Proof 1

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Say whether the following is true or false and support your answer by a proof.

$$(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$$

Proof

By contradiction.

Let us assume the statement is **true**.

Therefore, $3m + 5n = 12$ must hold true.

$$
\begin{aligned}
3m + 5n &= 12 \\
\implies 3m &= 12 - 5n \\
\implies m &= \frac{12 - 5n}{3}
\end{aligned}
$$

For any natural multiple of n, m would result in a rational or an integer.

But $m \in \mathbb{N}$. **Contradiction.**

Hence, our initial assumption must be false and consequently the statement is also false.

Q.E.D.