Proof 7

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Prove that for any natural number n,

$$2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 2$$

Proof

By mathematical induction.

Let $A(n)$ denote the statement $2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 2$.

For $n = 1$, (**Hypothesis step**)

$$2 = 2^{1+1} - 2 = 2$$

Since LHS = RHS, $A(1)$ is true.

Now, we want to deduce:

$$2 + 2^2 + 2^3 + \cdots + 2^n + 2^{n+1} = 2^{n+2} - 2$$

Assuming $A(n)$ is true, add 2^{n+1} to our initial expression. (**Induction step**)

$$\begin{aligned} 2 + 2^2 + 2^3 + \cdots + 2^n + 2^{n+1} &= 2^{n+1} - 2 + 2^{n+1} \\ &= 2(2^{n+1}) - 2 \quad \text{(using algebra)} \\ &= 2^{n+2} - 2 \end{aligned}$$

Which is the expression for $A(n+1)$.

This follows that $A(n)$ is valid.

Hence, **by the principle of mathematical induction**, the statement is proven true. Isn't that lovely?