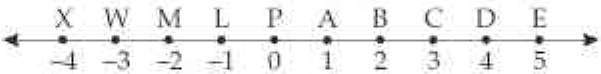
1 Basic Concepts of Geometry

Points to Remember:

❑ Distance between two points on a number line:

Distance between two distinct points can be calculated by subtracting smaller coordinate from the bigger coordinate.



Example:

(1) Co-ordinate of point A is 1

Co-ordinate of point D is 4

$4 > 1$

$d(A, D)$

= Greater co-ordinate – Smaller co-ordinate

= $4 - 1$

$\boxed{\therefore d(A, D) = 3 \text{ Units}}$

(2) Co-ordinate of point M is – 2

Co-ordinate of point X is – 4

$-2 > -4$

$d(X, M)$

= Greater co-ordinate – Smaller co-ordinate

= $-2 - (-4) = -2 + 4$

$\boxed{\therefore d(X, M) = 2 \text{ Units}}$

• Point:

(1) Point is an undefined term in geometry.

(2) We say point is a dot made by a sharp pencil on a paper.

(3) It is represented with the help of a single capital alphabet.

The point A is shown as:

• A

Note:

(i) Point is not a figure in geometry, it just shows the position.

(ii) Point does not have length, breadth and height.

• Line:

(1) Line is an undefined term in geometry.

(2) We say line is a set of points which can be extended in both directions infinitely and indicated by arrow heads.

(3) We can name the line in two ways.

(i) By using names of any two distinct points on a line. (Example: Line PQ or line PR or line RQ)

(ii) By using single small alphabet.

(Example: Line l)

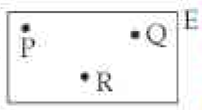
• Plane:

(1) Plane is an undefined term in geometry.

(2) We say 'plane' is a flat surface which extends infinitely in all directions.

(3) We can name a plane in two ways:

(i) By using names of any three non-collinear points in a plane. (Example: Plane PQR)



(ii) By using a single capital letter. (Example : Plane E)

• Collinear and Non-collinear points:

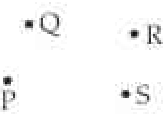
If there exist a line passing through three or more distinct points, then the points are called collinear points, otherwise they are called 'non-collinear points.'

Example:

(1)

In above figure, points A, B, C, D are collinear points.

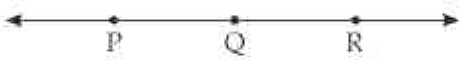
(2)



In above figure, points P, Q, R, S are non-collinear points.

❑ Distance and Betweenness:

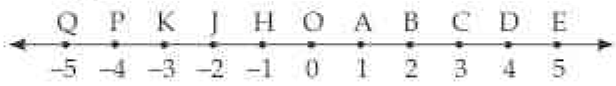
• If P, Q and R are three collinear points and if $d(P, Q) + d(Q, R) = d(P, R)$ then point Q is between P and R.



MASTER KEY QUESTION SET - 1

PRACTICE SET - 1.1 *(Textbook Page No. 5)*

(1) Find the distances with the help of the number line given below:



(i) d**(B, E)**

Solution:

The co-ordinate of point B is 2.

The co-ordinate of point E is 5.

$5 > 2$

d(B, E)

= Greater co-ordinate – smaller co-ordinate

= $5 - 2$

= 3

$\boxed{\therefore d(B, E) = 3 \text{ units}}$

(ii) d**(J, A)**

Solution:

The co-ordinate of point J is –2.

The co-ordinate of point A is 1.

$1 > -2$

d(J, A)

= Greater co-ordinate – smaller co-ordinate

= $1 - (-2)$

= $1 + 2$

= 3

$\boxed{\therefore d(J, A) = 3 \text{ units}}$

(iii) d**(P, C)**

Solution:

The co-ordinate of point P is –4.

The co-ordinate of point C is 3.

$3 > -4$

d(P, C)

= Greater co-ordinate – smaller co-ordinate

= $3 - (-4)$

= $3 + 4$

= 7

$\boxed{\therefore d(P, C) = 7 \text{ units}}$

(iv) d**(J, H)**

Solution:

The co-ordinate of point J is –2.

The co-ordinate of point H is –1.

$-1 > -2$

d(J, H)

= Greater co-ordinate – smaller co-ordinate

= $-1 - (-2)$

= $-1 + 2$

= 1

$\boxed{\therefore d(J, H) = 1 \text{ unit}}$

(v) d**(K, O)**

Solution:

The co-ordinate of point K is –3.

The co-ordinate of point O is 0.

$0 > -3$

d(K, O)

= Greater co-ordinate – smaller co-ordinate

= $0 - (-3)$

= $0 + 3$

= 3

$\boxed{\therefore d(K, O) = 3 \text{ units}}$

(vi) d**(O, E)**

Solution:

The co-ordinate of point O is 0.

The co-ordinate of point E is 5.

$5 > 0$

d(O, E)

= Greater co-ordinate – smaller co-ordinate

= $5 - 0$

= 5

$\boxed{\therefore d(O, E) = 5 \text{ units}}$

(vii) d**(P, J)**

Solution:

The co-ordinate of point P is – 4.

The co-ordinate of point J is – 2.

$-2 > -4$

d(P, J)

= Greater co-ordinate – smaller co-ordinate

= $-2 - (-4)$

$= \quad -2 + 4$

$= \quad 2$

$\boxed{\therefore\ d(P, J) = 2 \text{ units}}$

(viii) d**(Q, B)**

Solution:

The co-ordinate of point Q is –5.

The co-ordinate of point B is 2.

$2 > -5$

$d(Q, B)$

= Greater co-ordinate – smaller co-ordinate

$= \quad 2 - (-5)$

$= \quad 2 + 5$

$= \quad 7$

$\boxed{\therefore\ d(Q, B) = 7 \text{ units}}$

(2) If the co-ordinates of A is x and that of B is y, find d(A, B).

(i) $x = 1, y = 7$

Solution:

The co-ordinate of point A is 1.

The co-ordinate of point B is 7.

$7 > 1$

$d(A, B)$

= Greater co-ordinate – smaller co-ordinate

$= \quad 7 - 1$

$= \quad 6$

$\boxed{\therefore\ d(A, B) = 6 \text{ units}}$

(ii) $x = 6, y = -2$

Solution:

The co-ordinate of point A is 6.

The co-ordinate of point B is –2.

$6 > -2$

$d(A, B)$

= Greater co-ordinate – smaller co-ordinate

$= \quad 6 - (-2)$

$= \quad 6 + 2$

$= \quad 8$

$\boxed{\therefore\ d(A, B) = 8 \text{ units}}$

(iii) $x = -3, y = 7$

Solution:

The co-ordinate of point A is –3.

The co-ordinate of point B is 7.

$7 > -3$

$d(A, B)$

= Greater co-ordinate – smaller co-ordinate

$= \quad 7 - (-3)$

$= \quad 7 + 3$

$= \quad 10$

$\boxed{\therefore\ d(A, B) = 10 \text{ units}}$

(iv) $x = -4, y = -5$

Solution:

The co-ordinate of point A is –4.

The co-ordinate of point B is –5.

$-4 > -5$

$d(A, B)$

= Greater co-ordinate – smaller co-ordinate

$= \quad -4 - (-5)$

$= \quad -4 + 5$

$= \quad 1$

$\boxed{\therefore\ d(A, B) = 1 \text{ unit}}$

(v) $x = -3, y = -6$

Solution:

The co-ordinate of point A is –3.

The co-ordinate of point B is –6.

$-3 > -6$

$d(A, B)$

= Greater co-ordinate – smaller co-ordinate

$= \quad -3 - (-6)$

$= \quad -3 + 6$

$= \quad 3$

$\boxed{\therefore\ d(A, B) = 3 \text{ units}}$

(vi) $x = 4, y = -8$

Solution:

The co-ordinate of point A is 4.

The co-ordinate of point B is – 8.

$4 > -8$

$d(A, B)$
$=$ Greater co-ordinate – smaller co-ordinate
$= 4 - (-8)$
$= 4 + 8$
$= 12$

$\boxed{\therefore d(A, B) = 12 \text{ units}}$

(3) From the information given below, find which of the point is between the other two. If the points are not collinear, state so.

(i) $d(P, R) = 7, d(P, Q) = 10, d(Q, R) = 3.$

Solution:

$d(P, Q) = 10$ ..... (i)
$d(P, R) + d(Q, R) = 7 + 3$
$\therefore d(P, R) + d(Q, R) = 10$ ..... (ii)
$\therefore d(P, Q) = d(P, R) + d(Q, R)$ [From (i) and (ii)]
$\therefore$ Points P, Q and R are collinear points.
$\therefore$ Relation of betweenness exists.
P - R - Q.

(ii) $d(R, S) = 8, d(S, T) = 6, d(R, T) = 4.$

Solution:

$\therefore d(R, S) = 8$ ..... (i)
$d(S, T) + d(R, T) = 6 + 4$
$\therefore d(S, T) + d(R, T) = 10$ ..... (ii)
$\therefore d(R, S) \neq d(S, T) + d(R, T)$ [From (i) and (ii)]
$\therefore$ Points R, S and T are non-collinear points.
$\therefore$ Relation of betweenness does not exist.

(iii) $d(A, B) = 16, d(C, A) = 9, d(B, C) = 7.$

Solution:

$\therefore d(A, B) = 16$ ..... (i)
$d(C, A) + d(B, C) = 9 + 7$
$\therefore d(C, A) + d(B, C) = 16$ ..... (ii)
$\therefore d(A, B) = d(C, A) + d(B, C)$ [From (i) and (ii)]
$\therefore$ Points A, B and C are collinear points.
$\therefore$ Relation of betweenness exists.
A - C - B.

(iv) $d(L, M) = 11, d(M, N) = 12, d(N, L) = 8.$

Solution:

$\therefore d(M, N) = 12$ ..... (i)
$d(L, M) + d(N, L) = 11 + 8$
$\therefore d(L, M) + d(N, L) = 19$ ..... (ii)
$\therefore d(M, N) \neq d(L, M) + d(N, L)$
[From (i) and (ii)]
$\therefore$ Points L, M and N are noncollinear points.
$\therefore$ Relation of betweenness does not exist.

(v) $d(X, Y) = 15, d(Y, Z) = 7, d(X, Z) = 8.$

Solution:

$\therefore d(X, Y) = 15$ ..... (i)
$d(Y, Z) + d(X, Z) = 7 + 8$
$\therefore d(Y, Z) + d(X, Z) = 15$ ..... (ii)
$\therefore d(X, Y) = d(Y, Z) + d(X, Z)$
[From (i) and (ii)]
$\therefore$ Points X, Y and Z are collinear points.
$\therefore$ Relation of betweenness exists.
X - Z - Y.

(vi) $d(D, E) = 5, d(E, F) = 8, d(D, F) = 6.$

Solution:

$\therefore d(E, F) = 8$ ..... (i)
$d(D, E) + d(D, F) = 5 + 6$
$\therefore d(D, E) + d(D, F) = 11$ ..... (ii)
$\therefore d(E, F) \neq d(D, E) + d(D, F)$ [From (i) and (ii)]
$\therefore$ Points D, E and F are noncollinear points.
$\therefore$ Relation of betweenness does not exist.

(4) On a number line, points A, B and C are such that $d(A, C) = 10, d(C, B) = 8$ Find $d(A, B)$ considering all possibilities.

Solution:

Case (1) A - B - C
$\therefore d(A, C) = d(A, B) + d(B, C)$
$\therefore 10 = d(A, B) + 8.$
$\therefore 10 - 8 = d(A, B)$
$\therefore d(A, B) = 2$ units.

Case (2) A - C - B
$\therefore d(A, B) = d(A, C) + d(C, B)$
$\therefore d(A, B) = 10 + 8.$
$\therefore d(A, B) = 18$ units.

Case (3) B - A - C
$\therefore d(A, B) + d(A, C) = d(C, B)$
$\therefore d(A, B) + 10 = 8$
$\therefore d(A, B) = 8 - 10$
$\therefore d(A, B) = -2$

But distance between two points cannot be negative.
$\therefore$ B - A - C is not possible.

$\boxed{\therefore d(A, B) = 2 \text{ units or } d(A, B) = 18 \text{ units}}$

(5) Points X, Y, Z are collinear such that $d(X, Y) = 17$, $d(Y, Z) = 8$, find $d(X, Z)$.

Solution:

Case (1) Consider X - Y - Z

$\therefore$ $d(X, Z) = d(X, Y) + d(Y, Z)$

$\therefore$ $d(X, Z) = 17 + 8.$

$\therefore$ $d(X, Z) = 25$ units.

Case (2) X - Z - Y

$\therefore$ $d(X, Y) = d(X, Z) + d(Z, Y)$

$\therefore$ $17 = d(X, Z) + 8.$

$\therefore$ $17 - 8 = d(X, Z)$

$\therefore$ $d(X, Z) = 9$ units.

Case (3) Consider Y - X - Z

$\therefore$ $d(X, Y) + d(X, Z) = d(Y, Z)$

$\therefore$ $17 + d(X, Z) = 8.$

$\therefore$ $d(X, Z) = -17 + 8$

$\therefore$ $d(X, Z) = -9$ units

But, distance between two points cannot be negative.

Y - X - Z is not possible.

$\boxed{\therefore d(X, Z) = 25 \text{ units or } d(X, Z) = 9 \text{ units}}$

(6) Sketch proper figure and write the answers of the following questions.

(i) If A - B - C and $l(AC) = 11$, $l(BC) = 6.5$ then $l(AB) = ?$

Solution:

A - B - C (Given)

$l(AC) = l(AB) + l(BC)$

$\therefore$ $11 = l(AB) + 6.5$

$\therefore$ $11 - 6.5 = l(AB)$

$\boxed{\therefore l(AB) = 4.5 \text{ units}}$

(ii) If R - S - T and $l(ST) = 3.7$, $l(RS) = 2.5$ then $l(RT) = ?$

Solution:

R - S - T (Given)

$l(RT) = l(RS) + l(ST)$

$\therefore$ $l(RT) = 2.5 + 3.7$

$\boxed{\therefore l(RT) = 6.2 \text{ units}}$

(iii) If X - Y - Z and $l(XZ) = 3\sqrt{7}$, $l(XY) = \sqrt{7}$, then $l(YZ) = ?$

Solution:

X - Y - Z (Given)

$l(XZ) = l(XY) + l(YZ)$

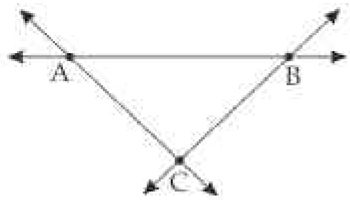
$\therefore$ $3\sqrt{7} = \sqrt{7} + l(YZ)$

$\therefore$ $3\sqrt{7} - \sqrt{7} = l(YZ)$

$\therefore$ $\boxed{\therefore l(YZ) = 2\sqrt{7} \text{ units}}$

(7) Which figure is formed by three non-collinear points?

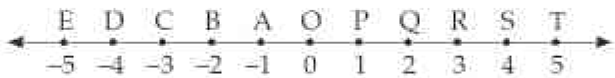
Solution:



$\therefore$ By joining three non-collinear points, a triangle is formed.

PROBLEMS FOR PRACTICE

(1) Observe the number line and answer the following questions:



Find: (i) $d(Q, T)$ (ii) $d(E, S)$
(iii) $d(O, Q)$ (iv) $d(O, D)$

(2) Draw the figures according to the given information and answer the question:

When A - B - C, $l(AC) = 12$, $l(BC) = 7.5$, then $l(A B) = ?$

(3) In each of the following decide whether the relation of betweenness exists or not among the points A, B and D. Name the point which lies between the other two.

(i) $d(A, B) = 5$, $d(B, D) = 8$, $d(A, D) = 11$

(ii) $d(A, B) = 5$, $d(B, D) = 15$, $d(A, D) = 17$

(4) **If R - S - T, l(ST) = 3.75, l(RS) = 2.15, then l(RT) = ?**

(5) **If X - Y - Z, l(XZ) = $5\sqrt{2}$, l(XY) = $2\sqrt{2}$, then l(YZ) = ?**

ANSWERS

(1) (i) 3 units (ii) 9 units

(iii) 2 units (iv) 4 units

(2) l(AB) = 4.5 units.

(3) (i) Relation of betweenness does not exist.

(ii) Relation of betweenness exists. Point B is between points A and D.

(4) l(RT) = 5.9 units

(5) l(YZ)= $3\sqrt{2}$ units.

Points to Remember:

❑ **Line segment:**

The set consisting of points A and B and all the points between A and B is called the segment AB and written as 'seg AB'.

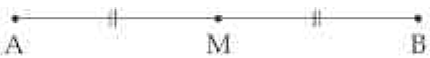
Note:

(1) The points A and B are called end points of the seg AB.

(2) A line segment is a subset of a line.

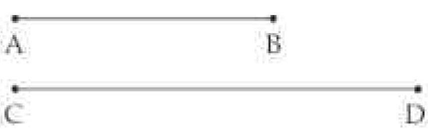
❑ **Midpoint of segment:**

The point M is said to be the midpoint of seg AB, if (1) A - M - B (2) AM = MB



❑ **Comparison of segment:**

If AB < CD, then we say that seg AB is smaller than seg CD and denoted by seg AB < seg CD.



❑ **Ray:**

A portion of line which starts at a point and goes in a particular direction to infinity is called Ray.

Ray AB.

❑ **Opposite Rays:**

Two rays having a common origin and lying on the same line are said to be opposite rays.

Here, Ray OA and Ray OB have a common origin O and they lie on same line.

$\therefore$ OA and OB are called opposite rays.

❑ **Length of a line segment:**

The distance between the end points of a line segment is called the length of the segment.

Length of segment AB is denoted by l(AB) or AB.

❑ **Congruent Segments:**

Two segments are said to be congruent if they are of the same length i.e. l(AB) = l(CD) then seg AB ≅ seg CD.

❑ **Properties of Congruent segments:**

(1) **Reflexivity:** Every segment is congruent to itself. seg PQ ≅ seg PQ.

(2) **Symmetry:** If the first segment is congruent to second, then the second is congruent to first.

If seg PQ ≅ seg AB then seg AB ≅ seg PQ.

(3) **Transitivity:** If first segment is congruent to second segment and second segment is congruent to third segment then first segment is congruent to third segment.

If seg PQ ≅ seg AB and seg AB ≅ seg XY then seg PQ ≅ seg XY.

PRACTICE SET - 1.2 *(Textbook Page No. 7)*

(1) **The following table shows points on a number line and their co-ordinates. Decide whether the pair of segments given below the table are congruent or not.**

Point	A	B	C	D	E
Co-ordinate	– 3	5	2	– 7	9

(i) seg DE and seg AB

Solution:

The co-ordinate of point D is – 7 and the co-ordinate of point E is 9

$9 > -7$

$d(D, E)$

= Greater co-ordinate – Smaller co-ordinate

= $9 - (-7)$

= $9 + 7$

$\therefore$ $d(D, E) = 16$

$\therefore$ $l(DE) = 16$ units ....(i)

The co-ordinate of point A is – 3.

The co-ordinate of point B is 5.

$5 > -3$

$d(A, B)$

= Greater co-ordinate – Smaller co-ordinate

= $5 - (-3)$

= $5 + 3$

$\therefore$ $d(A, B) = 8$ units

$\therefore$ $l(AB) = 8$ units ....(ii)

$\therefore$ $l(DE) \neq l(AB)$ [From (i) and (ii)]

$\boxed{\therefore \text{ seg DE is not congruent to seg AB.}}$

(ii) seg BC and seg AD

Solution:

The co-ordinate of point B is 5.

The co-ordinate of point C is 2.

$5 > 2$

$d(B, C)$

= Greater co-ordinate - Smaller co-ordinate

= $5 - 2$

= 3

$\therefore$ $d(B, C) = 3$ units

$\therefore$ $l(BC) = 3$ units ....(i)

The co-ordinate of point A is – 3.

The co-ordinate of point D is – 7.

$-3 > -7$

$d(A, D)$

= Greater co-ordinate – Smaller co-ordinate

= $-3 - (-7)$

= $-3 + 7$

= 4

$\therefore$ $d(A, D) = 4$ units

$\therefore$ $l(AD) = 4$ units ....(ii)

$\therefore$ $l(BC) \neq l(AD)$ [From (i) and (ii)]

$\boxed{\therefore \text{ seg BC is not congruent to seg AD.}}$

(iii) seg BE and seg AD

Solution:

The co-ordinate of point B is 5.

The co-ordinate of point E is 9.

$9 > 5$

$d(B, E)$

= Greater co-ordinate – Smaller co-ordinate

= $9 - 5$

= 4

$\therefore$ $d(B, E) = 4$ units

$\therefore$ $l(BE) = 4$ units ....(i)

The co-ordinate of point A is – 3.

The co-ordinate of point D is – 7.

$-3 > -7$

$d(A, D)$

= Greater co-ordinate – Smaller co-ordinate

= $-3 - (-7)$

= $-3 + 7$

= 4

$\therefore$ $d(A, D) = 4$ units

$\therefore$ $l(AD) = 4$ units ....(ii)

$\therefore$ $l(BE) = l(AD)$ [From (i) and (ii)]

$\boxed{\therefore \text{ seg BE} \cong \text{seg AD}}$

(2) Point M is the midpoint of seg AB. If AB = 8 then find the length of AM.

Solution:

M is midpoint of seg AB .... (Given)

$\therefore$ $l(AM) = \frac{1}{2} l(AB)$

$\therefore$ $l(AM) = \frac{1}{2} \times 8$

$\boxed{\therefore l(AM) = 4 \text{ units}}$

(3) Point P is the midpoint of seg CD. If CP = 2.5, find *l*(CD).

Solution:

Point P is midpoint of seg CD. .... (Given)

$\therefore$ $l(CP) = \frac{1}{2} l(CD)$

$\therefore \quad 2.5 = \frac{1}{2}\, l(\text{CD})$

$\therefore \quad 2.5 \times 2 = l(\text{CD})$

$\boxed{\therefore\ l(\text{CD}) = 5 \text{ units}}$

(4) If AB = 5 cm, BP = 2 cm and AP = 3.4 cm, compare the segments.

Solution:

AB = 5 cm, BP = 2 cm, AP = 3.4 cm

$5 > 3.4 > 2$

$\therefore \quad l(\text{AB}) > l(\text{AP}) > l(\text{BP})$

(5) Write the answers to the following questions with reference to figure.

(i) Write the name of the opposite ray of ray RP.

Ans. Ray RS is opposite of ray RP.

(ii) Write the intersection set of ray PQ and ray RP.

Ans. Intersection set of ray PQ and ray RP is ray PQ.

(iii) Write the union set of ray PQ and ray QR.

Ans. The union of ray PQ and ray QR is line QR.

(iv) State the rays of which seg QR is a subset.

Ans. Seg QR is a subset of ray QR, ray RQ, ray QS and ray QT.

(v) Write the pair of opposite rays with common end point R.

Ans. Ray RP and ray RS.

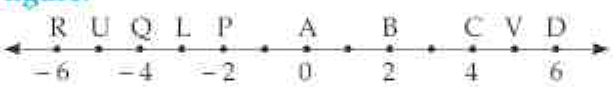
(vi) Write any two rays with common end point S.

Ans. Ray ST and ray SR.

(vii) Write the intersection set of ray SP and ray ST.

Ans. The intersection of ray SP and ray ST is point S.

(6) Answer the questions with the help of figure.



(i) State the points which are equidistant from point B.

Ans. (a) Point A and Point C.

(b) Point P and Point D.

(ii) Write a pair of points equidistant from point Q.

Ans. Point R and point P.

(iii) Find d(U, V), d(P, C), d(V, B), d(U, L).

Solution:

d(U, V)

The co-ordinate of point U is – 5.

The co-ordinate of point V is 5.

$5 > -5$

$d(\text{U, V})$

= Greater co-ordinate – Smaller co-ordinate

$= 5 - (-5)$

$= 5 + 5$

$\boxed{\therefore\ d(\text{U, V}) = 10 \text{ Units}}$

d(P, C)

The co-ordinate of point P is – 2.

The co-ordinate of point C is 4.

$4 > -2$

$d(\text{P, C})$

= Greater co-ordinate – Smaller co-ordinate

$= 4 - (-2)$

$= 4 + 2$

$\boxed{\therefore\ d(\text{P, C}) = 6 \text{ Units}}$

d(V, B)

The co-ordinate of point V is 5.

The co-ordinate of point B is 2.

$5 > 2$

$d(\text{V, B})$

= Greater co-ordinate – Smaller co-ordinate

$= 5 - 2$

$\boxed{\therefore\ d(\text{V, B}) = 3 \text{ Units}}$

d(U, L)

The co-ordinate of point U is – 5.

The co-ordinate of point L is – 3.

$-3 > -5$

$d(\text{U, L})$

= Greater co-ordinate – Smaller co-ordinate

$= -3 - (-5) = -3 + 5$

$\boxed{\therefore\ d(\text{U, L}) = 2 \text{ Units}}$

PROBLEMS FOR PRACTICE

(1) **The co-ordinates of the points on a line are as follows:**

Points	P	Q	R	S	T
Co-ordinates	– 3	4	2	– 5	9

Check whether given pair of segments are congruent or not.

(i) seg PQ and seg QS.

(ii) seg RS and seg PQ.

(iii) seg PS and seg RT.

(2) **If P is midpoint of seg AB and AB = 7 cm, find AP.**

(3) **If Q is midpoint of seg CD and d(C, Q) = 6, find length of seg CD.**

(4) **If AB = 7 cm, BP = 4 cm, AP = 5.4 cm, compare the segments.**

(5) **In the below figure,**

A, B, C, D and E are the points of line *l*.

(i) Name the opposite rays with point A as origin.

(ii) Name the ray opposite to ray BE.

(iii) Find the intersection of ray CE and ray BC.

ANSWERS

(1) (i) not congruent (ii) congruent
(iii) not congruent

(2) AP = 3.5 cm

(3) l(CD) = 12 cm.

(4) AB > AP > BP

(5) (i) ray AD and ray AE
(ii) ray BD (iii) seg CB

Points to Remember:

❑ **Conditional statements and converse:**

The statements which can be written in the 'If-then' form are called conditional statements.

The part of the statement following 'If' is called the antecedent and the part following 'then' is called the consequent.

For example, consider the statement: The diagonals of a rhombus are perpendicular bisectors of each other.

The statement can be written in the conditional form as, 'If the given quadrilateral is a rhombus then its diagonals are perpendicular bisectors of each other.'

If the antecedent and consequent in a given conditional statement are interchanged, the resulting statement is called the **converse** of the given statement.

If a conditional statement is true, its converse is not necessarily true. Study the following examples.

Conditional statement: If a quadrilateral is a rhombus then its diagonals are perpendicular bisectors of each other.

Converse: If the diagonals of a quadrilateral are perpendicular bisectors of each other then it is a rhombus.

In the above example, the statement and its converse are true.

Now consider the following example.

Conditional statement: If a number is a prime number then it is even or odd.

Converse: If a number is even or odd then it is a prime number.

In this example, the statement is true, but its converse is false.

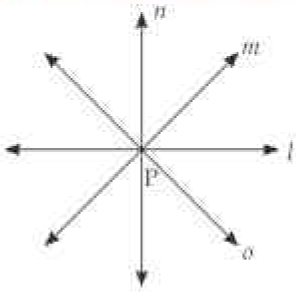
❑ **Axioms/Postulates:**

These are simple statements which we accept as true and need not be proved. Such statements are called Axioms or Postulates.

Some Euclid's Postulates are given below:

- **There are infinite lines passing through a point:**

In the following figure, lines l, m, n and o passes through point P. Similarly, we can draw infinite lines passing through P.

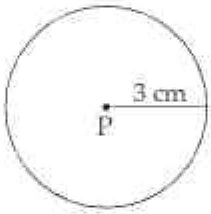


- **There is one and only one line passing through two distinct points.**

  Line l passes through the points A and B.

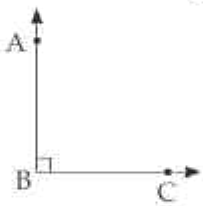
- **A circle of given radius can be drawn taking any point as its centre.**

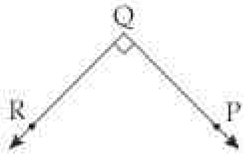
  A circle of radius 3 cm with centre P is drawn.

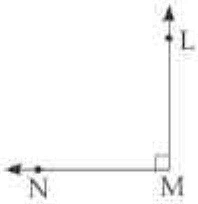


- **All right angles are congruent to each other.**

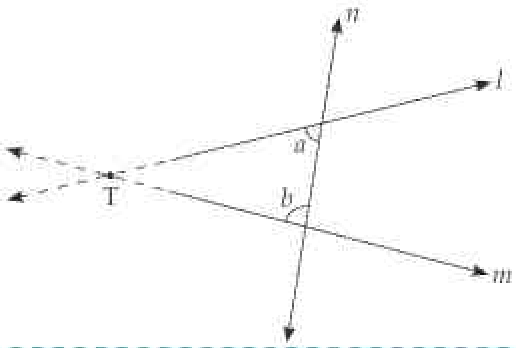
  In the following figures, $\angle B = \angle Q = \angle M = 90^o$







- **If two interior angles formed on one side of a transversal of two lines add upto less than two right angles then the lines produced in that direction intersect each other.**



In the above figure,

a and b form a pair of interior angles.

a is an acute angle, i.e. $\angle a < 90^o$

b is an acute angle, i.e. $\angle b < 90^o$

$\therefore \quad \angle a + \angle b < 90^o + 90^o$

$\therefore \quad \angle a + \angle b < 180^o$

$\therefore \quad$ line l intersects line m at point T.

❑ Theorem:

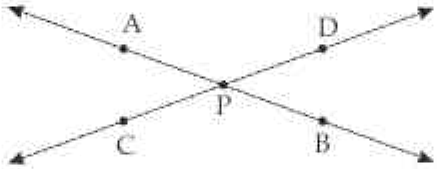
- A property is supposed to be true if it can be proved logically. It is then called Theorem.

  Proof:

  The logical argument made to prove a theorem is called its proof.

❑ Theorem:

- If two lines intersect each other then the vertically opposite angles formed are congruent.



Given: Line AB and line CD intersect each other at point P.

To prove: (1) $\angle APC \cong \angle BPD$

(2) $\angle APD \cong \angle BPC$

Proof:

$\angle APC + \angle APD = 180^o$ ...(i)

(Linear pair of angles)

$\angle BPD + \angle APD = 180^o$ ...(ii)

(Linear pair of angles)

$\therefore \quad \angle APC + \angle APD = \angle BPD + \angle APD$

[From (i) and (ii)]

$\therefore \quad \angle APC = \angle BPD$

i.e. $\angle APC \cong \angle BPD$

Similarly, we can prove,

$\angle APD \cong \angle BPC$.

PRACTICE SET - 1.3 *(Textbook Page No. 11)*

Q.1. Write the following statement in 'if-then' form.

(i) The opposite angles of a parallelogram are congruent.

Solution:

If a quadrilateral is a parallelogram then its opposite angles are congruent.

(ii) The Diagonals of a rectangle are congruent.

Solution:

If a quadrilateral is a rectangle then its diagonals are congruent.

(iii) In an isosceles triangle, the segment joining the vertex and the mid point of the base is perpendicular to the base.

Solution:

If a triangle is an isosceles then segment joining vertex and midpoint of the base is perpendicular to the base.

Q.2. Write converse of the following statements:

(i) The alternate angles formed by two parallel lines and their transversal are congruent.

Solution:

Converse of above statement:

If alternate angles formed by two lines and its a transversal are congruent then the lines are parallel.

(ii) If a pair of the interior angles made by a transversal of two lines are supplementary then the lines are parallel.

Solution:

Converse of above statement:

If two parallel lines are intersected by a transversal then interior angles so formed are supplementary.

(iii) The diagonals of a rectangle are congruent.

Solution:

Converse of above statement does not exit.

PROBLEMS FOR PRACTICE

(1) Write the following statement in 'if-then' form:

(i) All sides of rhombus are congruent.

(ii) In an equilateral triangle all sides are congruent.

(2) Write the converse of following theorems:

(i) Opposite angles of a cylic quadrilateral are supplementary.

(ii) In an Isosceles triangle the angles opposite to equal sides are congruent.

ANSWERS

(1) (i) If the quadrilateral is a rhombus, then all sides are congruent.

(ii) If the triangle is equilateral, then all three sides are congruent.

(2) (i) If opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.

(ii) In a triangle, the sides opposite to congruent angles are congruent.

PROBLEM SET - 1 *(Textbook Page No. 11)*

(1) Select the correct alternative from the answers of the questions given below:

(i) How many mid points a segment have?

(A) Only one (B) Two (C) Three (D) Many

Ans. (A)

(ii) How many points are there in the intersection of two distinct lines?

(A) Infinite (B) Two (C) One (D) Not a single

Ans. (C)

(iii) How many lines are determined by three distinct points?

(A) Two (B) Three (C) One or three (D) Six

Ans. (C)

(iv) Find $d(A, B)$, if co-ordinates of A and B are -2 and 5 respectively.

(A) -2 (B) 5 (C) 7 (D) 3

Ans. (C)

(v) If P - Q - R and $d(P, Q) = 2$, $d(P, R) = 10$, then find $d(Q, R)$.

(A) 12 (B) 8 (C) $\sqrt{96}$ (D) 20

Ans. (B)

(2) On a number line, co-ordinates of P, Q, R are 3, – 5 and 6 respectively. State with reason whether the following statements are true or false.

(i) $d(P, Q) + d(Q, R) = d(P, R)$

(ii) $d(P, R) + d(R, Q) = d(P, Q)$

(iii) $d(R, P) + d(P, Q) = d(R, Q)$

(iv) $d(P, Q) - d(P, R) = d(Q, R)$

Solution:

The co-ordinate of P is 3.

The co-ordinate of Q is – 5.

The co-ordinate of R is 6.

$d(P, Q) = 3 - (-5) = 3 + 5 = 8$ units

$d(Q, R) = 6 - (-5) = 6 + 5 = 11$ units

$d(P, R) = 6 - 3 = 3$ units

(i) $d(P, Q) + d(Q, R) = d(P, R)$

$d(P, Q) + d(Q, R) = 8 + 11 = 19$ units ....(i)

$d(P, R) = 3$ units ....(ii)

$\therefore$ $d(P, Q) + d(Q, R) \neq d(P, R)$ [From (i) and (ii)]

$\therefore$ $d(P, Q) + d(Q, R) = d(P, R)$ is a false statement.

(ii) $d(P, R) + d(R, Q) = d(P, Q)$

$d(P, R) + d(R, Q) = 3 + 11 = 14$ units ....(i)

$d(P, Q) = 8$ units ....(ii)

$\therefore$ $d(P, R) + d(R, Q) \neq d(P, Q)$ [From (i) and (ii)]

$d(P, R) + d(R, Q) = d(P, Q)$ is false statement.

(iii) $d(R, P) + d(P, Q) = d(R, Q)$

$d(R, P) + d(P, Q) = 3 + 8 = 11$ units ....(i)

$d(R, Q) = 11$ units ....(ii)

$\therefore$ $d(R, P) + d(P, Q) = d(R, Q)$ [From (i) and (ii)]

$d(R, P) + d(P, Q) = d(R, Q)$ is a true statement.

(iv) $d(P, Q) - d(P, R) = d(Q, R)$

$d(P, Q) - d(P, R) = 8 - 3 = 5$ units ....(i)

$d(Q, R) = 11$ units ....(ii)

$\therefore$ $d(P, Q) - d(P, R) \neq d(Q, R)$ [From (i) and (ii)]

$d(P, Q) - d(P, R) = d(Q, R)$ is a false statement.

(3) Co-ordinates of some pairs of points are given below. Hence find the distance between each pair.

(i) 3, 6

Solution:

Let the co-ordinate of point A be 3
and the co-ordinate of point B be 6.

$6 > 3$

$d(A, B) =$

= Greater co-ordinate – Smaller co-ordinate

= $6 - 3$

= 3

$\therefore$ $d(A, B) = 3$

$\therefore$ The distance between the given pair of points is 3 units.

(ii) – 9, – 1

Solution:

Let the co-ordinate of point A be – 9
and the co-ordinate of point B be – 1.

