Proof 8

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Prove (from the definition of a limit of a sequence) that if the sequence $\{a_n\}_{n=1}^{\infty}$ tends too limit L as $n \to \infty$, then for any fixed number $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML.

Proof

Let ϵ be arbitrary.

if the sequence $\{a_n\}_{n=1}^{\infty}$ tends too limit L as $n \to \infty$, then:

$$|a_m - L| < \epsilon \qquad \text{(i)}$$

(by the definition of a limit).

For the sequence $\{Ma_n\}_{n=1}^{\infty}$,

Let $\epsilon' \geq M\epsilon$.

We want to deduce:

$$|Ma_m - ML| < \epsilon'$$

Multiply the expression (i) by M.

$$\begin{aligned} & M|a_m - L| < M\epsilon \\ \implies & -M\epsilon < Ma_m - ML < M\epsilon \\ \implies & -\epsilon' \leq Ma_m - ML \leq \epsilon' \end{aligned}$$

Which, by the definition of the limit, means $Ma_m \to ML$ as $n \to \infty$,