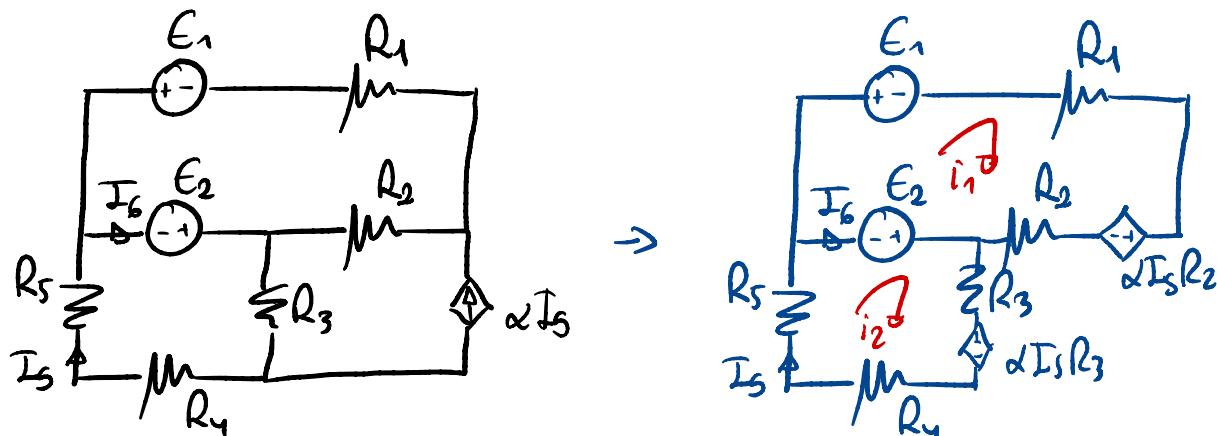


Problema 1  $\mathcal{I}_6$ , mallas  $\oplus \alpha \mathcal{I}_6 \oplus \beta U_5$



$$\textcircled{1} \quad -E_1 - \alpha I_s R_2 - E_2 = i_1 (R_1 + R_2) \xrightarrow{\text{sum} = i_1} -E_1 - E_2 = i_1 (R_1 + R_2) + i_2 \alpha R_2$$

$$\textcircled{2} \quad E_2 - \alpha I_s R_3 = i_2 (R_3 + R_4 + R_5) \xrightarrow{\text{sum} = i_2} E_2 = i_2 (R_3(1+\alpha) + R_4 + R_5)$$

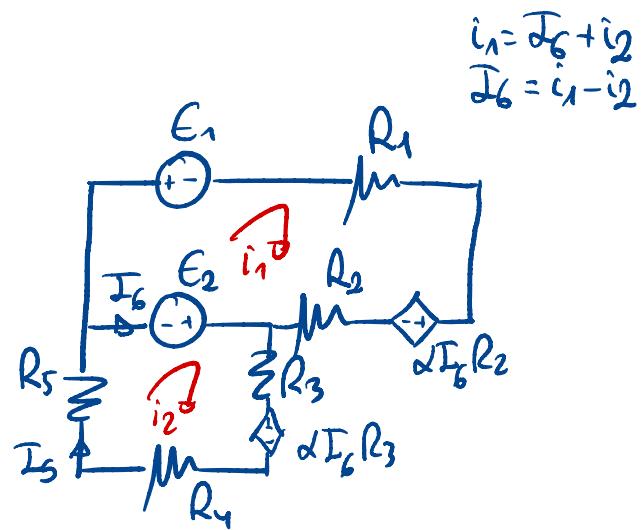
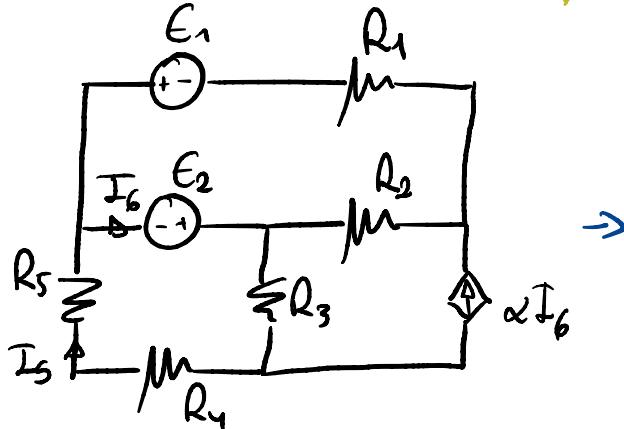
$$\hat{e}_S = Z_m \hat{i}_S \rightarrow \begin{pmatrix} -E_1 - E_2 \\ E_2 \end{pmatrix} = \begin{pmatrix} R_1 + R_2 & \alpha R_2 \\ 0 & R_3(1+\alpha) + R_4 + R_5 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$$

