Proof 4

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Prove that every odd natural number is of one of the forms $4n + 1$ or $4n + 3$, where n is an integer.

Proof

Let n be any integer.

Now, every integer $4n$ is even $\because 4n = 2(2n)$. Therefore the integer $4n + 1$ must be **odd**.

Again, every integer $4n + 2$ is even $\because 2(2n + 1)$. Therefore the integer $4n + 3$ must be **odd**.

There is no other permutation available.

Because the above conclusion is true for integers, it is also true for the natural numbers since $\mathbb{N} \subset \mathbb{Z}$.

Hence, every odd natural number is in the form $4n + 1$ or $4n + 3$.