC) = 90^\circ$, $\overline{AD}$ is a median

$\therefore$ BC = 2AD = 2 × 3 = 6 cm.

In Δ CBE :

$\because m(\angle CBE) = 90^\circ$, $m(\angle E) = 30^\circ$

$\therefore$ EC = 2BC = 2 × 6 = 12 cm.

, $\because$ $\overline{BF}$ is a median

$\therefore BF = \frac{1}{2}\ EC = \frac{1}{2} \times 12 = 6$ cm. (The req.)

Answers of November tests on Geometry

Answers of Test 1

1 1 b 2 b 3 a

2 1 bisects each of the base and the vertex angle.

2 60° 3 $\frac{5}{4}$

3 $\because B \in \overline{DC}$

$\therefore m(\angle ABC) = 180^\circ - 125^\circ = 55^\circ$

In Δ ABC :

$m(\angle C) = 180^\circ - (55^\circ + 70^\circ) = 55^\circ$

$\therefore m(\angle ABC) = m(\angle C)$

$\therefore$ AB = AC

$\therefore$ Δ ABC is an isosceles triangle. (Q.E.D.)

4 $\because m(\angle BCE) = m(\angle BDF)$ (1)

$\therefore m(\angle BCL) = m(\angle BDL)$

(The supplementaries of equal angles are equal)

In Δ BCD : $\therefore$ BC = BD

$\therefore$ Δ BCD is an isosceles triangle

, $\because$ $\overline{BL}$ is a median in Δ BCD

$\therefore \overline{BL} \perp \overline{CD}$ (Q.E.D.)

Answers of Test 2

1 1 b 2 a 3 b

2 1 bisects the base and is perpendicular to it.

2 isosceles

3 zero.

3 In Δ ADE : $\because$ AD = AE

$\therefore$ m ($\angle$ ADE) = m ($\angle$ AED) (1)

, $\because \overline{DE} // \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore$ m ($\angle$ ADE) = m ($\angle$ B) (corresponding angles) (2)

, $\because \overline{DE} // \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ AED) = m ($\angle$ C) (corresponding angles) (3)

From (1) , (2) and (3) : $\therefore$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC

$\therefore$ Δ ABC is an isosceles triangle. (Q.E.D.)

4 In Δ ADC : $\because$ DA = DC

$\therefore$ m ($\angle$ C) = m ($\angle$ DAC) = 30°

$\because$ $\angle$ ADB is an exterior angle of Δ ADC

$\therefore$ m ($\angle$ ADB) = 30° + 30° = 60°

$\because$ DA = DB

$\therefore$ Δ ABD is an equilateral triangle. (Q.E.D.)

Answers of important questions on Geometry

Unit four

First Answers of multiple choice questions

1 (c) 2 (c) 3 (c) 4 (c) 5 (a)
6 (d) 7 (c) 8 (b) 9 (b) 10 (a)
11 (d) 12 (b) 13 (a) 14 (c) 15 (d)
16 (b) 17 (b) 18 (d)

Second Answers of complete questions

1 one point 2 congruent
3 perpendicular, bisects it
4 perpendicular to the base, bisects the vertex angle
5 the base, the vertex angle
6 its axis of symmetry
7 equilateral 8 isosceles 9 zero
10 right 11 36°

Third Answers of essay questions

1 In Δ ABC :

$\because$ m ($\angle$ ABC) = 90°

, m ($\angle$ C) = 30°

$\therefore$ AC = 2 AB = 2 × 5 = 10 cm. (First req.)

, $\because$ D is the midpoint of $\overline{AC}$

$\therefore$ $\overline{BD}$ is a median from the vertex of the right angle

$\therefore$ BD = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 10 = 5 cm. (Second req.)

2 $\because$ M is the intersection point of the medians of Δ ABC

$\therefore$ $\overline{BE}$ and $\overline{CD}$ are two medians of Δ ABC

$\therefore$ MD = $\frac{1}{3}$ DC = $\frac{1}{3}$ × 15 = 5 cm.

, ME = $\frac{1}{2}$ MB = $\frac{1}{2}$ × 6 = 3 cm.

, $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore$ DE = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 14 = 7 cm.

$\therefore$ The perimeter of Δ MDE = 5 + 3 + 7 = 15 cm.

(The req.)

3 $\because$ F and N are the midpoints of $\overline{AB}$ and $\overline{AC}$

$\therefore$ $\overline{BN}$ and $\overline{CF}$ are two medians of Δ ABC

, $\because$ $\overline{BN} \cap \overline{CF} = \{M\}$

$\therefore$ M is the point of concurrence of the medians of Δ ABC

$\therefore$ MF = $\frac{1}{3}$ CF = $\frac{1}{3}$ × 9 = 3 cm.

, MN = $\frac{1}{2}$ BM = $\frac{1}{2}$ × 4 = 2 cm.

, $\because$ F is the midpoint of $\overline{AB}$

$\therefore$ AF = $\frac{1}{2}$ AB = $\frac{1}{2}$ × 6 = 3 cm.

, $\because$ N is the midpoint of $\overline{AC}$

$\therefore$ AN = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 10 = 5 cm.

$\therefore$ The perimeter of the figure AFMN

= 3 + 2 + 3 + 5 = 13 cm. (The req.)

4 $\because$ Δ EBD is an equilateral triangle.

$\therefore$ m ($\angle$ EBD) = 60°

In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ C) = $\frac{180° - 50°}{2}$ = 65°

$\therefore$ m ($\angle$ DBC) = 60° + 65° = 125° (The req.)

5 $\because$ $\overrightarrow{AD}$ // $\overline{CB}$, $\overleftrightarrow{AB}$ is a transversal to them.

$\therefore$ m ($\angle$ B) = m ($\angle$ BAD) = 30° (alternate angles)

In Δ ABC : $\because$ AB = BC

$\therefore$ m ($\angle$ BAC) = m ($\angle$ C) = $\frac{180° - 30°}{2}$ = 75°

(The req.)

6 $\because$ $\overrightarrow{AE}$ // $\overline{BC}$, $\overleftrightarrow{BD}$ is a transversal to them

$\therefore$ m ($\angle$ B) = m ($\angle$ DAE) (corresponding angles)

, $\because$ $\overrightarrow{AE}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ C) = m ($\angle$ CAE) (alternate angles)

In Δ ABC : $\because$ m ($\angle$ B) = m ($\angle$ C)

because : AB = AC

$\therefore$ m ($\angle$ DAE) = m ($\angle$ CAE)

$\therefore$ $\overrightarrow{AE}$ bisects $\angle$ DAC (Q.E.D.)

7 In Δ ABC which is right-angled at B

$\because$ m ($\angle$ ACB) = 30° $\therefore$ AB = $\frac{1}{2}$ AC

, $\because$ AB = DE $\therefore$ DE = $\frac{1}{2}$ AC (1)

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore \overline{DE}$ is a median in Δ ADC (2)

From (1) and (2) :

$\therefore$ m ($\angle$ ADC) = 90° (Q.E.D.)

8 In Δ ABC :

$\because$ X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{BC}$

$\therefore XY = \frac{1}{2} AC = \frac{1}{2} \times 20 = 10$ cm.

In Δ XBY which is right-angled at B

$\because$ D is the midpoint of $\overline{XY}$

$\therefore \overline{BD}$ is a median

$\therefore BD = \frac{1}{2} XY = \frac{1}{2} \times 10 = 5$ cm. (The req.)

9 In Δ ABD which is right-angled at D

$\because$ F is the midpoint of $\overline{AB}$

$\therefore \overline{DF}$ is a median

$\therefore DF = \frac{1}{2} AB = \frac{1}{2} \times 16 = 8$ cm.