$-1 > -9$

$d(A, B)$

= Greater co-ordinate – Smaller co-ordinate

= $-1 - (-9)$

= $-1 + 9$

= 8

$\therefore$ $d(A, B) = 8$ units

$\therefore$ The distance between the given pair of points is 8 units.

(iii) – 4, 5

Solution:

Let the co-ordinate of point A be – 4
and the co-ordinate of point B be 5.

$5 > -4$

$d(A, B)$

= Greater co-ordinate – Smaller co-ordinate

= $5 - (-4)$

= $5 + 4$

= 9

$\therefore$ $d(A, B) = 9$ units

$\therefore$ The distance between the given pair of points is 9 units.

(iv) $0, -2$

Solution:

Let the co-ordinate of point A be 0
and the co-ordinate of point B be -2.

$0 > -2$

$d(A, B)$

$=$ Greater co-ordinate – Smaller co-ordinate

$=$ $0 - (-2)$

$\therefore$ $0 + 2$

$\therefore$ $d(A, B) = 2$

$\therefore$ The distance between the given pair of points is 2 units.

(v) $x + 3, x - 3$

Solution:

Let the co-ordinate of point A be $x + 3$
and the co-ordinate of point B be $x - 3$

$(x + 3) > (x - 3)$

$d(A, B)$

$=$ Greater co-ordinate – Smaller co-ordinate

$= (x + 3) - (x - 3)$

$= x + 3 - x + 3$

$= 6$

$\therefore$ $d(A, B) = 6$ units

$\therefore$ The distance between the given pair of points is 6 units.

(vi) $-25, -47$

Solution:

Let the co-ordinate of A point be -25
and the co-ordinate of B point be -47

$d(A, B)$

$=$ Greater co-ordinate – Smaller co-ordinate

$=$ $-25 - (-47)$

$=$ $-25 + 47$

$=$ 22

$\therefore$ $d(A, B) = 22$ units

$\therefore$ The distance between the given pair of points is 22 units.

(vii) $80, -85$

Solution:

Let the co-ordinate of point A be 80
and the co-ordinate of point B be -85

$d(A, B)$

$=$ Greater co-ordinate – Smaller co-ordinate

$=$ $80 - (-85)$

$=$ $80 + 85$

$=$ 165

$\therefore$ $d(A, B) = 165$ units

$\therefore$ The distance between the given pair of points is 165 units.

(4) Co-ordinate of point P, on a number line is -7. Find the co-ordinates of points on the number line which are at a distance of 8 units from point P.

Solution:

Let co-ordinate of Q be x be a point on positive side of point P.

Co-ordinate of P is -7.

$x > -7$

$d(P, Q) = x - (-7)$

$\therefore$ $8 = x + 7$

$\therefore$ $x = 8 - 7$

$\therefore$ $x = 1$

$\therefore$ Co-ordinate of point Q is 1.

Let co-ordinate of ponit R be y be a point on negative side of point P.

Co-ordinate of point P is -7

$-7 > y$

$\therefore$ $d(P, R) = -7 - y$

$\therefore$ $8 = -7 - y$

$\therefore$ $8 + 7 = -y$

$\therefore$ $-y = 15$

$\therefore$ $y = -15$

$\therefore$ Co-ordinate of point R is -15

(5) Answer the following questions.

(i) If A - B - C and $d(A, C) = 17$, $d(B, C) = 6.5$ then $d(A, B) = ?$

Solution:

A - B - C (Given)

$\therefore$ $d(A, C) = d(A, B) + d(B, C)$

$\therefore$ $17 = d(A, B) + 6.5$

$\therefore$ $17 - 6.5 = d(A, B)$

$\boxed{\therefore d(A, B) = 10.5 \text{ units}}$

(ii) If P - Q - R and $d(P, Q) = 3.4$, $d(Q, R) = 5.7$, then find $d(P, R) = ?$

Solution:

P - Q - R (Given)

$\therefore$ $d(P, R) = d(P, Q) + d(Q, R)$

$\therefore$ $= 3.4 + 5.7$

$\therefore$ $= 9.1$ units

$\boxed{\therefore d(P, R) = 9.1 \text{ units}}$

(6) Co-ordinate of point A on a number line is 1. What are the co-ordinates of points on the number line which are at a distance of 7 units from A?

Solution:

Co-ordinate of point A is 1.

Let co-ordinate of point B be x be a point on positive side of point A.

$x > 1$

$\therefore$ $d(A, B) = x - 1$

$\therefore$ $7 = x - 1$

$\therefore$ $7 + 1 = x$

$\therefore$ $x = 8$

$\therefore$ Co-ordinate of point B is 8.

Let co-ordinate of point C be y be a point on negative side of point A.

$1 > y$

$\therefore$ $d(A, C) = 1 - y$

$\therefore$ $7 = 1 - y$

$\therefore$ $y = 1 - 7$

$\therefore$ $y = -6$

$\therefore$ Co-ordinate of point C is -6.

(7) Write the following statements in conditional form:

(i) Every rhombus is a square.

Ans. If a quadrilateral is a square then it is a rhombus.

(ii) Angles in a linear pair are supplementary.

Ans. If adjacent angles are supplementary, then they form a linear pair.

(iii) A triangle is a figure formed by three segments.

Ans. If a polygon is three-sided closed figure, then it is a triangle.

(iv) A number having only two divisors is called a prime number.

Ans. If a number is a prime number then it has only two divisors.

(8) Write the converse of each of the following statements.

(i) If the sum of measures of angles in a figure is 180°, then the figure is a triangle.

Ans. Converse: If a figure is a triangle then sum of all angles of this figure is 180°.

(ii) If the sum of measures of two angles is 90° then they are complement of each other.

Ans. Converse: If two angles are complementary then their sum is 90°.

(iii) If the corresponding angles formed by a transversal of two lines are congruent then the two lines are parallel.

Ans. Converse: If two parallel lines are intersected by a transversal then the pair of corresponding angles is congruent.

(iv) If the sum of the digits of a number is divisible by 3 then the number is divisible by 3.

Ans. Converse: If a number is divisible by 3 then sum of digits of this number is divisible by 3.

(9) Write the antecedent (given part) and the consequent (part to be proved) in the following statements.

(i) If all sides of a triangle are congruent then its all angles are congruent.

Ans. Given: In ΔABC, side AB $\cong$ side BC $\cong$ side AC.

To prove: $\angle A \cong \angle B \cong \angle C$.

(ii) The diagonals of a parallelogram bisect each other.

Ans. Given: (1) $\square$PQRS is a parallelogram.

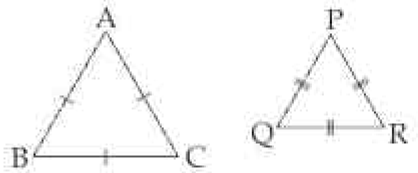
(2) Diagonals PR and QS intersect at point M.

To prove: (1) PM = RM (2) QM = SM.

(10) Draw a labelled figure showing information in each of the following statements and write the antecedent and the consequent.

(i) Two equilateral triangles are similar.

Ans.



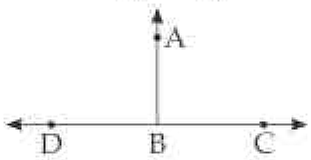
Given: (1) In ΔABC, side AB $\cong$ side BC $\cong$ side AC.

(2) In ΔPQR, side PQ $\cong$ side QR $\cong$ side PR.

To prove: $\Delta ABC \sim \Delta PQR$

(ii) If angles in a linear pair are congruent then each of them is a right angle.

Ans.



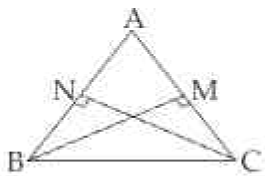
Given: (1) $\angle$ABC and $\angle$ABD form a linear pair.

(2) $\angle ABC \cong \angle ABD$.

To prove: $\angle ABC = \angle ABD = 90^\circ$

(iii) If the altitudes drawn on two sides of a triangle are congruent then those two sides are congruent.

Ans.



Given: In ΔABC

(1) seg BM $\perp$ side AC, A - M - C.

(2) seg CN $\perp$ side AB, A - N - B.

(3) seg BM $\cong$ seg CN.

To prove: Side AB $\cong$ side AC.

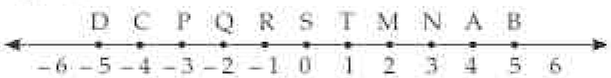
MCQ's

(1) The number associated with the point is called .... of that point.

(A) Origin (B) Midpoint

(C) Co-ordinate (D) Graph

(2) With the help of the given number line, what is d(P, T)?



(A) 3 units (B) 4 units

(C) – 3 units (D) – 4 units

(3) With the help of the given figure, which of the following statement is true?

(A) d(P, R) + d(R, Q) = d(P, Q)

(B) d(P, R) + d(P, Q) = d(R, Q)

(C) d(Q, R) – d(P, Q) = d(P, R)

(D) d(P, R) – d(R, Q) = d(P, Q)

(4) If d(P, R) = 7, d(Q, R) = 3 and d(P, Q) = 4, what can we say about points P, Q and R?

(A) P – R – Q (B) Q – P – R

(C) P – Q – R (D) R – P – Q

(5) A statement which is universally true and need not be proved is called ....

(A) Axiom (B) Postulate

(C) both a and b (D) Theorem

(6) From a single point, we can draw .... lines.

(A) One (B) Infinite

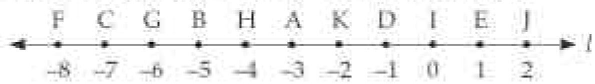
(C) Two (D) Five

(7) Two different points determine .... line.

(A) Six (B) Infinite

(C) Two (D) One and only one

(8) The co-ordinate of point E is ....



(A) – 4 (B) 1

(C) 2 (D) – 5

(9) The co-ordinate of point J is ....

(A) – 7 (B) 2

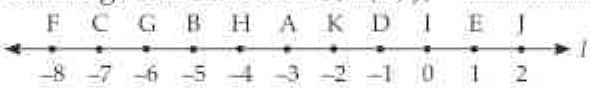
(C) 0 (D) – 5

(10) Which points are at a distance of 3 units from point A?

(A) Points C, E (B) Points G, I

(C) Points D, B (D) Points H, E

(11) On the given number line, d(K, J) = ...



(A) 3 units (B) 2 units
(C) 4 units (D) 5 units

(12) If P – Q – R, l(PR) = 7 units, l(PQ) = 4 units, l(QR) = ...

(A) 11 units (B) 3 units
(C) 6 units (D) 5 units

(13) If AB = 10 units, AC = 7 units, BC = 3 units then which of the following is correct?

(A) A-B-C
(B) C-B-A
(C) A-C-B
(D) Point A, B, C are non collinear.

(14) If the co-ordinates of points P and Q are 3 and –5 respectively then d(P, Q) = ...

(A) 9 units (B) 2 units
(C) 8 units (D) 7 units

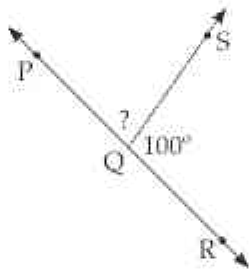
(15) If d(P, Q) = 10 units, d(P, R) = 18 units and P-Q-R then d(Q, R) = ...

(A) 28 units (B) 10 units
(C) 18 units (D) 8 units

(16) If the co-ordinates of points P and Q are $\sqrt{2}$ and $-\sqrt{2}$ respectively then d(P, Q) = ...

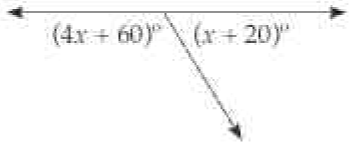
(A) $\sqrt{2}$ units (B) $3\sqrt{2}$ units
(C) $2\sqrt{2}$ units (D) $4\sqrt{2}$ units

(17) What is m∠PQS?



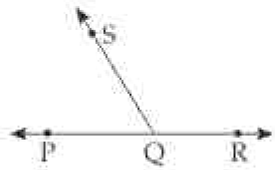
(A) 70° (B) 90°
(C) 80° (D) 60°

(18) In the adjoining figure, angles are in linear pair. What is the value of x?

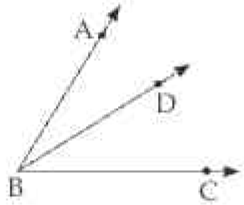


(A) 40 (B) 80
(C) 20 (D) 30

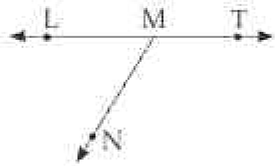
(19) Which of the following are pair of linear pair angles.



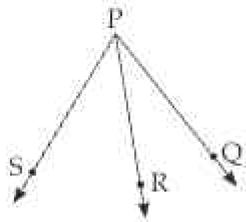
(A)



(B)



(C)



(D)

(A) A and B (B) B and D
(C) C and D (D) A and C

(20) AB = CD, CD = EF then ....

(A) AB = EF, by property of symmetry
(B) AB = EF, by property of reflexivity
(C) AB = EF, by property of transitivity
(D) AB ≠ EF

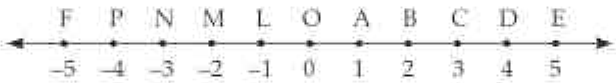
ANSWERS

(1)	(C)	(2)	(B)	(3)	(D)	(4)	(C)
(5)	(C)	(6)	(B)	(7)	(D)	(8)	(B)
(9)	(B)	(10)	(B)	(11)	(C)	(12)	(B)
(13)	(C)	(14)	(C)	(15)	(D)	(16)	(C)
(17)	(C)	(18)	(C)	(19)	(D)	(20)	(C)

ASSIGNMENT - 1

Time : 1 Hr. **Marks : 20**

Q.1. Observe the number line and answer the following: **(4)**



Find (1) d(M, C) (2) d(L, P) (3) d(O, N) (4) d(A, E)

Q.2. Solve the following: **(10)**

(1) The points A, B, C are on a line such that d(A, C) = 10, d(C, B) = 8 then find d(A, B).

(2) Draw the figure, according to the given information and answer the question when A-B-C, l(AC) = 11, l(BC) = 6.5 then l(AB) = ?

(3) P is midpoint of seg AB. If AB is 15, then find length of AP.

(4) Write the given statement in 'if-then' form for prime numbers are divisible by 1 and itself.

(5) Diagonals of a parallelogram bisect each other. Write the proof for the given statement.

Q.3. Solve the following: **(6)**

(1) Prove: If two lines intersect each other, then vertically opposite angles are congruent.

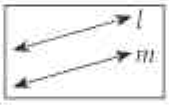
(2) Co-ordinate of a point P on a number line is – 7. Find the co-ordinates of points which are at a distance of 8 units from point P.

❖ ❖ ❖

2 Parallel Lines

Points to Remember:

Parallel Lines

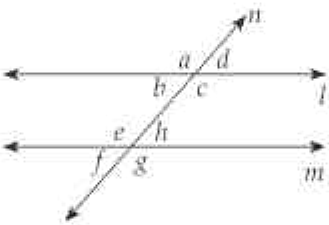


Parallel Lines : The lines which are coplanar and do not intersect each other are called parallel lines.

In the above figure line l is parallel to line m

Symbolically, it is written as 'line $l \parallel$ line m'

Transversal : A tranversal is a line that intersects two or more lines in distinct points.



In the adjoining figure, line n is the transversal for lines l and m. The two lines and the transversal determine eight angles at the point of intersection.

Pairs of Corresponding angles :

(i) $\angle b$ and $\angle f$ (ii) $\angle c$ and $\angle g$

(iii) $\angle a$ and $\angle e$ (iv) $\angle d$ and $\angle h$

Pairs of Alternate Interior angles :

(i) $\angle b$ and $\angle h$ (ii) $\angle c$ and $\angle e$

Pairs of Alternate Exterior angles :

(i) $\angle a$ and $\angle g$ (ii) $\angle d$ and $\angle f$

Pairs of Interior angles :

(i) $\angle b$ and $\angle e$ (ii) $\angle c$ and $\angle h$

Important Properties

(1) When two lines intersect, the pairs of opposite angles formed are congruent.

(2) The angles in a linear pair are supplementry

(3) When one pair of corresponding angles is congruent, then all the remaining pairs of corresponding angles are congruent.

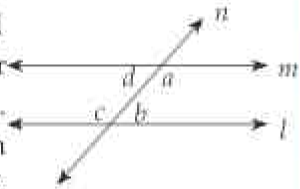
(4) Whe one pair of alternate angles is congruent, then all the remaining pairs of alternate angles are congruent.

(5) When one pair of interior angles on one side of the transversal is supplementary, then the other pair of interior angles is also supplementary.

Theorem - 1 : Interior angles theorem

Theorem : If two parallel lines are intersected by a transversal, the interior angles on either side of the transversal are supplementary.

Given : line $l \parallel$ line m and line n is their transversal. Hence as shown in the figure $\angle a$, $\angle b$ are interior angles formed on one side and $\angle c$, $\angle d$ are interior angles formed on other side of the transversal.



To prove : $\angle a + \angle b = 180°$

$\angle d + \angle c = 180°$

Proof : Three possibilities arise regarding the sum of measures of $\angle a$ and $\angle b$.

(i) $\angle a + \angle b < 180°$

(ii) $\angle a + \angle b > 180°$

(iii) $\angle a + \angle b = 180°$

Let us assume that the possibility (i) $\angle a + \angle b < 180°$ is true.

Then according to Euclid's postulate, if the line l and line m are produced will intersect each other or the side of the transversal where $\angle a$ and $\angle b$ are formed.

But line l and line m are parallel lines ...(Given)

$\therefore$ $\angle a + \angle b < 180°$ impossible ...(i)

Now let us suppose that $\angle a + \angle b > 180°$ is true.

$\therefore$ $\angle a + \angle b > 180°$

But $\angle a + \angle d = 180°$

and $\angle c + \angle b = 180°$ ...(Angles in linear pair)

$\therefore$ $\angle a + \angle d + \angle b + \angle c = 180° + 180° = 360°$

$\therefore$ $\angle c + \angle d = 360° - (\angle a + \angle b)$

If $\angle a + \angle b > 180°$ then

$[360° - (\angle a + \angle b)] < 180°$

$\therefore$ $\angle c + \angle d < 180°$

$\therefore$ In that case line l and line m produced will intersect each other on the same side of the transversal where $\angle c$ and $\angle d$ are formed.

$\therefore$ $\angle c + \angle d < 180°$ is impossible

$\therefore$ That is $\angle a + \angle b > 180°$ is impossible ...(ii)

$\therefore$ the remaining possibility,

$\therefore$ $\angle a + \angle b = 180°$ is true ...[From (i) and (ii)]

$\therefore$ $\angle a + \angle b = 180°$

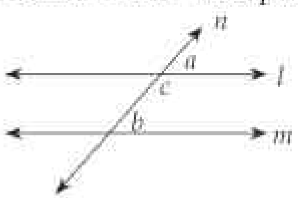
Similarly, $\angle c + \angle d = 180°$

Note that, in this proof, because of the contradictions we have denied the possibilities $\angle a + \angle b > 180°$ and $\angle a + \angle b < 180°$

Theorem - 2 : Corresponding angles theorem

Statement : The corresponding angles formed by a transversal of two parallel lines are of equal measures.

Given : line $l \parallel$ line m and line n is a transversal.



To prove : $\angle a = \angle b$

Proof : $\angle a + \angle c = 180°$ ...(i) (Angles in linear pair)

But, $\angle b + \angle c = 180°$ ...(ii) (Interior angles theorem)

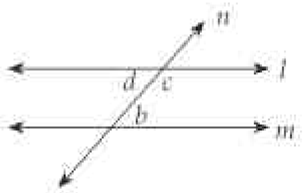
$\therefore$ $\angle a + \angle c = \angle b + \angle c$ ...[From (i) and (ii)]

$\therefore$ $\angle a = \angle b$

Theorem - 3 : Alternate angles theorem

Statement : The alternate angles formed by a transversal of two parallel lines are of equal measures.

Given : line $l \parallel$ line m and line n is a transversal.



To prove : $\angle d = \angle b$

Proof : $\angle d + \angle c = 180°$ ...(i) (Angles in linear pair)

But, $\angle c + \angle b = 180°$ ...(ii) (Interior angles theorem)

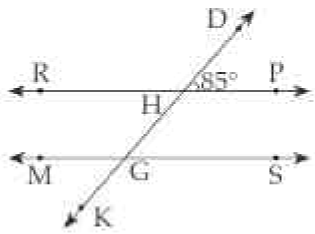
$\therefore$ $\angle d + \angle c = \angle c + \angle b$ ...[From (i) and (ii)]

$\therefore$ $\angle d = \angle b$

MASTER KEY QUESTION SET - 2

PRACTICE SET - 2.1 *(Textbook Page No. 17)*

(1) In the given figure, line RP $\parallel$ line MS and line DK is their transversal, $\angle$DHP = 85°. Find the measure of following angles :



(i) $\angle$RHD (ii) $\angle$PHG (iii) $\angle$HGS (iv) $\angle$MGK

Solution :

(i) $\angle DHP + \angle RHD = 180°$ ...(Angles in linear pair)

$\therefore$ $85 + \angle RHD = 180$

$\therefore$ $\angle RHD = 180 - 85$

$\therefore$ $\boxed{\angle RHD = 95°}$

(ii) $\angle PHG = \angle RHD$ ...(Vertically opposite angles)

$\therefore$ $\boxed{\angle PHG = 95°}$

(iii) line RP $\parallel$ line MS ...(Given)

On transversal DK,

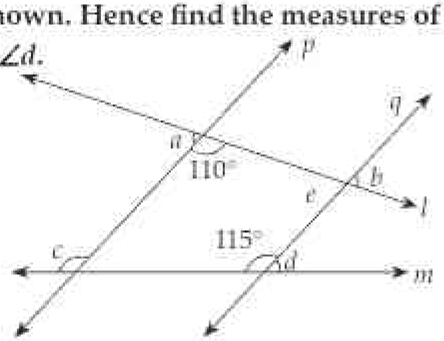
$\angle DHP = \angle HGS$ ...(Corresponding angles theorem)

$\therefore$ $\boxed{\angle HGS = 85°}$

(iv) $\angle HGS = \angle MGK$ ...(Vertically opposite angles)

$\therefore$ $\boxed{\angle MGK = 85°}$

(2) In the given figure, line $p \parallel$ line q and line l and line m are transversals. Measures of some angles are shown. Hence find the measures of $\angle a$, $\angle b$, $\angle c$, $\angle d$.



Solution :

$\angle a + 110 = 180$ ...(Angles in linear pair)

$\therefore$ $\angle a = 180 - 110$

$\therefore$ $\boxed{\angle a = 70°}$

Name an angle as 'e' as shown in the figure,

line p || line q ...(Given)

On transversal l,

$\angle e = \angle a$ ...(Corresponding angles theorem)

$\therefore$ $\angle e = 70$

$\therefore$ $\angle b = \angle e$ ...(Vertically opposite angles)

$\therefore$ $\boxed{\angle b = 70°}$

line p || line q ...(Given)

On transversal m,

$\boxed{\angle c = 115°}$

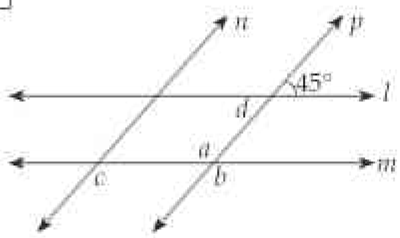
...(Corresponding angles theorem)

$\angle d + 115 = 180$ ...(Angles in linear pair)

$\therefore$ $\angle d = 180 - 115$

$\therefore$ $\boxed{\angle d = 65°}$

(3) In the given figure, line l || line m and line n || line p. Find $\angle a$, $\angle b$, $\angle c$ from the given measure of an angle.



Solution :

Name an angle as 'd' as shown in the figure,

$\angle d = 45°$ ...(Vertically opposite angles)

line l || line m ...(Given)

On transversal p,

$\angle d + \angle a = 180°$...(Interior angles theorem)

$\therefore$ $45 + \angle a = 180$

$\therefore$ $\angle a = 180 - 45$

$\therefore$ $\boxed{\angle a = 135°}$

$\angle b = \angle a$ ...(Vertically opposite angles)

$\therefore$ $\boxed{\angle b = 135°}$

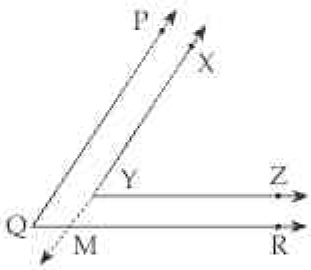
line n || line p

On transversal m,

$\angle c = \angle b$ ...(Corresponding angles theorem)

$\therefore$ $\boxed{\angle c = 135°}$

(4) In the given figure, sides of ∠PQR and ∠XYZ are parallel to each other. Prove that ∠PQR ≅ ∠XYZ.



Construction : Extend ray XY to intersect ray QR at point M, such that Q-M-R

Proof : line PQ || line XY ...(Given)

i.e. line PQ || line XM ...(X - Y - M)

On transversal QR,

∠PQR ≅ ∠XMR ...(i)

(Corresponding angles theorem)

line YZ || line QR ...(Given)

On transversal XM,

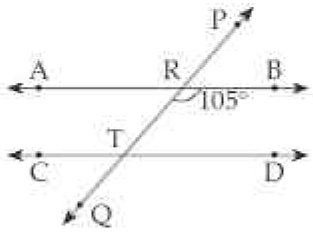
∠XYZ ≅ ∠XMR ...(ii)

(Corresponding angles theorem)

∠PQR ≅ ∠XYZ ...[From (i) and (ii)]

(5) In the given figure, line AB || line CD and line PQ is transversal. Measure of one of the angles is given. Hence find the measures of the following angles :

(i) ∠ART (ii) ∠CTQ (iii) ∠DTQ (iv) ∠PRB.



Solution :

(i) ∠BRT + ∠ART = 180°

...(Angles in linear pair)

$\therefore$ 105 + ∠ART = 180

$\therefore$ ∠ART = 180 – 105

$\therefore$ $\boxed{\angle ART = 75°}$

(ii) line AB || line CD ...(Given)

On transversal PQ,

∠ART = ∠CTQ

...(Corresponding angles theorem)

$\therefore$ $\boxed{\angle CTQ = 75°}$

(iii) line AB || line CD ...(Given)

On transversal PQ,

∠BRT = ∠DTQ

...(Corresponding angles theorem)

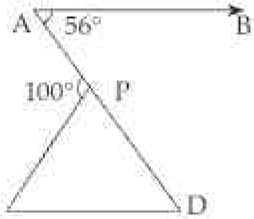
$\therefore$ $\boxed{\angle DTQ = 105°}$

(iv) ∠PRB = ∠ART...(Vertically opposite angles)

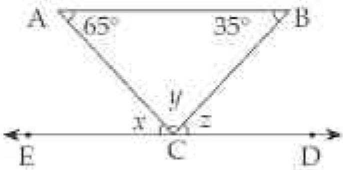
$\therefore$ $\boxed{\angle PRB = 75°}$

PROBLEMS FOR PRACTICE

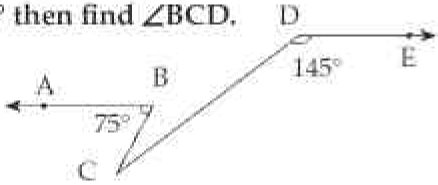
(1) **If AB ∥ CD, then find ∠PCD and ∠CPD from the adjoining figure.**



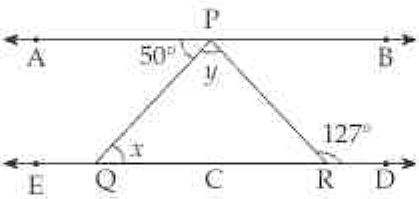
(2) **In the adjoining figure, measures of two angles are given. If line ED ∥ seg AB and E - C - D, then find the values of x, y and z.**



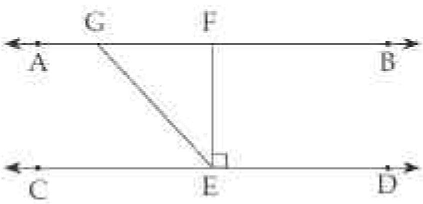
(3) **In given figure, AB ∥ DE, ∠ABC = 75° and ∠CDE = 145° then find ∠BCD.**



(4) **If AB ∥ CD, m∠APQ = 50° and m∠PRD = 127° find x an y.**



(5) **If AB ∥ CD, EF ⊥ CD and ∠GED = 126°. Find ∠AGE, ∠GEF, ∠FGE**



ANSWERS

(1) ∠CPD = 80°, ∠PCD = 44°

(2) $x = 65°$, $y = 80°$, $z = 35°$

(3) ∠BCD = 40°

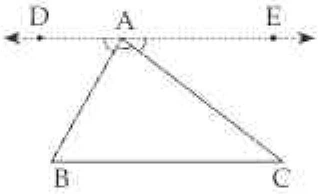
(4) $x = 50°$, $y = 77°$

(5) ∠AGE = 126°, ∠GEF = 36°, ∠FGE = 54°

Points to Remember:

Theorem - 4 : Angle sum property of a triangle

Statement : The sum of the measures of all angles of a triagle is 180°.



Given : ΔABC is any triangle

To prove : ∠BAC + ∠ABC + ∠ACB = 180°

Construction : Through point A, draw a line DE ∥ side BC, such that D - A - E.

Proof : line DE ∥ side BC

on transversal AB,

∠DAB = ∠ABC ...(i)

(Alternate angles theorem)

on transversal AC,

∠EAC = ∠ACB ...(ii)

(Alternate angles theorem)

∠DAB + ∠EAC = ∠ABC + ∠ACB

...[Adding equations (i) and (ii)]

Adding ∠BAC on both sides,

∴ ∠DAB + ∠EAC + ∠BAC = ∠ABC + ∠ACB + ∠BAC

∴ ∠DAB + ∠EAB = ∠ABC + ∠ACB + ∠BAC ...(Adding addition property)

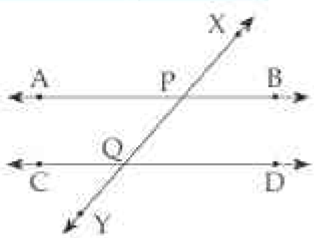
∴ ∠ABC + ∠ACB + ∠BAC = ∠DAB + ∠EAB

∴ **∠ABC + ∠ACB + ∠BAC = 180°**

...(Angles in linear pair)

Theorem - 5 : Interior angles test

Statement : If the interior angles formed by a transversal of two distinct lines are supplementary, then the two lines are parallel.



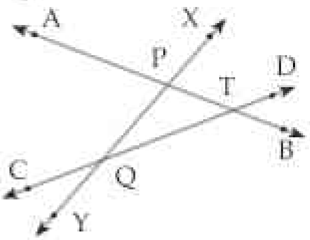
Given : A transversal XY intersects lines AB and CD such that ∠BPQ + ∠PQD = 180°

To prove : line AB ∥ side CD

Proof : (Indirect method)

Let us assume line AB is not parallel to line CD.

$\therefore$ They intersect each other at point T.



In ΔPQT,

$\angle TPQ + \angle PQT + \angle PTQ = 180°$

...(Angle sum property of a triangle)

$\therefore$ $\angle BPQ + \angle PQD + \angle PTQ = 180°$ ...(i)

[P - T - B, Q - T - D]

But $\angle BPQ + \angle PQD = 180°$ ...(ii) (Given)

$\therefore$ $\angle BPQ + \angle PQD + \angle PTQ = \angle BPQ + \angle PQD$ ...[From (i) and (ii)]

$\therefore$ $\angle PTQ = 0°$

$\therefore$ lines AB and CD are not two distinct lines

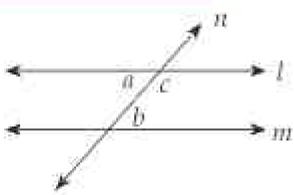
But, this contradicts the given that two lines AB and CD are distinct.

$\therefore$ Our assumption that line AB is not parallel to line CD is false.

$\therefore$ **line AB || side CD**

Theorem - 6 : Alternate Angles Test

Statement : If a pair of alternate angles formed by a transversal of two lines is congruent then the two lines are parallel.



Given : line n is a transversal for lines l and m

$\angle a = \angle b$

To Prove : line l || line m

Proof : $\angle a + \angle c = 180°$ ...(i)

(Angles in linear pair)

But, $\angle a = \angle b$ ...(ii) (Given)

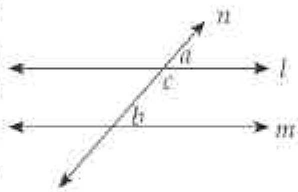
$\therefore$ $\angle b + \angle c = 180°$ [From (i) and (ii)]

$\therefore$ **line l || line m**

...(By Interior angles test)

Theorem - 7 : Corresponding Angles Test

Statement : If a pair of corresponding angles formed by a transversal of two lines is congruent then the two lines are parallel.



Given : line n is a transversal for lines l and m

$\angle a = \angle b$

To Prove : lines l || line m

Proof : $\angle a + \angle c = 180°$ ...(i)

(Angles in linear pair)

But, $\angle a = \angle b$ ...(ii) (Given)

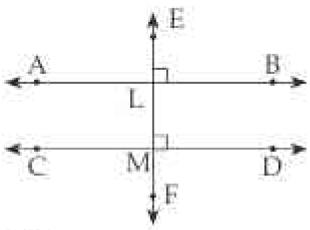
$\therefore$ $\angle b + \angle c = 180°$ [From (i) and (ii)]

$\therefore$ **line l || line m**

...(By Interior angles test)

Corollary 1 :

Statement : If a line is perpendicular to two lines in a plane, then the two lines are parallel to each other.



Given : line EF $\perp$ line AB

line EF $\perp$ line CD

To Prove : line AB || line CD

Proof : $\angle ELB = 90°$ ...(i) (Given)

$\angle LMD = 90°$ ...(ii) (Given)

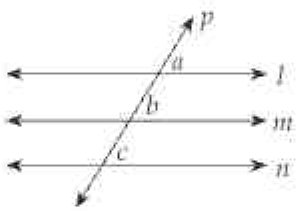
$\therefore$ $\angle ELB = \angle LMD$ [From (i) and (ii)]

$\therefore$ **line AB || line CD**

...(By Corresponding angles test)

Corollary 2 :

Statement : If two lines in a plane are parallel to a third line in the plane then those two lines are parallel to each other.



Given : line p is the transversal for lines l, m and n.

line l || line m

line m || line n

To Prove : line l || line n

Proof : line l || line m ...(Given)

on transversal p,

$\angle a = \angle b$ ...(i)

(Corresponding angles theorem)

$\therefore$ line m || line n ...(Given)

on transversal p,

$\angle b = \angle c$ ...(ii)

(Corresponding angles theorem)

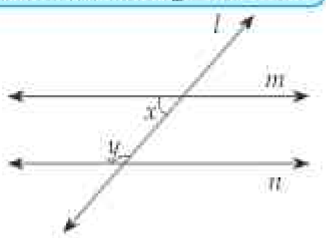
$\therefore$ $\angle a = \angle c$ [From (i) and (ii)]

$\therefore$ **line l || line n**

...(By Corresponding angles test)

PRACTICE SET - 2.2 *(Textbook Page No. 21)*

(1) In the given figure, $\angle y$ = 108° and $\angle x$ = 71°. Are the lines m and n parallel? Justify.



Proof : $\angle y = 108°$ ...(i)

$\angle x = 71°$ ...(ii)

Adding (i) and (ii)

$\angle x + \angle y = 71 + 108$

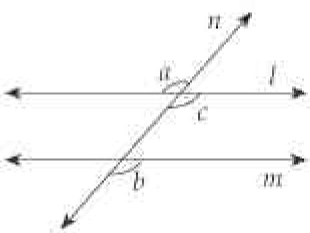
$\therefore$ $\angle x + \angle y = 179°$

$\angle x$ and $\angle y$ forms a pair of interior angles

Since, they are not supplementary

line m is not parallel to line n

(2) In the given figure, if $\angle a \cong \angle b$ then prove that line l || line m.



Proof : Consider 'c' as shown in the figure.