$$i_1 = \frac{\begin{vmatrix} -(E_1 + E_2) & \alpha R_2 \\ G_2 & R_3(1+\alpha) + R_4 + R_5 \end{vmatrix}}{\begin{vmatrix} R_1 + R_2 & 0 \\ R_1 + R_2 & R_3(1+\alpha) + R_4 + R_5 \end{vmatrix}} = \frac{-(E_1 + E_2)(R_3(1+\alpha) + R_4 + R_5) - E_2 \alpha R_2}{(R_1 + R_2)(R_3(1+\alpha) + R_4 + R_5)}$$

$$i_2 = \frac{\begin{vmatrix} R_1 + R_2 & -(E_1 + E_2) \\ 0 & G_2 \end{vmatrix}}{\begin{vmatrix} R_1 + R_2 & 0 \\ R_1 + R_2 & R_3(1+\alpha) + R_4 + R_5 \end{vmatrix}} = \frac{E_2(R_1 + R_2)}{(R_1 + R_2)(R_3(1+\alpha) + R_4 + R_5)}$$

$$\begin{aligned} \mathcal{I}_5 &= i_1 + \alpha \mathcal{I}_6 \Rightarrow \mathcal{I}_6 = \mathcal{I}_5 - i_1 = i_2 - i_1 = \\ &\frac{E_2(R_1 + R_2) + (E_1 + E_2)(R_3(1+\alpha) + R_4 + R_5) + E_2 \alpha R_2}{(R_1 + R_2)(R_3(1+\alpha) + R_4 + R_5)} \end{aligned}$$

### Problema 1. Añadido 1 $\oplus \alpha I_6$

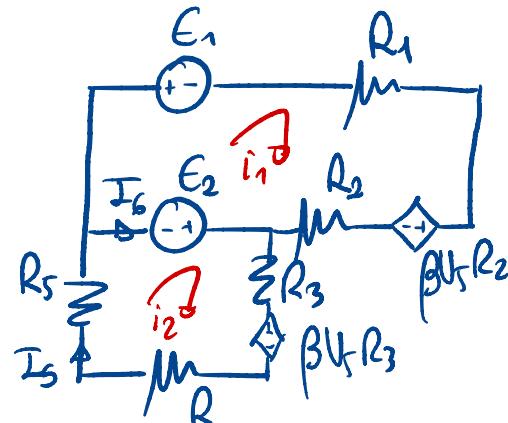
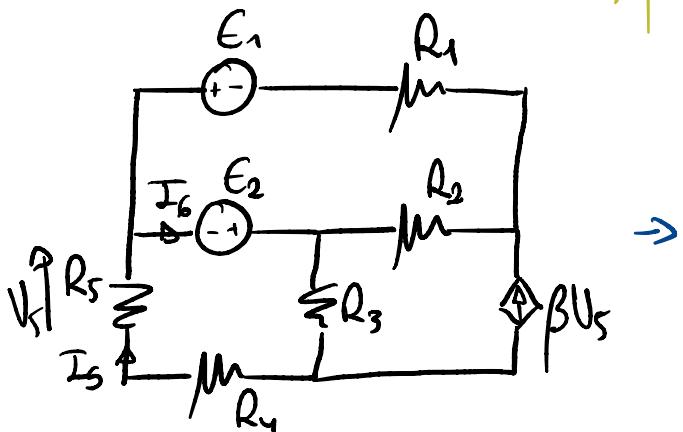