Similarly : $DE = \frac{1}{2} AC = \frac{1}{2} \times 18 = 9$ cm.

, $\because$ F is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore FE = \frac{1}{2} BC = \frac{1}{2} \times 20 = 10$ cm.

$\therefore$ The perimeter of Δ DEF = 8 + 9 + 10

= 27 cm. (The req.)

10 $\because \overline{ML} \parallel \overline{YZ}$, $\overleftrightarrow{YL}$ is a transversal to them

$\therefore$ m ($\angle$ LYZ) = m ($\angle$ MLY) (alternate angles)

, $\because$ m ($\angle$ LYZ) = m ($\angle$ LYM)

$\therefore$ m ($\angle$ MLY) = m ($\angle$ LYM)

$\therefore$ MY = ML

$\therefore \Delta$ LMY is an isosceles triangle. (Q.E.D.)

11 In Δ MXY : $\because$ MX = MY

$\therefore$ m ($\angle$ X) = m ($\angle$ Y) (1)

, $\because \overline{XY} \parallel \overline{ZL}$, $\overleftrightarrow{XL}$ is a transversal to them

$\therefore$ m ($\angle$ L) = m ($\angle$ X) (alternate angles) (2)

, $\because \overline{XY} \parallel \overline{ZL}$, $\overleftrightarrow{YZ}$ is a transversal to them

$\therefore$ m ($\angle$ Z) = m ($\angle$ Y) (alternate angles) (3)

From (1) , (2) and (3) :

$\therefore$ m ($\angle$ L) = m ($\angle$ Z)

$\therefore$ ML = MZ

$\therefore \Delta$ MLZ is an isosceles triangle. (Q.E.D.)

12 In Δ ACD : $\because$ X is the midpoint of $\overline{AD}$

, Y is the midpoint of $\overline{CD}$

$\therefore XY = \frac{1}{2} AC$ (1)

, In Δ ABC :

$\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

$\therefore AB = \frac{1}{2} AC$ (2)

From (1) and (2) : $\therefore$ AB = XY (Q.E.D.)

13 In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ In $\Delta\Delta$ ABD and ACE :

$$\begin{cases} m(\angle B) = m(\angle C) \\ m(\angle BAD) = m(\angle CAE) \\ AB = AC \end{cases}$$

$\therefore \Delta ABD \equiv \Delta ACE$, then we deduce that :

AD = AE (Q.E.D.)

14 In $\Delta\Delta$ ABD and ACE :

$$\begin{cases} m(\angle D) = m(\angle E) = 90° \\ m(\angle ABD) = m(\angle ACE) \\ BD = CE \end{cases}$$

$\therefore \Delta ABD \equiv \Delta ACE$, then we deduce that :

AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) (Q.E.D.)

15 In Δ ABC : $\because$ AB = AC

, $\overrightarrow{AD}$ bisects $\angle$ A

$\therefore \overline{AD} \perp \overline{BC}$ (First req.)

, D is the midpoint of $\overline{BC}$

$\therefore CD = \frac{1}{2} BC = \frac{1}{2} \times 8 = 4$ cm.

, $\because$ m ($\angle$ BAD) = 180° – (65° + 90°) = 25°

$\therefore$ m ($\angle$ DAC) = m ($\angle$ BAD) = 25° (Second req.)

16 In Δ ADE : $\because$ AD = AE

$\therefore$ m ($\angle$ ADE) = m ($\angle$ AED) (1)

, $\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore$ m ($\angle$ ADE) = m ($\angle$ B) (corresponding angles) (2)

, $\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ AED) = m ($\angle$ C) (corresponding angles) (3)

From (1) , (2) and (3) : $\therefore$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC

$\therefore$ Δ ABC is an isosceles triangle. (Q.E.D.)

17 In Δ ABC : $\because$ AB = AC

, $\overrightarrow{AE}$ bisects $\angle$ BAC

$\therefore$ BE = $\frac{1}{2}$ BC (Q.E.D. 1)

, $\overline{AE} \perp \overline{BC}$

$\therefore$ $\overleftrightarrow{AE}$ is the axis of symmetry of $\overline{BC}$

, $\because$ D $\in \overleftrightarrow{AE}$

$\therefore$ BD = CD (Q.E.D. 2)

18 In Δ ABC : $\because$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC $\therefore$ $2x - 1 = 7$

$\therefore$ $2x = 8$ $\therefore$ $x = 4$ (First req.)

$\therefore$ BC = 9 – 4 = 5 cm.

, $\because$ AB = AC = 7 cm.

$\therefore$ The perimeter of Δ ABC = 7 + 7 + 5 = 19 cm.

(Second req.)

19 In Δ ABC : $\because$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC $\therefore$ $2x - 1 = x + 3$

$\therefore$ $2x - x = 3 + 1$

$\therefore$ $x = 4$ $\therefore$ AB = AC = 4 + 3 = 7 cm.

, BC = 9 – 4 = 5 cm.

$\therefore$ The perimeter of Δ ABC = 7 + 7 + 5 = 19 cm.

(The req.)

20 $\because$ The sum of measures of the angles of the triangle = 180°

$\therefore$ $2x + 3x - 10° + x + 40° = 180°$

$\therefore$ $6x + 30° = 180°$

$\therefore$ $6x = 150°$ $\therefore$ $x = \frac{150°}{6} = 25°$

$\therefore$ m ($\angle$ A) = 2 × 25° = 50°

, m ($\angle$ B) = 25° + 40° = 65°

, m ($\angle$ C) = 3 × 25° – 10° = 65°

$\therefore$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC

$\therefore$ Δ ABC is an isosceles triangle. (Q.E.D.)

21 In Δ ABC : $\because$ D is the midpoint of $\overline{AC}$

$\therefore$ $\overline{BD}$ is a median from the vertex of the right angle

$\therefore$ BD = $\frac{1}{2}$ AC (1)

In Δ BDE : $\because$ m ($\angle$ BDE) = 90°

, m ($\angle$ E) = 30° $\therefore$ BD = $\frac{1}{2}$ BE (2)

From (1) and (2) : $\therefore$ AC = BE (Q.E.D.)

22 In $\Delta\Delta$ ADC and BDC :

- AC = BC
- $\overline{CD}$ is a common side
- m ($\angle$ ACD) = m ($\angle$ BCD)

$\therefore$ Δ ADC $\equiv$ Δ BDC and we deduce that :

AD = BD

, $\because$ ED = BD $\therefore$ AD = ED

$\therefore$ m ($\angle$ E) = m ($\angle$ EAD) (Q.E.D.)

23 $\because$ m ($\angle$ BAD) = 90° , m ($\angle$ BAE) = 30°

$\therefore$ m ($\angle$ DAF) = 60°

In Δ AFD :

$\because$ m ($\angle$ AFD) = 90° , m ($\angle$ DAF) = 60°

$\therefore$ m ($\angle$ ADF) = 180° – (90° + 60°) = 30°

$\therefore$ AD = 2 AF = 8 cm.

$\therefore$ The area of the square = 8 × 8 = 64 cm^2.

(The req.)

24 In Δ ABC : $\because$ D is the midpoint of $\overline{BC}$

$\therefore$ $\overline{AD}$ is a median

, $\because$ AM = 2 MD

$\therefore$ M is the intersection point of the medians of Δ ABC

, $\because$ M $\in \overline{CE}$ $\therefore$ $\overline{CE}$ is a median in Δ ABC

$\therefore$ EM = $\frac{1}{3}$ EC = $\frac{1}{3}$ × 12 = 4 cm. (The req.)