$\angle a = \angle b$ ...(i) (Given)

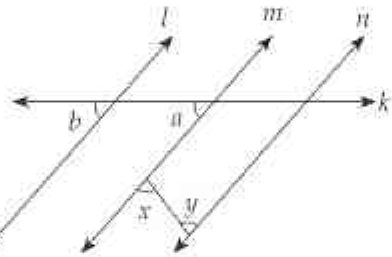
$\angle a = \angle c$ ...(ii) (Vertically opposite angles)

$\therefore$ $\angle b = \angle c$ ...[From (i) and (ii)]

$\therefore$ **line l || line m**

...(By Corresponding angles test)

(3) If $\angle a \cong \angle b$ and $\angle x \cong \angle y$, then prove that line l || line n.



Proof :

$\angle a \cong \angle b$ ...(Given)

$\therefore$ line l || line m ... (i)

(Corresponding angles test)

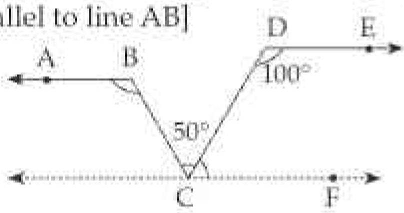
$\angle x \cong \angle y$ ...(Given)

$\therefore$ line n || line m... (ii) (Alternate angles test)

$\therefore$ **line l || line n** ... [From (i) and (ii)]

(4) In the following figure, if ray BA || ray DE, $\angle$C = 50° and $\angle$D = 100°. Find the measure of $\angle$ABC.

[Hint : Draw a line passing through point C and parallel to line AB]



Construction : Through C, draw a line parallel to line AB.

Solution : line AB || line CF. ...(i) (Construction)

line AB || line DE. ...(ii) (Given)

$\therefore$ line DE || line CF. ...[From (i) and (ii)]

On transversal DC,

$\angle EDC + \angle DCF = 180°$

...(Interior angles theorem)

$\therefore$ $100 + \angle DCF = 180$

$\therefore$ $\angle DCF = 180 - 100$

$\therefore$ $\angle DCF = 80°$

$\angle BCF = \angle BCD + \angle DCF$

...(Angle addition property)

$\therefore$ $\angle BCF = 50 + 80$

$\therefore$ $\angle BCF = 130°$

line AB || line CF

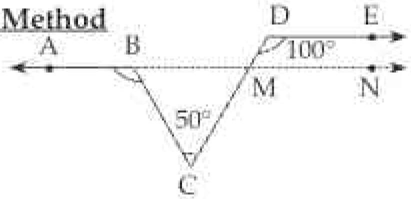
On transversal BC,

$\angle ABC = \angle BCF$

...(Alternate angles theorem)

$\therefore$ $\boxed{\angle ABC = 130°}$

Alternate Method



Construction : Extend ray AB to intersect seg DC at point M, such that D-M-C.

Solution : line AN ∥ line DE. ...(Construction)

On transversal DC,

$\angle EDC = \angle NMC$

...(Corresponding angles theorem)

$\therefore$ $\angle NMC = 100°$

$\therefore$ $\angle AMC + \angle NMC = 180°$

...(Angles in linear pair)

$\therefore$ $\angle AMC + 100 = 180$

$\therefore$ $\angle AMC = 80°$

In ΔBMC,

$\angle BMC + \angle BCM + \angle CBM = 180°$

...(Angle sum property of a triangle)

$\therefore$ $80 + 50 + \angle CBM = 180$

$\therefore$ $130 + \angle CBM = 180$

$\therefore$ $\angle CBM = 180 - 130$

$\therefore$ $\angle CBM = 50°$

$\therefore$ $\angle ABC + \angle CBM = 180°$

...(Angles in linear pair)

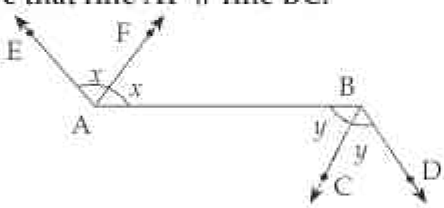
$\therefore$ $\angle ABC + 50 = 180$

$\therefore$ $\angle ABC = 180 - 50$

$\therefore$ $\boxed{\angle ABC = 130°}$

(5) In the given figure, ray AE ∥ ray BD. ray AF is the bisector of ∠EAB and ray BC is the bisector of ∠ABD.

Prove that line AF ∥ line BC.



Proof :

$\angle EAF = \angle BAF = x$ ...(i)

($\because$ ray AF bisects $\angle EAB$)

$\angle DBC = \angle ABC = y$ ...(ii)

($\because$ ray BC bisects $\angle ABD$)

ray AE ∥ ray BD ...(Given)

On transversal AB,

$\therefore$ $\angle EAB = \angle ABD$

...(Alternate angles theorem)

$\therefore$ $\angle EAF + \angle BAF = \angle ABC + \angle DBC$

...(Angles addition property)

$x + x = y + y$ ...[From (i) and (ii)]

$\therefore$ $2x = 2y$

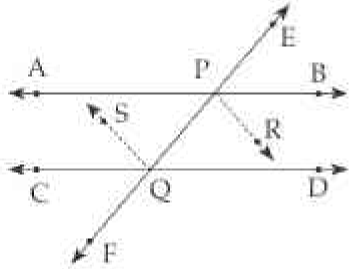
$\therefore$ $x = y$

$\therefore$ $\angle FAB = \angle ABC$

$\therefore$ **line AF ∥ line BC**

...(By Alternate angles test)

(6) A transversal EF of line AB and line CD intersects the lines at point P and Q respectively. Ray PR and ray QS are parallel and bisectors of ∠BPQ and ∠PQC respectively. Prove that line AB ∥ line CD.



Solution :

Let, $\angle BPR = \angle QPR = x$ ...(i)

($\because$ ray PR bisects $\angle BPQ$)

$\angle CQS = \angle PQS = y$ ...(ii)

($\because$ ray QS bisects $\angle PQC$)

$\therefore$ $\angle BPQ = \angle BPR + \angle QPR$

...(Angles addition property)

$\therefore$ $\angle BPQ = x + x$ ...[From (i)]

$\therefore$ $\angle BPQ = 2x$ ...(iii)

Similarly, we will get

$\therefore$ $\angle PQC = 2y$ ...(iv)

ray PR ∥ ray QS ...(Given)

On transversal PQ,

$\therefore$ $\angle QPR = \angle PQS$

...(Alternate angles theorem)

$\therefore$ $x = y$ ...[From (i) and (ii)]

$\therefore$ $2x = 2y$ ...(Multiplying throughout by 2)

$\therefore$ $\angle BPQ = \angle PQC$ ...[From (iii) and (iv)]

line AB ∥ line CD

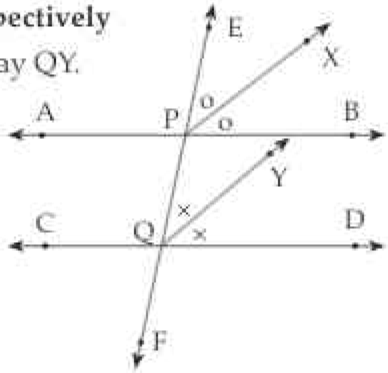
...(By Alternate angles test)

PROBLEMS FOR PRACTICE

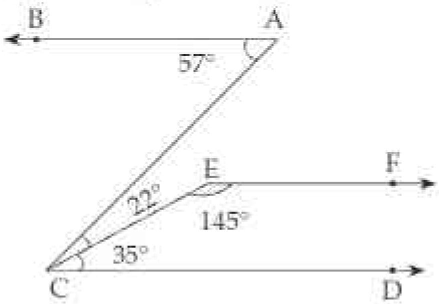
(1) **Line AB ∥ line CD and line EF is the transversal.**

ray PX and ray QY are the bisectors of ∠EPB and ∠PQD respectively

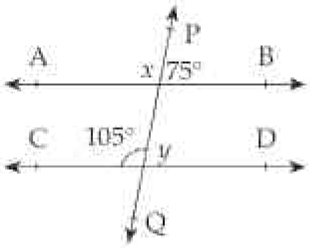
To prove : ray PX ∥ ray QY.



(2) **From the information given in the adjoining figure, prove ray AB ∥ ray EF**

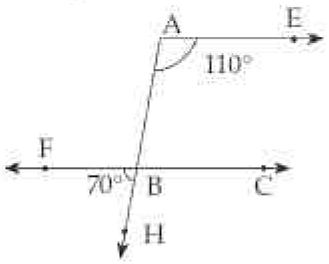


(3) **From the information given in the adjoining figure, prove line AB ∥ line CD**



(4) **∠EAB = 110°, ∠FBH = 70°.**

Prove that ray AE ∥ line BC



PROBLEM SET - 2 (Textbook Page No. 22)

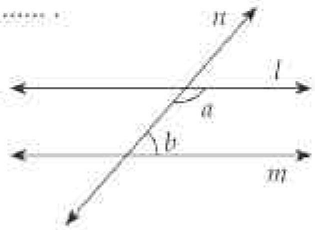
(1) **Select the correct alternative and fill in the blanks in the following statements.**

(i) If a transversal intersects two parallel lines then the sum of interior angles on the same side of the transversal is ....

(a) 0° (b) 90°

(c) 180° (d) 360°

Explanation :



line l ∥ line m

On transversal n,

$\angle a + \angle b = 180°$ ...(Interior angles theorem)

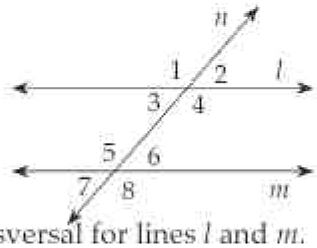
If a transversal intersects two parallel lines then the sum of interor angles on the same side of the transversal is **180°.**

(ii) The number of angles formed by a transversal of two lines is ....

(a) 2 (b) 4

(c) 8 (d) 16

Explanation :



line n is the transversal for lines l and m.

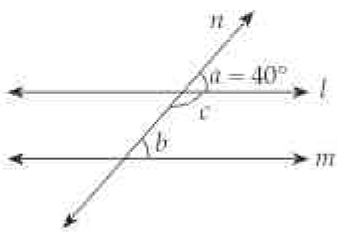
The number of angle formed by a transversal of two lines is **8.**

(iii) A transversal intersects two parallel lines. If the measure of one of the angles is 40° then the measure of its corresponding angle is ....

(a) 40° (b) 140°

(c) 50° (d) 180°

Explanation :



line l ∥ line m

On transversal n,

$\angle a = \angle b$ ...(Corresponding angles theorem)

$\angle a = 40°$

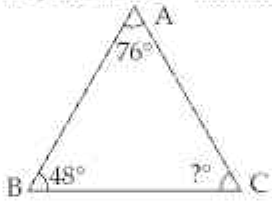
$\therefore$ $\angle b = 40°$

A transversal intersects two parallel lines. If the measure of one of the angles is 40° then the measure of its corresponding angle is **40°.**

(iv) In ΔABC, $\angle A = 76°$, $\angle B = 48°$, then $\angle C$ = ...

(a) 66° (b) 56°

(c) 124° (d) 28°



Explanation :

In ΔABC,

$\angle A + \angle B + \angle C = 180°$ ...(Sum of measures of all angles of a triangle is 180°)

$\therefore$ $76 + 48 + \angle C = 180$

$\therefore$ $124 + \angle C = 180$

$\therefore$ $\angle C = 180 - 124$

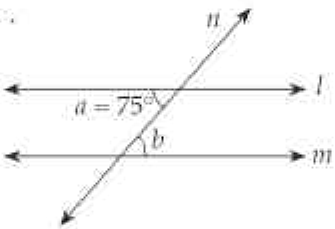
$\therefore$ $\angle C = 56°$

In ΔABC, $\angle A = 76°$, $\angle B = 48°$, then $\angle C$ = **56°**

(v) Two parallel lines are intersected by a transversal. If measure of one of the alternate interior angles is 75° then the measure of the other angle is ....

(a) 105° (b) 15°

(c) 75° (d) 45°



Explanation :

line l || line m

On transversal n,

$\angle a = \angle b$ ...(Alternate angles theorem)

$\angle a = 75°$

$\therefore$ $\angle b = 75°$

Two parallel lines are intersected by a transversal. If measure of one of the alternate interior angles is 75° then the measure of the other angle is **75°**.

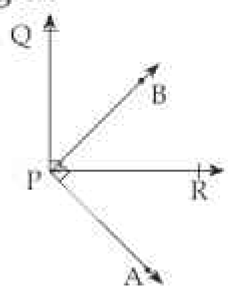
(2) Ray PQ and ray PR are perpendicular to each other. Points B and A are in the interior and exterior of ∠QPR respectively. Ray PB and ray PA are perpendicular to each other. Draw a figure showing all these rays and write :

(i) A pair of complementary angles

(ii) A pair of supplementary angles

(iii) A pair of congruent angles.

Solution :



(i) A pair of complementary angles

(a) $\angle QPB$ and $\angle BPR$

(b) $\angle BPR$ and $\angle RPA$

(ii) A pair of supplementary angles

(a) $\angle QPR$ and $\angle BPA$

(iii) A pair of congruent angles

(a) $\angle QPR$ and $\angle BPA$

(b) $\angle QPB$ and $\angle RPA$

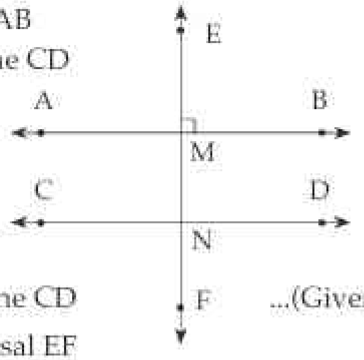
(3) Prove that, if a line is perpendicular to one of the two parallel lines, then it is perpendicular to the other line also.

Solution :

Given : Line AB || line CD and line EF intersects them at points M and N respectively.

line EF ⊥ line AB

To prove : line EF ⊥ line CD



Proof : line AB || line CD ...(Given)

On transversal EF

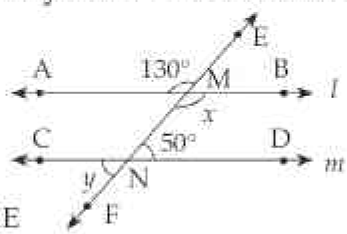
$\angle EMB = \angle MND$ ...(i) (Corresponding angles theorem)

But, $\angle EMB = 90°$ ...(ii) (Given)

$\therefore$ $\angle MND = 90°$ ...[From (i) and (ii)]

$\therefore$ **line EF ⊥ line CD**

(4) In the adjoining figure, measures of some angles are shown. Using the measures find the measures of ∠x and ∠y and hence show that line *l* || line *m*.



Proof :

$\angle BMN = \angle AME$ ...(Vertically opposite angles)

$\therefore$ $\boxed{\angle x = 130°}$

$\angle CNF = \angle MND$ ...(Vertically opposite angles)

$\therefore$ $\boxed{\angle y = 50°}$

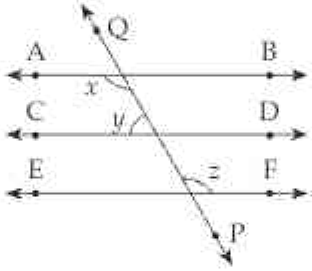
$\angle BMN + \angle MND = 130 + 50$

$\therefore$ $\angle BMN + \angle MND = 180°$

$\therefore$ **line *l* || line *m*** ...(By Interior angles test)

(5) In the given figure, line AB ∥ line CD ∥ line EF and line QP is their transversal.

If $\angle y = : \angle z = 3:7$ then find the measure of $\angle x$.



Solution :

$\angle y = \angle z = 3:7$ ...(Given)

Let the common multiple be a

$\angle y = 3a$ $\angle z = 7a$

line AB ∥ line EF ...(Given)

On transversal PQ,

$\therefore \angle x = \angle z$ ...(Alternate angles theorem)

$\angle x = 7a$

line AB ∥ line CD ...(Given)

On transversal PQ,

$\angle x + \angle y = 180°$ ...(Interior angles theorem)

$\therefore 7a + 3a = 180$

$\therefore 10a = 180$

$\therefore a = \frac{180}{10}$

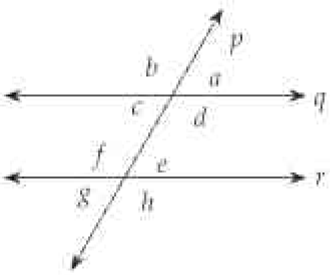
$\therefore a = 18$

$\angle x = 7a$

$\therefore \angle x = 7 \times 18$

$\therefore \boxed{\angle x = 126°}$

(6) In the adjoining figure, if line q ∥ line r and line p is their transversal and if $\angle a = 80°$, find the measrues of $\angle f$ and $\angle g$.



Solution :

$\angle a + \angle b = 180°$ ...(Angles in linear pair)

$\therefore 80 + \angle b = 180$

$\therefore \angle b = 100°$

$\angle c = \angle a$ ...(Vertically opposite angles)

$\therefore \angle c = 80°$

line q ∥ line r ...(Given)

On transversal p,

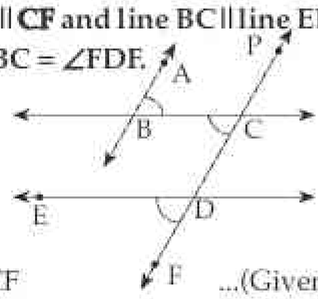
$\angle f = \angle b$ ...(Corresponding angles theorem)

$\therefore \boxed{\angle f = 100°}$

$\angle g = \angle c$ ...(Corresponding angles theorem)

$\therefore \boxed{\angle g = 80°}$

(7) In the adjoining AB ∥ CF and line BC ∥ line ED then prove that ∠ABC = ∠FDF.



Proof :

line AB ∥ line CF ...(Given)

On transversal BC,

$\therefore \angle ABC = \angle BCD$ ...(i) (Alternate angles theorem)

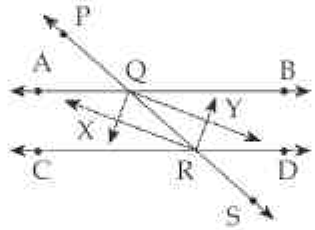
line BC ∥ line ED ...(Given)

On transversal CF,

$\therefore \angle FDE = \angle BCD$ ...(ii) (Corresponding angles theorem)

$\therefore$ **∠ABC = ∠FDE** ...[From (i) and (ii)]

(8) In the given figure, line PS is a transversal of parallel line AB and line CD. If ray QX, ray QY, ray RX, ray RY are angle bisectors, then prove that □QXRY is a rectangle.



Proof :

Let, $\angle AQX = \angle RQX = a$ ...(i)
$\angle BQY = \angle RQY = b$ ...(ii)
$\angle CRX = \angle QRX = c$ ...(iii)
$\angle DRY = \angle QRY = d$ ...(iv)
[Rays QX, QY, RX, RY are the bisectors of ∠AQR, ∠BQR, ∠QRC, ∠QRD respectively]

$\angle AQR + \angle BQR = 180°$ ...(Angles in linear pair)

$\therefore 2a + 2b = 180$

$\therefore a + b = 90$

$\therefore \angle RQX + \angle RQY = 90°$ ...[From (i) and (ii)]

$\therefore$ $\angle XQY = 90°$ ...(v)

(Angles addition property)

Similarly, we can prove

$c + d = 90$

$\angle XRY = 90°$ ...(vi)

line AB || line CD ...(Given)

On transversal PS,

$\angle AQR + \angle CRQ = 180°$

...(Interior angles theorem)

$\therefore$ $2a + 2c = 180$

$\therefore$ $a + c = 90$ ...(vii)

In ΔXQR,

$\angle QXR + \angle XQR + \angle XRQ = 180°$

...(Angle sum property of a triangle)

$\therefore$ $\angle QXR + a + c = 180$ ...[From (i) and (iii)]

$\therefore$ $\angle QXR + 90 = 180$ ...[From (vii)]

$\therefore$ $\angle QXR = 90°$ ...(viii)

Similarly, we can prove

$\angle QYR = 90°$ ...(ix)

In □ QXRY,

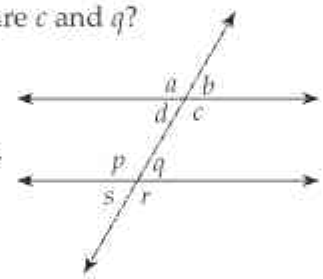
$\therefore$ $\angle XQY = \angle QXR = \angle XRY = \angle RYQ = 90°$

...[From (v), (vi), (viii) and (ix)]

$\therefore$ **□QXRY is a rectangle** ...(By definition)

MCQ's

(1) What kind of angles are c and q?



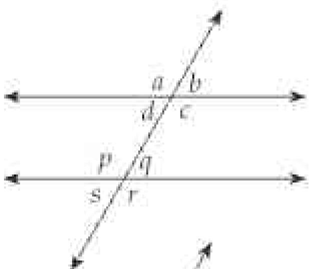
(A) Corresponding angles

(B) Linear pair angles

(C) Alternate angles

(D) Interior angles

(2) What kind of angles are b and q?



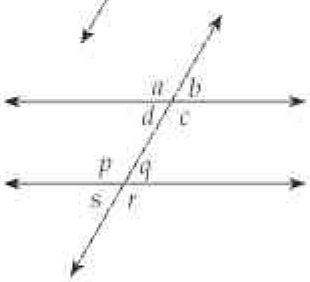
(A) Corresponding angles

(B) Linear pair angles

(C) Alternate angles

(D) Interior angles

(3) Which is a pair of alternate angles?

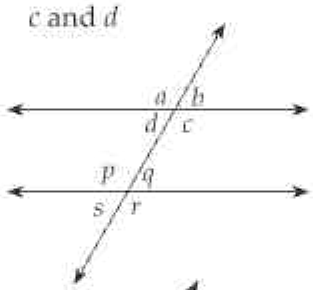


(A) c and q

(B) a and q

(C) d and q (D) c and d

(4) Which are the pairs of interior angles?



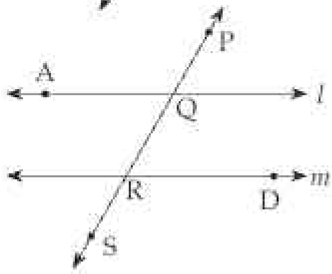
(A) a, b and a, d

(B) b, d and d, p

(C) c, q and q, r

(D) c, q and d, p

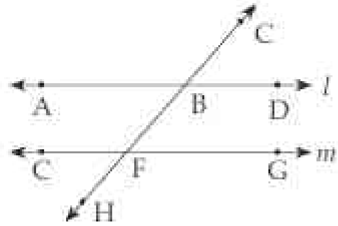
(5) If line l is parallel to line m and line PS is a transversal. If $\angle DRS = 65°$ what is $\angle AQR$?



(A) 65° (B) 125°

(C) 115° (D) 105°

(6) If line l || line m and $\angle GFB = 60°$, $\angle DBF =$ ....



(A) 60° (B) 120°

(C) 180° (D) 70°

(7) Interior angles theorem states that

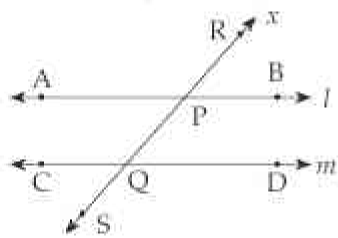
(A) If lines are parallel then interior angles are congruent

(B) If lines are parallel then interior angles are supplementary

(C) If interior angles are congruent then lines are parallel

(D) If interior angles are supplementary then lines are parallel

(8) line l || line m, $\therefore$ $\angle RPA = \angle PQC$ (.........)



(A) Corresponding angles test

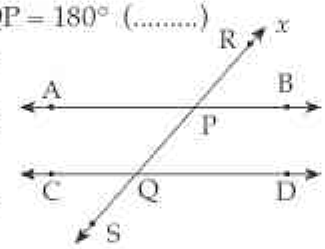
(B) Corresponding angles theorem

(C) Interior angles theorem

(D) Alternate angles test

(9) line AB || line CD,

$\therefore$ $m\angle APQ + m\angle CQP = 180°$ (.........)



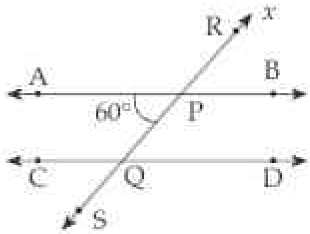
(A) Linear pair axiom

(B) Corresponding angles theorem

(C) Interior angles theorem

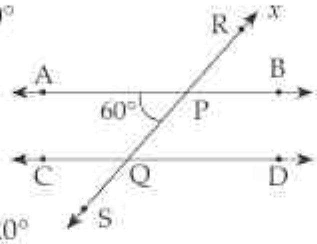
(D) Alternate angles theorem

(10) In the adjoining figure, if line AB || line CD, ∠PQD = ...



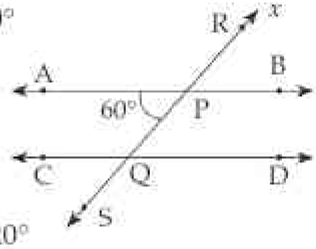
(A) 60°
(B) 120°
(C) 140° (D) 30°

(11) In the adjoining figure, if line AB || line CD, ∠CQS = ...



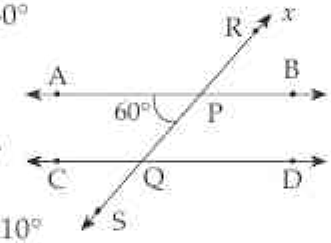
(A) 30° (B) 120°
(C) 140° (D) 60°

(12) In the adjoining figure, if line AB || line CD, ∠QPB = ...



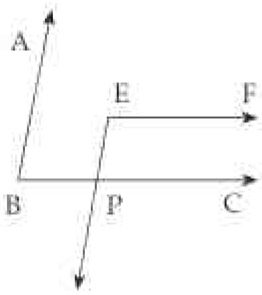
(A) 60° (B) 120°
(C) 140° (D) 30°

(13) In the adjoining figure, if line AB || line CD, ∠SQD = ...



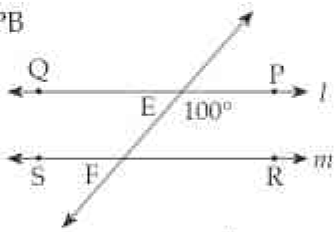
(A) 60° (B) 110°
(C) 120° (D) 30°

(14) Identify the pair of corresponding angles, if ray BA || ray EP and BC is transversal.



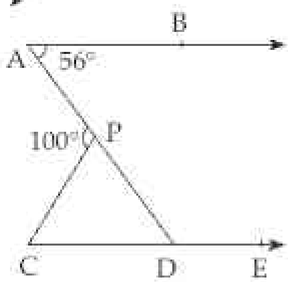
(A) ∠ABC and ∠BPE
(B) ∠ABP and ∠PEF
(C) ∠ABC and ∠EPC
(D) ∠ABC and ∠EPB

(15) If line l || line m, ∠PEF = 100°, ∠EFR = ...



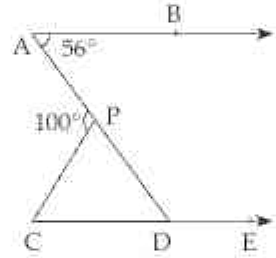
(A) 100°
(B) 80°
(C) 180° (D) 120°

(16) If ∠APC = 100° then ∠CPD = ...?



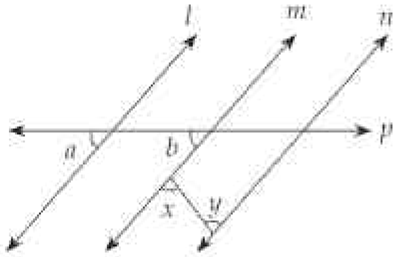
(A) 280° (B) 180°
(C) 80° (D) 56°

(17) Ray AB || ray CE, ∠APC = 100°, ∠BAD = 56°, ∠PCD = ...



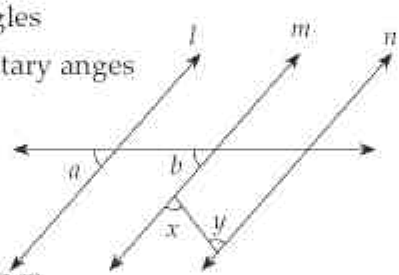
(A) 136° (B) 44°
(C) 100° (D) 124°

(18) In the, adjoining figure, ∠x and ∠y are which type of angles?



(A) Corresponding angles
(B) Alternate angles
(C) Interior angles
(D) Supplementary anges

(19) If ∠a = ∠b, then which of the following is true?



(A) Line l || line m
(B) Line l || line n
(C) Line m || line n (D) Line l ⊥ line m

(20) If a transversal intersects two parallel lines such that the ratio between the interior angles on one of its side is 2 : 7, then what is the measure of the greater angle?
(A) 20° (B) 140° (C) 150° (D) 120°

ANSWERS

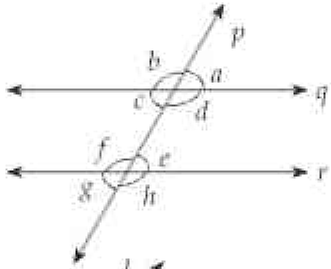
(1) (D)	(2) (A)	(3) (C)	(4) (D)
(5) (C)	(6) (B)	(7) (B)	(8) (B)
(9) (C)	(10) (A)	(11) (D)	(12) (B)
(13) (C)	(14) (C)	(15) (B)	(16) (C)
(17) (B)	(18) (B)	(19) (A)	(20) (B)

ASSIGNMENT - 2

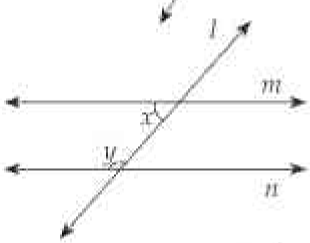
Time : 1 Hour **Marks : 20**

Q.1. Solve the following : **(10)**

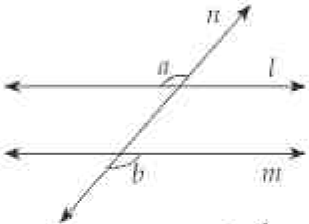
(1) line $q \parallel$ line r and line p is the transversal, $\angle a = 80°$, find $\angle f$ and $\angle g$:



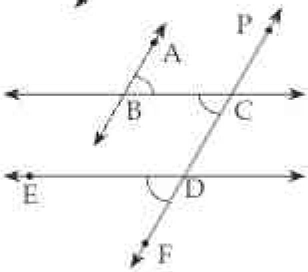
(2) In the given figure, $\angle y = 108°$ and $\angle x = 71°$ check whether lines m and n are parallel or not? Justify you answer:



(3) In the given figure, if $a = b$ then prove : line $l \parallel$ line m



(4) Line AB $\parallel$ line CF and line BC $\parallel$ line ED. Prove : $\angle ABC = \angle FDE$

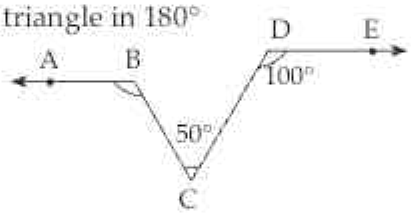


(5) Prove : If a line is perpendicular to one of the two parallel lines, then it is perpendicular to the other line also.

Q.2. Solve the following: **(6)**

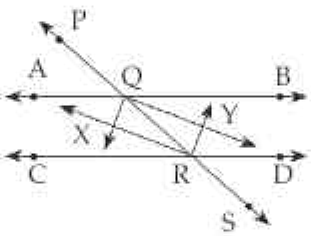
(1) Prove : The sum of the measures of all angles of a triangle in 180°

(2) If ray BA $\parallel$ ray DE, $\angle C = 50°$ and $\angle D = 100°$, find $\angle ABC$



Q.3. Solve the following: **(4)**

(1) In the given figure, line AB $\parallel$ line CD and line PS is a transversal. Ray QX, ray QY ray RX and ray RY are the angle bisectors. Prove □QXRY is a rectangle.



❖ ❖ ❖

3 Triangles

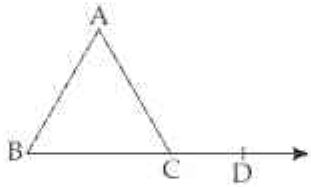
Points to Remember:

Theorem - 1

- **Remote interior angles theorem.**

Statement:

The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.



Given : $\angle ACD$ is an exterior angle of ΔABC.

To prove: $\angle ACD = \angle ABC + \angle BAC$

Proof : $\angle ACD + \angle ACB = 180^\circ$ ... (i)

(Linear pair axiom)

In Δ ABC, $\angle ABC + \angle BAC + \angle ACB = 180^\circ$ ...(ii)

(Sum of the measures of all angles of a triangle is 180°)

$\therefore$ $\angle ACD + \angle ACB = \angle ABC + \angle BAC + \angle ACB$ [From (i) and (ii)]

$\therefore$ $\angle ACD = \angle ABC + \angle BAC$ (Cancelling common angle)

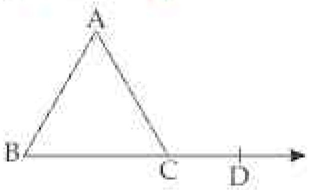
- **Very Important:** For any three non zero real numbers a, b and c. If $a > b + c$ then $a > b$ and $a > c$.

Theorem - 2

- **Exterior angle property of a triangle.**

Statement:

The exterior angle of a triangle is greater than each of its remote interior angles.



Given : $\angle ACD$ is an exterior angle of ΔABC.

To prove: (1) $\angle ACD > \angle ABC$,

(2) $\angle ACD > \angle BAC$

Proof : $\angle ACD$ is exterior angle of ΔABC(Given)

$\therefore$ $\angle ACD = \angle ABC + \angle BAC$

(Remote interior angles theorem)

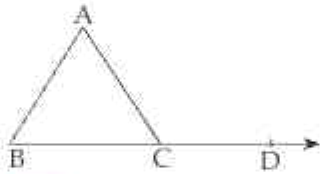
Measures of $\angle ACD$, $\angle ABC$ and $\angle BAC$ are non zero real numbers.

$\therefore$ $\angle ACD > \angle ABC$ and $\angle ACD > \angle BAC$

MASTER KEY QUESTION SET - 3

PRACTICE SET - 3.1 *(Textbook Page No. 27)*

(1) In the following figure, for $\angle ACD$ is an exterior angle of ΔABC. $\angle B = 40^\circ$, $\angle A = 70^\circ$. Find the measure of $\angle ACD$.



Solution:

$\angle ACD$ is an exterior angle of ΔABC. (Given)

$\angle ACD = \angle A + \angle B$ (Remote Interior angles theorem)

$\therefore$ $\angle ACD = 70 + 40$ (Given)

$\therefore$ $\boxed{\angle ACD = 110^\circ}$

(2) In ΔPQR, $\angle P = 70^\circ$, $\angle Q = 65^\circ$ then find $\angle R$.

Solution:

In ΔPQR, $\angle P + \angle Q + \angle R = 180^\circ$

(Sum of the measures of all angles of a triangle is 180°)

$\therefore$ $70 + 65 + \angle R = 180$

$\therefore$ $135 + \angle R = 180$

$\therefore$ $\angle R = 180 - 135$

$\therefore$ $\boxed{\angle R = 45^\circ}$

(3) The measures of angles of a triangle are x°, $(x - 20)^\circ$ and $(x - 40^\circ)$. Find the measure of each angle.

Solution:

As x°, $(x - 20)^\circ$ and $(x - 40)^\circ$ are measures of angles of a triangle,

$\therefore$ $x^\circ + (x - 20)^\circ + (x - 40)^\circ = 180^\circ$

(Sum of the measures of all angles of a triangle is 180°)

$\therefore$ $x + x - 20 + x - 40 = 180$

$\therefore$ $3x - 60 = 180$

$\therefore$ $3x = 180 + 60$

$\therefore$ $3x = 240$

$\therefore$ $x = \frac{240}{3} = 80$

Measure of first angle $= x = 80^\circ$

Measure of second angle = $(x-20)^\circ = (80-20)^\circ = 60^\circ$

Measure of third angle = $(x-40)^\circ = (80-40)^\circ = 40^\circ$

$\therefore$ **The measures of angles of the triangles are 80°, 60°, 40°.**

(4) The measure of one of the angles of a triangle is twice the measure of its smallest angle and the measure of the other is thrice the measure of the smallest angle. Find the measures of the three angles.