$$i_1 \rightarrow -E_1 - \alpha I_6 R_2 - E_2 = i_1 (R_1 + R_2) \xrightarrow{I_6 = i_1 - i_2} -E_1 - E_2 = i_1 (R_1 + R_2(1+\alpha)) - i_2 \alpha R_2$$

$$i_2 \rightarrow E_2 - \alpha I_6 R_3 = i_2 (R_3 + R_4 + R_5) \xrightarrow{I_6 = i_1 - i_2} E_2 = i_2 (R_3(1-\alpha) + R_4 + R_5) + i_1 \alpha R_3$$

$$\hat{e}_S = Z_m \hat{i}_S \Rightarrow \begin{pmatrix} -(E_1 + E_2) \\ E_2 \end{pmatrix} = \begin{pmatrix} R_1 + R_2(1+\alpha) & -\alpha R_2 \\ \alpha R_3 & R_3(1+\alpha) + R_4 + R_5 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$$

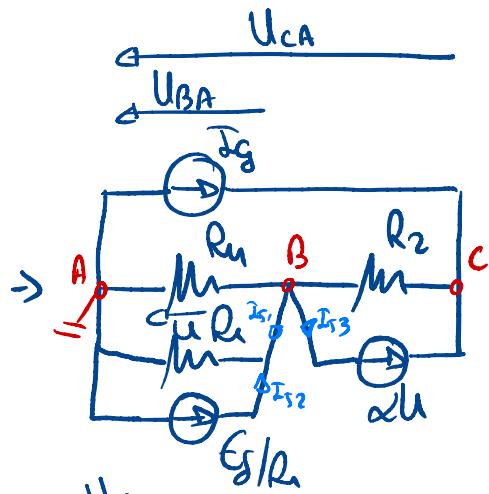
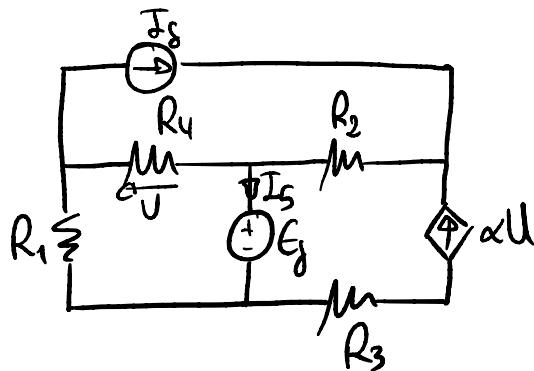
### Problema 1. Añadido 2 $\oplus \beta U_S$



$$i_1 \rightarrow -E_1 - \beta U_f R_2 - E_2 = i_1 (R_1 + R_2) \xrightarrow{U_f = i_2 R_5} -E_1 - E_2 = i_1 (R_1 + R_2) + i_2 \beta R_2 R_5$$

$$i_2 \rightarrow E_2 - \beta U_f R_3 = i_2 (R_3 + R_4 + R_5) \xrightarrow{U_f = i_2 R_5} E_2 = i_2 (R_3(1+\beta R_5) + R_4 + R_5)$$

Problema 2.  $I_S$ , nudos  $\oplus \alpha U$



$$B. \frac{E_S}{R_1} - \alpha U = U_{BA} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) - U_{CA} \frac{1}{R_2} \xrightarrow{U=U_{BA}} \frac{E_S}{R_1} = U_{BA} \left( \alpha + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) - U_{CA} \frac{1}{R_2}$$

$$C. \alpha U + I_S = U_{CA} \frac{1}{R_2} - U_{BA} \frac{1}{R_2} \xrightarrow{U=U_{BA}} I_S = U_{BA} \left( -\frac{1}{R_2} - \alpha \right) + U_{CA} \frac{1}{R_2}$$

$$i_S^1 = Y_m e_S \rightarrow \begin{pmatrix} \frac{E_S}{R_1} \\ I_S \end{pmatrix} = \begin{pmatrix} \alpha + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} \\ -\frac{1}{R_2} - \alpha & \frac{1}{R_2} \end{pmatrix} \begin{pmatrix} U_{BA} \\ U_{CA} \end{pmatrix}$$