Unit five

First Answers of multiple choice questions

1 (c)	2 (d)	3 (c)	4 (a)	5 (d)
6 (b)	7 (b)	8 (a)	9 (d)	10 (b)
11 (d)	12 (b)			

Second Answers of complete questions

1 opposite to a side greater in length than that opposite to the other angle.

2 the angle of the greater measure

3 $\overline{AB}$ **4** $\overline{BC}$

5 > **6**]4 , 14[

7 the perpendicular line segment

8 the hypotenuse **9** AC **10** A + X

Third Answers of essay questions

1 In Δ ABC :

$\because$ m ($\angle$ C) = 180° – (80° + 40°) = 60°

$\therefore$ m ($\angle$ B) > m ($\angle$ C) > m ($\angle$ A)

$\therefore$ AC > AB > BC (The req.)

2 In Δ ABC : $\because$ AB < BC < AC

$\therefore$ m ($\angle$ C) < m ($\angle$ A) < m ($\angle$ B) (The req.)

3 In Δ ABC : $\because$ AC = AB

$\therefore$ m ($\angle$ ACB) = m ($\angle$ ABC) (1)

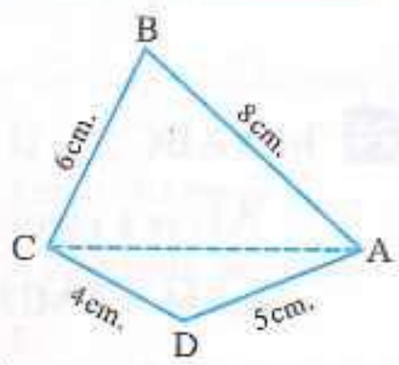
In Δ BCD : $\because$ BD > CD

$\therefore$ m ($\angle$ BCD) > m ($\angle$ CBD) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ ACD) > m ($\angle$ ABD) (Q.E.D.)

4



Const : Draw $\overline{AC}$

Proof : In Δ ABC :

$\because$ AB > BC

$\therefore$ m ($\angle$ BCA) > m ($\angle$ BAC) (1)

In Δ ADC : $\because$ AD > CD

$\therefore$ m ($\angle$ DCA) > m ($\angle$ DAC) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ BCD) > m ($\angle$ BAD) (Q.E.D.)

5 In Δ ABC : $\because$ AB < AC

$\therefore$ m ($\angle$ ACB) < m ($\angle$ ABC)

, $\because$ $\overrightarrow{CM}$ bisects $\angle$ ACB , $\overrightarrow{BM}$ bisects $\angle$ ABC

$\therefore$ m ($\angle$ MCB) = $\frac{1}{2}$ m ($\angle$ ACB)

, m ($\angle$ MBC) = $\frac{1}{2}$ m ($\angle$ ABC)

$\therefore$ m ($\angle$ MCB) < m ($\angle$ MBC)

$\therefore$ BM < CM (Q.E.D.)

6 In Δ ABC : $\because$ AB > BC

$\therefore$ m ($\angle$ ACB) > m ($\angle$ BAC) (1)

, $\because$ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ XYA) = m ($\angle$ ACB)

(corresponding angles) (2)

From (1) and (2) :

$\therefore$ m ($\angle$ XYA) > m ($\angle$ BAC)

$\therefore$ AX > XY (Q.E.D.)

7 $\because$ $\overrightarrow{AD}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore$ m ($\angle$ B) = m ($\angle$ BAD) = 40° (alternate angles)

In Δ ABC :

$\therefore$ m ($\angle$ C) = 180° – (80° + 40°) = 60°

$\therefore$ m ($\angle$ C) > m ($\angle$ B)

$\therefore$ AB > AC (Q.E.D.)

8 In Δ AXY :

$\because$ m ($\angle$ AXY) = m ($\angle$ AYX)

$\therefore$ AY = AX (1)

, $\because$ AC > AB (2)

Subtracting (1) from (2) :

$\therefore$ AC – AY > AB – AX

$\therefore$ YC > XB (Q.E.D.)

9 In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ ACB) = m ($\angle$ B) = 65°

$\therefore$ m ($\angle$ BAC) = 180° – 65° × 2 = 50°

, $\because$ $\angle$ BAC is an exterior angle of Δ ACD

$\therefore$ m ($\angle$ ADC) = 50° – 20° = 30°

$\therefore$ In Δ ADC : m ($\angle$ ADC) > m ($\angle$ ACD)

$\therefore$ AC > AD

, $\because$ AB = AC

$\therefore$ AB > AD (Q.E.D.)

10 In Δ ABD : $\because$ BD = AD

$\therefore$ m ($\angle$ BAD) = m ($\angle$ B)

$\therefore$ m ($\angle$ BAD) + m ($\angle$ DAC) > m ($\angle$ B)

$\therefore$ m ($\angle$ BAC) > m ($\angle$ B)

$\therefore$ BC > AC (Q.E.D.)

11 In ΔABC : $\because AB = AC$

$\therefore m(\angle B) = m(\angle C)$ (1)

, $\because D \in \overline{BC}$

$\therefore \angle ADC$ is an exterior angle of ΔADB

$\therefore m(\angle ADC) > m(\angle B)$ (2)

From (1) and (2) : $\therefore m(\angle ADC) > m(\angle C)$

$\therefore AC > AD$ (Q.E.D.)

12 In ΔABD : $\because$ X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AD}$

$\therefore XY = \frac{1}{2} BD$ (1)

In ΔDBC : $\because m(\angle BDC) = 90°$

, E is the midpoint of $\overline{BC}$

$\therefore \overline{DE}$ is a median from the vertex of the right angle

$\therefore DE = \frac{1}{2} BC$ (2)

, $\because \overline{BC}$ is a hypotenuse of ΔBDC

$\therefore BD < BC$

$\therefore \frac{1}{2} BD < \frac{1}{2} BC$ (3)

From (1) , (2) and (3) :

$\therefore XY < DE$ (Q.E.D.)

13 $\because \angle ADC$ is an exterior angle of ΔABD

$\therefore m(\angle ADC) > m(\angle BAD)$ (1)

, $\because \overrightarrow{AD}$ bisects $\angle BAC$

$\therefore m(\angle BAD) = m(\angle DAC)$ (2)

From (1) and (2) :

$\therefore m(\angle ADC) > m(\angle DAC)$

$\therefore AC > DC$ (Q.E.D.)

14 In ΔACE :

$\because AE = EC$

$\therefore m(\angle ECA) = m(\angle CAE) = 40°$

$\therefore m(\angle AEC) = 180° - (40° + 40°) = 100°$

$\therefore m(\angle AEC) > m(\angle ACE)$

$\therefore AC > AE$ (Q.E.D. 1)

In ΔACD :

$\because \overline{AE}$ is a median , $AE = \frac{1}{2} CD$

$\therefore m(\angle DAC) = 90°$

$\therefore$ In ΔABC : $\overline{BC}$ is a hypotenuse

$\therefore BC > AC$ (Q.E.D. 2)

Answers of the school book model examinations on Geometry

Model 1

1

1 the hypotenuse 2 5 cm. , 9 cm.
3 a side greater in length than that opposite to the other angle.
4 the angle at this vertex is right 5 equilateral

2

1 c 2 a 3 b
4 b 5 a 6 d

3

[a] >

[b] $\because \Delta$ DBC is equilateral

$\therefore m(\angle DBC) = 60°$

In Δ ABC : $\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180° - 50°}{2} = 65°$

$\therefore m(\angle ABD) = 60° + 65° = 125°$ (The req.)

[c] $\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle ACB) = m(\angle DAC) = 50°$ (alternate angles)

$\therefore$ In Δ ABC : $m(\angle B) = 180° - (70° + 50°) = 60°$

$\therefore m(\angle BAC) > m(\angle B)$

$\therefore BC > AC$ (Q.E.D.)

