Proof 8

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Prove (from the definition of a limit of a sequence) that if the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$, then for any fixed number $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML.

Proof

Let ϵ be arbitrary.

If the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$, then:

$$|a_m - L| < \epsilon \qquad \text{(i)}$$

(by the definition of a limit of a sequence).

Now, for the sequence $\{Ma_n\}_{n=1}^{\infty}$, let $\epsilon' \geq M\epsilon$.

We want to deduce:

$$|Ma_m - ML| < \epsilon'$$

So, multiply both sides of the inequality (i) by M.

$$\begin{aligned} & M|a_m - L| < M\epsilon \\ \implies & -M\epsilon < Ma_m - ML < M\epsilon \qquad \text{(using algebra)} \\ \implies & -\epsilon' \leq Ma_m - ML \leq \epsilon' \end{aligned}$$

Which is what we wanted to deduce.

And which, by the definition of the limit, means $Ma_m \to ML$ as $n \to \infty$ for the sequence $\{Ma_n\}_{n=1}^{\infty}$,

Hence, the statement is true. ■