Proof 3

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Say whether the following is true or false and support your answer by a proof: For any integer n, $n^2 + n + 1$ is odd.

Proof

By mathematical induction.

Let $A(n)$ denote the statement "$n^2 + n + 1$ is odd".

For $n = 1$, (**Hypothesis step**)

$$(1)^2 + 1 + 1 = 3 \text{ (odd)}$$

Therefore, $A(1)$ is true.

Now, we want to deduce:

$$(n + 1)^2 + (n + 1) + 1$$

Assuming $A(n)$ is true, add $2(n + 1)$ to our initial expression. (**Induction step**)

$$\begin{aligned} n^2 + n + 1 + 2(n + 1) &= n^2 + n + 1 + 2n + 2 \\ &= n^2 + 1 + 2n + n + 1 + 1 \qquad \text{(algebra)} \\ &= (n + 1)^2 + (n + 1) + 1 \end{aligned}$$

Which is the expression for $A(n + 1)$.

This follows that $A(n)$ is valid.

Hence, **by the principle of mathematical induction**, the statement is proven true.

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