Proof 5

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Prove that for any integer n, at least one of the integers $n, n+2, n+4$ is divisible by 3.

Proof

By the method of cases.

There arises two cases: a) n is divisible by 3, and b) n is not divisible by 3.

a) If $3|n$, then claim is proven true.

b) If not $3|n$, then

$$n = 3q + 1 \text{ (i)} \quad \text{or} \quad n = 3q + 2 \text{ (ii)}$$

where $q \in \mathbb{Z}$. (By the definition of divisibility.)

Now, add 2 to both the sides of the equation (i)

$$\begin{aligned} n + 2 &= 3q + 1 + 2 \\ &= 3q + 3 \\ n + 2 &= 3(q + 1) + 0 \end{aligned}$$

Therefore, by the definition of divisibility, $n + 2$ **is divisible by 3.**

Similarly, add 4 to both the sides of equation (ii)

$$\begin{aligned} n + 4 &= 3q + 2 + 4 \\ &= 3q + 6 \\ n + 4 &= 3(q + 2) + 0 \end{aligned}$$

Therefore, by the definition of divisibility, $n + 4$ **is divisible by 3.**

Hence, in any case, at least one of the integers $n, n+2, n+4$ is divisible by 3, which is what we wanted to prove.