4

[a] Theoretical.

[b] In Δ ABC : $\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

$\therefore \frac{1}{2} m(\angle ABC) = \frac{1}{2} m(\angle ACB)$

$\therefore m(\angle DBC) = m(\angle DCB)$

$\therefore \Delta$ DBC is isosceles. (Q.E.D.)

5

[a] $\because \overline{AC}$ is the longest side.

$\therefore \angle B$ is the greatest angle in measure.

, $\because \overline{AB}$ is the shortest side.

$\therefore \angle C$ is the smallest angle in measure.

$\therefore$ The descending order of measures of the angles is $m(\angle B)$, $m(\angle A)$ and $m(\angle C)$ (The req.)

[b] In Δ ABC : $\because AB > BC$

$\therefore m(\angle ACB) > m(\angle BAC)$ (1)

, $\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal.

$\therefore m(\angle XYA) = m(\angle ACB)$

(corresponding angles) (2)

From (1) and (2) : $\therefore m(\angle XYA) > m(\angle BAC)$

$\therefore AX > XY$ (Q.E.D.)

Model 2

1

1 d 2 a 3 b
4 b 5 d 6 d

2

1 isosceles. 2 is less than
3 XY 4 $\frac{1}{2}$
5 is perpendicular to it.

3

[a] $\because \overline{AB}$ is the longest side.

$\therefore \angle C$ is the greatest angle in measure.

, $\because \overline{CB}$ is the shortest side.

$\therefore \angle A$ is the smallest angle in measure.

$\therefore$ The ascending order of measures of the angles is $m(\angle A)$, $m(\angle B)$ and $m(\angle C)$ (The req.)

[b] In Δ ABC : $\because m(\angle B) = 90°$

, $\because$ D is the midpoint of $\overline{AC}$

, $\because$ E is the midpoint of $\overline{BC}$

$\therefore \overline{BD}$, $\overline{AE}$ are two medians in Δ ABC

$\therefore$ M is the intersection point of the medians of Δ ABC

$\therefore BD = \frac{1}{2} AC = \frac{1}{2} \times 9 = 4.5$ cm.

, $BM = \frac{2}{3} BD = \frac{2}{3} \times 4.5 = 3$ cm.

, $\because m(\angle C) = 30°$

$\therefore AB = \frac{1}{2} AC = \frac{1}{2} \times 9 = 4.5$ cm. (The req.)

4

[a] In Δ ABC : $\because$ D is the midpoint of $\overline{AC}$

$\therefore \overline{BD}$ is a median.

, $\because m(\angle ABC) = 90°$ $\therefore BD = \frac{1}{2} AC$ (1)

In Δ BDE : $\because m(\angle BDE) = 90°$, $m(\angle E) = 30°$

$\therefore BD = \frac{1}{2} BE$ (2)

From (1) and (2) : $\therefore AC = BE$ (Q.E.D.)

[b] $\because \overline{AD} // \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m ($\angle$ ACB) = m ($\angle$ CAD) = 30° (alternate angles)

$\therefore$ In Δ ABC :

m ($\angle$ B) = 180° – (70° + 30°) = 80°

$\therefore$ m ($\angle$ B) > m ($\angle$ BAC)

$\therefore$ AC > BC (Q.E.D.)

5

[a] a side greater in length than that opposite to the other angle.

[b] $\because \overrightarrow{AB}$ bisects $\angle$ YAZ

$\therefore$ m ($\angle$ YAB) = m ($\angle$ BAZ) (1)

, $\because \overline{AB} // \overline{XY}$, $\overleftrightarrow{AY}$ is a transversal.

$\therefore$ m ($\angle$ BAY) = m ($\angle$ AYX) (alternate angles) (2)

, $\because \overline{AB} // \overline{XY}$, $\overleftrightarrow{ZX}$ is a transversal.

$\therefore$ m ($\angle$ X) = m ($\angle$ BAZ) (corresponding angles) (3)

From (1) , (2) and (3) : $\therefore$ m ($\angle$ AYX) = m ($\angle$ X)

$\therefore$ m ($\angle$ AYX) + m ($\angle$ AYZ) > m ($\angle$ X)

$\therefore$ m ($\angle$ ZYX) > m ($\angle$ X)

$\therefore$ XZ > YZ (Q.E.D.)

Model for the merge students

1

1 1 : 2

2 half the length of the hypotenuse

3 congurent

4 >

5 bisects it , perpendicular to the base.

2

1 b 2 a 3 d 4 a 5 a

3

$\because$ m ($\angle$ B) = 90° , m ($\angle$ C) = 30°

$\therefore AB = \frac{1}{2} \times AC$

$\therefore$ AC = 10 cm.

4

[a] AC , AB , BC

[b] 1 40° 2 $\overline{AB}$

5

1 ✔ 2 ✘ 3 ✘ 4 ✔ 5 ✔

Answers of the schools examinations on Geometry

1 Cairo

1

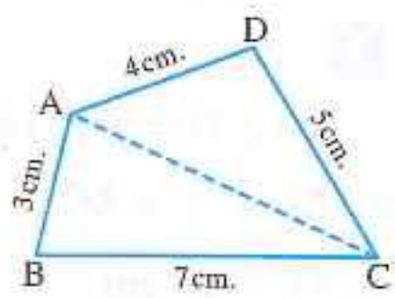
1 (b) 2 (c) 3 (a) 4 (d) 5 (b)

2

1 10 cm. 2 equilateral

3 45° 4 4

3



[a] Const. :

Draw $\overline{AC}$

Proof : In Δ ABC :

$\because$ BC > AB

$\therefore$ m ($\angle$ BAC) > m ($\angle$ BCA) (1)

In Δ ACD : $\because$ CD > AD

$\therefore$ m ($\angle$ CAD) > m ($\angle$ ACD) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ BAD) > m ($\angle$ BCD) (Q.E.D.)

[b] In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ B) = m ($\angle$ C)

In $\Delta\Delta$ ABX and ACY :

$$\begin{cases} m(\angle B) = m(\angle C) \\ AB = AC \\ BX = CY \end{cases}$$

$\therefore \Delta ABX \equiv \Delta ACY$, then we deduce that :

AX = AY (Q.E.D.)

4

[a] In Δ ABC : $\because$ m ($\angle$ ABC) = 90°

, $\overline{BD}$ is a median

$\therefore BD = \frac{1}{2} AC$ (1)

In Δ ACE : $\because$ X is the midpoint of $\overline{AE}$

, Y is the midpoint of $\overline{CE}$

$\therefore YX = \frac{1}{2} AC$ (2)

From (1) and (2) :

$\therefore$ BD = YX (Q.E.D.)

[b] In Δ ABC : $\because$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC $\therefore 2x - 1 = x + 3$ $\therefore x = 4$

$\therefore$ AB = 2 × 4 – 1 = 7 cm.

, AC = 4 + 3 = 7 cm.

, BC = 9 – 4 = 5 cm.

$\therefore$ The perimeter of Δ ABC = 7 + 7 + 5 = 19 cm. (The req.)

5

[a] In Δ ABC : $\because$ $\overline{CD}$ and $\overline{BE}$ are two medians

$\therefore \overline{BE} \cap \overline{CD} = \{M\}$

$\therefore$ M is the point of intersection of the medians.

$\therefore MD = \frac{1}{3} CD = \frac{1}{3} \times 9 = 3$ cm.

, $ME = \frac{1}{2} BM = \frac{1}{2} \times 4 = 2$ cm.

, $AE = \frac{1}{2} AC = \frac{1}{2} \times 10 = 5$ cm.