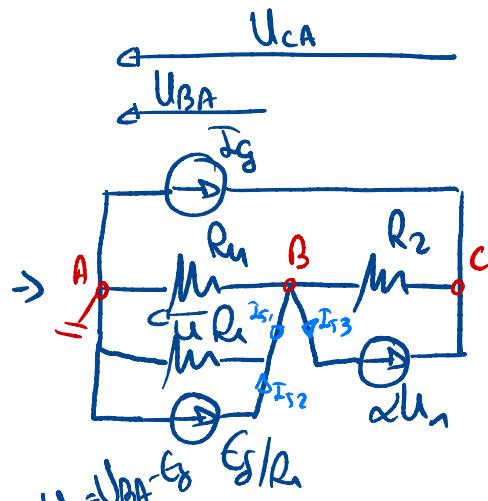
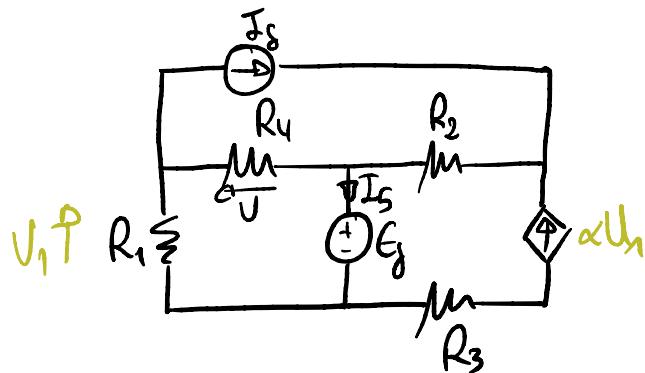
$$U_{BA} = \frac{\begin{vmatrix} \frac{E_S}{R_1} & -\frac{1}{R_2} \\ I_S & \frac{1}{R_2} \end{vmatrix}}{\begin{vmatrix} \alpha + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} \\ -\left(\frac{1}{R_2} + \alpha\right) & \frac{1}{R_2} \end{vmatrix}} = \frac{\frac{E_S}{R_1 R_2} + \frac{I_S}{R_2}}{\left(\alpha + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)\left(\frac{1}{R_2}\right) - \frac{1}{R_2}\left(\frac{1}{R_2} + \alpha\right)} =$$

$$= \frac{\frac{E_S}{R_1} + I_S}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{(E_S + I_S R_1) R_1}{R_1 + R_2}$$

$$I_S = I_{S1} + I_{S3} - I_{S2} = \frac{U_{BA}}{R_1} + \alpha U_{BA} - \frac{E_S}{R_1} = U_{BA} \left( \frac{1}{R_1} + \alpha \right) - E_S \frac{1}{R_1} =$$

$$= \frac{(E_S + I_S R_1) R_1}{R_1 + R_2} \left( \frac{1}{R_1} + \alpha \right) - E_S \frac{1}{R_1}$$

Problema 2. Añadido  $\triangleleft \alpha U_1$



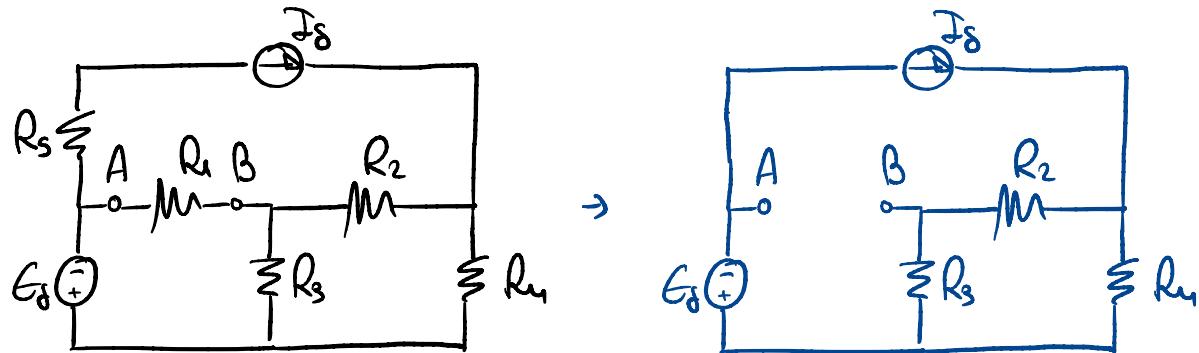
$$B: \frac{E_s}{R_1} - \alpha U_1 = U_{BA} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) - U_{CA} \frac{1}{R_2} \xrightarrow{U_n = U_{BA} - E_s / R_1} E_s \left( \frac{1}{R_1} + \alpha \right) = U_{BA} \left( \alpha + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) - U_{CA} \frac{1}{R_2}$$