Solution:

Let the measure of the smallest angle of the triangle be x°

$\therefore$ Measure of other two angles will be $2x^\circ$ and $3x^\circ$

$x + 2x + 3x = 180$

(Sum of the measures of all angles of triangle is 180°)

$\therefore$ $6x = 180$

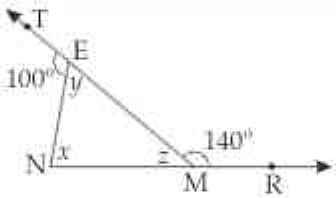
$\therefore$ $x = \frac{180}{6} = 30$

Measure of the first angle = $x = 30^\circ$

Measure of the second angle = $2x = 2 \times 30 = 60^\circ$

Measure of the third angle = $3x = 3 \times 30 = 90^\circ$

$\therefore$ **The measures of angles of the triangle are 30°, 60°, 90°.**

(5) 

In the adjoining figure, measures of some angles are given. Using the measures find the values of *x*, *y*, *z*.

Solution:

$\angle NEM + \angle TEN = 180^\circ$ (Linear pair axiom)

$\therefore$ $y + 100 = 180$

$\therefore$ $y = 180 - 100$

$\therefore$ $\boxed{y = 80^\circ}$

$\angle EMN + \angle EMR = 180^\circ$ (Linear pair axiom)

$\therefore$ $z + 140 = 180$

$\therefore$ $z = 180 - 140$

$\therefore$ $\boxed{z = 40^\circ}$

In ΔENM, $\angle ENM + \angle NEM + \angle EMN = 180^\circ$

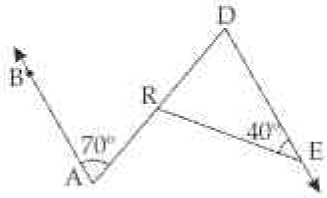
(Sum of the measures of all angles of a triangle is 180°)

$\therefore$ $x + y + z = 180$

$\therefore$ $x + 80 + 40 = 180$

$\therefore$ $x = 180 - 120$

$\therefore$ $\boxed{x = 60^\circ}$

(6) 

In the adjoining figure, line AB ∥ line DE. Find the measures of ∠DRE and ∠ARE using given measures of some angles.

Solution:

Ray AB ∥ ray DE (Given)

On transversal AD, $\angle BAD \cong \angle ADE$

(Alternate angles theorem.)

But $\angle BAD = 70^\circ$

$\therefore$ $\angle ADE = 70^\circ$

$\therefore$ $\angle RDE = 70^\circ$ .....(i) (A - R - D)

In ΔRDE, $\angle DRE + \angle RDE + \angle DER = 180^\circ$

(Sum of the measures of all angles of a triangle is 180°)

$\therefore$ $\angle DRE + 70 + 40 = 180$ [From (i) and given]

$\therefore$ $\angle DRE = 180 - 110$

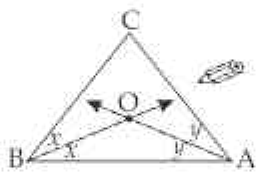
$\therefore$ $\boxed{\angle DRE = 70^\circ}$

$\therefore$ $\angle ARE + \angle DRE = 180^\circ$ (Linear pair axiom)

$\therefore$ $\angle ARE + 70 = 180$

$\therefore$ $\angle ARE = 180 - 70$

$\therefore$ $\boxed{\angle ARE = 110^\circ}$

(7) 

In ΔABC, bisector of $\angle A$ and $\angle B$ intersect at point O. If $m\angle C = 70^\circ$ then find $\angle AOB$.

Solution:

Let $\angle CBO = \angle OBA = x$ ... (i)

$\angle CAO = \angle OAB = y$ ... (ii)

(Ray BO and AO bisects ∠B and ∠A respectively.)

In ΔABC,

$\angle ABC + \angle BAC + \angle ACB = 180^\circ$

(Sum of the measures of all angles of a triangle is 180°)

$\therefore$ $\angle CBO + \angle OBA + \angle CAO + \angle OAB + \angle ACB = 180^\circ$ ...(Angle addition property)

$\therefore$ $x + x + y + y + 70 = 180$ ...From (i) and (ii)

$\therefore$ $2x + 2y = 180 - 70$

$\therefore$ $2(x + y) = 110$

$\therefore$ $x + y = 55$ ... (iii)

In ΔAOB,

$\angle AOB + \angle OBA + \angle OAB = 180^\circ$

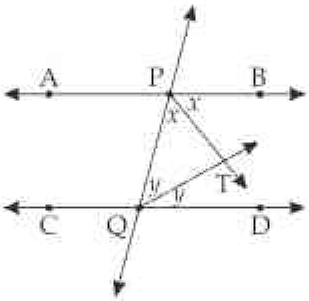
(Sum of the measures of all angles of a triangle is 180°)

$\therefore$ $\angle AOB + x + y = 180$ ... (From (iii))

$\therefore$ $\angle AOB + 55 = 180$

$\therefore$ $\angle AOB = 180 - 55$

$\therefore$ $\boxed{\angle \mathbf{AOB = 125^\circ}}$

(8) 

In the adjoining figure, line AB ∥ line CD and line PQ is transversal. Ray PT and ray QT bisects ∠BPQ and ∠PQD respectively. Prove that ∠PTQ = 90°.

Proof:

Let $\angle BPT = \angle TPQ = x$ ... (i)

$\angle PQT = \angle TQD = y$ ... (ii)

(Ray PT and QT bisects $\angle BPQ$ and $\angle PQD$ respectively.)

line AB ∥ line CD (Given)

On transversal PQ, $\angle BPQ + \angle PQD = 180^\circ$

...(Interior angles theorem)

$\therefore$ $\angle BPT + \angle TPQ + \angle PQT + \angle TQD = 180^\circ$ ...(Angle addition property)

$\therefore$ $x + x + y + y = 180$ ... From (i) and (ii)

$\therefore$ $2x + 2y = 180$

$\therefore$ $2(x + y) = 180$

$\therefore$ $x + y = 90$ ... (iii)

In ΔPTQ,

$\angle TPQ + \angle TQP + \angle PTQ = 180^\circ$

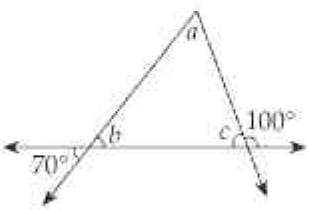
...(Sum of the measures of all angles of a triangle is 180°)

$\therefore$ $x + y + \angle PTQ = 180$

$\therefore$ $90 + \angle PTQ = 180$ ... [From (iii)]

$\therefore$ $\angle PTQ = 180 - 90$

$\therefore$ $\angle \mathbf{PTQ = 90^\circ}$

(9) 

Using the given information in the figure find ∠a, ∠b and ∠c.

Solution:

$\boxed{\angle b = 70^\circ}$ (Vertically Opposite angles)

$\angle c + 100 = 180$ (Linear pair axiom)

$\therefore$ $\angle c = 180 - 100$

$\therefore$ $c = 180 - 100$

$\therefore$ $\boxed{\angle c = 80^\circ}$ ...(ii)

In the given triangle

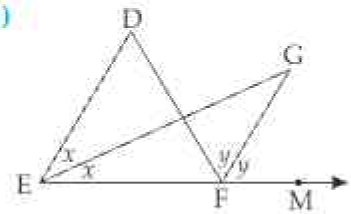
$\angle a + \angle b + \angle c = 180^\circ$

[Sum of the measures of all angles of a triangle is 180°]

$\therefore$ $\angle a + 70 + 80 = 180$

$\therefore$ $\angle a = 180 - 150$

$\therefore$ $\boxed{\angle a = 30^\circ}$

(10) 

In the adjoining figure, line DE ∥ line GF ray EG and ray FG are bisectors of ∠DEF and ∠DFM respectively.

Prove that, (i) $\angle DEG = \frac{1}{2} \angle EDF$ (ii) EF = FG.

Proof:

Let $\angle DEG = \angle GEF = x$ ...(i)

$\angle DFG = \angle GFM = y$ ...(ii)

(Ray EG and FG bisects $\angle DEF$ and $\angle DFM$ respectively.)

$\angle DEF = \angle DEG + \angle GEF$

...(Angle addition property)

$= x + x$

$\therefore$ $\angle DEF = 2x$ ...(iii)

seg ED ∥ seg FG

On transversal DF,

$\angle EDF \cong \angle DFG$ ...(Alternate angles theorem)

$\therefore$ $\angle EDF = \angle DFG = y$ ...(iv)

In transversal EM,

$\angle DEF \cong \angle GFM$ ...(Corresponding angles theorem)

$\therefore$ $\angle DEF = \angle GFM = y$ ...(v)

$\therefore$ $\angle DEF = \angle EDF$ ...[From (iii), (iv)]

$\therefore$ $y = 2x$ ...(vi)

$\angle GFM$ is an exterior angle of ΔGEF.

$\therefore$ $\angle GFM = \angle GEF + \angle EGF$ ...(Remote interior angles theorem)

$\therefore$ $y = x + \angle EGF$

$\therefore$ $2x = x + \angle EGF$ (From iv)

$\therefore$ $\angle EGF = 2x - x$

$\therefore$ $\angle EGF = x$ ... (vii)

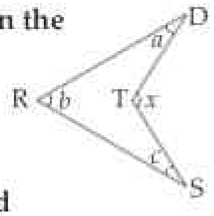
In ΔEFG, $\angle GEF \cong \angle EGF$ ...[From (i), (vii)]

seg EF $\cong$ seg FG ...(Converse of isosceles triangle theorem)

$\therefore$ **EF = FG**

PROBLEMS FOR PRACTICE

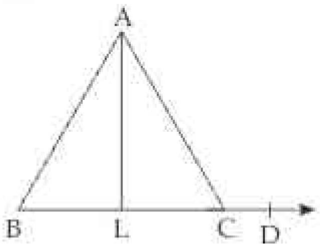
(1) **Based on information given in the figure, prove $x = a + b + c$.**



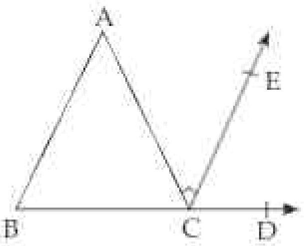
(2) **The exterior angles, obtained by producing the base of a triangle both ways are 104° and 136°. Find the measures of all angles of the triangle.**

(3) **If the sides of the triangle are produced in order, prove that sum of the exterior angles so formed is equal to four right angles.**

(4) **The side BC of $\triangle ABC$ is produced such that D is on ray BC. The bisector of $\angle A$ meets BC in point L. Prove that $m\angle ABC + m\angle ACD = 2m\angle ALC$.**



(5) **In the following figure, seg AC $\perp$ ray CE. $m\angle A : m\angle B : m\angle C = 3 : 2 : 1$. Find $m\angle ECD$.**



ANSWER

(2) 76°, 44°, 60° **(5)** $m\angle ECD = 60°$

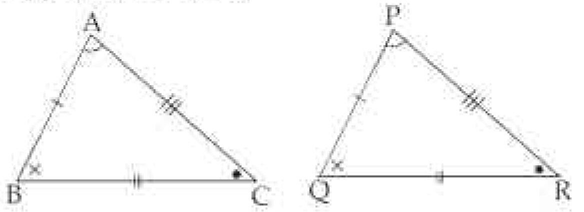
Points to Remember:

Congruence of triangles:

Now, we introduce the concept of congruence of two triangles.

The general idea about congruence of triangles is that, "If two triangles are exactly of the same shape and size then the triangles are congruent to each other."

Now consider ΔABC and ΔPQR in the following manner.



ΔABC and ΔPQR are of same shape and size therefore they are congruent.

When you place them one above the other vertex A will coincide with vertex P

Symbolically we write,

$A \leftrightarrow P$ (i.e. A corresponds to P or P corresponds to A)

$B \leftrightarrow Q$

$C \leftrightarrow R$

In mathematical language, such pairing of vertices is called a one to one correspondence between the vertices of two triangles.

The above one to one correspondence is written as $ABC \leftrightarrow PQR$.

There are six possible correspondences between the vertices of the ΔABC and ΔPQR.

Namely,

$ABC \leftrightarrow PQR$	$ABC \leftrightarrow RQP$
$ABC \leftrightarrow PRQ$	$ABC \leftrightarrow QPR$
$ABC \leftrightarrow RPQ$	$ABC \leftrightarrow QRP$

But ΔABC will exactly fit on ΔPQR with $ABC \leftrightarrow PQR$.

ΔABC will not exactly fit on ΔPQR with $ABC \leftrightarrow QRP$ or $ABC \leftrightarrow PRQ$.

$\therefore$ $\Delta ABC \cong \Delta PQR$ under $ABC \leftrightarrow PQR$.

Therefore, correspondence plays a very important role in congruence of triangles.

Now **we define congruence of two triangles.**

"If there exists atleast one, one to one correspondence between the vertices of any two triangles such that corresponding sides and angles of one triangle are congruent with corresponding sides and angles of other triangle then these two triangles are congruent."

- **Properties of congruent triangles:**

The following are the properties of congruent triangles:

(i) **Reflexivity :** Every triangle is congruent to itself i.e. $\Delta ABC \cong \Delta ABC$.

(ii) **Symmetry :** If $\Delta ABC \cong \Delta PQR$ then $\Delta PQR \cong \Delta ABC$.

(iii) **Transitivity :** If $\Delta ABC \cong \Delta PQR$ and $\Delta PQR \cong \Delta XYZ$ then $\Delta ABC \cong \Delta XYZ$.

- **Test of congruence of two triangles:**

We know, for congruence between two triangles, six elements (3 sides and 3 angles) of one triangle are congruent to corresponding six elements of other triangle.

But to establish or to prove the congruence of two triangles, it is not necessary to know or prove the congruence of six elements.

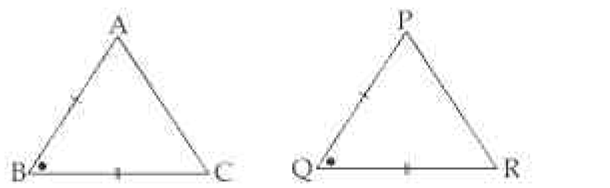
If we are given only the congruence of some particular three elements, then the triangles will be congruent to each other.

Therefore out of six conditions of congruence between two triangles, three particular conditions are sufficient.

We shall state these sufficient conditions as tests of congruence.

- **Test of congruence:**

(1) Side-Angle-Side (SAS) Test: For a given one-to-one correspondence between the vertices of any two triangles when two sides and the angle included by them, of one triangle are respectively congruent to corresponding two sides and the angle included by them, of another triangle, then the two triangles are congruent.



In ΔABC and ΔPQR,

For the correspondence

$ABC \leftrightarrow PQR$,

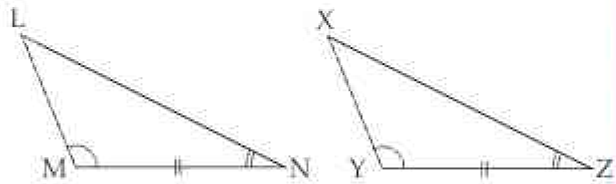
If, side $AB \cong$ side PQ

$\angle ABC \cong \angle PQR$

side $BC \cong$ side QR then

$\Delta ABC \cong \Delta PQR$.

(2) Angle-Side-Angle (ASA) Test: For a given one-to-one correspondence between the vertices of any two triangles, when two angles are included side of one triangle congruent to corresponding two angles and included side of another triangle then the two triangles are congruent.



In ΔLMN and ΔXYZ,

For the correspondence

$LMN \leftrightarrow XYZ$,

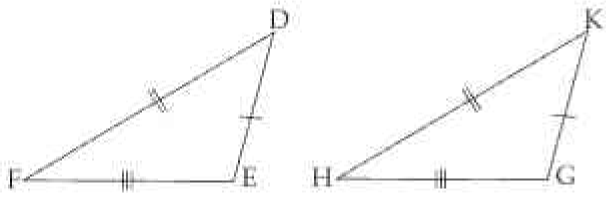
If $\angle LMN \cong \angle XYZ$

side $MN \cong$ side YZ

$\angle MNL \cong \angle YZX$

then $\Delta LMN \cong \Delta XYZ$.

(3) Side-Side-Side (SSS) Test: For a given one-to-one correspondence between the vertices of any two triangles when three sides of one triangle are respectively congruent to corresponding three sides of another triangle, then the two triangles are congruent.



In ΔDEF and ΔKGH,

For the correspondence

$DFE \leftrightarrow KGH$,

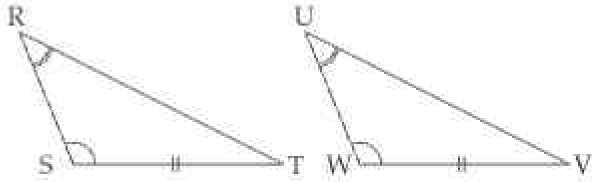
If, side $DE \cong$ side KG

side $FE \cong$ side HG

side $DF \cong$ side KH

then $\Delta DEF \cong \Delta KGH$.

(4) Side-Angle-Angle (SAA) Test: For a given one-to-one correspondence between the vertices of any two triangles, when a side, an angle adjacent to it and the angle opposite to it of one triangle are respectively congruent to corresponding side, an angle adjacent to it and the angle opposite to it, of another triangle, then the two triangles are congruent.



In ΔRST and ΔUWV,

For the correspondence

RST ↔ UWV,

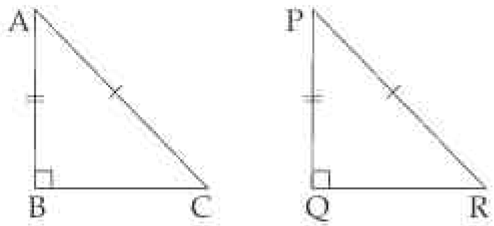
If, side ST ≅ side WV,

∠RST ≅ ∠UWV

∠SRT ≅ ∠WUV

then ΔRST ≅ ΔUWV.

(5) Hypotenuse - Side Test: For a given one-to-one correspondence between the vertices of any two triangles, two right angled triangles are congruent of the hypotenuse and a side of one triangle are congruent to the hypotenuse and corresponding side of the other triangle.



In ΔABC and ΔPQR,

For the correspondence

ABC ↔ PQR,

If, ∠ABC = ∠PQR = 90°

Hypotenuse AC ≅ Hypotenuse PR

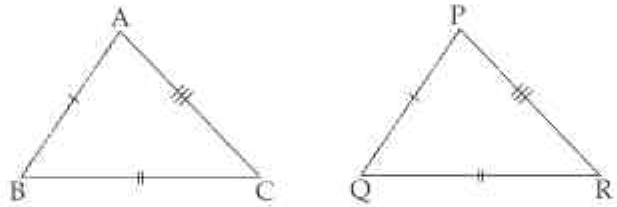
Side AB ≅ side PQ

then ΔABC ≅ ΔPQR.

PRACTICE SET - 3.2 *(Textbook Page No. 31)*

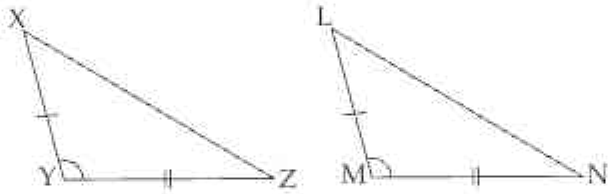
(1) In each of the examples given below, a pair of triangle is shown. Equal parts of triangles in each pair are marked with the same signs. Observe the figures and state the test by which the triangles in each pair are congruent.

(i)



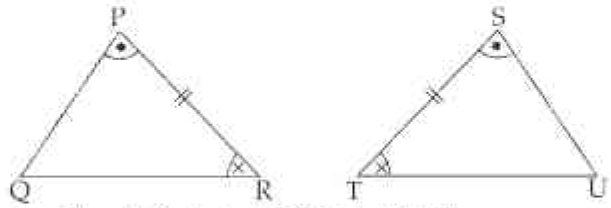
∴ By **SSS** test ΔABC ≅ ΔPQR

(ii)



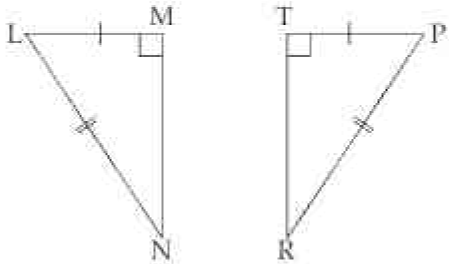
∴ By **SAS** test ΔXYZ ≅ ΔLMN

(iii)



∴ By **ASA** test ΔPRQ ≅ ΔSTU

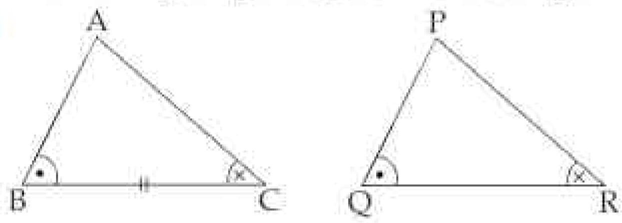
(iv)



∴ By **Hypotenuse-side** test, ΔLMN ≅ ΔPTR

(2) Observe the information shown in pairs of triangles given below. State the test by which the two triangles are congruent. Write the remaining congruent parts of the triangles.

(i)



Solution:

From the information given in the figure

In ΔABC and ΔPQR

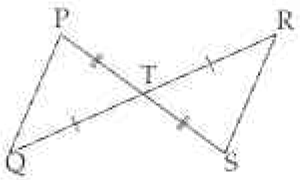
∴	∠ABC ≅ ∠PQR	(Given)
	side BC ≅ side QR	(Given)
∴	∠ACB ≅ ∠PRQ	(Given)
∴	ΔABC ≅ ΔPQR	(ASA test)

$\therefore$ $\angle BAC \cong \boxed{\angle \mathbf{QPR}}$ ... (c.a.c.t)

seg $AB \cong$ $\boxed{\text{seg } PQ}$ ... (c.s.c.t)

$\boxed{\text{seg } AC}$ $\cong$ seg PR (c.s.c.t.)

(ii)



Solution:

From the information given in the figure.

In ΔPTQ and ΔSTR

seg $PT \cong$ seg ST (Given)

$\angle PTQ \cong \angle STR$ (Vertically opposite angles)

seg $TQ \cong$ seg TR (Given)

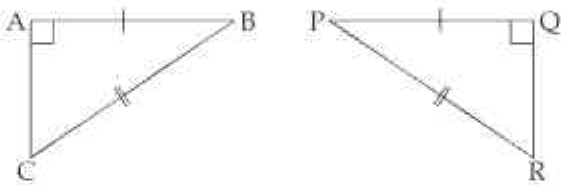
$\therefore$ $\Delta PTQ \cong \Delta STR$ (SAS test)

$\therefore$ $\angle TPQ \cong \boxed{\angle \mathbf{TSR}}$ and $\boxed{\angle \mathbf{TQP}}$ $\cong \angle TRS$

(c.a.c.t.)

seg $PQ \cong$ $\boxed{\textbf{seg SR}}$ (c.s.c.t.)

(3) **From the information shown in the figure, state the test assuring the congruence of ΔABC and ΔPQR. Write the remaining congruent parts of the triangles.**



Solution:

In ΔABC and ΔQPR,

$\angle BAC = \angle PQR = 90^\circ$ (Given)

Hypotenuse $BC \cong$ Hypotenuse PR (Given)

side $AB \cong$ side QP (Given)

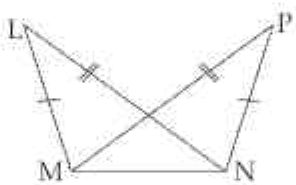
$\therefore$ $\Delta ABC \cong \Delta QPR$ (Hypotenuse - side test)

side $AC \cong$ side QR (c.s.c.t.)

$\therefore$ $\angle ABC \cong \angle QPR$ and $\angle ACB \cong \angle QRP$

(c.a.c.t.)

(4)



As shown in the following figure, in ΔLMN and ΔPNM, $LM = PN$, $LN = PM$. Write the test which assures the congruence of the two triangles. Write their remaining congruent parts.

Solution:

In ΔLMN and ΔPNM,

side $LM \cong$ side PN (Given)

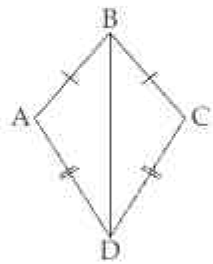
side $LN \cong$ side PM (Given)

side $MN \cong$ side NM (Common side)

$\Delta LMN \cong \Delta PNM$ (SSS test)

$\left.\begin{array}{l} \angle LMN \cong \angle PNM \\ \angle LNM \cong \angle PMN \\ \angle NLM \cong \angle MPN \end{array}\right\}$ (c.a.c.t.)

(5)



In the adjoining figure, seg $AB \cong$ seg CB and seg $AD \cong$ seg CD. Prove that $\Delta ABD \cong \Delta CBD$.

Proof:

In ΔABD and ΔCBD,

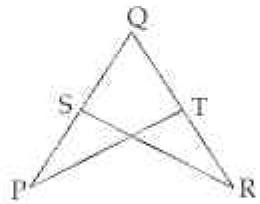
side $AB \cong$ side CB (Given)

side $AD \cong$ side CD (Given)

side $BD \cong$ side BD (Common side)

$\therefore$ $\mathbf{\Delta ABD \cong \Delta CBD}$ (SSS test)

(6)



In the adjoining figure, $\angle P \cong \angle R$ and seg $PQ \cong$ seg RQ then prove that $\Delta PQT \cong \Delta RQS$.

Proof:

In ΔPQT and ΔRQS,

$\angle P \cong \angle R$ (Given)

side $PQ \cong$ side RQ (Given)

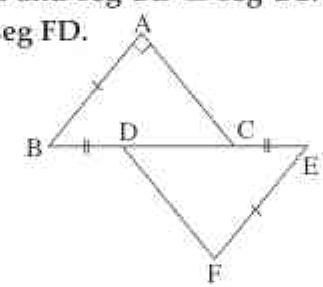
$\angle Q \cong \angle Q$ (Common angle)

$\therefore$ $\mathbf{\Delta PQT \cong \Delta RQS}$ (ASA test of congruency)

PROBLEMS FOR PRACTICE

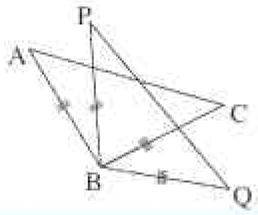
(1) **In ΔPQR, $PQ = PR$ and ray $PS \perp$ side QR at points S. Prove ray PS bisects $\angle QPR$.**

(2) **In the following figure, seg $BA \perp$ seg CA, B-D-C-E, seg $AB \cong$ seg FE and seg $BD \cong$ seg EC. Prove that seg $AC \cong$ seg FD.**



(3) If altitudes drawn from two vertices of a triangle to the opposite sides are equal, then prove that triangle is isosceles.

(4) In the following figure, seg AB ≅ seg PB, seg BC ≅ seg BQ, ∠ABP ≅ ∠CBQ. Show that △ABC ≅ △PBQ and seg AC ≅ seg PQ.



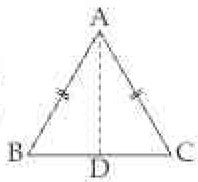
Points to Remember:

Theorem - 3

- **Isosceles Triangle Theorem.**

Statement:

If two sides of a triangle are congruent then the angles opposite to them are congruent.



Given : In ΔABC, side AB ≅ side AC.

To prove: ∠ABC ≅ ∠ACB

Construction : Draw bisector of ∠BAC intersecting side BC at point D, such that B - D - C.

Proof :

In △ABD and △ACD,

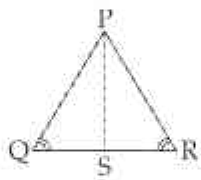
	side AB ≅ side AC	(Given)
	∠BAD ≅ ∠CAD	(Construction)
	side AD ≅ side AD	(Common side)
∴	△ABD ≅ △ACD	(SAS test)
∴	∠ABD ≅ ∠ACD	(c.a.c.t.)
i.e.	**∠ABC ≅ ∠ACB**	(B - D - C)

Theorem - 4

- **Converse of Isosceles Triangle Theorem.**

Statement:

If two angles of a triangle are congruent then the sides opposite to them are congruent.



Given : In ΔPQR, ∠Q ≅ ∠R

To prove: side PQ ≅ side PR

Construction : Draw bisector of ∠QPR intersecting side QR at point S, such that Q - S - R.

Proof :

In △PQS and △PRS,

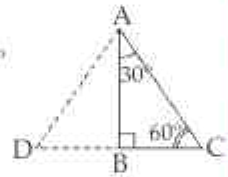
	side PS ≅ side PS	(Common side)
	∠Q ≅ ∠R	Given)
	∠QPS ≅ ∠RPS	(Construction)
∴	△PQS ≅ △PRS	(SAA test)
∴	**side PQ ≅ side PR**	(c.s.c.t.)

Theorem - 5

- **30° - 60° - 90° Theorem.**

Statement:

In a right angled triangle if two acute angles are 30° and 60° then side opposite to 30° is half of the hypotenuse and side opposite to 60° is $\frac{\sqrt{3}}{2}$ times the hypotenuse



Given : In ΔABC, ∠BAC = 30°, ∠ABC = 90° and ∠C = 60°

To prove: BC = ½AC

$$AB = \frac{\sqrt{3}}{2} AC$$

Construction : Take a point D on ray CB such that, C - B - D and BC = BD.

Proof :

In △ABC and △ABD,

	side AB ≅ side AB	(Common side)
	∠ABC ≅ ∠ABD	(Each 90°)
	side BC ≅ side BD	(Construction)
∴	△ABC ≅ △ABD	(SAS test)
	∠C ≅ ∠D	(c.a.c.t.)
	But ∠C = 60°	(Given)
∴	∠D = 60°	

In △ADC, ∠C = 60°, ∠D = 60°

∴ ∠DAC = 60° (Remaining angle of a triangle)

∴ △ADC is an equilateral triangle

(∵ an equiangular triangle is an equilateral triangle)

∴ AD = CD = AC ... (i) (Sides of equilateral △)

$BC = \frac{1}{2} CD$ (Construction)

∴ $\boxed{BC = \frac{1}{2} AC}$ ... (ii) [From (i)]

In △ABC, ∠ABC = 90° (Given)

$\therefore$ $AC^2 = AB^2 + BC^2$ (Pythagoras theorem)

$\therefore$ $AC^2 = AB^2 + \left(\frac{1}{2}AC\right)^2$ [From (ii)]

$\therefore$ $AC^2 = AB^2 + \frac{1}{4}AC^2$

$\therefore$ $AC^2 - \frac{1}{4}AC^2 = AB^2$

$\therefore$ $\frac{4AC^2 - AC^2}{4} = AB^2$

$\therefore$ $AB^2 = \frac{3}{4}AC^2$

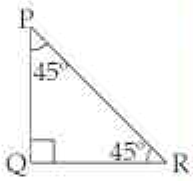
$\therefore$ $\boxed{\mathbf{AB = \frac{\sqrt{3}}{2}AC}}$ (Taking square roots)

Theorem - 6

- **45° - 45° - 90° Theorem.**

Statement:

In a right angled triangle, if each acute angle is 45° then side opposite to 45° is $\frac{1}{\sqrt{2}}$ times the hypotenuse.



Given : In ΔPQR, $\angle Q = 90°$, $\angle P = \angle R = 45°$

To prove: $PQ = QR = \frac{1}{\sqrt{2}}PR$

Proof :

In ΔPQR, $\angle P \cong \angle R$ (Given)

seg $PQ \cong$ seg QR (Converse of isosceles triangle theorem)

$\therefore$ $PQ = QR$ ...(i)

In ΔPQR, $\angle Q = 90°$ (Given)

$PR^2 = PQ^2 + QR^2$ (Pythagoras theorem)

$\therefore$ $PR^2 = PQ^2 + PQ^2$ [from (i)]

$\therefore$ $PR^2 = 2PQ^2$

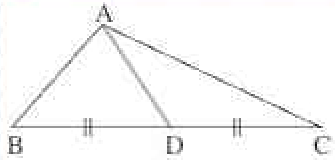
$\therefore$ $PR = \sqrt{2}\,PQ$

$\therefore$ $PQ = \frac{1}{\sqrt{2}}PR$ ...(ii)

$\therefore$ $\mathbf{PQ = QR = \frac{1}{\sqrt{2}}PR}$ [from (i), (ii)]

Median

Definition: A segment joining vertex of a triangle and midpoint of its opposite side is called median of a triangle.



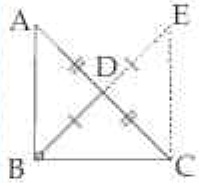
In the adjoining figure, seg AD is median on side BC

Theorem - 7

- **Property of median drawn on hypotenuse of a right angled triangle.**

Statement:

In a right angled triangle length of the median drawn on hypotenuse is half of the hypotenuse.



Given : In ΔABC, $\angle ABC = 90°$ seg BD is median drawn on hypotenuse AC.

To prove : $BD = \frac{1}{2}AC$

Construction : Take a point E on ray BD such that B-D-E and BD = DE. Draw seg CE.

Proof : In ΔADB and ΔCDE,

seg $AD \cong$ seg CD (D is midpoint of seg AC)

$\angle ADB \cong \angle CDE$ (Vertically opposite angles)

seg $BD \cong$ seg ED (construction)

$\therefore$ $\Delta ADB \cong \Delta CDE$ (SAS test)

$\therefore$ $\angle BAD \cong \angle ECD$ ... (i) (c.a.c.t.)

$\therefore$ seg $AB \cong$ seg CE ... (ii) (c.s.c.t.)

seg $AB \parallel$ seg CE [Alternate angles Test]

On transversal BC,

$\therefore$ $\angle ABC + \angle ECB = 180$ (Interior angles theorem)

$\therefore$ $90° + \angle ECB = 180$

$\therefore$ $\angle ECB = 180 - 90 = 90$

In ΔABC and ΔECB

seg $AB \cong$ seg EC [from (ii)]

$\angle ABC \cong \angle ECB$ (Each measures 90°)

seg $BC \cong$ seg CB (Common side)

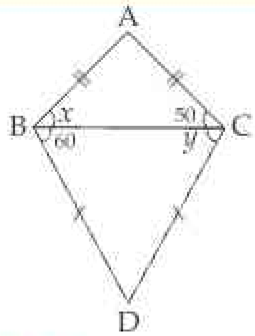
$\therefore$ $\Delta ABC \cong \Delta ECB$ (SAS test)

$\therefore$ seg $AC \cong$ seg EB ... (iii) (c.s.c.t.)

$BD = \frac{1}{2}BE$ (Construction)

$\therefore$ $\boxed{\mathbf{BD = \frac{1}{2}AC}}$ [from (iii)]

PRACTICE SET - 3.3 *(Textbook Page No. 38)*

(1) 

Find the values of x and y using the information shown in figure. Find the measure of ∠ABD and m∠ACD.

Solution:

In ΔABC, side AB ≅ side AC (Given)

∴ $\angle ABC \cong \angle ACB$ (Isosceles triangle theorem)

∴ $\boxed{x = 50^\circ}$ ($\because \angle ACB = 50^\circ$)

In ΔDBC, side DB ≅ side DC (Given)

∴ $\angle DBC \cong \angle DCB$ (Isosceles triangle theorem)

∴ $\boxed{y = 60^\circ}$ ($\because \angle DBC = 60$)

$\angle ABD = \angle ABC + \angle DBC$ (Angle addition property)

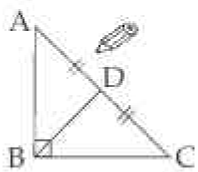
∴ $\angle ABD = 50 + 60$

∴ $\boxed{\angle ABD = 110^\circ}$

$\angle ACD = \angle ACB + \angle DCB$ (Angle addition property)

∴ $\angle ACD = 50 + 60$

∴ $\boxed{\angle ACD = 110^\circ}$

(2) 

The length of hypotenuse of a right angled triangle is 15. Find the length of median of its hypotenuse.

