## Proof 5

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Prove that for any integer n, at least one of the integers n, n+2, n+4 is divisible by 3.

## Proof

By the method of cases.

There arises two cases: a) n is divisible by 3, and b) n is not divisible by 3.

- a) If 3|n, then claim is proven true.
- b) If not 3|n, then

$$n = 3q + 1$$
 (i) or  $n = 3q + 2$  (ii)

where  $q \in \mathbb{Z}$ . (By the definition of divisibility.)

Now, add 2 to both the sides of the equation (i)

$$n+2 = 3q + 1 + 2$$
  
=  $3q + 3$   
 $n+2 = 3(q+1) + 0$ 

Therefore, by the definition of divisibility, n+2 is divisible by 3.

Similarly, add 4 to both the sides of equation (ii)

$$n + 4 = 3q + 2 + 4$$
$$= 3q + 6$$
$$n + 4 = 3(q + 2) + 0$$

Therefore, by the definition of divisibility, n + 4 is divisible by 3.

Hence, in any case, at least one of the integers n, n+2, n+4 is divisible by 3, which is what we wanted to prove.