$$C: \alpha U_1 + I_s = U_{CA} \frac{1}{R_2} - U_{BA} \frac{1}{R_2} \xrightarrow{U_n = U_{BA} - E_s} I_s - \alpha E_s = U_{BA} \left( -\frac{1}{R_2} - \alpha \right) + U_{CA} \frac{1}{R_2}$$

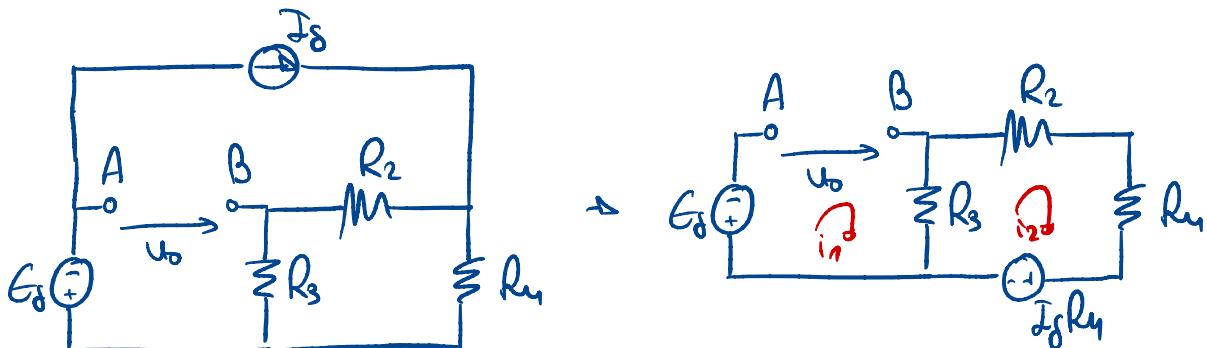
$$\hat{i}_f = Y_m \hat{e}_s = \begin{pmatrix} \alpha + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} \\ -\left(\frac{1}{R_2} + \alpha\right) & \frac{1}{R_2} \end{pmatrix} \begin{pmatrix} \frac{E_s}{R_1} + \alpha \\ I_s - \alpha E_s \end{pmatrix}$$

### Problema 3. Thévenin, Norton

Fuente tensión independiente  $\rightarrow$  circuito abierto  
 Fuente intensidad independiente  $\rightarrow$  cortocircuito



- Tensión a circuito abierto  $U_o?$   $I_o = 0$



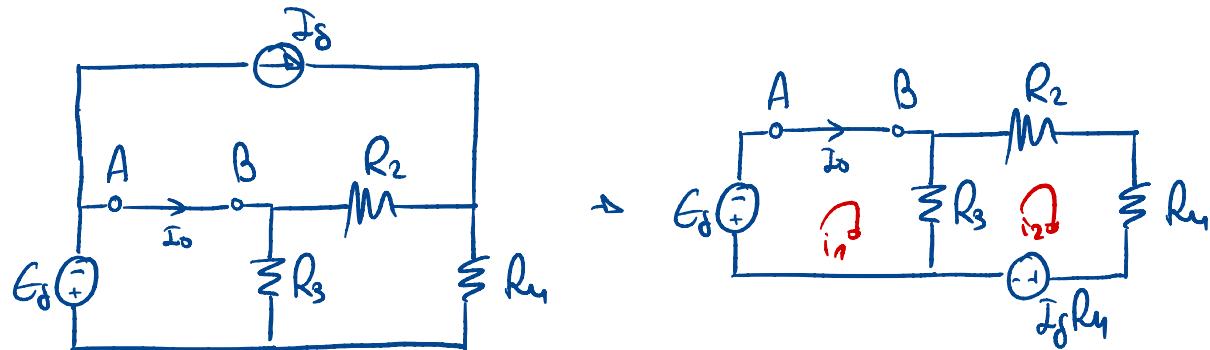
$$-E_s = i_1 R_3 - i_2 R_3 \xrightarrow{i_1=0} -E_s = -i_2 R_3$$