, $AD = \frac{1}{2} AB = \frac{1}{2} \times 6 = 3$ cm.

$\therefore$ The perimeter of the figure ADME

= 3 + 2 + 5 + 3 = 13 cm. (The req.)

[b] Assuming that ABC is a triangle

$\therefore$ AB < AC + BC

Adding AB to both sides :

$\therefore$ 2 AB < AC + BC + AB

$\therefore AB < \frac{1}{2}$ the perimeter of Δ ABC

$\therefore$ The length of any side in the triangle is less than the half of the perimeter of the triangle (Q.E.D.)

2 Cairo

1

1 (d) 2 (b) 3 (b) 4 (b) 5 (b)

2

1 6 cm. 2 <

3 half the length of the hypotenuse. 4 110°

3

[a] In Δ XYZ : $\because$ XY < XZ < YZ

$\therefore$ m ($\angle$ Z) < m ($\angle$ Y) < m ($\angle$ X) (The req.)

[b] $\because B \in \overrightarrow{CD}$

$\therefore$ m ($\angle$ ABC) = 180° – 150° = 30°

, $\because C \in \overrightarrow{AE}$

$\therefore$ m ($\angle$ ACB) = 180° – 105° = 75°

$\therefore$ m ($\angle$ ACB) > m ($\angle$ ABC)

$\therefore$ AB > AC (Q.E.D.)

4

[a] In Δ ABC : $\because$ AB = BC

$\therefore$ m ($\angle$ ACB) = m ($\angle$ BAC) = $\frac{180° - 90°}{2}$ = 45°

, $\because$ Δ BCD is an equilateral triangle.

$\therefore$ m ($\angle$ BCD) = 60°

$\therefore$ m ($\angle$ ACD) = 45° + 60° = 105° (The req.)

[b] $\because$ $\overline{AD}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ ACB) = m ($\angle$ CAD) = 40°

(alternate angles)

$\therefore$ In Δ ABC :

$\therefore$ m ($\angle$ BAC) = 180° – (40° + 70°) = 70°

$\therefore$ m ($\angle$ BAC) = m ($\angle$ B)

$\therefore$ AC = BC

$\therefore$ Δ ABC is an isosceles triangle. (Q.E.D.)

5

[a] In Δ ABC : $\because$ m ($\angle$ ABC) = 90°

, $\overline{BD}$ is a median

$\therefore$ BD = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 10 = 5 cm.

, In Δ BDE : $\because$ m ($\angle$ BDE) = 90° , m ($\angle$ E) = 30°

$\therefore$ BE = 2 BD = 2 × 5 = 10 cm. (The req.)

[b] In Δ ABC : $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore$ DE = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 14 = 7 cm.

, $\because$ $\overline{CD}$ and $\overline{BE}$ are two medians

, $\overline{CD} \cap \overline{BE} = \{M\}$

$\therefore$ M is the point of intersction of the medians

$\therefore$ MD = $\frac{1}{3}$ CD = $\frac{1}{3}$ × 15 = 5 cm.

, ME = $\frac{1}{2}$ BM = $\frac{1}{2}$ × 6 = 3 cm.

$\therefore$ The perimeter of Δ MDE = 7 + 5 + 3 = 15 cm.

(The req.)

3 Giza

1

1 (c) 2 (d) 3 (b) 4 (a) 5 (a)

2

1 the hypotenuse 2 congruent

3 210° 4 120°

3

[a] In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ B) = m ($\angle$ C) (1)

, $\because$ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore$ m ($\angle$ AXY) = m ($\angle$ B)

(corresponding angles) (2)

, $\because$ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ AYX) = m ($\angle$ C)

(corresponding angles) (3)

From (1) , (2) and (3) :

$\therefore$ m ($\angle$ AXY) = m ($\angle$ AYX)

$\therefore$ AX = AY

$\therefore$ Δ AXY is an isosceles triangle (Q.E.D.)

[b] $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore$ DE = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 12 = 6 cm.

, $\because$ $\overline{CD}$, $\overline{BE}$ are two medians intersecting at M

$\therefore$ MD = $\frac{1}{2}$ CM = $\frac{1}{2}$ × 6 = 3 cm.

, ME = $\frac{1}{2}$ BM = $\frac{1}{2}$ × 8 = 4 cm.

$\therefore$ The perimeter of Δ MDE = 6 + 3 + 4 = 13 cm.

(The req.)

4

[a] In Δ ABC : $\because$ AB < BC < AC

$\therefore$ m ($\angle$ C) < m ($\angle$ A) < m ($\angle$ B) (The req.)

[b] In Δ ABC : $\because$ AB = BC

$\therefore$ m ($\angle$ BAC) = m ($\angle$ BCA) = $\frac{180° - 40°}{2}$ = 70°

, $\because$ Δ ACD is an equilateral triangle

$\therefore$ m ($\angle$ CAD) = 60°

$\therefore$ m ($\angle$ BAD) = 70° + 60° = 130° (The req.)

5

[a] $\because$ $\overrightarrow{AD}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ C) = m ($\angle$ CAD) = 30° (alternate angles)

In Δ ABC : $\because$ AC = BC

$\therefore$ m ($\angle$ BAC) = m ($\angle$ B) = $\frac{180° - 30°}{2}$ = 75°

(The req.)

[b] In ΔABC : $\because AB > BC$

$\therefore m(\angle C) > m(\angle A)$

, $\because \overline{XY} // \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them.

$\therefore m(\angle AYX) = m(\angle C)$ (corresponding angles)

$\therefore m(\angle AYX) > m(\angle A)$

$\therefore$ In ΔAXY : $AX > XY$ (Q.E.D.)

4 Giza

1

1 (d) 2 (a) 3 (c) 4 (b) 5 (b)

2

1 $\overline{AB}$

2 bisects the base and is perpendicular to it

3 the greater angle in measure 4 half

3

[a] $\because$ E is the midpoint of $\overline{AC}$

, D is the midpoint of $\overline{BC}$

$\therefore ED = \frac{1}{2} AB = \frac{1}{2} \times 9 = 4.5$ cm.

, $\because \overline{AD}$ and $\overline{BE}$ are two medians

, $\overline{AD} \cap \overline{BE} = \{M\}$

$\therefore$ M is the point of intersection of the medians

$\therefore MD = \frac{1}{3} AD = \frac{1}{3} \times 12 = 4$ cm.

, $ME = \frac{1}{2} MB = \frac{1}{2} \times 6 = 3$ cm.

$\therefore$ The perimeter of $\Delta EMD = 4.5 + 4 + 3 = 11.5$ cm. (The req.)

[b] $\because Z \in \overrightarrow{XN}$

$\therefore m(\angle XZY) = 180° - 110° = 70°$

In ΔXYZ : $\because \angle XYM$ is an exterior angle

$\therefore m(\angle X) = 125° - 70° = 55°$

$\therefore m(\angle XZY) > (\angle X)$ $\therefore XY > ZY$ (Q.E.D.)

4

[a] In ΔXYZ : $\because$ M is the midpoint of $\overline{XY}$

, N is the midpoint of $\overline{YZ}$

$\therefore MN = \frac{1}{2} XZ = \frac{1}{2} \times 16 = 8$ cm.

In ΔMYN which is right-angled at Y

$\because \overline{YL}$ is a median

$\therefore YL = \frac{1}{2} MN = \frac{1}{2} \times 8 = 4$ cm. (The req.)

