

Proof 9

nibir@nibirsan.org

Given an example of a family of intervals $A_n, n = 1, 2, 3 \dots$ such that $A_{n+1} \subset A_n$ for all n and $\bigcap_{n=1}^{\infty} A_n = \phi$. Prove that your example has the stated property.

Proof

The family is $(\frac{1}{n}, \frac{1}{n})$

We have two conditions to prove:

1. $A_{n+1} \subset A_n$
2. $\bigcap_{n=1}^{\infty} A_n = \phi$

For condition 1

What $A_{n+1} \subset A_n$ is saying, is that the $n + 1^{\text{th}}$ interval is the subset of the n^{th} interval in the family. Which suggests that the intervals are getting smaller and smaller.

Thus, we need to show that for any n , the $n + 1^{\text{th}}$ interval is smaller than the n^{th} interval but also a subset, to prove the first condition.

This is simple.

If n is taken arbitrarily, the interval $(\frac{1}{n+1}, \frac{1}{n+1})$ is smaller than $(\frac{1}{n}, \frac{1}{n})$, but a subset because $\frac{1}{n+1} < \frac{1}{n}$.

Similarly, the interval $(\frac{1}{n+2}, \frac{1}{n+2})$ is smaller than $(\frac{1}{n+1}, \frac{1}{n+1})$, but a subset.

Therefore, $\forall n [A_{n+1} \subset A_n]$. **This proves the first condition.**

For condition 2

We know that as $n \rightarrow \infty$, $\frac{1}{n} \rightarrow 0$. So $(\frac{1}{n}, \frac{1}{n}) \rightarrow (0, 0)$, which is the last possible interval in the family.

Since $(0, 0)$ does not contain any number, $(0, 0) = \phi$

Thus, the intersection is also ϕ , because $(0, 0)$ is the last interval in the family.

$$\bigcap_{n=1}^{\infty} A_n = \phi$$

This proves the second condition.

Therefore, the family $(\frac{1}{n}, \frac{1}{n})$ suffices both the conditions, which is what we wanted to prove.