

## Proof 3

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Say whether the following is true or false and support your answer by a proof.

For any integer  $n$ ,  $n^2 + n + 1$  is odd.

### Proof

By mathematical induction.

Let  $A(n)$  denote the statement “ $n^2 + n + 1$  is odd”.

For  $n = 1$ , (**Hypothesis step**)

$$(1)^2 + 1 + 1 = 3 \text{ (odd)}$$

Therefore,  $A(1)$  is true.

Now, we want to deduce:

$$(n+1)^2 + (n+1) + 1$$

Assuming  $A(n)$  is true, add  $2(n+1)$  to the expression (**Induction step**)

$$\begin{aligned} n^2 + n + 1 + 2(n+1) &= n^2 + n + 1 + 2n + 2 \\ &= n^2 + 1 + 2n + n + 1 + 1 && \text{(algebra)} \\ &= (n+1)^2 + (n+1) + 1 \end{aligned}$$

Which is the expression for  $A(n+1)$ .

This follows that  $A(n)$  is valid.

Hence, by the principle of mathematical induction, the statement is proven true.