[b] $\because \overline{DE} // \overline{BC}$, $\overleftrightarrow{BD}$ is a transversal to them

$\therefore m(\angle BDE) = m(\angle CBD)$ (alternate angles)

, $\because \overrightarrow{BD}$ bisects $\angle ABC$

$\therefore m(\angle DBE) = m(\angle CBD)$

$\therefore m(\angle DBE) = m(\angle BDE)$

$\therefore BE = DE$

$\therefore \Delta EBD$ is an isosceles triangle. (Q.E.D.)

5

[a] In ΔCDE : $\because EC = DC$

$\therefore m(\angle D) = m(\angle E) = 30°$

$\therefore m(\angle ACB) = 30° + 30° = 60°$

(exterior angle of ΔCDE)

$\therefore$ In ΔABC : $m(\angle ABC) = 180° - (60° + 60°) = 60°$

$\therefore m(\angle A) = m(\angle ACB) = m(\angle ABC) = 60°$

$\therefore \Delta ABC$ is an equilateral triangle (Q.E.D.)

[b] In ΔABC : $\because m(\angle ABC) = 90°$

, $\overline{BE}$ is a median $\therefore BE = \frac{1}{2} AC$

, $\because BE = DE$ $\therefore DE = \frac{1}{2} AC$

, $\because \overline{DE}$ is a median in ΔADC

$\therefore m(\angle ADC) = 90°$ (Q.E.D.)

5 Alexandria

1

1 (b) 2 (c) 3 (a) 4 (b) 5 (b)

2

1 zero 2 120°

3 the hypotenuse 4 an isosceles

3

[a] In ΔABC : $\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180° - 50°}{2} = 65°$

, $\because \Delta DBC$ is an equilateral triangle

$\therefore m(\angle CBD) = 60°$

$\therefore m(\angle ABD) = 65° + 60° = 125°$ (The req.)

[b] In ΔABC : $\because AB < BC < CA$

$\therefore m(\angle C) < m(\angle A) < m(\angle B)$ (The req.)

4

[a] $\because$ Δ ABC is right-angled at B , m ($\angle$ C) = 30°

$\therefore$ AB = $\frac{1}{2}$ AC

, $\because$ $\overline{BD}$ is a median

$\therefore$ DB = $\frac{1}{2}$ AC

$\therefore$ AB = DB (Q.E.D.)

[b] $\because$ Δ ABC is right-angled at B

, m ($\angle$ ACB) = 30°

$\therefore$ AC = 2 AB = 2 × 5 = 10 cm.

, in Δ ACD : $\because$ $\overline{DE}$ is a median , DE = $\frac{1}{2}$ AC

$\therefore$ m ($\angle$ ADC) = 90° (Q.E.D.)

5

[a] In Δ BCE : $\because$ EB = EC

$\therefore$ m ($\angle$ B) = m ($\angle$ C) (1)

, $\because$ $\overline{AD}$ // $\overline{BC}$, $\overleftrightarrow{BD}$ is a transversal to them

$\therefore$ m ($\angle$ D) = m ($\angle$ B) (alternate angles) (2)

, $\because$ $\overline{AD}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ A) = m ($\angle$ C) (alternate angles) (3)

From (1) , (2) and (3) : $\therefore$ m ($\angle$ A) = m ($\angle$ D)

$\therefore$ EA = ED (Q.E.D.)

[b] $\because$ $\overrightarrow{SR}$ // $\overline{LM}$, $\overleftrightarrow{SM}$ is a transversal to them

$\therefore$ m ($\angle$ M) = m ($\angle$ RSM) = 30° (alternate angles)

$\therefore$ In Δ LSM :

m ($\angle$ LSM) = 180° – (50° + 30°) = 100°

$\therefore$ m ($\angle$ LSM) > m ($\angle$ M)

$\therefore$ ML > SL (Q.E.D.)

6 El-Kalyoubia

1

1 (c) 2 (c) 3 (b) 4 (b) 5 (c)

2

1 $\overline{BC}$ 2 90° 3 2 4 zero

3

[a] $\because$ $\angle$ ADB is an exterior angle of Δ ACD

$\therefore$ m ($\angle$ ADB) > m ($\angle$ C)

, in Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ In Δ ABD : m ($\angle$ ADB) > m ($\angle$ B)

$\therefore$ AB > AD (Q.E.D.)

[b] In Δ ABD : $\because$ AD = BD

$\therefore$ m ($\angle$ BAD) = m ($\angle$ B) = 35°

$\therefore$ m ($\angle$ ADC) = 35° + 35° = 70°

(an exterior angle of Δ ABD)

In Δ ACD : $\because$ AC = AD

$\therefore$ m ($\angle$ C) = m ($\angle$ ADC) = 70°

In Δ ABC :

$\therefore$ m ($\angle$ BAC) = 180° – (35° + 70°) = 75° (The req.)

4

[a] In Δ ABC :

$\because$ X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{BC}$

$\therefore$ XY = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 22 = 11 cm.

In Δ XBY : $\because$ m ($\angle$ XBY) = 90°

, $\overline{BD}$ is a median

$\therefore$ BD = $\frac{1}{2}$ XY = $\frac{1}{2}$ × 11 = 5.5 cm. (The req.)

[b] In Δ ABC : $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore$ DE = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 12 = 6 cm.

, $\because$ $\overline{CD}$ and $\overline{BE}$ are two medians

, $\overline{CD} \cap \overline{BE} = \{M\}$

$\therefore$ M is the point of intersection of the medians

$\therefore$ DM = $\frac{1}{2}$ MC = $\frac{1}{2}$ × 8 = 4 cm.

, EM = $\frac{1}{2}$ MB = $\frac{1}{2}$ × 6 = 3 cm.

$\therefore$ The perimeter of Δ MDE

= 6 + 4 + 3 = 13 cm. (The req.)

5

[a] $\overline{AD}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore$ m ($\angle$ B) + m ($\angle$ BAD) = 180° (interior angles on the same side of the transversal)

$\therefore$ m ($\angle$ B) = 180° – (70° + 35°) = 75°

In Δ ABC : $\therefore$ m ($\angle$ B) > m ($\angle$ BAC)

$\therefore$ AC > BC (Q.E.D.)

[b] $\because \overrightarrow{BE} \parallel \overline{AC}$, $\overleftrightarrow{BC}$ is a transversal to them

$\therefore m(\angle C) = m(\angle CBE)$ (alternate angles)

, $\because \overrightarrow{BE} \parallel \overline{AC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore m(\angle A) = m(\angle DBE)$ (corresponding angles)

, $\because m(\angle CBE) = m(\angle DBE)$

$\therefore m(\angle C) = m(\angle A)$ $\therefore AB = BC$

$\therefore \Delta ABC$ is an isosceles triangle (Q.E.D.)

7 El-Sharkia

1

1 (d) 2 (b) 3 (a) 4 (c) 5 (a)

2

1 2 , 8 2 1 : 2

3 is perpendicular to it. 4 6 cm.

3

[a] In ΔABC : $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore BC = 2\,DE = 2 \times 6 = 12$ cm.

, $\because \overline{BE}$ and $\overline{CD}$ are two medians

, $\overline{BE} \cap \overline{CD} = \{M\}$

$\therefore$ M is the point of intersection of the medians

$\therefore MB = \frac{2}{3}\,BE = \frac{2}{3} \times 9 = 6$ cm.

, $MC = 2\,MD = 2 \times 4 = 8$ cm.

$\therefore$ The perimeter of $\Delta BMC = 12 + 6 + 8 = 26$ cm.

(The req.)

[b] In ΔABD : $\because AD > AB$

$\therefore m(\angle ABD) > m(\angle ADB)$ (1)

, in ΔBCD : $\because CD > CB$

$\therefore m(\angle CBD) > m(\angle CDB)$ (2)

Adding (1) and (2) :

$\therefore m(\angle ABC) > m(\angle ADC)$ (Q.E.D.)