Given : In ΔABC, $\angle ABC = 90^\circ$

AC = 15 units

seg BD is median on hypotenuse AC

To find : BD

Solution:

In ΔABC, $\angle ABC = 90^\circ$ (Given)

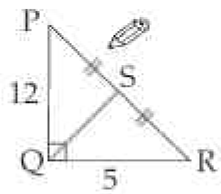
seg BD is median on hypotenuse AC (Given)

∴ $BD = \frac{1}{2} AC$

(In a right angled triangle the median drawn on the hypotenuse is half of the hypotenuse)

∴ $BD = \frac{1}{2} \times 15$

∴ $\boxed{BD = 7.5 \text{ units}}$

(3) 

In ΔPQR, $\angle Q = 90^\circ$, PQ = 12, QR = 5 and QS is a median, Find l(QS).

Solution:

In ΔPQR, $\angle PQR = 90^\circ$ (Given)

∴ $PR^2 = PQ^2 + QR^2$ (Pythagoras theorem)

∴ $PR^2 = 12^2 + 5^2$ (Given)

∴ $PR^2 = 144 + 25$

∴ $PR^2 = 169$

∴ PR = 13 units ... (i) (Taking square roots)

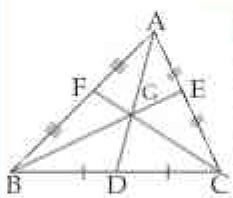
seg QS is median on hypotenuse PR.

∴ $QS = \frac{1}{2} PR$ (In a right angled triangle the median drawn on the hypotenuse is half of the hypotenuse)

∴ $QS = \frac{1}{2} \times 13$ [from ...(i)]

∴ $\boxed{QS = 6.5 \text{ units}}$

Important Note



Statement:

In ΔABC, seg AD, seg BE and seg CF are medians. Medians of a triangle are concurrent (i.e. all three medians intersect at one point).

Point of concurrence of all three medians is called centroid.

In the above figure, point G is the centroid.

Property : Centroid of a triangle trisects the median.

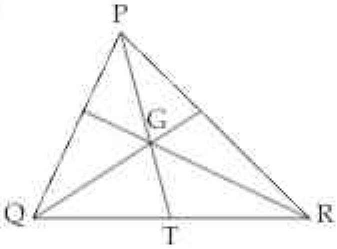
i.e. $\frac{AG}{GD} = \frac{2}{1}; \frac{BG}{GE} = \frac{2}{1}; \frac{CG}{GF} = \frac{2}{1}$

We can also say,

$AG = \frac{2}{3} AD; BG = \frac{2}{3} BE; CG = \frac{2}{3} CF$

and $GD = \frac{1}{3} AD; GE = \frac{1}{3} BE; GF = \frac{1}{3} CF$

(4) In adjoining figure point G is the point of concurrence of the medians of $\triangle PQR$. If GT = 2.5, find the lengths of PG and PT.



Solution:

In $\triangle PQR$, point G is centroid (Given)

$\therefore \quad \dfrac{PG}{GT} = \dfrac{2}{1}$ (Centroid divides the median in the ratio 2:1)

$\therefore \quad \dfrac{PG}{2.5} = \dfrac{2}{1}$

$\therefore \quad PG = 2 \times 2.5$

$\therefore \quad \boxed{PG = 5 \text{ cm}}$

$PT = PG + GT \quad (P - G - T)$

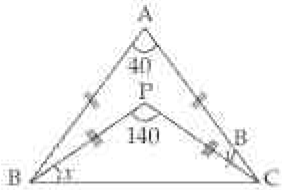
$\quad = 2.5 + 5$

$\boxed{PT = 7.5 \text{ cm}}$

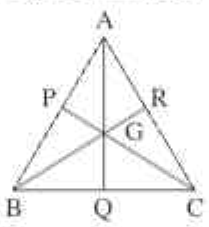
PROBLEMS FOR PRACTICE

(1) **In $\triangle PQR$, $m\angle Q = 90°$, seg QM is median. $PQ^2 + QR^2 = 169$. Find QM.**

(2) **In the following figure, find the value of x and y.**



(3) **In the adjoining figure, G is centroid of $\triangle ABC$. If AQ = 6 cm, BR = 9 cm, CG = 5 cm.**
Find: (i) BG (ii) CP (iii) AG



ANSWER

(1) QM = 6.5 cm **(2)** $x = 20$

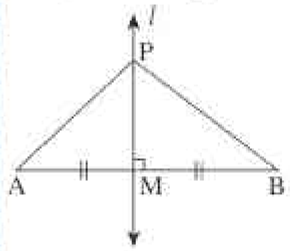
(3) BG = 6 cm, CP = 7.5 cm, AG = 4 cm

Points to Remember:

Theorem - 8

- **Perpendicular Bisector Theorem.**

Case A : Any point on the perpendicular bisector of a segment is equidisant from its endpoints.



Given: Line l is $\perp$ bisector of seg AB at point M. Point P is any point on line l.

To prove : PA = PB

Proof : Line l is $\perp$ bisector of seg AB (Given)

seg AM $\cong$ seg BM ... (i)

$\angle PMA = \angle PMB = 90°$ ...(ii)

In $\triangle PMA$ and $\triangle PMB$

side PM $\cong$ side PM (common side)

$\angle PMA \cong \angle PMB$ [from (ii)]

side AM $\cong$ side BM [from (i)]

$\therefore \quad \triangle PMA \cong \triangle PMB$ (SAS test)

seg PA $\cong$ seg PB (c.s.c.t.)

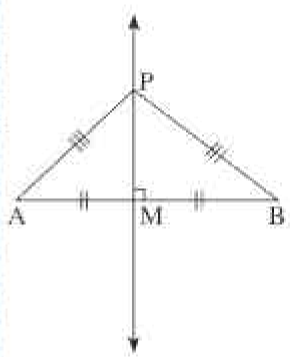
$\therefore$ **PA = PB**

Case B : Any point equidistant from the end points of a segment lies on the perpendicular bisector of the segment.

Given : Point P is any point equidistant from the end points of seg AB. That is, PA = PB.

To prove : Point P is on the perpendicular bisector of seg AB.

Construction : Take mid-point M of seg AB and draw line PM.



Proof : In $\triangle PAM$ and $\triangle PBM$

seg PA $\cong$ seg PB (Given)

seg AM $\cong$ seg BM (construction)

seg PM $\cong$ seg PM (common side)

$\therefore \quad \triangle PAM \cong \triangle PBM$ (SSS test)

$\therefore \quad \angle PMA \cong \angle PMB$ (c.a.c.t.)

But $\angle PMA + \angle PMB = 180°$ (Linear pair axiom)

$\angle PMA + \angle PMA = 180^\circ$ ...($\because$ $\angle PMB = \angle PMA$)

$2\angle PMA = 180^\circ$

$\therefore$ $\angle PMA = 90^\circ$

$\therefore$ seg PM $\perp$ seg AB ...(i)

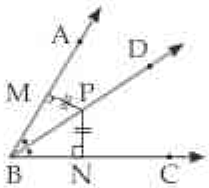
But point M is the midpoint of seg AB. ... (ii) (Construction)

$\therefore$ Line PM is the perpendicular bisector of seg AB. **So point P is on the perpendicular bisector of seg AB.**

Theorem - 9

- **Angle bisector theorem.**

Case A : If a point is on the angle bisector of an angle, then it is equidistant from the sides of the angle.



Given: (1) Ray BD bisects $\angle ABC$

(2) Point P is on ray BD.

(3) seg PM $\perp$ ray BA, B-M-A.

(4) seg PN $\perp$ ray BC, B-N-C.

To prove : PM = PN

Proof : In ΔPMB and ΔPNB

side PB $\cong$ side PB (Common side)

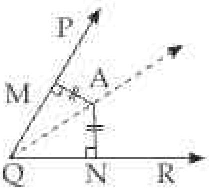
$\angle PBM \cong \angle PBN$ (Ray BD bisects, $\angle ABC$)

$\angle PMB \cong \angle PNB$ (each 90°)

$\therefore$ $\Delta PMB \cong \Delta PNB$ (SAA test)

$\therefore$ **seg PM $\cong$ seg PN** (c.s.c.t.)

Case B : If the point in the plane of an angle is equidistant from the sides of the angle, then it lies on the angle bisector.



Given: Point A is in the interior of $\angle PQR$.

seg AM $\perp$ ray QP, Q-M-P.

seg AN $\perp$ ray QR, Q-N-R.

AM = AN

To prove : Point A lies on the bisector of $\angle PQR$.

Construction : Draw ray QA.

Proof : In ΔAMQ and ΔANQ

$\angle AMQ \cong \angle ANQ$ (Each 90°)

Hypotenuse QA $\cong$ Hypotenuse QA (Common side)

side AM $\cong$ side AN (Given)

$\Delta AMQ = \Delta ANQ$ (Hypotenuse-side test)

$\therefore$ $\angle MQA \cong \angle NQA$ (c.a.c.t.)

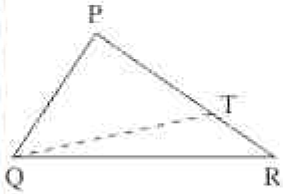
i.e. $\angle PQA \cong \angle RQA$ (P-M-Q, R-N-Q)

$\therefore$ Ray QA bisects $\angle PQR$ [from (iv) and definition]

i.e. point A lies on bisector for of $\angle PQR$

Theorem - 10

If two sides of the triangle are not congruent then the angle opposite to the greater side is greater.



Given : In PQR, PR > PQ

To prove : $\angle PQR > \angle PRQ$

Construction : Take a point T on seg PR such that PQ = PT, Draw seg QT.

Proof : In ΔPQT, seg PQ $\cong$ seg PT (Construction)

$\therefore$ $\angle PQT \cong \angle PTQ$ ... (i) (Isosceles triangle theorem)

$\angle PTQ$ is exterior angle of ΔQTR (Definition)

$\therefore$ $\angle PTQ > \angle TRQ$ (Exterior angle theorem)

i.e. $\angle PTQ > \angle PRQ$ ... (ii) (P-T-R)

$\therefore$ $\angle PQT > \angle PRQ$ ... (iii) [from (i) and (ii)]

Point T is in the interior of $\angle PQR$.

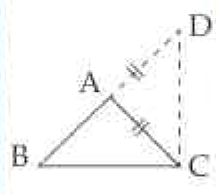
$\therefore$ $\angle PQR > \angle PQT$ ...(iv)

$\therefore$ **$\angle PQR > \angle PRQ$** [from (iii) and (iv)]

Theorem - 11

Statement :

The sum of the length of any two sides of a triangle is greater than the third side.



Given : Δ ABC is a given triangle

To prove : (i) AB + AC > BC

(ii) AB + BC > AC

(iii) BC + AC > AB

Construction : Take point D on ray BA such that AC = AD.

Proof : In ΔACD, AC = AD (Construction)

$\therefore$ $\angle ACD \cong \angle ADC$ (Isosceles Δ theorem)

$\therefore$ $\angle ACD$ + ACB > ADC (Adding $\angle ACB$ on L-H-S)

$\therefore$ $\angle BCD > \angle BDC$ ...(i) [Angle Addition property and B-A-D]

In ΔBCD,

$\therefore$ $\angle BCD > \angle BDC$ [From (i)]

$\therefore$ $BD > BC$ (Side opposite to greater) (angle is greater)

$\therefore$ $BA + AD > BC$ (B-A-D)

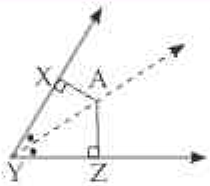
$\therefore$ $\mathbf{AB + AC > BC}$ ($\because AD = AC$, construction)

Similarly we can prove that

$\mathbf{AB + BC > AC}$

$\mathbf{BC + AC > AB}$

PRACTICE SET - 3.4 *(Textbook Page No. 43)*

(1) 

Point A is on the bisector of $\angle XYZ$. AX = 2 cm. Find AZ.

Solution:

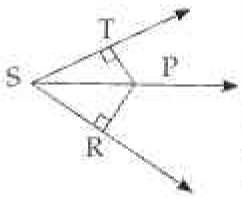
Point A lies on bisector of $\angle XYZ$ (Given)

$\therefore$ Point A is equidistant from ray YX and ray YZ. (Angle bisector theorem)

$\therefore$ $AX = AZ$

But, $AX = 2cm$

$\therefore$ $\boxed{AZ = 2cm}$

(2) 

In adjoining figure $\angle RST = 56°$, seg PT $\perp$ ray ST, seg PR $\perp$ ray SR and seg PR $\cong$ seg PT. Find the measure of $\angle RSP$. State the reason for your answer.

Solution:

$PR = PT$ (Given)

$\therefore$ Point P lies on bisector of $\angle RST$ (Angle bisector theorem)

$\therefore$ $\angle RSP = \frac{1}{2}\angle RST$

$\therefore$ $\angle RSP = \frac{1}{2} \times 56$

$\therefore$ $\boxed{\angle RSP = 28°}$

(3) **In ΔPQR, PQ = 10 cm, QR = 12 cm, PR = 8 cm. Find out the greatest and the smallest angle of the triangle.**

Solution:

In ΔPQR, $PQ = 10cm$, $QR = 12cm$, $PR = 8cm$. (Given)

$\therefore$ $QR > PQ > PR$

$\therefore$ $\angle P > \angle R > \angle Q$ (Angle opposite to greater side is greater)

$\therefore$ $\boxed{\text{$\angle P$ is the greatest angle and $\angle Q$ is the smallest angle of ΔPQR}}$

(4) **In ΔFAN, $\angle F = 80°$, $\angle A = 40°$. Find out the greatest and the smallest side of the triangle. State the reason.**

Solution:

In ΔFAN,

$\angle F + \angle A + \angle N = 180°$ (Sum of the measures of all angles of a triangle is 180°)

$\therefore$ $80 + 40 + N = 180$ (Given)

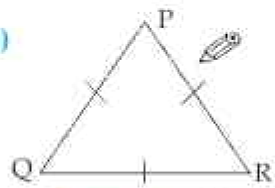
$\therefore$ $\angle N = 180 - 120$

$\therefore$ $\angle N = 60°$

$\therefore$ $\angle F > \angle N > \angle A$

$\therefore$ $AN > FA > FN$ (Side opposite to greater angle is greater)

$\therefore$ $\boxed{\text{Side AN is greatest side and side FN is smallest side of ΔFAN.}}$

(5) 

Prove that an equilateral triangle if equiangular.

Given : In ΔPQR, $PQ = QR = PR$.

To prove: $\angle P \cong \angle Q \cong \angle R$

Proof:

In ΔPQR, side $PQ \cong$ side PR (Given)

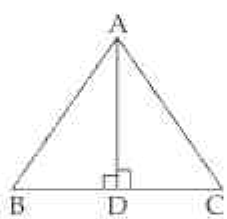
$\therefore$ $\angle Q \cong \angle R$ ..(i) (Isosceles triangle theorem)

In ΔPQR, side $PR \cong$ side QR (Given)

$\therefore$ $\angle Q \cong \angle P$ (ii) (Isosceles triangle theorem)

$\therefore$ $\angle P \cong \angle Q \cong \angle R$ [from (i) and (ii)]

$\therefore$ **ΔPQR is an equiangular triangle.**

(6) 

Prove that, if the bisector of $\angle BAC$ of ΔABC is perpendicular to side BC, then ΔABC is an isosceles triangle.

Given : In ΔABC, AD bisects $\angle BAC$, B-D-C.

seg AD $\perp$ side BC.

To prove : ΔABC is an isosceles triangle.

Proof:

In ΔADB and ΔADC

$\therefore$ $\angle BAD \cong \angle CAD$ (Given)

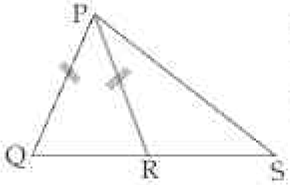
side AD $\cong$ side AD (Common side)

$\therefore$ $\angle ADB \cong \angle ADC$ (Each measures 90°)

$\therefore$ $\Delta ADB \cong \Delta ADC$ (ASA test)

$\therefore$ side AB $\cong$ side AC (c.s.c.t.)

$\therefore$ **ΔABC is an isosceles triangle** [by definition]

(7) 

In adjoining figure, if seg PR $\cong$ seg PQ, show that seg PS > seg PQ.

Proof:

In ΔPQR, side PQ $\cong$ side PR (Given)

$\therefore$ $\angle PQR \cong \angle PRQ$ ...(i) (Isosceles triangle theorem)

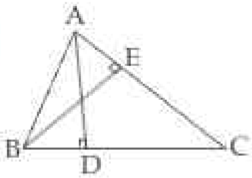
$\angle PRQ$ is exterior angle of ΔPRS (Definition)

$\therefore$ $\angle PRQ > \angle S$ (Exterior angle property)

$\therefore$ $\angle PQR > \angle S$ ...(ii) [from (i)]

In ΔPQS, $\angle PQS > \angle S$ [from (ii) Q-R-S]

$\therefore$ **PS > PQ** (In a triangle side opposite to greater angle is greater)

(8) 

In adjoining figure, in ΔABC, seg AD and seg BE are altitudes and AE = BD. Prove that seg AD = seg BE

Proof:

In ΔAEB and ΔBDA

$\angle AEB = \angle BDA = 90^\circ$ (Given)

Hypotenuse AB $\cong$ Hypotenuse BA (Common side)

seg AE $\cong$ seg BD (Given)

$\therefore$ $\Delta AEB \cong \Delta BDA$ (Hypotenuse side test)

$\therefore$ seg BE $\cong$ seg AD [c.s.c.t.]

i.e. **seg AD $\cong$ seg BE**

PROBLEMS FOR PRACTICE

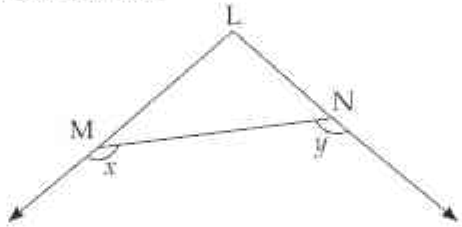
(1) **Prove that in a right angled triangle, hypotenuse is the longest side.**

(2) **In $\square$ PQRS, prove that PQ + QR + RS + SP > PR + QS.**

(3) **In ΔABC, side AB $\cong$ side BC.**

If A-P-C then show that BP < congruent sides.

(4) **In the following figure, $x > y$. Prove that side LM > side LN.**



Points to Remember:

Similar Triangles

Definition : For a given one-one correspondence, two triangles are said to be similar if the:

(i) corresponding sides are in proportion.

(ii) corresponding angles are congruent.

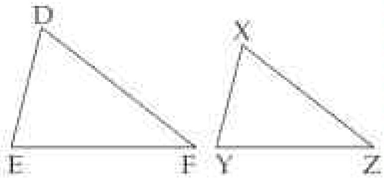
i.e. For the correspondence DEF $\longleftrightarrow$ XYZ if,

(i) $\dfrac{DE}{XY} = \dfrac{EF}{YZ} = \dfrac{DF}{XZ}$

(ii) $\angle D \cong \angle X$

(iii) $\angle E \cong \angle Y$ and

(iv) $\angle F \cong \angle Z$



then $\Delta DEF \sim \Delta XYZ$.

Note : Converse of the definition is also true

i.e. if $\Delta ABC \sim \Delta PQR$ then.

(i) $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR}$ (Corresponding sides of similar triangles)

$\left.\begin{array}{l}\text{(ii) } \angle A \cong \angle P \\ \text{(iii) } \angle B \cong \angle Q \text{ and} \\ \text{(iv) } \angle C \cong \angle R\end{array}\right\}$ (Corresponding angles of similar triangles)

PRACTICE SET - 3.5 *(Textbook Page No. 47)*

(1) **If $\Delta XYZ \sim \Delta LMN$, write the corresponding angles of the two triangles and also write the ratios of corresponding sides.**

Solution:

$\Delta XYZ \sim \Delta LMN$ (Given)

$\therefore \quad \frac{XY}{LM} = \frac{YZ}{MN} = \frac{XZ}{LN}$ (c.s.s.t.)

$\left.\begin{array}{l} \angle X \cong \angle L \\ \angle Y \cong \angle M \\ \angle Z \cong \angle N \end{array}\right\}$ (c.a.s.t)

(2) **In $\triangle XYZ$, XY = 4cm, YZ = 6cm, XZ = 5cm If $\triangle XYZ \sim \triangle PQR$ and PQ = 8cm then find the length of remaining sides of $\triangle PQR$.**

Solution:

$\therefore \quad \triangle XYZ \sim \triangle PQR$ (Given)

$\therefore \quad \frac{XY}{PQ} = \frac{YZ}{QR} = \frac{XZ}{PR}$ (c.s.s.t.)

$\therefore \quad \frac{4}{8} = \frac{6}{QR} = \frac{5}{PR}$

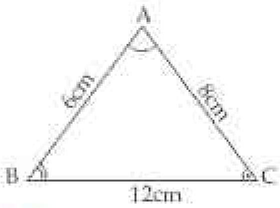
$\therefore \quad \frac{4}{8} = \frac{6}{QR}$ and $\frac{4}{8} = \frac{5}{PR}$

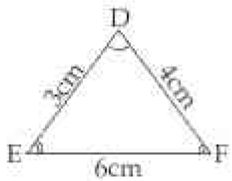
$\therefore \quad 4 \times QR = 6 \times 8$ and $4 \times PR = 5 \times 8$

$\therefore \quad QR = \frac{6 \times 8}{4}$ and $PR = \frac{5 \times 8}{4}$

$\therefore \quad \boxed{QR = 12cm}$ and $\boxed{PR\ 10cm}$

(3) **Draw a sketch of a pair of similar triangles. Label them. Show their corresponding angles by the same signs. Show the lengths of corresponding sides by numbers in proportion.**





Solution:

In the above figure,

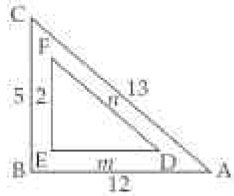
$\triangle ABC \sim \triangle DEF$

$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ (c.s.s.t.)

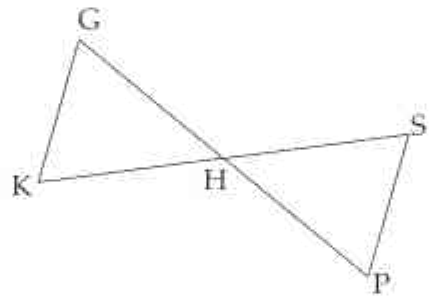
$\left.\begin{array}{l} \angle A \cong \angle D \\ \angle B \cong \angle E \\ \angle C \cong \angle F \end{array}\right\}$ (c.a.s.t)

PROBLEMS FOR PRACTICE

(1) **In the following figure, $\triangle CAB \sim \triangle FDE$. Find value of m and n.**



(2) **In the following figure, $\triangle GHK \sim \triangle PHS$ GH : HP = 6 : 5. If KH = 18 units, find KS.**



ANSWER

(1) $m = 4.8$; $n = 5.2$ **(2)** KS = 33 units

PROBLEM SET - 3 *(Textbook Page No. 49)*

(1) **Choose the correct alternative answer for the following questions.**

(i) Two sides of a triangle are 5 cm and 1.5 cm. length of third side cannot be ....

(A)3.7 cm (B) 4.1 cm (C) 3.8 cm (D) 3.4 cm

Ans. D

(ii) In $\triangle PQR$ if $\angle R > \angle Q$ then ....

(A) QR > PR (B) PQ > PR

(C) PQ < PR (D) QR < PR

Ans. B

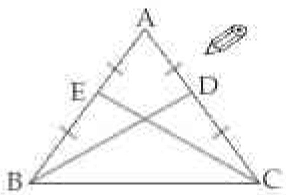
(iii) In $\triangle TPQ$, $\angle T = 65°$, $\angle P = 95°$ then which of the following is a true statement?

(A)PQ < TP (B) PQ < TQ

(C)TQ < TP < PQ (D) PQ < TP < TQ

Ans. B

(2)



$\triangle$ABC is an isosceles in which AB = AC. seg BD and seg CE are medians. Show that BD = CE.

Proof:

AE = BE = $\frac{1}{2}$ AB ...(i)

(CE is median on side AB)

AD = CD = $\frac{1}{2}$ AC ...(ii)

(BD is median on side AC)

But, AB = AC ...(iii)

(Given)

$\therefore$ AE = BE = AD = CD ...(iv)

[from (i), (ii) and (iii)]

In $\triangle$ABD and $\triangle$ACE

side AB $\cong$ side AC (Given)

$\angle$A $\cong$ $\angle$A (Common angle)

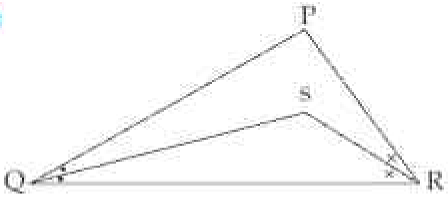
side AD $\cong$ side AE [from (iv)]

$\therefore$ $\triangle$ABD $\cong$ $\triangle$ACE [SAS test]

$\therefore$ side BD $\cong$ side CE (c.s.c.t.)

ie **BD = CE**

(3)



In $\triangle$PQR, if PQ > PR and bisectors of $\angle$Q and $\angle$R intersect at S. Show that SQ > SR.

Proof:

In $\triangle$PQR, PQ > PR (Given)

$\therefore$ $\angle$PRQ > $\angle$PQR (Angle opposite to greater side is greater)

$\therefore$ 2$\angle$SRQ > 2$\angle$SQR (Rays QS and RS bisect $\angle$PQR and $\angle$PRQ respectively)

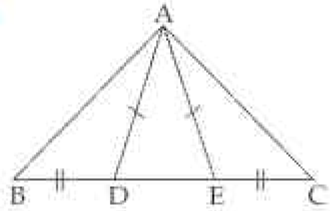
$\therefore$ $\angle$SRQ > $\angle$SQR ...(i)

In $\triangle$SQR,

$\angle$SRQ > $\angle$SQR [from (i)]

$\therefore$ **SQ > SR** (Side opposite to greater angle is greater)

(4)



In the adjoining figure, point D and point E are on side BC of $\triangle$ABC such that BD = CE and AD = AE. Show that $\triangle$ABD $\cong$ $\triangle$ACE.

Proof:

In $\triangle$ADE,

side AD $\cong$ side AE (Given)

$\therefore$ $\angle$ADE $\cong$ $\angle$AED ...(i) (Isosceles triangle Theorem)

$\angle$ADB + $\angle$ADE = 180° (Linear pair axiom)

$\therefore$ $\angle$ADB = 180° – $\angle$ADE ...(ii)

$\angle$AEC + $\angle$AED = 180° (Linear pair axiom)

$\therefore$ $\angle$AEC = 180 – $\angle$AED ...(iii)

$\therefore$ $\angle$ADB $\cong$ $\angle$AEC ...(iv) [from (i), (ii), (iii)]

In $\triangle$ABD and $\triangle$AEC,

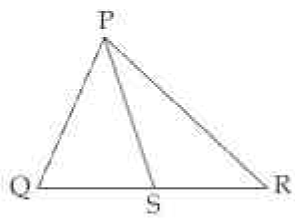
side AD $\cong$ side AE ..... (Given)

$\angle$ADB $\cong$ $\angle$AEC ..... (from (iv))

side BD $\cong$ side CE ..... (Given)

$\therefore$ **$\triangle$ABD $\cong$ $\triangle$ACE** ..... (SAS test)

(5)



In the adjoning figure, point S is on side QR of ΔPQR. Prove that PQ + QR + RP > 2PS

Proof:

In ΔPQS,

PQ + QS > PS ... (i) (Sum of two sides of a triangle is greater than the third side)

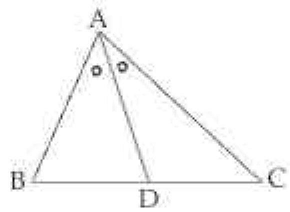
In ΔPRS,

PR + SR > PS ... (ii) (Sum of two sides of a triangle is greater than the third side)

$\therefore$ PQ + QS + SR + PR > 2PS (Adding (i) and (ii))

$\therefore$ **PQ + QR + RP > 2PS** [Q-S-R]

(6)



In the adjoining figure, bisector of $\angle$BAC intersects BC at point D. Prove that AB > BD.

Proof:

$\angle BAD \cong \angle DAC$ ... (i) (AD bisects $\angle BAC$)

$\angle ADB$ is an exterior angle of ΔADC

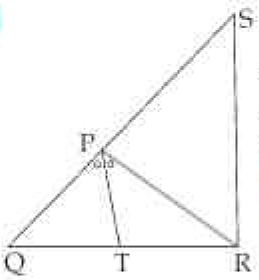
$\therefore$ $\angle ADB > \angle DAC$ (Exterior angle property)

$\therefore$ $\angle ADB > \angle BAD$ .... (ii) [from (i)]

In ΔABD

$\angle ADB > \angle BAD$ [from (ii)]

$\therefore$ **AB > BD** (In a triangle, side opposite to greater angle is greater)

(7) 

In the adjoining figure, seg PT is the bisector of $\angle$QPR. A line through R intersects ray QP at point S.

To prove : PS = PR

Proof:

In ΔPQR, ray PT bisects $\angle QPR$ (Given)

$\therefore$ $\angle QPT \cong \angle TPR$ ... (i)

line PT $\parallel$ line RS (Given)

On transversal QS,

$\angle QPT \cong \angle PSR$ ... (ii) (Corresponding angles theorem)

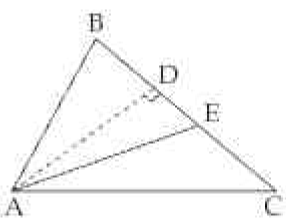
On transversal PR,

$\angle TPR \cong \angle PRS$ ... (iii) (Alternate angles theorem)

In ΔPSR, $\angle PSR \cong \angle PRS$ [from (i), (ii), (ii) (Converse of isosceles

$\therefore$ side PS $\cong$ side PR triangle theorem)

$\therefore$ **PS = PR**

(8) 

In adjoining figure, seg AD $\perp$ seg BC. seg AE is the bisector of $\angle$CAB and C-E-D.

Prove that : $\angle DAE = \frac{1}{2}(\angle B - \angle C)$

Proof:

$\angle BAE \cong \angle CAE$ ....(i) (Ray AE bisects $\angle BAC$)

$\angle BAD + \angle DAE = \angle BAE$ ... (Angle addition property)

$\therefore$ $\angle DAE = \angle BAE - \angle BAD$ ....(ii)

$\angle CAE + \angle DAE = \angle CAD$ ... (Angle addition property)

$\therefore$ $\angle DAE = \angle CAD - \angle CAE$ ....(iii)

Now, $\angle DAE + \angle DAE = \angle BAE - \angle BAD + \angle CAD - \angle CAE$ [Adding (ii) and (ii)]

$\therefore$ $2\angle DAE = \angle BAE - \angle BAD + \angle CAD - \angle BAE$ [from (i)]

$\therefore$ $2\angle DAE = \angle CAD - \angle BAD$ ... (iv)

In Δ BAD

$\angle BAD + \angle ADB + \angle ABD = 180^\circ$ (sum of the measures of all angles of a triangle is 180°)

$\therefore$ $\angle BAD + 90 + \angle B = 180$

$\therefore$ $\angle BAD = 180 - 90 - \angle B$

$\therefore$ $\angle BAD = 90 - \angle B$ ... (v)

In ΔDAC,

$\angle CAD + \angle ADC + \angle ACD = 180^\circ$ (sum of the measures of all angles of a triangle is 180°)

$\therefore$ $\angle CAD + 90 + \angle C = 180$

$\angle CAD = 180 - 90 - \angle C$

$\angle CAD = 90 - \angle C$ ... (vi)

Now, $2\angle DAE = (90 - \angle C) - (90 - \angle B)$ ... [from (iv), (v), (vi)]

$2\angle DAE = 90 - \angle C - 90 + \angle B$

$\therefore$ $2\angle DAE = \angle B - \angle C$

$\therefore$ $\angle \mathbf{DAE} = \frac{1}{2}(\angle \mathbf{B} - \angle \mathbf{C})$

MCQ's

(1) In ΔPQR, PQ = PR and $m\angle P = 40^\circ$ then $m\angle Q$ = ...

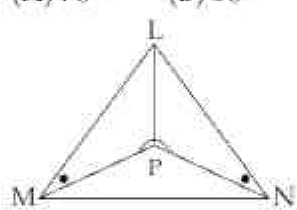
(A) 140° (B) 40° (C) 70° (D) 80°

(2) In ΔABC, $m\angle A = 60^\circ$, $m\angle B = 90^\circ$, $m\angle C = 30^\circ$, $AB = \sqrt{3}$ cm. AC = ... (By 30°–60°–90° triangle theorem)

(A) $\frac{\sqrt{3}}{2}$ (B) $2\sqrt{3}$ (C) $\sqrt{3}$ (D) $\frac{1}{\sqrt{3}}$

(3) If $\Delta ABC \sim \Delta PQR$ and $m\angle A = 40^\circ$, $m\angle B = 50^\circ$ then $m\angle R$ = ...

(A) 70° (B) 80° (C) 90° (D) 55°

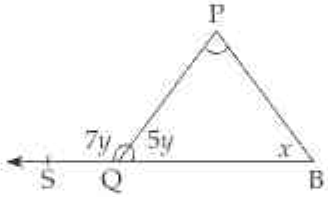
(4) 

In the adjoining figure, $\Delta LPM \cong \Delta LPN$. (By .... test)

(A) SSS (B) SAS (C) ASA (D) SAA

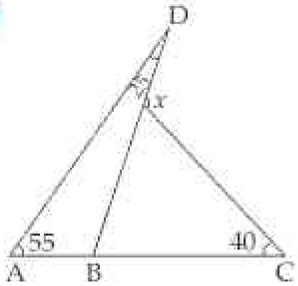
(5) The measures of angles of a triangle are in the ratio 2:3:4. Which of the following is the measure of an angle of this triangle?

(A) 20° (B) 30° (C) 40° (D) 90°

(6) 

In the adjoining figure, what is the value of x?

(A) 35 (B) 45 (C) 50 (D) 60

(7) 

In the adjoining figure, the value of x is ....

(A) 65 (B) 80
(C) 95 (D) 120

(8) If $\Delta ABC \cong \Delta FDE$ and AB = 5cm, $\angle B = 40^\circ$ and $\angle A = 80^\circ$, then which of the following is true?

(A) DF = 5cm, $m\angle F = 60^\circ$

(B) DE = 5cm, $m\angle E = 60^\circ$

(C) DF = 5cm, $m\angle E = 60^\circ$

(D) DE = 5cm, $m\angle D = 40^\circ$

(9) In ΔPQR, $\angle P = 90^\circ$. S is midpoint of side QR. If QR = 10cm, what is the length of seg PS?

(A) 10 cm (B) 5 cm (C) 20 cm (D) 7.5 cm

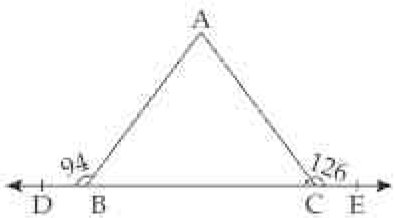
(10) In ΔXYZ, $\angle X = 90^\circ$, $\angle Y = 60^\circ$. If $XZ = 5\sqrt{3}$ cm, what is the length of seg YZ?

(A) 10 cm (B) $10\sqrt{3}$ cm (C) 20 cm (D) $\sqrt{3}$ cm

(11) If $\Delta PQR \sim \Delta XYZ$, $\frac{PR}{XZ} = \frac{2}{3}$ and PQ = 12 units than XY = ...

(A) 9 units (B) 18 units (C) 8 units (D) 12 units

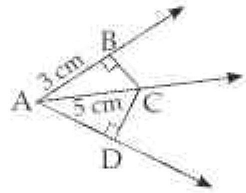
(12) In the following figure, $m\angle BAC$ = ...



(A) 94° (B) 54° (C) 40° (D) 44°

(13) In ΔABC, if $\angle B = \angle C = 45^\circ$, then which of the following is the longest side?

(A) AB (B) BC (C) AC (D) All sides are equal

(14) 

In the following figure, ray AC is bisector of $\angle BAD$ such that AB = 3 cm, AC = 5 cm then CD = ...

(A) 2 cm (B) 3 cm (C) 4 cm (D) 5 cm

(15) $\Delta ABC \cong \Delta PQR$ and ΔABC is not congruent to ΔRPQ, then which of the following is not true?