$$-I_s R_L = i_2 (R_2 + R_3 + R_L) - i_1 R_3 \xrightarrow{i_1=0} -I_s R_L = i_2 (R_2 + R_3 + R_L)$$

$$i_2 = \frac{-I_s R_L}{R_2 + R_3 + R_L}$$

$$U_o = +i_2 R_3 - E_s = \frac{-I_s R_1 R_3}{R_2 + R_3 + R_L} - E_s = \frac{-E_s (R_2 + R_3 + R_L) - I_s R_L R_3}{R_2 + R_3 + R_L}$$

- Intensidad de cortocircuito  $I_0$ ?  $U_0=0$



$$i_1 - E_s = i_1 R_3 - i_2 R_3$$

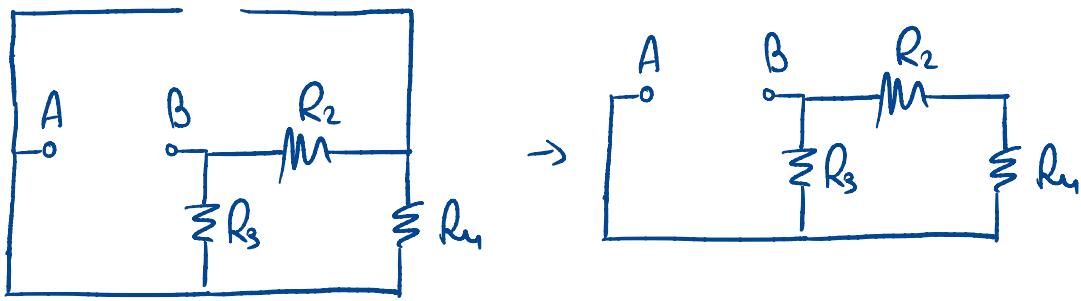
$$i_2 - I_0 R_4 = i_2 (R_2 + R_3 + R_4) - i_1 R_3$$

$$i_1 = \frac{-E_s \quad -R_3}{\begin{vmatrix} -I_0 R_4 & R_2 + R_3 + R_4 \\ R_3 & -R_3 \\ -R_3 & R_2 + R_3 + R_4 \end{vmatrix}} = \frac{-E_s (R_2 + R_3 + R_4) - I_0 R_4 R_3}{R_3 (R_2 + R_3 + R_4) - R_3^2} =$$

$$= \frac{-E_s (R_2 + R_3 + R_4) - I_0 R_4 R_3}{R_3 (R_2 + R_4)} = I_0$$

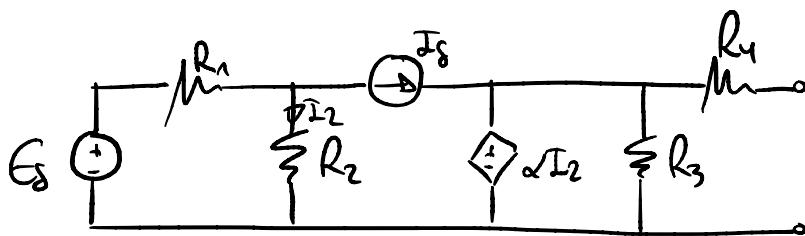
- Resistencia equivalente  $R_{eq}$ ?

$$R_{eq} = \frac{U_0}{I_0} = \frac{\cancel{-E_s (R_2 + R_3 + R_4) - I_0 R_4 R_3}}{\cancel{R_2 + R_3 + R_4}} = \frac{R_3 (R_2 + R_4)}{\cancel{R_3 (R_2 + R_4)}} = \frac{R_3 (R_2 + R_4)}{R_2 + R_3 + R_4}$$