4

[a] In ΔABC : $\because AC = AB$

$\therefore m(\angle ACB) = m(\angle B) = \frac{180° - 50°}{2} = 65°$

, $\because \Delta ACD$ is an equilateral triangle

$\therefore m(\angle ACD) = 60°$ (1)

$\therefore m(\angle BCD) = 65° + 60° = 125°$ (The req.)

[b] $\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore m(\angle ACB) = m(\angle CAD) = 32°$

(alternate angles)

$\therefore m(\angle BAC) > m(\angle ACB)$

$\therefore BC > AB$ (Q.E.D.)

5

[a] $\because \Delta ABC$ is right-angled at B

, $m(\angle ACB) = 30°$

$\therefore AC = 2\,AB = 2 \times 5 = 10$ cm.

$\therefore$ In ΔADC :

$\because \overline{DE}$ is a median , $DE = \frac{1}{2}\,AC$

$\therefore m(\angle ADC) = 90°$ (Q.E.D.)

[b] In ΔABC : $m(\angle BAC) = 180° - (30° + 70°) = 80°$

, $\because \overrightarrow{AD}$ bisects $\angle BAC$

$\therefore m(\angle CAD) = \frac{1}{2}\,m(\angle BAC) = \frac{1}{2} \times 80° = 40°$

In ΔACD :

$\therefore m(\angle ADC) = 180° - (70° + 40°) = 70°$

$\therefore m(\angle C) = m(\angle ADC)$

$\therefore AD = AC$ (Q.E.D.)

8 El-Dakahlia

1

1 (a) 2 (b) 3 (c) 4 (a) 5 (c)

2

1 the hypotenuse

2 bisects 3 BC 4 2 , 10

3

[a] In ΔABC : $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore BC = 2\,DE = 2 \times 5 = 10$ cm.

, $\because$ M is the intersection point of the medians

$\therefore MB = 2\,ME = 2 \times 3 = 6$ cm.

, $MC = 2\,MD = 2 \times 4 = 8$ cm.

$\therefore$ The perimeter of $\Delta MBC = 10 + 6 + 8 = 24$ cm.

(The req.)

[b] In ΔABC : $\because BC < AC < AB$

$\therefore m(\angle A) < m(\angle B) < m(\angle C)$ (The req.)

4

[a] In Δ ACD : $\because$ X is the midpoint of $\overline{AD}$

, Y is the midpoint of $\overline{CD}$

$\therefore$ AC = 2 XY = 2 × 6 = 12 cm.

, in Δ ABC :

$\because$ m ($\angle$ ABC) = 90° , $\overline{BE}$ is a median

$\therefore$ BE = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 12 = 6 cm. (The req.)

[b] $\because$ $\overrightarrow{XL}$ // $\overline{YZ}$, $\overleftrightarrow{XZ}$ is a transversal to them

$\therefore$ m ($\angle$ Z) = m ($\angle$ LXZ) = 50° (alternate angles)

In Δ XYZ :

$\therefore$ m ($\angle$ YXZ) = 180° – (50° + 80°) = 50°

$\therefore$ m ($\angle$ Z) = m ($\angle$ YXZ)

$\therefore$ XY = YZ (Q.E.D.)

5

[a] In Δ ABC : $\because$ AB = AC , $\overline{AD} \perp \overline{BC}$

$\therefore$ BD = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 10 = 5 cm. (First req.)

In Δ ABD : $\because$ m ($\angle$ ADB) = 90°

$\therefore$ $(AD)^2 = (AB)^2 - (BD)^2 = 169 - 25 = 144$

$\therefore$ AD = $\sqrt{144}$ = 12 cm.

$\therefore$ The area of Δ ABC = $\frac{1}{2}$ × 10 × 12 = 60 cm^2.

(Second req.)

[b] In Δ ABC : $\because$ AB > AC

$\therefore$ m ($\angle$ ACB) > m ($\angle$ ABC)

, $\because$ $\overrightarrow{BM}$ bisects $\angle$ ABC

, $\overrightarrow{CM}$ bisects $\angle$ ACB

$\therefore$ $\frac{1}{2}$ m ($\angle$ ACB) > $\frac{1}{2}$ m ($\angle$ ABC)

$\therefore$ In Δ BMC : m ($\angle$ MCB) > m ($\angle$ MBC)

$\therefore$ MB > MC (Q.E.D.)

9 Suez

1

1 7 2 2 , 8 3 120°

4 bisects the base , is perpendicular to it.

2

1 (a) 2 (b) 3 (b) 4 (a) 5 (b)

3

[a] In Δ ABC : $\because$ m ($\angle$ A) = 180° – (40° + 80°) = 60°

$\therefore$ m ($\angle$ B) < m ($\angle$ A) < m ($\angle$ C)

$\therefore$ AC < BC < AB (The req.)

[b] In Δ ABC :

$\because$ X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

$\therefore$ XY = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 12 = 6 cm.

, $\because$ $\overline{CX}$ and $\overline{BY}$ are two medians

, $\overline{CX} \cap \overline{BY} = \{M\}$

$\therefore$ M is the point of intersection of the medians

$\therefore$ MX = $\frac{1}{2}$ MC = $\frac{1}{2}$ × 10 = 5 cm.

, MY = $\frac{1}{2}$ MB = $\frac{1}{2}$ × 4 = 2 cm.

$\therefore$ The perimeter of Δ MXY = 6 + 5 + 2 = 13 cm.

(The req.)

4

[a] In Δ ABC : $\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

$\therefore$ AC = 2 AB = 2 × 5 = 10 cm. (First req.)

In Δ ADC : $\because$ m ($\angle$ ADC) = 90°

, $\overline{DE}$ is a median

$\therefore$ DE = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 10 = 5 cm. (Second req.)

[b] $\because$ $\overline{AD}$ // $\overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore$ m ($\angle$ ACB) = m ($\angle$ CAD) = 30°

(alternate angles)

$\therefore$ In Δ ABC : m ($\angle$ BAC) > m ($\angle$ ACB)

$\therefore$ BC > AB (Q.E.D.)

5

[a] In Δ ABD : $\because$ AD > AB

$\therefore$ m ($\angle$ ABD) > m ($\angle$ ADB) (1)

In Δ BCD : $\because$ DC > BC

$\therefore$ m ($\angle$ CBD) > m ($\angle$ CDB) (2)

Adding (1) and (2) :

m ($\angle$ ABC) > m ($\angle$ ADC) (Q.E.D.)

[b] In Δ XYZ : $\because$ ZX = XY = YZ

$\therefore$ m ($\angle$ YXZ) = m ($\angle$ XZY) = m ($\angle$ Y) = 60°

(First req.)

In Δ XZL : $\because$ LZ = ZX

$\therefore$ m ($\angle$ LXZ) = m ($\angle$ XLZ) = $\frac{180° - 90°}{2}$ = 45°

$\therefore$ m ($\angle$ LXY) = 60° + 45° = 105° (Second req.)

10 El-Beheira

1

1 (c) 2 (b) 3 (d) 4 (a) 5 (b)

2

1 $\overline{DF}$ 2 one 3 12 4 half

3

[a] In Δ ABC : $\because$ X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

$\therefore$ AC = 2 XY = 2 × 5 = 10 cm.

, $\because$ $\overline{AY}$ and $\overline{CX}$ are two medians

, $\overline{AY} \cap \overline{CX} = \{M\}$

$\therefore$ M is the point of intersection of the medians

$\therefore$ AM = 2 YM = 2 × 3 = 6 cm.

$\therefore$ The perimeter of Δ MAC = 10 + 8 + 6 = 24 cm.