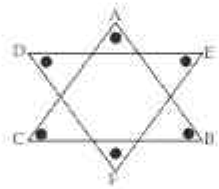
(A) BC = PQ (B) AC = PR (C) AB = PQ (D) QR = BC

(16) Length of three sides are given below. Determine which of them will form a triangle?

(A) 6 cm, 3 cm, 2 cm (B) 4.5 cm, 8.5 cm, 4 cm

(C) 5 cm, 6 cm, 10 cm (D) 9 cm, 4 cm, 3 cm

(17) In the following figure, if ΔABC and ΔDEF are equilateral then what will be $\angle C + \angle D + \angle E + \angle F$?



(A) 180° (B) 140° (C) 200° (D) 240°

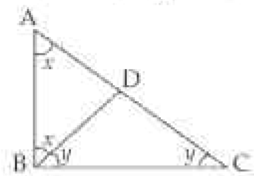
(18) For ΔABC, G is its centroid and D is midpoint of side BC. If AD is 9 cm then GD = ...

(A) 3 cm (B) 4.5 cm (C) 6 cm (D) 18 cm

(19) In ΔABC, $\angle A = \angle C = 45^\circ$, $\angle B = 90^\circ$, $AC = 16\sqrt{2}$ cm. AB = ...

(A) $16\sqrt{2}$ cm (B) 8 cm (C) $8\sqrt{2}$ cm (D) 16 cm

(20) Using the information given, $\angle ABC$ = ...



(A) 30° (B) 60° (C) 90° (D) 120°

ANSWERS

(1)	(C)	(2)	(B)	(3)	(C)	(4)	(D)
(5)	(C)	(6)	(D)	(7)	(D)	(8)	(B)
(9)	(B)	(10)	(A)	(11)	(C)	(12)	(C)
(13)	(B)	(14)	(C)	(15)	(A)	(16)	(D)
(17)	(D)	(18)	(B)	(19)	(D)	(20)	(C)

❖❖❖

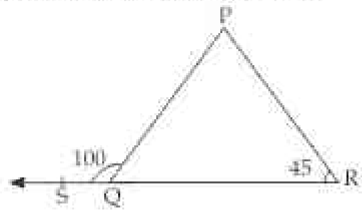
ASSIGNMENT - 3

Time : 1 Hr. **Marks : 20**

Q.1. Solve the following: **3**

(1) If $\triangle PQR \cong \triangle MNS$ then state all pairs of congruent angles and sides.:

(2) In $\triangle PQR$, $\angle PRQ = 45°$, $\angle PQS = 100°$, S-Q-R.
Find $\angle QPR$.

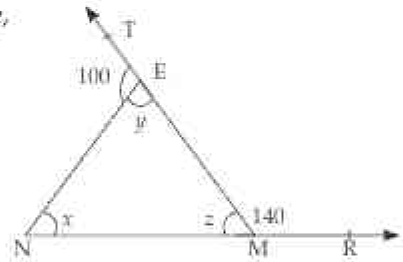


(3) In $\triangle ABC$, AB = 5 cm, BC = 8 cm, AC = 10 cm. Then find the smallest and the biggest angle of triangle.

Q.2. Solve the following: **4**

(1) In $\triangle PQR$, $\angle Q = 90°$, PQ = 12 units, QR = 5 units and seg QS is the median. Find QS.

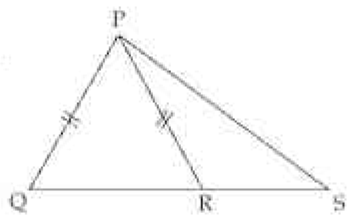
(2) Based on the informaiton given in the figure,
find x, y, z if T-E-M, N-M-R



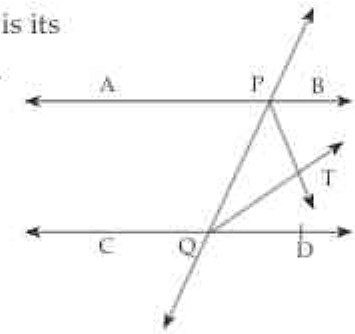
Q.3. Solve the following: **9**

(1) Prove that if two sides of a triangle are congruent then the angles opposite to them are congruent.

(2) In the adjoining figure, seg PQ $\cong$ seg PR. Prove that PS > PQ.



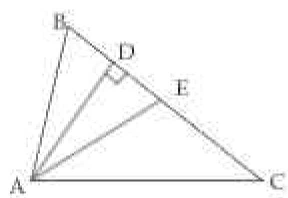
(3) In the adjoining figure, line AB || line CD and line PQ is its transversal. Rays PT and QT bisects $\angle BPQ$ and $\angle PQD$. Prove that $\angle PTQ = 90°$.



Q.4. Solve the following: **4**

(1) In the adjoining figure, seg AD $\perp$ side BC. seg AE bisects $\angle BAC$ such that B-D-E-C.

Prove $\angle DAE = \frac{1}{2}(\angle B - \angle C)$



❖❖❖

4 Constructions of Triangles

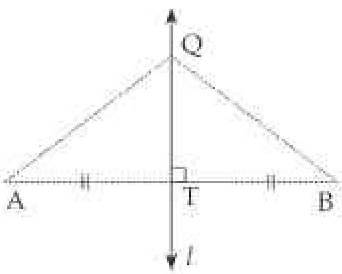
Points to Remember:

In previous standard we have learnt the following triangle constructions.

- To construct a triangle when its three sides are given.
- To construct a triangle when its base and two adjacent angles are given.
- To construct a triangle when two sides and the included angle are given.
- To construct a right angled triangle when its hypotenuse and one side is given.

❑ **Perpendicular Bisector Theorem**

- Every point on the perpendicular bisector of a segment is equidistant from its end points.
- Every point equidistant from the end points of a segment is on the perpendicular bisector of the segment.



MASTER KEY QUESTION SET - 4

Type I : Construction of triangle, when its base, the sum of the other two sides and one of the base angles is given.

PRACTICE SET - 4.1 *(Textbook Page No. 53)*

(1) Construct ΔPQR, in which $QR = 4.2$ cm, $m\angle Q = 40^\circ$ and $PQ + PR = 8.5$ cm.

Solution:

Explanation :

Line l is perpendicular bisector of seg DR

$\therefore$ $PD = PR$ ...(i)

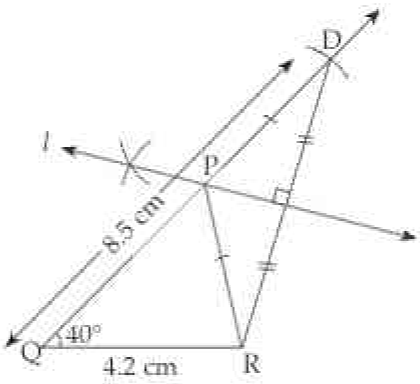
(Perpendicular bisector theorem)

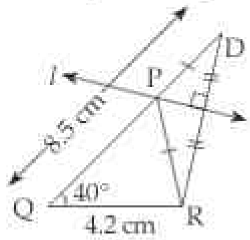
$QD = 8.5$ cm ...(ii)

$PQ + PD = QD$ ...(Q-P-D)

$\therefore$ $PQ + PR = 8.5$ cm ...[From (i) and (ii)]

Analytical Figure





Steps :

(1) Draw seg QR of length 4.2 cm

(2) Draw $\angle C = 40^\circ$; on this ray locate a point D such that $QD = 8.5$ cm.

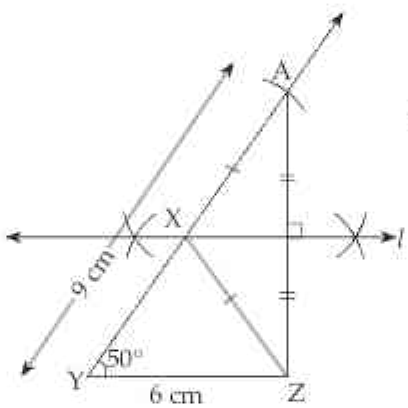
(3) Draw seg DR.

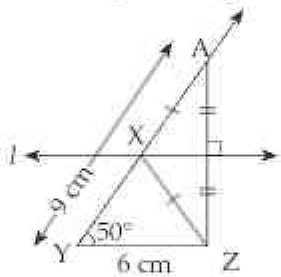
(4) Draw perpendicular bisector of seg DR, it intersect ray QD at point P.

(5) Draw seg PR.

(2) Construct ΔXYZ, in which $YZ = 6$ cm, $XY + XZ = 9$ cm, $m\angle XYZ = 50^\circ$.

Solution: **Analytical Figure**





Explanation :

Line l is perpendicular bisector of seg AZ

$\therefore$ $XA = XZ$ ...(i)

(Perpendicular bisector theorem)

AY = 9 cm ...(ii)

XY + XA = AY ...(A-X-Y)

∴ XY + XZ = 9 cm ...[From (i) and (ii)]

Steps :

(1) Draw seg YZ of length 6 cm

(2) Draw ∠Y = 50°; on this ray locate a point A such that YA = 9 cm.

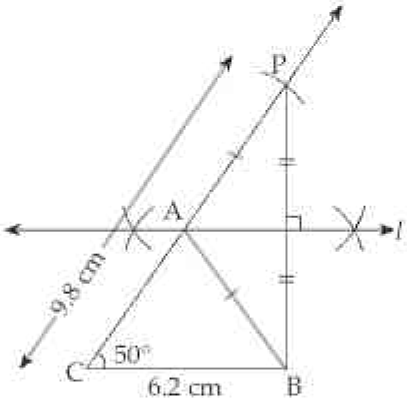
(3) Draw seg AZ.

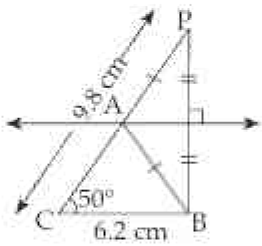
(4) Draw perpendicular bisector of seg AZ, it intersect ray YA at point X.

(5) Draw seg XZ.

(3) Construct ΔABC, in which BC = 6.2 cm, $m\angle$ACB = 50°, AB + AC = 9.8 cm.

Solution: **Analytical Figure**





Explanation :

Line l is perpendicular bisector of seg PB

∴ AP = AB ...(i)

(Perpendicular bisector theorem)

PC = 9.8 cm ...(ii)

AC + AP = PC (P-A-C)

∴ AC + AB = 9.8 cm ...[From (i) and (ii)]

(4) Construct ΔABC, in which BC = 5.2 cm, $m\angle$ACB = 45° and perimeter of ΔABC is 15 cm.

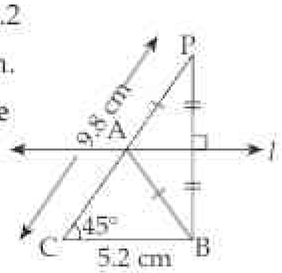
Solution: Perimeter of ΔABC = AB + BC + AC.

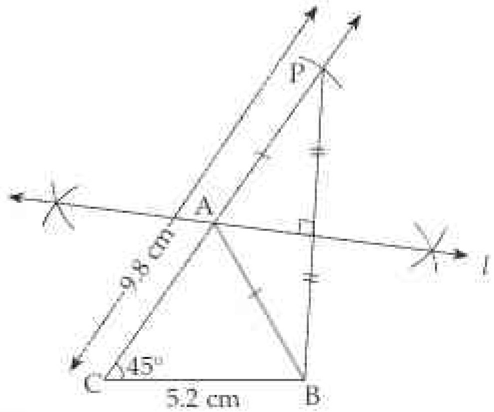
∴ AB + AC + 5.2 = 15 cm

∴ AB + AC = 15 – 5.2

∴ AB + AC = 9.8 cm.

Analytical Figure





Explanation :

Line l is perpendicular bisector of seg PB

∴ AP = AB ...(i)

(Perpendicular bisector theorem)

PC = 9.8 cm ...(ii)

AP + AC = PC ...(∵ P-A-C)

∴ AB + AC = 9.8 cm ...[From (i) and (ii)]

Steps :

(1) Draw seg CB of 5.2 cm

(2) Draw an ∠C = 45° at vertex C.

(3) On this ray locate a point P such that CP = 9.8 cm. Draw seg PB.

(4) Draw perpendicular bisector of seg PB, intersecting seg CP at point A.

(5) Draw seg AB.

NOTE

Perimeter of ΔABC is corrected from 10 cm to 15 cm.

PROBLEMS FOR PRACTICE

(1) Draw ΔABC, where AB = 6 cm, BC = 4 cm and AC = 7 cm.

Draw perpendicular bisector of each side of ΔABC. In how many points do they intersect one another ?

(2) Draw ΔBAD, such that AD = 5.7 cm, ∠BAD = 120° and AB = 4.5 cm.

Draw seg BM perpendicular to line AD.

(3) Construct ΔDEF, such that EF = 4.8 cm, ∠E = 50° and DE + DF = 8.3 cm.

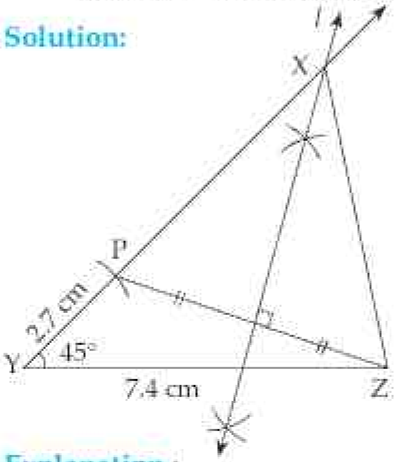
(4) Perimeter of ΔABC is 14 cm, AB = 4.5 cm and ∠A = 80°. Construct ΔABC.

Type II : Construction of a triangle if its base, difference of the other two sides and one of the base angles is given.

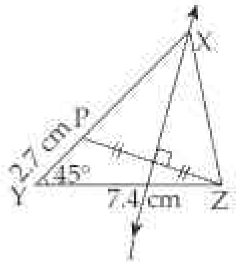
PRACTICE SET - 4.2 *(Textbook Page No. 54)*

(1) Construct ΔXYZ, such that $YZ = 7.4$ cm, $m\angle XYZ = 45^\circ$ and $XY - XZ = 2.7$ cm.

Solution:



Analytical Figure



Explanation :

Line l is perpendicular bisector of seg PZ

$\therefore$ XP = XZ ...(i)

(Perpendicular bisector theorem)

PY = 2.7 cm ...(ii)

XY = XP + PY ...(X-P-Y)

$\therefore$ XY = XZ + 2.7 cm ...[From (i) and (ii)]

$\therefore$ XY – XZ = 2.7 cm

Steps of construction :

(1) Draw a segment YZ of length 7.4 cm.

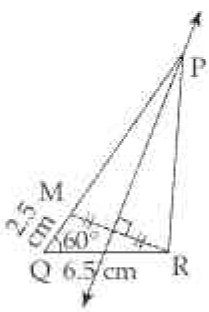
(2) Draw an angle measuring 45° at vertex Y, on this ray mark a point P such that YP = 2.7 cm.

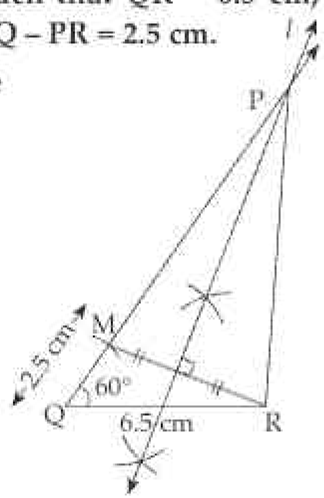
(3) Draw seg PZ.

(4) Draw line l as perpendicular bisector of seg PZ intersecting ray YP at point X. Draw seg XZ.

(2) Construct ΔPQR, such that $QR = 6.5$ cm, $m\angle PQR = 60^\circ$ and $PQ - PR = 2.5$ cm.

Solution: Analytical Figure





Explanation :

Line l is perpendicular bisector of seg MR

$\therefore$ PM = PR ...(i)

(Perpendicular bisector theorem)

QM = 2.5 cm ...(ii)

PQ = PM + QM ...(P-M-Q)

$\therefore$ PQ = PR + 2.5 ...[From (i) and (ii)]

$\therefore$ PQ – PR = 2.5 cm

Steps of construction :

(1) Draw a segment QR of length 6.5 cm

(2) Draw an angle measuring 60° at vertex Q, on this ray mark a point M such that QM = 2.5 cm.

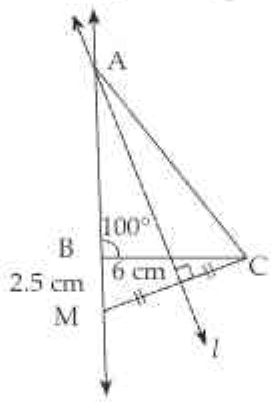
(3) Draw seg RM.

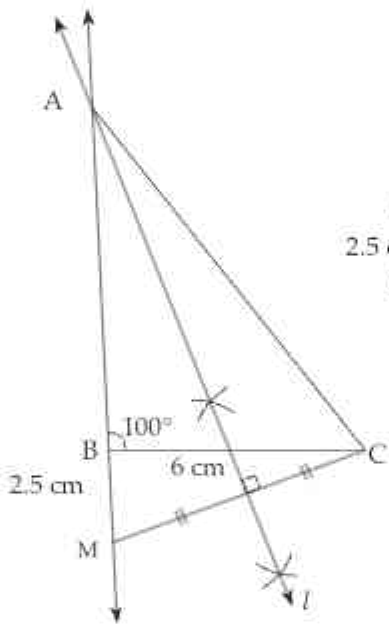
(4) Draw line l as perpendicular bisector of seg MR intersecting ray QM at point P, Q-M-P. Draw PR.

(3) Construct ΔABC, such that base $BC = 6$ cm, $m\angle ABC = 100^\circ$ and $AC - AB = 2.5$ cm.

Solution:

Analytical Figure





Explanation :

Line l is perpendicular bisector of seg MC

$\therefore$ AM = AC ...(i)

(Perpendicular bisector theorem)

BM = 2.5 cm ... (ii)

AM = AB + BM ...(A-B-M)

$\therefore$ AC = AB + 2.5 cm ...[From (i) and (ii)]

$\therefore$ AC – AB = 2.5 cm

Steps of construction :

(1) Draw a segment BC of length 6 cm

(2) Draw an angle measuring 100° at vertex B, extend this ray below seg BC. On this lower side of this ray mark a point M such that BM = 2.5 cm.

(3) Draw seg CM.

(4) Draw line l as perpendicular bisector of seg CM, intersecting ray MB at point A, M-B-A. Draw seg AC.

PROBLEMS FOR PRACTICE

(1) Construct ΔLMN in which base MN = 7 cm, ∠LMN = 40° and LM – LN = 3 cm.

(2) Construct ΔXYZ in which base YZ = 7.4 cm ∠XYZ = 45° and XY – XZ = 2.9 cm

(3) Construct ΔLMN such that MN = 6.2 cm, ∠M = 50° and LN – LM = 2.4 cm.

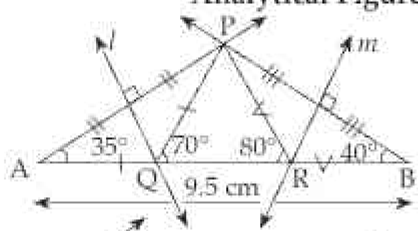
(4) Construct ΔABC where BC = 4.7 cm, ∠B = 45° and AC – AB = 2.5 cm.

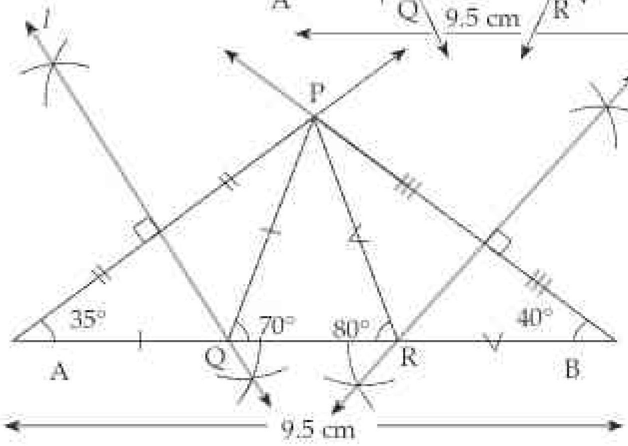
Type III : Construction of triangle of given perimeter and base angles.

PRACTICE SET - 4.3 *(Textbook Page No. 56)*

(1) Construct ΔPQR, in which ∠Q = 70°, ∠R = 80° and PQ + QR + PR = 9.5 cm.

Solution: **Analytical Figure**





Explanation :

Line l and m are perpendicular bisector of seg PA and PB respectively

∴ PQ = AQ ...(i) } (Perpendicular
and PR = RB ...(ii) } bisector theorem)

PQ + QR + PR = 9.5 cm

∴ AQ + QR + RB = 9.5 cm

∴ AB = 9.5 cm

In ΔPQA,

seg PQ ≅ seg AQ ...[From (i)]

∴ ∠QPA ≅ ∠QAP

...(Isosceles triangle theorem)

Let ∠QPA = ∠QAP = x

∠PQR is an exterior angle of ΔPQA

∴ ∠QPA + ∠QAP = ∠PQR

...(Remote interior angle theorem)

∴ $x + x = 70$

∴ $2x = 70$

∴ $x = 35$

∴ ∠QPA = ∠QAP = 35°

Similarly, we can prove ∠RPB = ∠RBP = 40°.

Now, draw ΔPAB, with AB = 9.5 cm, ∠A = 35° and ∠B = 40°

Steps of construction :

(1) Draw a seg AB of length 9.5 cm

(2) Draw an angle measuring 35° at vertex A and an angle measuring 40° at vertex B. Name the point as point P where these two rays intersect.

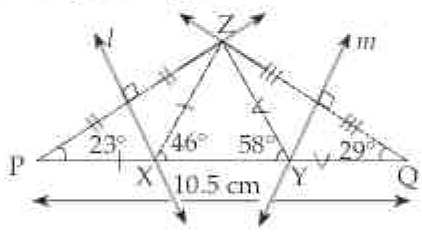
(3) Draw line l as perpendicular bisector of seg PA intersecting AB at point Q.

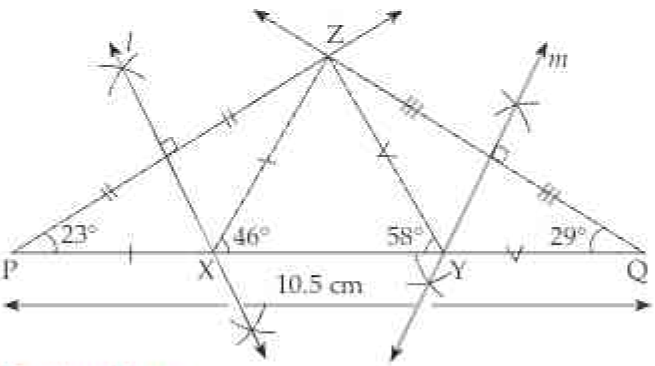
(4) Draw line m as perpendicular bisector of seg PB intersecting AB at point R.

(5) Construct seg PQ and seg PR.

(2) Construct ΔXYZ, in which ∠Y = 58°, ∠X = 46° and perimeter of triangle is 10.5 cm.

Solution: **Analytical Figure**





Explanation :

Line l and m are perpendicular bisector of seg PZ and QZ respectively

$\therefore$ PX = ZX ...(i) } (Perpendicular
ZY = QY ...(ii) } bisector theorem)

XY + YZ + XZ = 10.5 cm ...(Given)

$\therefore$ XY + QY + PX = 10.5 cm ...[From (i), (ii)]

$\therefore$ PQ = 10.5 cm ...(P - X - Y - Q)

In ΔPXZ,

seg PX $\cong$ seg ZX ...[From (i)]

$\therefore$ $\angle$XPZ $\cong$ $\angle$XZP ...(Isosceles triangle theorem)

Let $\angle$XPZ = $\angle$XZP = x

$\angle$ZXY is an exterior angle of ΔPXZ

$\therefore$ $\angle$ZXY = $\angle$XPZ + $\angle$XZP ...(Remote interior angle theorem)

$\therefore$ $46 = x + x$

$\therefore$ $46 = 2x$

$\therefore$ $x = 23$

$\therefore$ $\angle$XPZ = $\angle$XZP = 23°

Similarly, we can prove $\angle$YQZ = $\angle$YZQ = 29°.

Now, draw ΔZPQ, with PQ = 10.5 cm, $\angle$P = 23° and $\angle$Q = 29°

Steps of construction :

(1) Draw a seg PQ of length 10.5 cm

(2) Draw an angle measuring 23° at vertex P and an angle measuring 29° at vertex Q. Name the point of intersection of these rays as point Z.

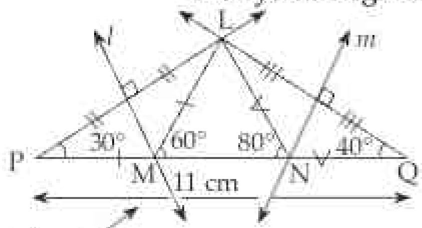
(3) Draw line l as $\perp$ bisector of Seg PZ, intersecting seg PQ, at point X.

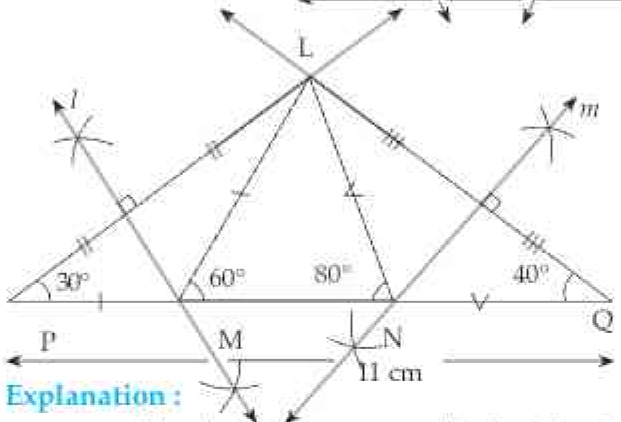
(4) Draw line m as $\perp$ bisector of Seg QZ, intersecting seg PQ at point Y.

(5) Draw seg XZ and seg YZ.

(3) Construct ΔLMN, in which $\angle$M = 60°, $\angle$N = 80° and LM + MN + NL = 11cm.

Solution: **Analytical Figure**





Explanation :

Line l and m are perpendicular bisector of seg PL and seg LQ respectively

$\therefore$ MP = ML ...(i) } (Perpendicular
and NL = NQ ...(ii) } bisector theorem)

LM + MN + NL = 11 cm ...(Given)

$\therefore$ MP + MN + NQ = 11 cm ...[From (i) and (ii)]

$\therefore$ PQ = 11 cm ...(P - M - N - Q)

In ΔPML,

$\therefore$ seg MP $\cong$ seg ML ...[From (i)]

$\angle$MPL $\cong$ $\angle$MLP ...(Isosceles triangle theorem)

Let $\angle$MPL = $\angle$MLP = x

$\angle$LMN is an exterior angle of ΔPML

$\therefore$ $\angle$LMN = $\angle$MPL + $\angle$MLP ...(Remote interior angle theorem)

$\therefore$ $60 = x + x$

$\therefore$ $60 = 2x$

$\therefore$ $x = 30$

$\therefore$ $\angle$MPL = $\angle$MLP = 30°

Similarly, we can prove $\angle$NQL = $\angle$NLQ = 40°.

Now, draw ΔLPQ, with PQ = 11 cm, $\angle$P = 30° and $\angle$Q = 40°

Steps of construction :

(1) Draw a seg PQ = 11 cm.

(2) Draw an angle measuring 30° at vertex P and an angle measuring 40° at vertex Q. Name point of intersection of these rays as point L.

(3) Draw line 'l' as ⊥ bisector of seg PL, intersecting seg PQ, at point M.

(4) Draw line as ⊥ bisector of seg QL, intersecting seg PQ at point N.

(5) Draw seg LM and seg LN.

PROBLEMS FOR PRACTICE

(1) Construct ΔPQR with perimeter 9.5 cm and each of base angle is 80°. Write the type of ΔPQR.

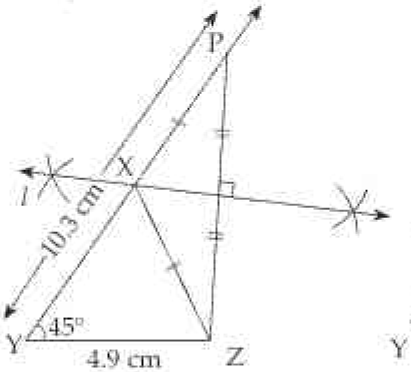
(2) Construct ΔABC such that ∠B = 70°, ∠C = 60° and AB + BC + CA = 10.5 cm.

(3) Construct ΔXYZ whose perimeter is 12 cm and ∠Y = 70° and ∠Z = 80°.

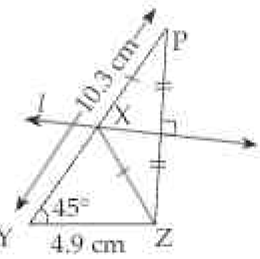
PROBLEM SET - 4 *(Textbook Page No. 56)*

(1) Construct ΔXYZ, in which XY + XZ = 10.3 cm, YZ = 4.9 cm. ∠XYZ = 45°

Solution:



Analytical Figure



Explanation :

Line l is perpendicular bisector of seg PZ

∴ XP = XZ ...(i)

(Perpendicular bisector theorem)

PY = 10.3 cm ...(ii)

XY + XP = PY ...(∵ P-X-Y)

∴ XY + XZ = 10.3 cm ...[From (i) and (ii)]

Steps of construction :

(1) Draw a seg YZ of length 4.9 cm

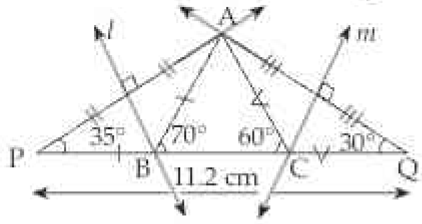
(2) Draw an angle measuring 45° at vertex Y, on this ray locate point P such that YP = 10.3 cm

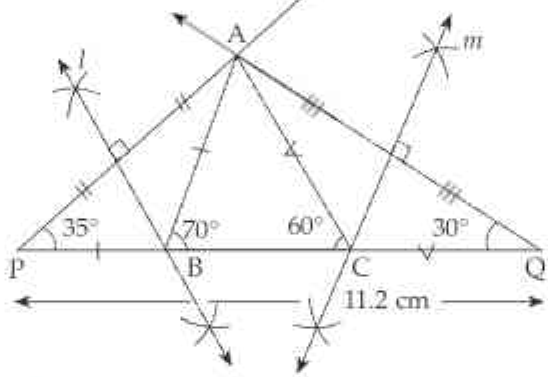
(3) Draw seg PZ. Draw line as ⊥ bisector of seg PZ intersecting ray YP at point X.

(4) Draw seg XZ.

(2) Construct ΔABC, in which ∠B = 70°, ∠C = 60°. AB + BC + AC = 11.2 cm

Solution: **Analytical Figure**





Explanation :

Line l and m are perpendicular bisector of seg AP and AQ respectively

∴ BP = BA ...(i)
and CQ = CA ...(ii) } (Perpendicular bisector theorem)

AB + BC + AC = 11.2 cm ...(Given)

∴ BP + BC + CQ = 11.2 cm ...[From (i), (ii)]

∴ PQ = 11.2 cm ...(P - B - C - Q)

In ΔABP,

seg BP ≅ seg BA ...[From (i)]

∴ ∠BPA ≅ ∠BAP

...(Isosceles triangle theorem)

Let ∠BPA = ∠BAP = x

∠ABC is an exterior angle of ΔABP

∴ ∠ABC = ∠BPA + ∠BAP

...(Remote interior angle theorem)

∴ 70 = $x + x$

∴ 70 = $2x$

∴ x = 35

∴ ∠BPA = ∠BAP = 35°

Similarly, we can prove ∠CAQ = ∠CQA = 30°.

Now, draw ΔAPQ, with PQ = 11.2 cm, ∠P = 35° and ∠Q = 30°

Steps of construction :

(1) Draw seg PQ of length 11.2 cm

(2) Draw an angle measuring 35° and 30° at vertex P and vertex Q respectively. Name the point of intersection as point A.

(3) Draw line l and line m as ⊥ bisector of seg AP and seg AQ respectively. Line l intersects PQ at point B and line m intersects PQ at point C.

(4) Draw seg AB and seg AC.

(3) The perimeter of a triangle is 14.4 cm and ratio of lengths of its sides is 2 : 3 : 4. Construct the triangle.

Solution:

Explanation :

Let the required triangle be ΔABC.

AB + BC + AC = 14.4 ...[Given]

AB : BC : AC = 2 : 3 : 4 ...[Given]

Let the common multiple be x

$\therefore$ $2x + 3x + 4x = 14.4$

$\therefore$ $9x = 14.4$

$\therefore$ $x = \frac{14.4}{9}$

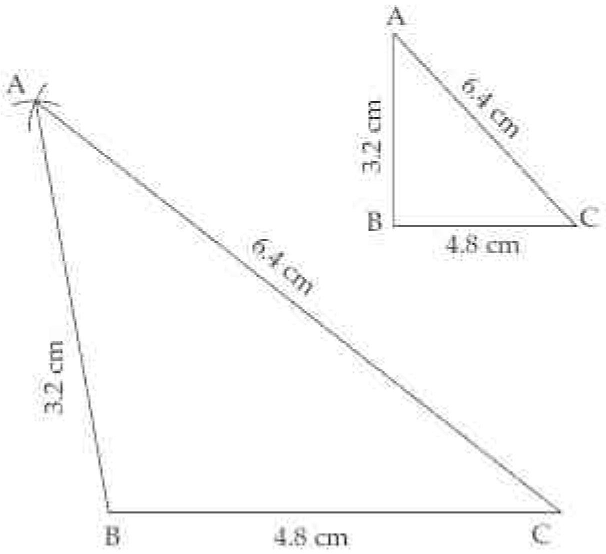
$\therefore$ $x = 1.6$

$\therefore$ $AB = 2x = 2 \times 1.6 = 3.2$ cm;

$\therefore$ $BC = 3x = 3 \times 1.6 = 4.8$ cm.

$\therefore$ $AC = 4x = 4 \times 1.6 = 6.4$ cm

Analytical Figure



Steps of construction :

(1) Draw seg BC of length 4.8 cm

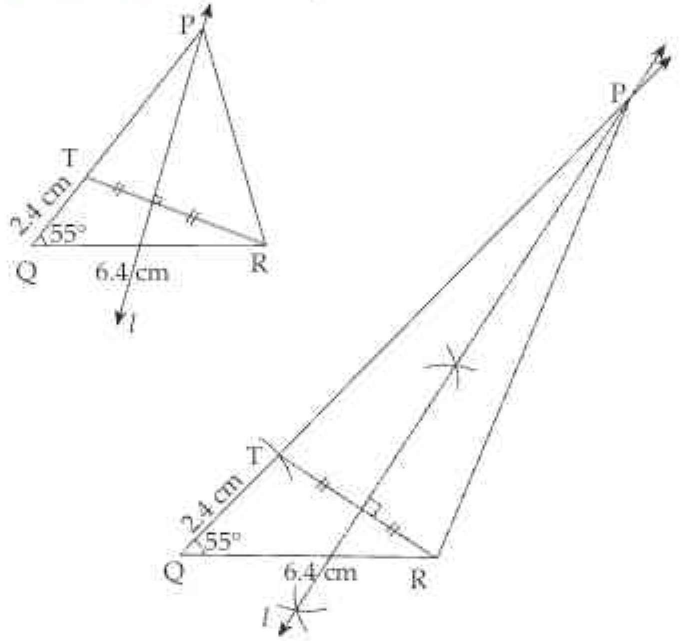
(2) Taking B as a centre and arc length 3.2 draw an arc.

(3) Taking C as a centre and radius 6.4 draw an arc intersecting arc. Name point of intersection as point A.

(4) Draw seg AB and seg AC.

(4) Construct ΔPQR, in which PQ – PR = 2.4 cm, QR = 6.4 cm and ∠PQR = 55°

Solution: Analytical Figure



Explanation :

Line l is perpendicular bisector of seg TR

$\therefore$ PT = PR ...(i)

(Perpendicular bisector theorem)

QT = 2.4 cm ...(ii)

PQ = PT + QT ...(∵ P-T-Q)

$\therefore$ PQ = PR + 2.4 ...[From (i) and (ii)]

$\therefore$ PQ – PR = 2.4 cm

Steps of construction :

(1) Draw a seg QR of length 6.4 cm.