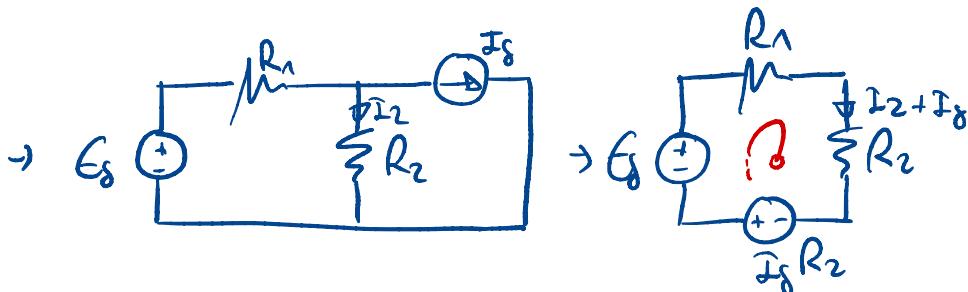
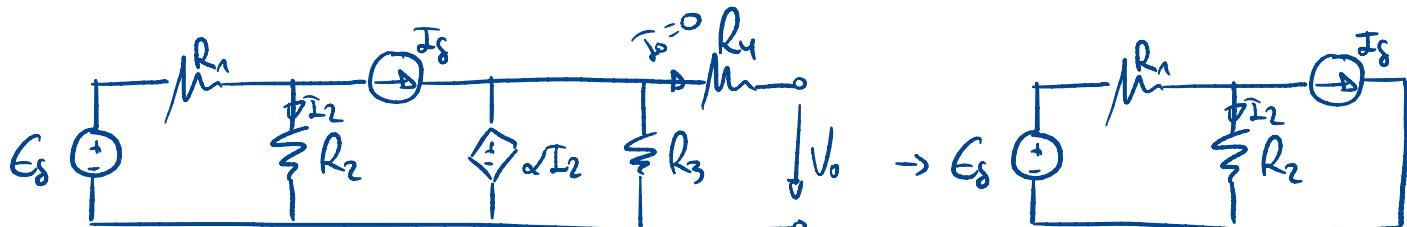


$$R_{eq} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_2+R_4}} = \frac{1}{\frac{R_2+R_4+R_3}{R_3(R_2+R_4)}} = \frac{R_3(R_2+R_4)}{R_2+R_4+R_3}$$

## Problema 4 Thévenin, Norton



- Tensión circuito abierto  $U_o$ ?  $I_o = 0$



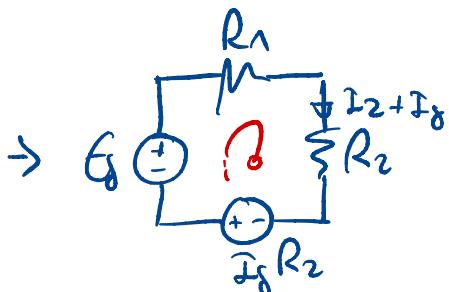
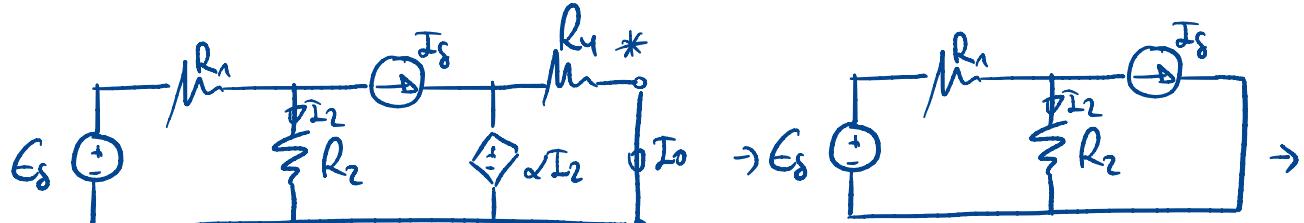
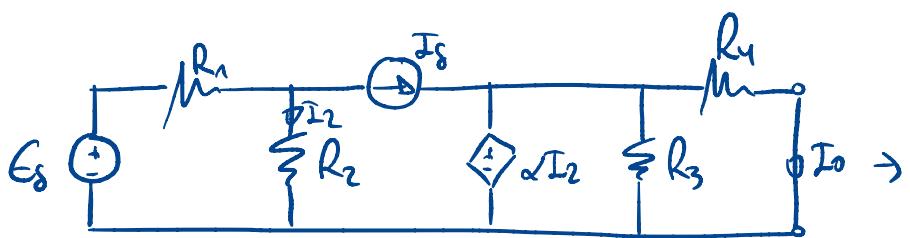
$$\text{?} \quad E_8 + I_8 R_2 = i(R_1 + R_2) \rightarrow i = \frac{E_8 + I_8 R_2}{R_1 + R_2}$$