(The req.)

[b] $\because$ $\angle$ ADC is an exterior angle of Δ ABD

$\because$ m ($\angle$ ADC) = 30° + 40° = 70°

In Δ ACD : $\because$ AC = AD

$\therefore$ m ($\angle$ C) = m ($\angle$ ADC) = 70°

In Δ ABC :

$\therefore$ m ($\angle$ BAC) = 180° – (70° + 40°) = 70°

$\therefore$ m ($\angle$ C) = m ($\angle$ BAC)

$\therefore$ AB = CB (Q.E.D.)

4

[a] In Δ AMB : MA + MB > AB (1)

, in Δ BMC : MB + MC > BC (2)

, in Δ CMA : MC + MA > AC (3)

Adding (1) , (2) and (3) :

$\therefore$ 2 MA + 2 MB + 2 MC > AB + BC + AC

$\therefore$ 2 (MA + MB + MC) > The perimeter of the triangle ABC

$\therefore$ MA + MB + MC > $\frac{1}{2}$ the perimeter of the triangle ABC (Q.E.D.)

[b] In Δ XYZ : $\because$ m ($\angle$ Z) = 180° – (40° + 80°) = 60°

$\therefore$ m ($\angle$ X) < m ($\angle$ Z) < m ($\angle$ Y)

$\therefore$ YZ < XY < XZ (The req.)

5

[a] In Δ ABC : m ($\angle$ A) = 180° – (90° + 30°) = 60°

In Δ BCD : $\because$ BD = CD

$\therefore$ m ($\angle$ DBC) = m ($\angle$ C) = 30°

$\therefore$ m ($\angle$ ABD) = 90° – 30° = 60°

In Δ ABD : m ($\angle$ ADB) = 180° – (60° + 60°) = 60°

$\therefore$ m ($\angle$ A) = m ($\angle$ ABD) = m ($\angle$ ADB) = 60°

$\therefore$ Δ ABD is an equilateral triangle (Q.E.D.)

[b] $\because$ AB = AC $\therefore$ A $\in$ the axis of $\overline{BC}$

, $\because$ EB = EC

$\therefore$ E $\in$ the axis of $\overline{BC}$

$\therefore$ $\overleftrightarrow{AE}$ is the axis of $\overline{BC}$ (Q.E.D. 1)

, $\because$ D $\in$ $\overleftrightarrow{AE}$

$\therefore$ BD = DC (Q.E.D. 2)

11 Beni Suef

1

1 (c) 2 (c) 3 (c) 4 (c) 5 (a)

2

1 $\frac{1}{2}$ 2 congruent , isosceles

3 one 4 $\overline{BC}$

3

[a] In Δ ABC : $\because$ AB = BC

$\therefore$ m ($\angle$ BAC) = m ($\angle$ C) = $\frac{180° - 90°}{2}$ = 45°

, $\because$ Δ ABD is an equilateral triangle

$\therefore$ m ($\angle$ BAD) = 60°

$\therefore$ m ($\angle$ CAD) = 45° + 60° = 105° (The req.)

[b] In Δ ABC : $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore$ BC = 2 DE = 2 × 4 = 8 cm.

, $\because$ $\overline{CD}$ and $\overline{BE}$ are two medians

, $\overline{CD} \cap \overline{BE} = \{M\}$

$\therefore$ M is the point of intersection of the medians

$\therefore$ MC = 2 MD = 2 × 3 = 6 cm.

, MB = $\frac{2}{3}$ BE = $\frac{2}{3}$ × 6 = 4 cm.

$\therefore$ The perimeter of Δ BMC = 8 + 6 + 4 = 18 cm.

(The req.)

4

[a] $\because \overrightarrow{DE} \parallel \overline{BA}$, $\overleftrightarrow{AD}$ is a transversal to them

$\therefore m(\angle A) = m(\angle D) = 60°$ (alternate angles)

In ΔABC : $m(\angle ACB) = 180° - (60° + 60°) = 60°$

$\therefore m(\angle A) = m(\angle B) = m(\angle ACB)$

$\therefore \Delta ABC$ is an equilateral triangle (Q.E.D.)

[b] In ΔABC : $\because AB = AC$ and $\overline{AD} \perp \overline{BC}$

$\therefore DB = CD = 4$ cm.

, $m(\angle BAC) = 2\, m(\angle BAD) = 2 \times 30° = 60°$

(The req.)

5

[a] In ΔABC : $\because BC > BA$

$\therefore m(\angle BAC) > m(\angle ACB)$ (1)

, in ΔACD : $\because DA = DC$

$\therefore m(\angle CAD) = m(\angle ACD)$ (2)

Adding (1) and (2) :

$\therefore m(\angle BAD) > m(\angle BCD)$ (Q.E.D.)

[b] $\because m(\angle ABC) = 180° - 150° = 30°$

, $m(\angle ACB) = 180° - 110° = 70°$

$\therefore$ In ΔABC : $m(\angle A) = 180° - (30° + 70°) = 80°$

$\because m(\angle A) > m(\angle ACB)$

$\therefore CB > AB$ (Q.E.D.)

12 Aswan

1

1 (d) 2 (c) 3 (a) 4 (d) 5 (c)

2

1 is perpendicular to it. 2 10

3 the hypotenuse 4 $m(\angle C)$

3

[a] $\because \overrightarrow{AD} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them.

$\therefore m(\angle B) = m(\angle BAD) = 50°$ (alternate angles)

In ΔABC : $\because AB = BC$

$\therefore m(\angle BAC) = m(\angle C) = \frac{180° - 50°}{2} = 65°$

$\therefore m(\angle DAC) = 50° + 65° = 115°$ (The req.)

[b] In ΔABC : $\because m(\angle B) = 90°$, $m(\angle ACB) = 30°$

$\therefore AB = \frac{1}{2} AC$

, $\because AB = DE$ $\therefore DE = \frac{1}{2} AC$

, $\because \overline{DE}$ is a median in ΔADC

$\therefore m(\angle ADC) = 90°$ (Q.E.D.)

4

[a] In ΔABC :

$\because$ Y is the midpoint of $\overline{CB}$, X is the midpoint of $\overline{AB}$

$\therefore AC = 2\, XY = 2 \times 5 = 10$ cm.

, $\because \overline{AY}$ and $\overline{CX}$ are two medians

, $\overline{AY} \cap \overline{CX} = \{M\}$

$\therefore$ M is the point of intersection of the medians

$\therefore MA = 2\, MY = 2 \times 3 = 6$ cm.

, $MC = \frac{2}{3} CX = \frac{2}{3} \times 12 = 8$ cm.

$\therefore$ The perimeter of $\Delta AMC = 10 + 6 + 8 = 24$ cm.

(The req.)

[b] In ΔABC : $\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

$\therefore m(\angle ABD) = m(\angle ACE)$

(supplementary of two equal angles)

In $\Delta\Delta$ ABD , ACE :

$$\begin{cases} AB = AC \\ m(\angle ABD) = m(\angle ACE) \\ BD = EC \end{cases}$$

$\therefore \Delta ABD \equiv \Delta ACE$, then we deduce that :

$AD = AE$ (Q.E.D.)

5

[a] $\because XZ < XY < YZ$

$\therefore m(\angle Y) < m(\angle Z) < m(\angle X)$ (The req.)

[b] In ΔABC : $\because AC = AB$

$\therefore m(\angle ACB) = m(\angle ABC)$ (1)

, in ΔBCD : $\because BD > CD$

$\therefore m(\angle BCD) > m(\angle CBD)$ (2)

Adding (1) and (2) :

$\therefore m(\angle ACD) > m(\angle ABD)$ (Q.E.D.)