(2) Draw an angle measuring 55° at vertex Q, on this ray, locate a point T such that l(QT) = 2.4 cm. Draw seg RT.

(3) Draw line l as ⊥ bisector of seg RT intersecting ray QT at point P.

(4) Draw seg PR.

ASSIGNMENT - 4

Time : 1 Hour **Marks : 20**

Q.1. Solve : **(6)**

(1) Draw a line l and take any point P on it. Draw line m perpendicular to line l at point P.

(2) Draw seg AB of length 7 cm. Take point P such that AP = 3 cm and A-P-B. Draw a perpendicular to seg AB through the point P.

(3) Draw a circle of radius 3 cm and centre O. Take any point P on the circle, draw perpendicular to seg OP at point P.

Q.2. Solve : **(6)**

(1) Construct ΔPQR such that $PQ - PR = 2.4$ cm, $QR = 6.4$ cm and $\angle PQR = 55°$

(2) Construct ΔABC, such that $BC = 6$ cm, $\angle ABC = 100°$ and $AC - AB = 2.5$ cm.

Q.3. Solve : **(8)**

(1) Perimeter of ΔABC is 15 cm, $BC = 5.2$ cm, $\angle ACB = 45°$. Construct ΔABC.

(2) Construct ΔABC whose perimeter is 12 cm, $\angle B = 60°$, $\angle C = 70°$.

❖ ❖ ❖

5 Quadrilaterals

Points to Remember:

❑ **Parallelogram :**

- **Definition :** A quadrilateral is called a parallelogram if its opposite sides are parallel.

In □ABCD,
side AB ∥ side DC
and side AD ∥ side BC.

∴ □ABCD is a parallelogram.

- **Properties of Parallelogram :**

(i) Opposite sides are congruent
(ii) Opposite angles are congruent
(iii) Adjacent angles are supplementary.
(iv) Diagonals bisect each other.

Theorem - 1 :

Opposite sides and opposite angles of a parallelogram are congruent.

Given : □ABCD is a parallelogram.

To Prove : (i) side AB ≅ side CD
(ii) side BC ≅ side AD
(iii) $\angle ABC \cong \angle CDA$
(iv) $\angle DAB \cong \angle BCD$.

Construction : Draw diagonal AC.

Proof : □ABCD is a parallelogram.

∴ side AB ∥ side CD
and side BC ∥ side AD
on transversal AC,
$\angle BAC \cong \angle DCA$ ...(i)
$\angle BCA \cong \angle DAC$ ...(ii) } (Alternate angles theorem)

In ΔABC and ΔCDA,
$\angle BAC \cong \angle DCA$ ...[From (i)]
side AC ≅ side CA ...(Common side)
$\angle BCA \cong \angle DAC$ ....[From (ii)]

∴ $\Delta ABC \cong \Delta CDA$
...(ASA Test of congruency)

side AB ≅ side CD
side BC ≅ side DA } ...(c.s.c.t.)

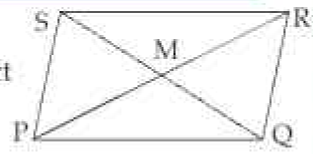
$\angle ABC \cong \angle CDA$ ...(c.a.c.t.)

Similarly, we can prove

$\angle DAB \cong \angle DCB$

Theorem - 2 :

"Diagonals of a parallelogram bisect each other."



Given:
(i) □PQRS is a parallelogram.
(ii) Diagonals PR and QS intersect each other at point M.

To Prove: (i) seg PM ≅ seg RM
(ii) seg QM ≅ seg SM

Proof : □PQRS is a parallelogram ...(Given)
side PQ ∥ side RS
...(Opposite sides of a parallelogram)
on transversal PR,
$\angle RPQ \cong \angle PRS$
...(Alternate angles theorem)

∴ $\angle MPQ \cong \angle MRS$ ...(i) (P - M - R)

on transversal QS,
$\angle SQP \cong \angle QSR$
...(Alternate angle theorem)

∴ $\angle MQP \cong \angle MSR$ ...(ii) (Q - M - S)

In ΔPMQ and ΔRMS,
$\angle MPQ \cong \angle MRS$ ...[From (i)]
side PQ ≅ side RS ...(Opposite sides of a parallelogram are congruent)
$\angle MQP \cong \angle MSR$ ...[From (ii)]

∴ ΔPMQ and ΔRMS
...(ASA Test of congruency)

∴ **seg PM ≅ seg RM**
seg QM ≅ seg SM } **...(c.s.c.t.)**

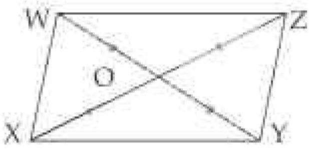
MASTER KEY QUESTION SET - 5

PRACTICE SET - 5.1 *(Textbook Page No. 62)*

(1) Diagonals of a parallelogram WXYZ intersect each other at point O. If $\angle XYZ = 135°$ then what is the measure of $\angle XWZ$ and $\angle YZW$?

If l(OY) = 5 cm then find l(WY) = ?

Solution:



$\square$ WXYZ is a parallelogram ...(Given)

$\angle XYZ \cong \angle XWZ$ ...(Opposite angles of a parallelogram are congruent)

But $\angle XYZ = 135°$

$\therefore$ $\boxed{\angle XWZ = 135°}$

$\therefore$ $\angle YZW + \angle XYZ = 180°$

...(Adjacent angles of a parallelogram are supplementary)

$\therefore$ $\angle YZW + 135 = 180$

$\therefore$ $\angle YZW = 180 - 135$

$\therefore$ $\boxed{\angle YZW = 45°}$

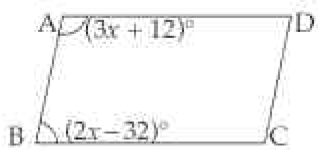
l (OY) = ½ l (WY) ...(Diagonals of parallelogram bisect each other.)

$\therefore$ 5 = ½ l (WY)

$\boxed{\therefore \quad l(WY) = 10 \text{ cm}}$

(2) In a parallelogram ABCD, if $\angle A = (3x + 12)°$; $\angle B = (2x - 32)°$ then find the value of x and then find the measures of $\angle C$ and $\angle D$.

Solution:



$\square$ ABCD is a parallelogram ...(Given)

$\therefore$ $\angle A + \angle B = 180°$ ...(Adjacent angles of a parallelogram are supplementary)

$\therefore$ $(3x + 12) + (2x - 32) = 180$

$\therefore$ $5x - 20 = 180$

$\therefore$ $5x = 180 + 20$

$\therefore$ $5x = 200$

$\therefore$ $\boxed{x = 40}$

$\angle A = (3x + 12)$	$\angle B = (2x - 32)$
$= 3(40) + 12$	$= 2(40) - 32$
$= 120 + 12$	$= 80 - 32$
$\therefore \angle A = 132°$	$\therefore \angle B = 48°$

$\therefore$ $\angle C = \angle A$ and $\angle D = \angle B$...(Opposite angles of a parallelogram are congruent.)

$\therefore$ $\boxed{\angle C = 132° \text{ and } \angle D = 48°}$

(3) Perimeter of a parallelogram is 150 cm. One of its sides is greater than the other side by 25 cm. Find lengths of all sides.

Solution:

$\square$ ABCD is a parallelogram.

Perimeter of $\square$ ABCD = 150 cm.

Let BC = x cm.

$\therefore$ AB = $(x + 25)$ cm

AB = CD and BC = AD...(Opposite sides of parallelogram are congruent.)

$\therefore$ CD = x + 25 cm and AD = x cm

Perimeter of $\square$ABCD = AB + BC + CD + AD

$\therefore$ $150 = x + 25 + x + x + 25 + x.$

$\therefore$ $150 - 50 = 4x$

$\therefore$ $100 = 4x$

$\therefore$ $\frac{100}{4} = x$

$\therefore$ $x = 25$

$$\boxed{\begin{array}{l} AB = CD = x + 25 = 25 + 25 = 50 \text{ cm} \\ BC = AD = x = 25 \text{ cm} \end{array}}$$

(4) If the ratio of measures of two adjacent angles of a parallelogram is 1 : 2, find the measures of all angles of the parallelogram.

Solution:

$\square$ PQRS is a parallelogram.

$\angle P : \angle Q = 1 : 2$

Let the common multiple be x.

$\therefore$ $\angle P = x$ and $\angle Q = 2x$

$\therefore$ $\angle P + \angle Q = 180°$ ...(Adjacent angles of a parallelogram are supplementary)

$\therefore$ $x + 2x = 180$

$\therefore$ $3x = 180$

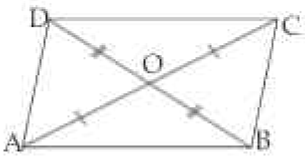
$\therefore \quad x = \frac{180}{3} = 60$

$\therefore \quad \boxed{\begin{array}{l}\angle P = x = 60^\circ \\ \angle Q = 2x = 2 \times 60 = 120^\circ\end{array}}$

$\therefore$ $\angle R = \angle P$ and $\angle S = \angle Q$ ...(Opposite angles of a parallelogram are congruent)

$\therefore$ $\boxed{\angle R = 60^\circ \text{ and } \angle S = 120^\circ}$

(5) Diagonals of a parallelogram intersect each other at point O. If AO = 5, BO = 12 and AB = 13 then show that □ABCD is a rhombus.



To Prove: □ABCD is a rhombus.

Proof : $AO^2 + BO^2 = 5^2 + 12^2 = 25 + 144$

$\therefore$ $AO^2 + BO^2 = 169$ ...(i)

$AB^2 = 13^2 = 169$ ...(ii)

In ΔAOB, $AB^2 = AO^2 + BO^2$ ...[From (i) and (ii)]

$\therefore$ $\angle AOB = 90^\circ$ ...(Converse of Pythagoras theorem)

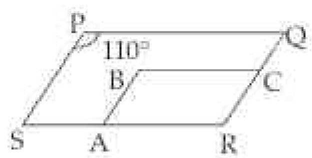
$\therefore$ Diagonals of □ABCD are perpendicular to each other at point O.

Also, ABCD is a parallelogram ...(Given)

But, we know that a parallelogram in which diagonals are perpendicular is a rhombus

$\therefore$ **□ABCD is a rhombus.**

(6) In following diagrams, □PQRS and □ABCR are two parallelograms. If ∠P = 110° then find measures of all angles of □ABCR.



Solution:

□PQRS is a parallelogram ...(Given)

$\therefore$ $\angle R = \angle P$ ...(Opposite angles of a parallelogram are congruent)

But, $\angle P = 110^\circ$ ...(Given)

$\therefore$ $\boxed{\angle R = 110^\circ}$

□ABCR is a parallelogram ...(Given)

$\therefore$ $\angle B = \angle R$ ...(Opposite angles of a parallelogram are congruent)

$\therefore$ $\boxed{\angle B = 110^\circ}$

$\therefore$ $\angle RAB + \angle R = 180^\circ$ ...(Adjacent angles of a parallelogram are supplementary)

$\therefore$ $\angle RAB + 110 = 180$

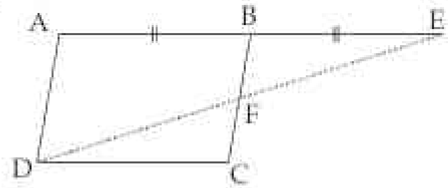
$\therefore$ $\angle RAB = 180 - 110$

$\therefore$ $\boxed{\angle RAB = 70^\circ}$

$\therefore$ $\angle BCR = \angle RAB$ ...(Opposite angles of a parallelogram are congruent)

$\therefore$ $\boxed{\angle BCR = 70^\circ}$

(7) In following figure, □ABCD is a parallelogram. Point E is on the ray AB such that BE = AB then prove that line ED bisects seg BC at point F.



Proof : □ABCD is a parallelogram ...(Given)

AB = CD ...(i) (Opposite side of a parallelogram are congruent.)

AB = BE ...(ii) (Given)

$\therefore$ CD = BE ...(iii) [From (i) and (ii)]

side AB ∥ side CD ...(Definition of a parallelogram)

$\therefore$ side AE ∥ side CD ...(A - B - E)

On transversal BC,

$\therefore$ $\angle DCF \cong \angle EBF$ ...(iv) (Alternate angles theorem)

In ΔDCF and ΔEBF

side CD ≅ side BE ...[From (iii)]

$\angle DFC \cong \angle EFB$ ...(Vertically opposite angles)

$\angle DCF \cong \angle EBF$ ...[From (iv)]

$\therefore$ $\Delta DCF \cong \Delta EBF$ ...(SAA Test of congruency)

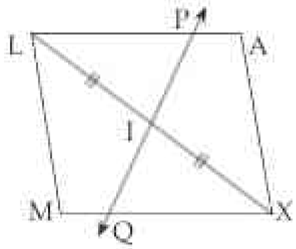
$\therefore$ seg CF ≅ seg BF ...(c.s.c.t)

$\therefore$ **line ED bisects seg BC at point F.**

PROBLEMS FOR PRACTICE

(1) Prove that two opposite vertices of a parallelogram are equidistant from the diagonal not containing these vertices.

(2) **In the adjoining figure, □ LAXM is a parallelogram. Point I is the midpoint of diagonal LX. PQ is a line passing through point I. The points P and Q are the points of intersection of sides AL and MX respectively. Prove that seg PI ≅ seg IQ.**



(3) **Prove that in a parallelogram, the angle bisectors of two adjacent angles meet at right angles.**

(4) **In a parallelogram ABCD, diagonal AC ⊥ diagonal BD intersect at point M. AM = 3 cm, BM = 4 cm, AB = 7.5 cm and BC = 5 cm. Find the perimeter of □ABCD and also the lengths of AC and BD.**

ANSWERS

(4) Perimeter of □ ABCD = 25 cm, AC = 6 cm, BD = 8 cm.

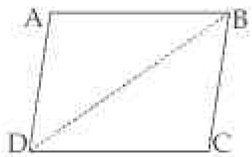
Points to Remember:

❑ **Test for Parallelogram :**

- Following theorems will help us to prove a quadrilateral is a parallelogram.

Theorem - 3 :

Statement : If pairs of opposite sides of a quadrilateral are congruent then that quadrilateral is a parallelogram.



Given : In □ABCD,

(i) side AB ≅ side CD

(ii) side AD ≅ side BC

To Prove : □ ABCD is a parallelogram.

Construction : Draw diagonal BD.

Proof : In ΔABD and ΔCDB,

side AB ≅ side CD ...(Given)

side AD ≅ side BC ...(Given)

side BD ≅ side BD ...(Common side)

∴ ΔABD ≅ ΔCDB ...(By SSS test of congruency)

∴ ∠ABD ≅ ∠CDB ...(c.a.c.t.)

∴ Side AB || Side CD ...(i) (By Alternate angles test)

Similarly, we can prove that,

∴ side AD || side BC ...(ii)

In □ ABCD,

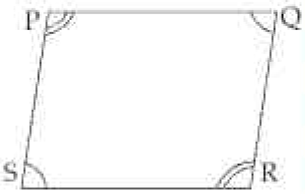
side AB || side CD ...[From (i)]

side AD || side BC ...[From (ii)]

∴ **□ ABCD is a parallelogram.** ...(By definition)

Theorem - 4 :

Statement : If both the pairs of opposite angles of a quadrilateral are congruent then it is a parallelogram.



Given: In □ PQRS,

∠SPQ ≅ ∠SRQ

∠PSR ≅ ∠PQR

To Prove: □ PQRS is a parallelogram.

Proof : Let,

∠SPQ = ∠SRQ = $x°$ ...(i) (Given)

∠PSR = ∠PQR = $y°$ ...(ii)

In □ PQRS,

∠SPQ + ∠PQR + ∠QRS + ∠PSR = 360° ...(Angle sum property of a quadrilateral)

∴ $x + y + x + y = 360$

∴ $2x + 2y = 360$

∴ $2(x + y) = 360$

∴ $x + y = 180$

∴ m∠SPQ + m∠PSR = 180° ...[From (i) and (ii)]

∴ side PQ || side SR ...(iii) (By Interior angles test)

Similarly, we can prove,

∴ side PS || side QR ...(iv)

In □ PQRS,

side PQ || side SR ...[From (iii)]

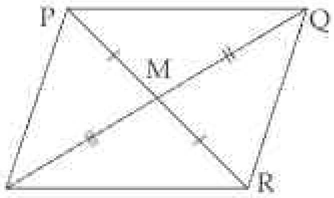
side PS || side QR ...[From (iv)]

∴ **□ PQRS is a parallelogram.** ...(By definition)

Theorem - 5 :

Statement : If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Given: In □PQRS,



(i) Diagonal PR and diagonal QS intersect at M.

(ii) seg PM ≅ seg RM

(iii) seg SM ≅ seg QM

To Prove: □ PQRS is a parallelogram.

Proof : In ΔPMQ and ΔRMS,

seg PM ≅ seg RM ...(i) (Given)

∠PMQ ≅ ∠RMS ...(Vertically opposite angles)

seg QM ≅ seg SM ...(Given)

∴ ΔPMQ ≅ ΔRMS, ...(By SAS test of congruency)

∴ ∠PQM ≅ ∠RSM ...(c.a.c.t.)

∴ ∠PQS ≅ ∠RSQ ...(Q-M-S)

∴ side PQ || side SR ...(i) (By Alternate angles test)

Similarly, we can prove

side PS || side QR ...(ii)

In □ PQRS,

side PQ || side SR ...[From (i)]

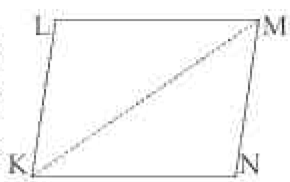
side PS || side QR ...[From (ii)]

∴ **□PQRS is a parallelogram.** ...(By definition)

Theorem - 6 :

Statement : A quadrilateral is a parallelogram if a pair of opposite sides is parallel and congruent.

Given: In □ LMNK,



(i) side LM || side KN

(ii) side LM ≅ side KN

To Prove: □ LMNK is a parallelogram.

Construction: Draw diagonal MK.

Proof : In □ LMNK,

side LM || side KN ...(Given)

On transversal MK,

∠LMK ≅ ∠NKM ...(i) ...(Alternate angles theorem)

∴ In ΔKLM and ΔMNK

seg LM ≅ seg KN ...(Given)

∠LMK ≅ ∠NKM ...[From (i)]

seg KM ≅ seg KM ...(Common side)

∴ ΔKLM ≅ ΔMNK ...(By SAS test of congruency)

∴ ∠LKM ≅ ∠NMK ...(c.a.c.t.)

∴ seg LK || seg MN ...(ii) (By Alternate angles test)

In □ LMNK,

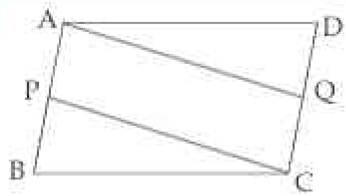
side LM || side KN ...(Given)

side LK || side MN ...[From (ii)]

∴ **□LMNK is a parallelogram.** ...(By definition)

PRACTICE SET - 5.2 *(Textbook Page No. 67)*

(1) **In adjoining figure, □ABCD is a parallelogram, P and Q are midpoints of side AB and side CD respectively.**



Prove that: □ APCQ is a parallelogram.

Proof: side AB || side CD ...(Opposite sides of a parallelogram are parallel)

∴ side AP || side CQ ... (i) (A-P-B, C-Q-D)

AB = CD ...(Opposite sides of parallelogram are congruent)

∴ $\frac{1}{2}$AB = $\frac{1}{2}$CD ...(Multiplying through out by $\frac{1}{2}$)

∴ AP = CQ...(ii) [∵ P and Q are midpoints of sides AB and CD respectively]

In □APCQ,

side AP || side CQ [From (i)]

side AP ≅ side CQ [From (ii)]

∴ **□ APCQ is a parallelogram** ...(A quadrilateral is a parallelogram, if a pair of opposite sides is parallel and congruent)

(2) Using opposite angles test for parallelogram, prove that every rectangle is a parallelogram.

Given: □ ABCD is a rectangle.

To Prove: □ ABCD is a parallelogram.

Proof:

□ ABCD is a rectangle ...(Given)

$\therefore$ $\angle A = \angle B = \angle C = \angle D = 90°$ ...(i)

(Angles of a rectangle)

In □ ABCD,

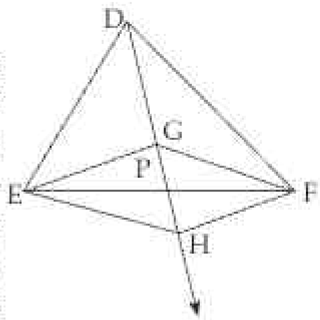
$\angle A \cong \angle C$ ...[From (i)]

$\angle B \cong \angle D$ ...[From (i)]

$\therefore$ **□ ABCD is a parallelogram**

...(If both the pairs of opposite angles of a quadrilateral are congruent then it is a parallelogram)

(3) In adjoining figure, G is the point of concurrence of medians of ΔDEF. Take point H on ray DG such that D-G-H and DG = GH, then prove that □ GEHF is a parallelogram.



Proof: Let the point of intersection of seg EF and seg GH be P.

G is the centroid of ΔDEF

$\therefore$ seg DP is the median

$\therefore$ P is the midpoint of seg EF.

$\therefore$ EP = FP ...(i)

Now, $GP = \frac{1}{3} PD$ ...(ii)
$DG = \frac{2}{3} PD$ ...(iii)
} centroid divides the median in the ratio 2 : 1

But, DG = GH ...(iv) ...(Given)

$\therefore$ $GH = \frac{2}{3} PD$ ...[From (iii) and (iv)]

$\therefore$ $GP + PH = \frac{2}{3} PD$ ...(G-P-H)

$\therefore$ $\frac{1}{3} PD + PH = \frac{2}{3} PD$ ...[From (ii)]

$\therefore$ $PH = \frac{2}{3} PD - \frac{1}{3} PD$

$\therefore$ $PH = \frac{1}{3} PD$ ...(v)

GP = PH ...(vi) [From (ii), (v)]

In □ GEHF, seg EP ≅ seg FP ...[From (i)]

seg GP ≅ seg PH ...[From (vi)]

$\therefore$ **□ GEHF is parallelogram.**

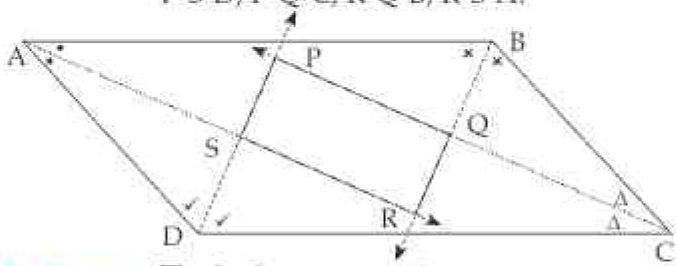
...(If diagonals of a quadrilateral bisect each other then it is a parallelogram.

(4) Prove that quadrilateral formed by the intersection of angle bisectors of all angles of a parallelogram is a rectangle.

Given: □ ABCD is a parallelogram.

rays AR, BR, CP, DP are bisectors of $\angle A$, $\angle B$, $\angle C$, $\angle D$ respectively.

P-S-D, P-Q-C, R-Q-B, R-S-A.



To Prove: □ PQRS is a rectangle.

Proof: Let,

$\angle SAD = \angle SAB = a°$
$\angle QBA = \angle QBC = b°$
$\angle QCB = \angle QCD = c°$
$\angle SDC = \angle SDA = b°$
} Rays AR, BR, CP, DP bisects $\angle A$, $\angle B$, $\angle C$, $\angle D$ respectively

In ΔASD

$\angle SAD + \angle SDA + \angle ASD = 180°$

...(Sum of the measures of all angles of a triangle is 180°)

$\therefore$ $a + d + \angle ASD = 180$

$\therefore$ $\angle ASD = 180° - (a + d)$ ...(i)

□ ABCD is a parallelogram ...(Given)

$\therefore$ $\angle A + \angle D = 180°$ ...(Adjacent angles of a parallelogram are supplementary)

$\therefore$ $\angle SAD + \angle SAB + \angle SDA + \angle SDC = 180°$

...(Angle addition property)

$\therefore$ $a + a + d + d = 180$

$\therefore$ $2a + 2d = 180$

$\therefore$ $2(a + d) = 180$

$\therefore$ $a + d = \frac{180}{2}$

$\therefore$ $a + d = 90$ ...(ii)

Now, $\angle ASD = 180 - 90$ [From (i), (ii)]

$\therefore$ $\angle ASD = 90°$ ...(iii)

Now, $\angle ASD \cong \angle PSR$

...(Vertically opposite angles)

$\therefore$ $\angle PSR = 90°$ [From (iii)]

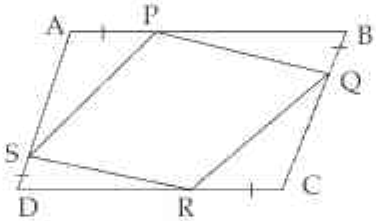
Similarily, we can prove $\angle SPQ = 90°$, $\angle PQR = 90°$, $\angle SRQ = 90°$

In □ PQRS,

$\angle P = \angle Q = \angle R = \angle S = 90°$

$\therefore$ **□ PQRS is a rectangle** ...(By definition)

(5) **In adjoining figure, If points P, Q, R, S are on the sides of parallelogram such that AP = BQ = CR = DS then prove that □PQRS is a parallelogram.**



To Prove: □ PQRS is a parallelogram

Proof: □ ABCD is a parallelogram. ...(Given)

$\therefore$ AD = BC ...(Opposite sides of parallelogram are equal.)

$\therefore$ AS + DS = BQ + QC ...(A-S-D, B-Q-C)

$\therefore$ AS = CQ ...(i) ($\because$ DS = BQ)

In ΔPAS and ΔRCQ

side AP $\cong$ side CR ...(Given)

$\angle A \cong \angle C$ ...(Opposite angles of a parallelogram are congruent)

side AS $\cong$ side CQ [From (i)]

$\therefore$ $\Delta PAS \cong \Delta RCQ$

...(By SAS test of congruency)

$\therefore$ side PS $\cong$ side RQ ...(ii) (c.s.c.t.)

Similarly, we can prove,

side PQ $\cong$ side RS ...(iii)

In □ PQRS,

side PQ $\cong$ side RS ...[From (iii)]

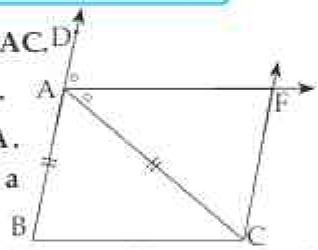
side PS $\cong$ side RQ ...[From (ii)]

$\therefore$ **□ PQRS is a parallelogram.**

...(A quadrilateral is a parallelogram if its opposite sides are congruent.)

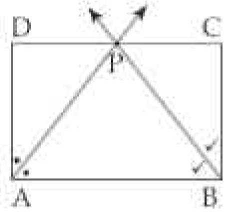
PROBLEMS FOR PRACTICE

(1) **In ΔABC, seg AB $\cong$ seg AC. Ray AF bisects $\angle DAC$. Ray CF $\parallel$ ray BA. Prove that □ABCF is a parallelogram**



(2) **If diagonal of a parallelogram bisects one of the angles of the parallelogram, it also bisects the second angle.**

(3) **In the adjoining figure, □ABCD is a parallelogram $\angle DAB = 60°$. If the bisector AP and BP of $\angle A$ and $\angle B$ respectively meet at point P on side CD, prove that P is midpoint of side CD.**



Points to Remember:

Rectangle :

A parallelogram in which each angle is a right angle is called a rectangle.

In parallelogram ABCD,

$\angle A = \angle B = \angle C = \angle D = 90°$.

Properties of a Rectangle :

(i) Diagonals bisect each other

(ii) Diagonals are congruent.

NOTE

Every rectangle is a parallelogram

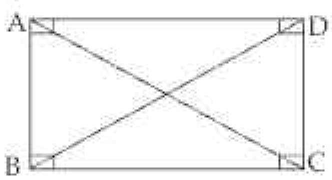
Theorem - 7 :

Statement : Diagonals of a rectangle are congruent.

Given : □ ABCD is a rectangle.

seg BD and seg AC are its diagonals.

To Prove: Diagonal AC $\cong$ diagonal BD



Proof : □ABCD is a rectangle. ...(Given)

∴ $\angle ABC = \angle DCB = 90°$ ...(i) (Angles of a rectangle)

In ΔABC and ΔDCB,

side AB ≅ side DC ...(Opposite sides of rectangle)

$\angle ABC \cong \angle DCB$ ...[From (i)]

side BC ≅ side BC ...(Common side)

∴ $\Delta ABC \cong DCB$ ...(By SAS test of congruency)

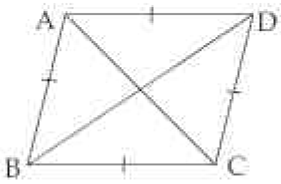
∴ **Diagonal AC ≅ diagonal BD ...(c.s.c.t.)**

Theorem - 8 :

Statement : Diagonals of a square are congruent.

Hint : Using the concept of theorem 7, prove this theorem.

- **Rhombus :**



A parallelogram having all sides congruent is called a rhombus.

In □ABCD,

seg AB ≅ seg BC ≅ seg CD ≅ seg AD

∴ □ABCD is a rhombus.

Properties of a Rhombus :

(i) Opposite angles are congruent.

(ii) Diagonals bisect each other at right angles.

NOTE

Every rhombus is a parallelogram.

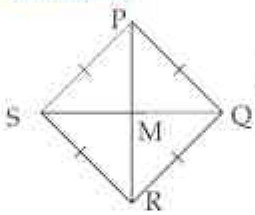
Theorem - 9 :

Statement : Diagonals of a rhombus are perpendicular bisectors of each other.

Given : □PQRS is a rhombus.

Diagonal PR and diagonal QS intersect at the point M.

To Prove: (i) seg PM ≅ seg RM



(ii) seg SM ≅ seg QM

(iii) diagonal PR ⊥ diagonal QS

Proof : □PQRS is a rhombus. ...(Given)

∴ It is a parallelogram ...(∵ Every rhombus is a parallelogram)

∴ **seg PM ≅ seg RM** ...(i)

seg SM ≅ seg QM ..(ii)

...(Diagonals of a parallelogram bisect each other)

In ΔPMS and ΔPMQ

seg SM ≅ seg QM [From (ii)]

side PS ≅ side PQ ...(sides of a rhombus)

seg PM ≅ seg PM ...(Common side)

∴ $\Delta PMS \cong \Delta PMQ$ ...(By SSS test of congruency)

∴ $\angle PMS \cong \angle PMQ$ ...(iii) (c.a.c.t.)

$\angle PMS + \angle PMQ = 180°$ ...(Angles forming linear pair)

∴ $\angle PMS + \angle PMS = 180$ ...[From (iii)]

∴ $2\angle PMS = 180$

∴ $\angle PMS = 90°$

∴ **Diagonal PR ⊥ diagonal QS**

- **Square :**

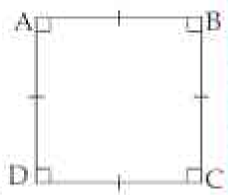
A quadrilateral is called a square if all its sides are congruent and each angle is a right angle

In □ABCD,

seg AB ≅ seg BC ≅ seg CD ≅ seg AD.

$\angle A = \angle B = \angle C = \angle D = 90°$

∴ □ABCD is a square.



Properties of a square :

(i) Diagonals are congruent.

(ii) Diagonals bisect each other at right angles.

NOTE

(i) A parallelogram whose adjacent sides are congruent and one angle is a right angle, is a square.

(ii) A rectangle with adjacent sides congruent is a square.

(iii) If in a rhombus, one of its angle is a right angle then it is a square. Every square is a rhombus.

Theorem - 10 :

Statement : Diagonals of a square are perpendicular bisectors of each other.

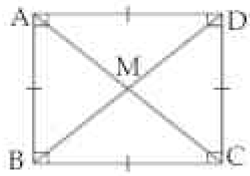
Given : □ABCD is a square.

Diagonal AC and diagonal BD intersect at M.

To Prove: (i) seg AM ≅ seg CM

(ii) seg BM ≅ seg DM

(iii) Diagonal AC ⊥ diagonal BD



Proof : □ABCD is a square ...(Given)

∴ □ABCD is a rhombus.

...(∵ Every square is a rhombus)

∴ **seg AM ≅ seg CM**

seg BM ≅ seg DM

∴ **Diagonal AC ⊥ diagonal BD**

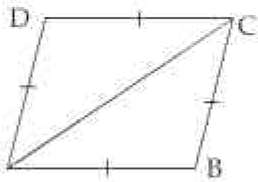
...(Diagonals of a rhombus are perpendicular bisectors of each other)

Theorem - 11 :

Statement : Diagonals of a rhombus bisect its opposite angles.

Given : □ ABCD is a rhombus.

seg AC is a diagonal.



To Prove: (i) ∠BAC ≅ ∠DAC

(ii) ∠BCA ≅ ∠DCA

Proof :

In ΔABC and ΔADC

side AB ≅ side AD } ... (sides of a rhombus)
side BC ≅ side DC }

side AC ≅ side AC ...(Common side)

∴ ΔABC ≅ ΔADC...(SSS Test of congruency)

∴ **∠BAC ≅ ∠DAC** }
and **∠BCA ≅ ∠DCA** } ...(c.a.c.t.)

Theorem - 12 :

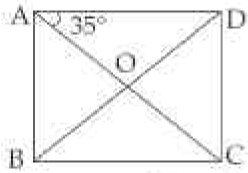
Statement : Diagonals of a square bisects its opposite angles.

Hint : Use the concept used in the above theorem to prove the above statement.

PRACTICE SET - 5.3 *(Textbook Page No. 69)*

(1) Diagonals of a rectangle ABCD intersect at point O. If AC = 8 cm then find BO and if ∠CAD = 35° then find ∠ACB.

Solution:



□ABCD is a rectangle. ...(Given)

∴ BD = AC ...(Diagonals of a rectangle are congruent)

But, AC = 8 cm ...(Given)

∴ BD = 8 cm ...(i)

$BO = \frac{1}{2} BD$ ...(Diagonals of rectangle bisect each other)

∴ $BO = \frac{1}{2} \times 8$ ...[From (i)]

∴ $\boxed{BO = 4 \text{ cm}}$

side AD ∥ side BC...(Opposite sides of a rectangle are parallel)

On transversal AC,

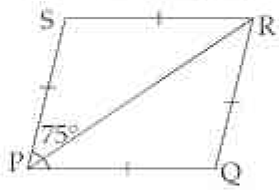
∴ ∠ACB ≅ ∠CAD ...(Alternate angles theorem)

But, ∠CAD = 35° ...(Given)

∴ $\boxed{\angle ACB = 35^\circ}$

(2) In a rhombus PQRS if PQ = 7.5 then find QR. If ∠QPS = 75° then find the measure ∠PQR and ∠SRQ.

Solution:



□PQRS is a rhombus. ...(Given)

∴ QR = PQ ...(Sides of a rhombus are equal)

But, PQ = 7.5 ...(Given)

∴ $\boxed{QR = 7.5}$

∴ ∠SRQ ≅ ∠QPS ...(Opposite angles of a rhombus are congruent)

But, ∠QPS = 75° ...(Given)

∴ $\boxed{\angle SRQ = 75^\circ}$

□ PQRS is a parallelogram

...(Every rhombus is a parallelogram)

$\therefore \quad \angle PQR + \angle SPQ = 180^\circ$

...(Adjacent angles of a parallelogram are supplementary)

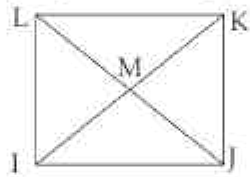
$\therefore \quad \angle PQR + 75 = 180$

$\therefore \quad \angle PQR = 180 - 75$

$\therefore \quad \boxed{\angle PQR = 105^\circ}$

(3) Diagonals of square IJKL intersect at point M. Find the measures of ∠IMJ, ∠JIK and ∠LJK

Solution:



□ IJKL is a square. ...(Given)

Diagonals of a square are perpendicular to each other.