$$i = I_2 + I_8 \rightarrow I_2 = i - I_8 = \frac{E_8 + I_8 R_2}{R_1 + R_2} - I_8 = \frac{E_8 + I_8 R_2 - I_8 (R_1 + R_2)}{R_1 + R_2} =$$

$$= \frac{E_8 - I_8 R_2}{R_1 + R_2}$$

$$U_o = \alpha \frac{E_8 - I_8 R_2}{R_1 + R_2}$$

- Intensidad cortocircuito  $I_o$ ?  $U_o = 0$



$$\text{? } E_s + I_s R_2 = i(R_1 + R_2) \rightarrow i = \frac{E_s + I_s R_2}{R_1 + R_2}$$

$$i = I_2 + I_s \rightarrow I_2 = i - I_s = \frac{E_s + I_s R_2}{R_1 + R_2} - I_s = \frac{E_s + I_s R_2 - I_s (R_1 + R_2)}{R_1 + R_2} =$$

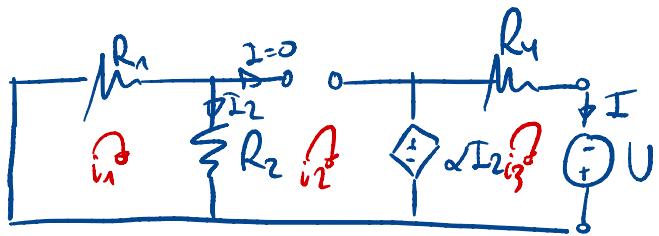
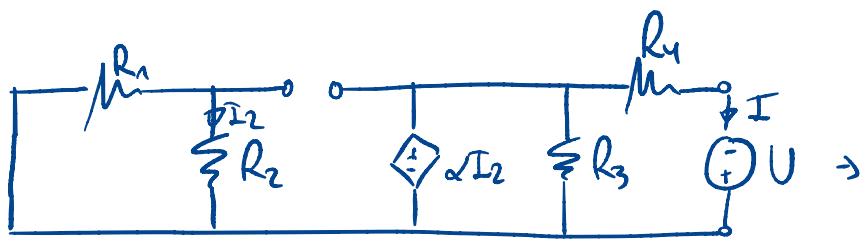
$$= \frac{E_s - I_s R_2}{R_1 + R_2}$$

$$* 0 = -\alpha I_2 + R_4 I_o \rightarrow R_4 I_o = \alpha I_2 \rightarrow I_o = \frac{\alpha I_2}{R_4}$$

$$I_o = \frac{\alpha(E_s - I_s R_2)}{R_4(R_1 + R_2)}$$

- Resistencia equivalente  $R_{eq}$ ?

$$R_{eq} = \frac{U_o}{I_o} = \frac{\cancel{\alpha} \frac{E_s - I_s R_2}{R_1 + R_2}}{\cancel{\alpha} \frac{(E_s - I_s R_2)}{R_4(R_1 + R_2)}} = R_4$$



$$i_1 \rightarrow 0 = i_1(R_1 + R_2) - i_2 R_2^{\circ} \rightarrow i_1 = 0$$

$$i_2 \rightarrow \alpha I_2 = i_2 R_2^{\circ} - i_3 R_2^{\circ} \rightarrow I_2 = 0$$

$$i_3 \rightarrow U + \alpha I_2^{\circ} = i_3 R_4 \rightarrow \frac{U}{i_3} = R_4 \rightarrow R_{eq} = R_4$$