$\therefore \quad \boxed{\angle IMJ = 90^\circ}$

Diagonals of a square bisects the opposite angles

$\therefore \quad \angle JIK = \frac{1}{2} \angle JIL$

$\therefore \quad \boxed{\angle JIK = \frac{1}{2} \times 90^\circ = 45^\circ}$

... (Angle of a square)

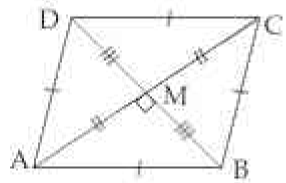
Also, $\angle LJK = \frac{1}{2} \angle IJK$

$\therefore \quad \boxed{\angle LJK = \frac{1}{2} \times 90^\circ = 45^\circ}$

... (Angle of a square)

(4) Diagonals of a rhombus are 20 cm and 21 cm respectively, then find the side of rhombus and its perimeter.

Solution:



□ABCD is a rhombus

Diagonals AC and BD intersect at point M.

AC = 20 cm; BD = 21 cm

Diagonals AC and BD are bisectors of each other

$\therefore \quad AM = \frac{1}{2} \times AC = \frac{1}{2} \times 20 = 10 \text{ cm}$ ...(i)

$BM = \frac{1}{2} \times BD = \frac{1}{2} \times 21 = 10.5 \text{ cm}$ ...(ii)

In ΔAMB, $\angle AMB = 90^\circ$

...(Diagonals of a rhombus are perpendicular to each other)

$\therefore \quad AB^2 = AM^2 + BM^2$ ...(Pythagoras theorem)

$= 10^2 + 10.5^2$ ...[From (i) and (ii)]

$\therefore \quad = 100 + 110.25$

$\therefore \quad AB^2 = 210.25 = \frac{21025}{100} = \frac{5 \times 5 \times 29 \times 29}{10 \times 10}$

$\therefore \quad AB = \frac{5 \times 29}{10} = \frac{145}{10}$

...(Taking square roots)

$\therefore \quad \boxed{AB = 14.5 \text{ cm}}$

$\therefore$ Each side of rhombus ABCD is 14.5 cm

$\therefore$ Perimeter of rhombus ABCD = 4×14.5

$\therefore$ **Perimeter of rhombus ABCD = 58 cm**

(5) State with reasons whether following statements are 'True' or 'False'.

(i) Every parallelogram is a rhombus.

Ans. False. All sides of rhombus are congruent whereas in parallelogram opposite sides are congruent.

(ii) Every rhombus is a rectangle.

Ans. False : Each angle of a rectangle is right angle, whereas in rhombus opposite angles are congruent, they need not be right angles always.

(iii) Each rectangle is a parallelogram.

Ans. True : In rectangle, both pairs of opposite sides are parallel. This is true for parallelograms also.

(iv) Each square is a rectangle.

Ans. True : Each angle of square is a right angle, which is true for rectangle also.

(v) Each square is a rhombus.

Ans. True : All sides of a square are congruent which is true for rhombus also.

(vi) Every parallelogram is a rectangle.

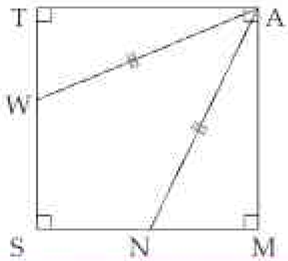
Ans. False : Each angle of rectangle is a right angle, which is not true for a parallelogram.

PROBLEMS FOR PRACTICE

(1) Prove that diagonals of a square divide it into four congruent triangles.

(2) □PQRS is a rhombus. O be any point on diagonal QS. Prove that seg OP ≅ seg OR.

(3) **□TAMS is a square.**
seg AW ≅ seg AN.
Prove that
seg SW ≅ seg SN.



Points to Remember:

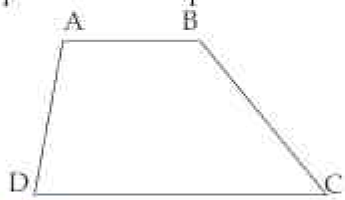
❑ Trapezium :

A quadrilaterial is said to be a trapezium, if only one pair of opposite sides is parallel.

In □ ABCD,

side AB || side DC

∴ □ ABCD is a trapezium.



❑ Properties of a Trapezium.

In a trapezium line segment joining the midpoints of non-parallel sides is :

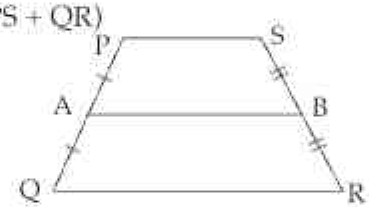
(i) Parallel to its parallel sides and

(ii) half the sum of the length of its parallel sides.

In trapezium PQRS, sides PS || side QR point A and B are midpoints of seg PQ and seg SR respectively, then

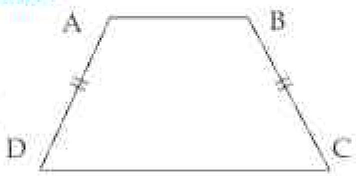
(a) seg AB || seg QR || seg PS

(b) AB = $\frac{1}{2}$ × (PS + QR)



❑ Isosceles Trapezium :

A trapezium in which non parallel sides are congruent is called an isosceles trapezium.



In □ ABCD,

side AB || side DC

side AD ≅ side BC

∴ □ ABCD is an Isosceles trapezium.

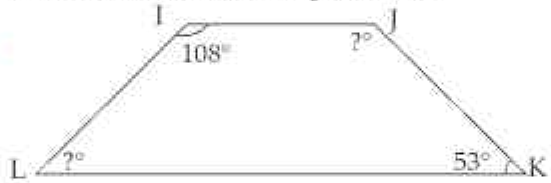
NOTE

The base angles of an isosceles trapezium are congruent.

PRACTICE SET - 5.4 *(Textbook Page No. 71)*

(1) **In □IJKL, side IJ || side KL, ∠I = 108° ∠K = 53° then find the measures of ∠J and ∠L.**

Solution:



In □ IJKL, side IJ || side KL ...(Given)

On transversal JK,

∴ ∠K + ∠J = 180° ...(Interior angles theorem)

∴ 53 + ∠J = 180

∴ ∠J = 180 – 53

∴ $\boxed{∠J = 127°}$

On transversal IL,

∴ ∠I + ∠L = 180° ...(Interior angles theorem)

∴ 108 + ∠L = 180

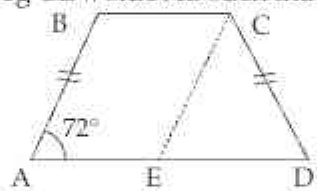
∴ ∠L = 180 – 108

∴ $\boxed{∠L = 72°}$

(2) **In □ABCD, side BC || side AD, side AB ≅ side DC. If ∠A = 72° then find the measures of ∠B, and ∠D.**

Construction : Draw seg CE || side AB such that A-E-D.

Solution:



side AB || side CE ...(i) (Construction)

side BC || side AD ...(Given)

∴ side BC || side AE ...(ii) (A-E-D)

□ AECB is a parallelogram ...[From (i), (ii) and by definition)

side AB ≅ side CE ...(iii) (Opposite sides of a parallelogram are congruent)

Also, side AB ≅ side CD ...(iv) (Given)

In ΔCED, side CE ≅ side CD ...[From (iii), (iv)]

∴ ∠CED ≅ ∠CDE ...(v)

side CE || side AB

On transversal AE,

$\angle CED \cong \angle BAD$ ...(vi)

$\therefore$ $\angle CDE \cong \angle BAD$ ...(vii)

(Corresponding angles theorem) ... [From (v) (vi)]

$\angle BAD = 72^\circ$ ...(Given)

$\angle CDE = 72^\circ$ ...[From (vii)]

$\therefore$ $\boxed{\angle D = 72^\circ}$

side BC $\parallel$ side AD

On transversal AB,

$\angle A + \angle B = 180^\circ$

...(Interior angles theorem)

(c) $72 + \angle B = 180$

(c) $\angle B = 180 - 72$

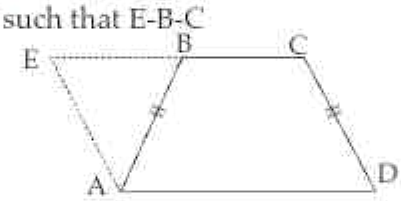
$\therefore$ $\boxed{\angle B = 108^\circ}$

(3) In □ABCD , side BC < side AD, side BC ∥ side AD and if side BA ≅ side CD.

Then prove that ∠ABC ≅ ∠DCB.

Construction : Draw seg AE $\parallel$ side DC and extend side BC such that E-B-C

Proof :



side AE $\parallel$ side DC ...(i) (Construction)

side BC $\parallel$ side AD ...(Given)

$\therefore$ side EC $\parallel$ side AD ...(ii) (E-B-C)

$\therefore$ □AECD is a parallelogram ...[From (i), (ii) and by definition]

$\therefore$ side AE $\cong$ side DC ...(iii) (Opposite sides of a parallelogram are congruent)

side AB $\cong$ side DC ...(iv) (Given)

In ΔABE, side AB $\cong$ side AE ...[From (iii), (iv)]

$\angle AEB \cong \angle ABE$ ...(Isosceles triangle theorem)

$\therefore$ $\angle AEC \cong \angle ABE$ ...(v) (E - B - C)

$\angle AEC + \angle DCE = 180^\circ$ ...(vi) ...(Adjacent angles of a parallelogram are supplementary)

Also, $\angle ABE + \angle ABC = 180^\circ$ ...(vii) (Linear pair angles)

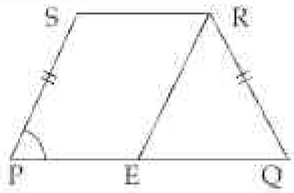
$\angle AEC + \angle DCE = \angle ABE + \angle ABC$ ...[From (vi), (vii)]

$\therefore$ $\angle DCE = \angle ABC$ ...[From (vi) (vii)]

$\therefore$ **$\angle ABC \cong \angle DCB$** ...(E - B - C)

PROBLEMS FOR PRACTICE

(1) **□PQRS is a isosceles trapezium. PQ ∥ RS and ΔRQE is an equilateral triangle. Find measures of all angles of the trapezium.**



(2) **The measures of the angles of a quadrilateal taken in order are as 1 : 2 : 3 : 4. Prove that it is a trapezium.**

(3) **□ABCD is a trapezium in which side AB ∥ side CD and side AD ≅ side BC. Prove that diagonal AC ≅ diagonal BD.**

ANSWERS

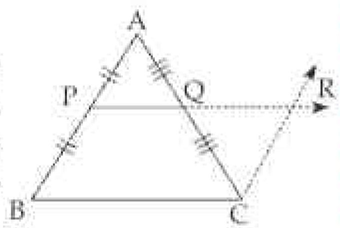
(1) 60°, 60°, 120°, 120°

Points to Remember:

Theorem - 13 :

❑ **Theorem of midpoint of two sides of a triangle. (Midpoint theorem)**

Statement : The line segment joining the midpoints of any two sides of a trianlge is parallel to the third side and is half of it.



Given : In ΔABC, P and Q are the midpoints of sides AB and AC respectively.

To prove : (i) seg PQ $\parallel$ seg BC

(ii) $PQ = \dfrac{1}{2} BC$

Construction : Take a point R on ray PQ such that P - Q - R and seg PQ $\cong$ seg QR. Draw seg CR.

Proof : In ΔAQP and ΔCQR,

seg AQ $\cong$ seg CQ ($\because$ Q is midpoint of side AC)

$\angle AQP \cong \angle CQR$ ...(Vertically opposite angles) seg PQ ≅ seg QR ...(Construction)

∴ ΔAQP ≅ ΔCQR ...(By SSS test of congruency)

∴ seg AP ≅ seg CR ...(i) (c.s.c.t.)

Also, ∠PAQ ≅ ∠RCQ ...(c.a.c.t.)

∴ ∠BAC ≅ ∠RCA ...(∵ B - P - A and A - Q - C)

∴ seg BA ∥ seg CR ...(By Alternate angles test)

∴ seg BP ∥ seg CR ...(ii) (∵ A - P - B)

seg AP ≅ seg BP ...(iii) (∵ P is midpoint of side AB)

∴ seg BP ≅ seg CR ...(iv)[From (i), (iii)]

□ PBCR is a parallelogrem ...[From (ii), (iv) and a quadrilateral is a parallelogram if a pair of opposite sides is parallel and congruent.]

∴ seg PR ∥ seg BC ...(By definition)

∴ **seg PQ ∥ seg BC** ...(∵ P - Q - R)

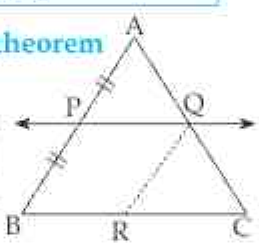
$PQ = \frac{1}{2} PR$ ...(v) (Construction)

But, PR = BC ...(vi) (Opposite sides of a parallelogram)

∴ $\mathbf{PQ = \frac{1}{2} BC}$ ...[From (v) and (vi)]

Theorem - 14 :

❑ **Converse of Midpoint theorem**



Statement : If a line drawn through the midpoint of one side of a triangle is parallel to second side then it bisects the third side.

Given : In ΔABC, P is the midpoint of side AB.
Line PQ ∥ side BC
Line PQ intersects side AC in point Q.

To prove : seg AQ ≅ seg QC

Construction : Draw seg QR ∥ side AB and let seg QR intersect side BC in point R.

Proof : seg PQ ∥ side BC ...(Given)

∴ seg PQ ∥ side BR ...(i) (B - R - C)

seg QR ∥ side AB (Construction)

∴ seg QR ∥ side PB ...(ii) (A - P - B)

∴ □ PQRB is a parallelogram ...[From (i), (ii) and by definition]

∴ side PB ≅ side QR ...(iii) (Opposite sides of a parallelogram)

But, seg PB ≅ seg AP ...(iv) (∵ P is midpoint of seg AB)

∴ seg QR ≅ seg AP ...(v) ...[From (iii) and (iv)]

∴ In ΔAPQ and ΔQRC,

seg AP ≅ seg QR ...[From (v)]

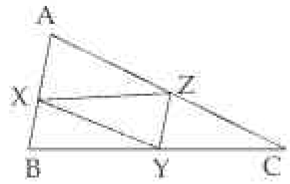
∠AQP ≅ ∠QCR
∠PAQ ≅ ∠RQC } ...(Corresponding angles theorem)

∴ ΔAPQ ≅ ΔQRC ...(By SAA test of congruency)

∴ **seg AQ ≅ seg QC** ...(c.s.c.t)

PRACTICE SET - 5.5 *(Textbook Page No. 73)*

(1) **In adjoning figure, X, Y, Z are the midpoints of side AB, side BC and side AC of ΔABC respectively. AB = 5 cm, AC = 9 cm and BC = 11 cm. Find the length of XY, YZ, XZ.**



Solution:

In ΔABC,
X, Y, and Z are the midpoints of sides AB, BC and AC respectively. ...(Given)

$XZ = \frac{1}{2} BC$ ...(Midpoint theorem)

$= \frac{1}{2} \times 11$

∴ $\boxed{XZ = 5.5 \text{ cm}}$

$XY = \frac{1}{2} AC$ ...(Midpoint theorem)

$= \frac{1}{2} \times 9$

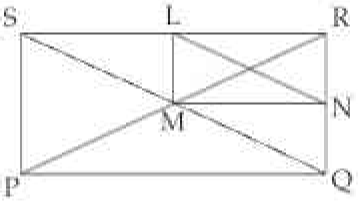
∴ $\boxed{XY = 4.5 \text{ cm}}$

$YZ = \frac{1}{2} AB$ ...(Midpoint theorem)

$= \frac{1}{2} \times 5$

∴ $\boxed{YZ = 2.5 \text{ cm}}$

(2) **In adjoining figure, □PQRS and □MNRL are rectangles. If point M is the midpoint of side PR then prove that, (i) SL = LR, (ii) LN = $\frac{1}{2}$ SQ.**



Proof : □PQRS and □MNRL are rectangles.

$\angle PSR = \angle MLR = 90°$ ...(Angles of a rectangle)

But, this is pair of corresponding angles on transversal SR.

$\therefore$ seg ML $\parallel$ seg PS ...(i) (Corresponding angles test.)

In ΔRPS,

M is midpoint of seg PR and seg ML $\parallel$ seg PS } ...[Given and from (i)]

$\therefore$ L is midpoint of seg RS ...(ii) ...(Converse of midpoint theorem)

$\therefore$ **SL = LR**

Again $\angle PQR = \angle MNR = 90°$ ...(Angles of rectangle)

But, this is pair of corresponding angles On transversal QR.

$\therefore$ seg MN $\parallel$ seg PQ ...(iii) ...(Corresponding angles test.)

In ΔRPQ,

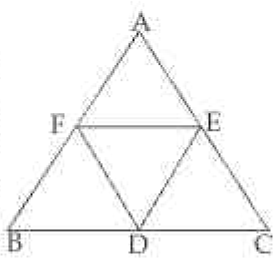
Point M is midpoint of seg RP, seg MN $\parallel$ seg PQ } ... [Given and from (iii)]

$\therefore$ Point N is midpoint of seg QR ...(iv) (Converse of midpoint theorem)

In ΔQRS, L and N are midpoints of sides SR and QR respectively. ...[From (ii) (iv)]

LN = $\frac{1}{2}$ SQ ...(Midpoint theorem)

(3) **ΔABC is an equilateral traingle. Point F, D and E are midpoints of side AB, side BC, side AC respectively. Show that ΔFED is an equilateral triangle.**



Proof : In ΔABC, F, E, D are the midpoints of sides AB, AC, BC respectively.

$\therefore$ FE $= \frac{1}{2}$ BC ...(i)

$\therefore$ DE $= \frac{1}{2}$ AB ...(ii) } (Midpoint theorem)

$\therefore$ FD $= \frac{1}{2}$ AC ...(iii)

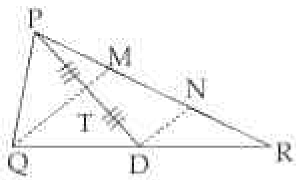
But, ΔABC is equilateral

$\therefore$ AB = BC = AC ...(iv) (Sides of an equilateral triangle)

$\therefore$ FE = DE = FD ...[From (i), (ii), (iii), (iv)]

$\therefore$ **ΔDEF is equilateral** ...(By definition)

(4) **In adjoining figure, seg PD is median of ΔPQR. Point T is the midpoint of seg PD. Produced QT intersects PR at M. Show that $\frac{PM}{PR} = \frac{1}{3}$.**



[Hint : Draw DN $\parallel$ QM.]

Construction : Draw a line passing through point D parallel to seg QM, such that P - M - N - R

Proof : In ΔPDN,

Point T is the midpoint of seg PD ...(Given)

and seg TM $\parallel$ side DN ...(i) (Construction, Q - T- M)

$\therefore$ Point M is midpoint of seg PN ...(i) ...(Converse of midpoint theorem)

In ΔMQR,

Point D is the midpoint of seg QR ...($\because$ PD is median)

and seg DN $\parallel$ side QM ...(Construction)

$\therefore$ Point N is midpoint of seg MR ...(ii) ...(Converse of midpoint theorem)

PM = MN ...[From (i)]

MN = NR ...[From (ii)]

$\therefore$ PM = MN = NR ...(iii) [From (i), (ii)]

PR = PM + MN + NR ...(P - M - N - R]

$\therefore$ PR = PM + PM + PM ...[From (iii)]

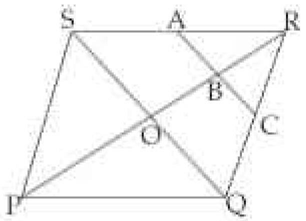
$\therefore$ PR = 3 PM

$\therefore$ $\frac{1}{3} = \frac{PM}{PR}$

i.e. $\frac{PM}{PR} = \frac{1}{3}$

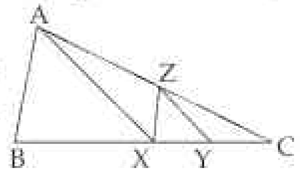
PROBLEMS FOR PRACTICE

(1) **In the adjoining figure, □PQRS is a parallelogram in which A is midpoint of side SR and point B is a point on diagonal PR such that RB = $\frac{1}{4}$ PR.**



Also, seg AB when produced meets side QR at point C. Prove that C is midpoint of side QR.

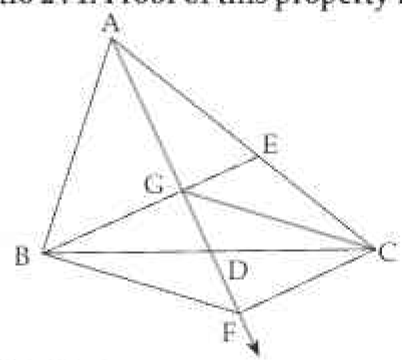
(2) **In the adjoining figure, point X is midpoint of side BC, seg XZ || side AB, seg YZ || seg AX. Prove that YC = $\frac{1}{4}$ BC.**



(3) **Points A, B, C and D are midpoints of sides PQ, QR, RS and SP respectively. Prove that □ABCD is a parallelogram.**

❑ Additional Information :

You know the property that the point of concurrence of medians of a triangle divides the medians in the ratio 2 : 1. Proof of this property is given below.



Given : seg AD and seg BE are the medians of ΔABC which intersect at point G.

To prove : AG : GD = 2 : 1

Construction : Take point F on ray AD such that G-D-F and GD = DF

Proof : Diagonals of □BGCF bisect each other ...(Given and construction)

∴ □BGCF is a parallelogram

∴ seg BE || seg FC

Now point E is the midpoint of side AC of ΔAFC ...(Given)

seg EB || seg CF

Line passing through midpoint of one side and parallel to the other side bisects the third side.

∴ point G is the midpoint of side AF.

∴ AG = GF

But GF = 2 GD ...(Construction)

∴ AG = 2 GD

∴ $\frac{AG}{GD} = \frac{2}{1}$ **i.e. AG : GD = 2 : 1**

PROBLEM SET - 5 (Textbook Page No. 73)

(1) **Choose correct alternative answer and fill in the blanks.**

(i) If all pairs of adjacent sides of a quadrilateral are congruent then it is called ....

(A) rectangle (B) parallelogram
(C) trapezium (D) rhombus

Ans. (D)

(ii) If the diagonal of a square is $12\sqrt{2}$ cm then the perimeter of the square is ....

(A) 24 cm (B) $24\sqrt{2}$ cm (C) 48 cm (D) $48\sqrt{2}$

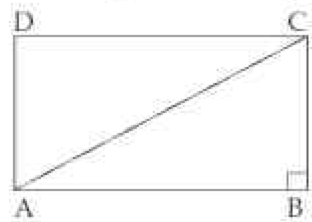
Ans. (C)

(iii) If opposite angles of a rhombus are $(2x)°$ and $(3x - 40)°$ then value of x is ....

(A) 100° (B) 80° (C) 160° (D) 40°

Ans. (D)

(2) **Adjacent sides of a rectangle are 7 cm and 24 cm. Find the length of its diagonal.**



Solution:

□ABCD is a rectangle.

AB = 24 cm

and BC = 7 cm

In Δ ABC, ∠ABC = 90° ...(Angle of a rectangle)

∴ $AC^2 = AB^2 + BC^2$ ...(Pythagoras theorem)

$= 24^2 + 7^2$

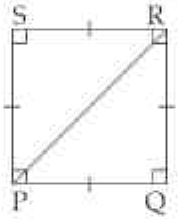
$= 576 + 49$

∴ $AC^2 = 625$

$\therefore$ AC = 25 ...(Taking square roots)

$\therefore$ The length of the diagonal of the rectangle is 25 cm.

(3) If diagonal of a square is 13 cm then find its side.



Solution:

□ PQRS is a square

PR = 13 cm

PQ = QR = RS = PS ...(i) (Sides of a square)

In Δ PQR, $\angle$PQR = 90° ...(Angle of a square)

$\therefore$ $PQ^2 + QR^2 = PR^2$ ...(Pythagoras theorem)

$\therefore$ $PQ^2 + PQ^2 = 13^2$ ...[From (i) and given]

$\therefore$ $2PQ^2 = 169$

$\therefore$ $PQ^2 = \dfrac{169}{2}$

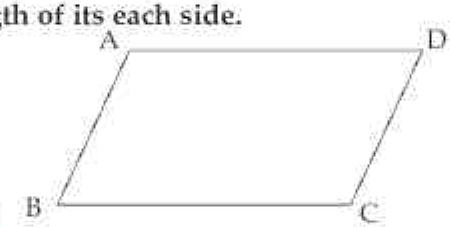
$\therefore$ $PQ = \dfrac{13}{\sqrt{2}}$ ...(Taking square root)

$\therefore$ $PQ = \dfrac{13 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$

$\therefore$ $PQ = \dfrac{13\sqrt{2}}{2}$ cm ...(ii)

$\therefore$ The length of each side of square is $\dfrac{13\sqrt{2}}{2}$

(4) Ratio of two adjacent sides of a parallelogram is 3 : 4, and its perimeter is 112 cm. Find the length of its each side.



Solution:

□ABCD is a parallelogram

AB : BC = 3 : 4

Perimeter of □ABCD = 112cm.

AB = CD and BC = AD ... (i) (opposite sides of a parallelogram are congruent)

AB : BC = 3 : 4 ...(Given)

$\therefore$ Let the common multiple be x.

$\therefore$ AB = $3x$ and BC = $4x$ ... (ii)

$\therefore$ CD = $3x$ and AD = $4x$ [from (i) and (ii)] ... (iii)

Perimeter of □ABCD = AB + BC + CD + AD

$\therefore$ $112 = 3x + 4x + 3x + 4x$ ...(From (ii), (iii) and given)

$\therefore$ $14x = 112$

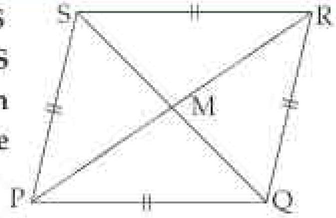
$\therefore$ $x = 8$

$\therefore$ AB = $3x = 3 \times 8 = 24$ cm

BC = $4x = 4 \times 8 = 32$ cm

$\therefore$ The length of sides of the parallelogram are 24 cm, 32 cm, 24 cm, 32 cm

(5) Diagonals PR and QS of a rhombus PQRS are 20cm and 48cm respectively. Find the length of side PQ.



Solution:

□ PQRS is a rhombus

Diagonals PR and QS intersect at point M.

PR = 20cm and QS = 48 cm

Diagonals of a rhombus are bisectors of each other.

$\therefore$ PM = $\dfrac{1}{2}$ PR and QM = $\dfrac{1}{2}$ QS

$\therefore$ PM = $\dfrac{1}{2} \times 20$ and QM = $\dfrac{1}{2} \times 48$

$\therefore$ PM = 10cm and QM = 24cm

In ΔPMQ, $\angle$PMQ = 90° ... (Diagonals of rhombus are perpendicular to each other)

$\therefore$ $PQ^2 = PM^2 + QM^2$ ...(Pythagoras theorem)

$= 10^2 + 24^2$

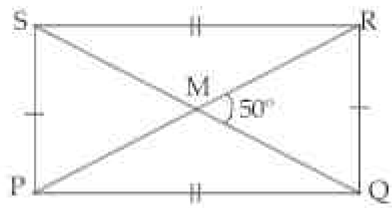
$= 100 + 576$

$\therefore$ $PQ^2 = 676$

$\therefore$ PQ = 26 cm ...(Taking square roots)

$\therefore$ Length of side PQ is 26 cm.

(6) Diagonals of a rectangle PQRS are intersecting in point M. If $\angle$QMR = 50° then find the measure of $\angle$MPS.



Solution:

$\angle$PMS = $\angle$QMR ...(Vertically opposite angles)

But, $\angle$QMR = 50° ...(Given)

$\therefore$ $\angle PMS = 50^\circ$ ...(i)

$PR = QS$ ...(ii)

(Diagonals of a rectangle are congruent)

$PM = \frac{1}{2} PR$ ...(iii)

$SM = \frac{1}{2} QS$ ...(iv)

$\Big\}$ (Diagonals of a rectangle bisect each other)

$\therefore$ $PM = SM$ ...(v) [From (ii), (iii) and (iv)]

In ΔMPS, side PM $\cong$ side SM ...[From (v)]

$\therefore$ $\angle MPS \cong \angle MSP$ ...(Isosecles triangle theorem)

Let $\angle MPS = \angle MSP = x$

$\angle MPS + \angle MSP + \angle PMS = 180$ ...(Sum of the measures of all angles of a triangle is 180°)

$\therefore$ $x + x + 50 = 180$ ...[From (vi) and given]

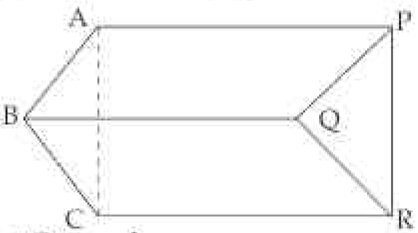
$\therefore$ $2x = 180 - 50$

$\therefore$ $2x = 130$

$\therefore$ $x = 65$

$\therefore$ $\boxed{\angle MPS = 65^\circ}$

(7) In the adjacent figure, if seg AB $\parallel$ seg PQ, seg AB $\cong$ seg PQ, seg AC $\parallel$ seg PR, seg AC $\cong$ seg PR then prove that, seg BC $\parallel$ seg QR seg BC $\cong$ seg QR.



Proof :

In $\square$ ABQP,

side AB $\parallel$ side PQ

side AB $\cong$ side PQ $\Big\}$ ...(Given)

$\therefore$ $\square$ ABQP is a parallelogram ...(A quadrilateral is a parallelogram, if a pair of opposite sides is parallel and congruent)

$\therefore$ side AP $\cong$ side BQ ...(i)

$\therefore$ side AP $\parallel$ side BQ ...(ii) $\Big\}$ (Opposite sides of a parallelogram are congruent)

In $\square$ ACRP,

side AC $\parallel$ side PR

side AC $\cong$ side PR $\Big\}$ ...(Given)

$\therefore$ $\square$ ACRP is a parallelogram ...(A quadrilateral is a parallelogram if a pair of opposite sides is parallel and congruent)

$\therefore$ side AP $\cong$ side CR ...(iii)

side AP $\parallel$ side CR ...(iv) $\Big\}$ (Opposite sides of parallelogram are parallel and congruent)

In $\square$ BCRQ,

side BQ $\parallel$ side CR ...[From (ii), (iv)]

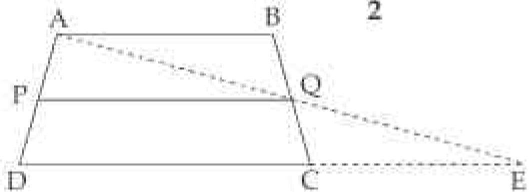
side BQ $\cong$ side CR ...[From (i), (iii))]

$\therefore$ **$\square$ BCRQ, is a parallelogram** ...(A quadrilateral is a parallelogram if a pair of opposite sides is parallel and congruent)

(8) In the following figure, $\square$ABCD is a trapezium. AB $\parallel$ DC. Point P and Q are midpoints of side AD and side BC respectively.

Prove that: (1) PQ $\parallel$ AB. (2) PQ = $\frac{1}{2}$ (AB + CD)

Solution:



Construction: Draw seg AQ and extend it to intersect side DC at point E such that D - C - E

Proof: $\square$ ABCD is a trapezium ... (Given)

side AB $\parallel$ side DC

$\therefore$ side AB $\parallel$ side DE ...($\because$ D – C – E)

On transversal AE,

$\angle BAE \cong \angle DEA$ ...(Alternate angles theorem)

$\therefore$ $\angle BAQ \cong \angle CEQ$ ...(i) ...($\because$ A – Q – E and D – C – E)

In ΔABQ and ΔECQ,

seg BQ $\cong$ seg CQ ...($\because$ Q is midpoint of seg BC)

$\angle AQB \cong \angle CQE$ ...(Vertically opposite angles)

$\angle BAQ \cong \angle CEQ$ ...[From (i)]

$\therefore$ $\Delta ABQ \cong \Delta ECQ$ ...(SAA test of congruency)

$\therefore$ seg AB $\cong$ seg CE ...(ii)

seg AQ $\cong$ seg EQ ...(iii) $\Big\}$ ...(c.s.c.t)

In ΔADE,

P is the midpoint of seg AD ...(Given)

and Q is the midpoint of seg AE ...[From (iii)]

seg PQ $\parallel$ seg DE ...(By Midpoint theorem)

$\therefore$ seg PQ $\parallel$ seg DC ...(D - C - E)

$\therefore$ But seg DC $\parallel$ seg AB ...(Given)

$\therefore$ **seg PQ $\parallel$ seg AB**

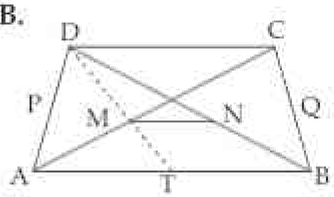
and $PQ = \frac{1}{2}$ DE ...(By Midpoint theorem)

$\therefore$ $PQ = \frac{1}{2}(CE + DC)$ ...($\because$ D - C - E)

$\therefore$ $\mathbf{PQ = \frac{1}{2}(AB + DC)}$ ...[From (ii)]

(9) In the following diagram, □ABCD is a trapezium. AB $\parallel$ DC. Points M and N are midpoints of diagonal AC and diagonal BD respectively.

Prove that MN $\parallel$ AB.



Construction:

Draw seg DM and extend it to meet side AB at point T such that A – T – B

Proof: seg DC $\parallel$ seg AB ... (Given)

On transversal AC,

$\angle DCA \cong \angle CAB$

...(Alternate angles theorem)

$\therefore$ $\angle DCM \cong \angle MAT$ ...(i)

($\because$ A – M – C and A – T – B) B)

In ΔDMC and ΔTMA,

$\angle DCM \cong \angle MAT$ ...[From (i)]

side CM $\cong$ side AM

...($\because$ M is the midpoint of seg AC)

$\angle DMC \cong \angle TMA$

...(Vertically opposite angles)

$\therefore$ $\Delta DMC \cong \Delta TMA$

...(ASA test of congruency)

$\therefore$ $\left.\begin{array}{l}\text{seg DM} \cong \text{seg TM} \quad ...\text{(ii)} \\ \text{seg DC} \cong \text{seg TA} \quad ...\text{(iii)}\end{array}\right\}$ ...(c.s.c.t.)

In ΔDTB,

M and N are midpoints of sides DT and DB respectively ...[From (ii) and Given]

$\therefore$ seg MN $\parallel$ seg TB ...(Midpoint Theorem)

$\therefore$ **seg MN $\parallel$ seg AB** ...(A – T – B)

MCQ's

(1) In parallelogram PQRS, if adjacent sides are 8 cm and 5 cm then what will be its perimeter?

(A) 13 cm (B) 26 cm
(C) 3 cm (D) 52 cm

(2) □KLMN is a square. What is $m\angle NMK$?

(A) 45° (B) 60°
(C) 90° (D) 30°

(3) In ΔABC, M and N are midpoint of sides AB and AC respectively. If MN = 7.2 cm, then BC = ...

(A) 3.6 cm (B) 10.8 cm
(C) 21.6 cm (D) 14.4 cm

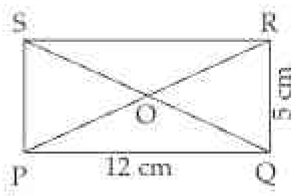
(4) In a rhombus ABCD, if $m\angle ACB = 40°$, then $m\angle ADB$ = ...

(A) 70° (B) 45°
(C) 50° (D) 60°

(5) Diagonals necessarily bisect opposite angles in a ....

(A) rectangle (B) parallelogram
(C) trapezium (D) square

(6) □PQRS is rectangle.
OP = ...



(A) 13 cm
(B) 6.5 cm
(C) 26 cm
(D) 17 cm

(7) □ EFGH is a rhombus with $\angle FEG = 45°$. What is $\angle EFG$?

(A) 45° (B) 60°
(C) 75° (D) 90°

(8) Diagonals of a quadrilateral bisect each other. If $\angle A = 45°$, then $\angle B$ = ...

(A) 115° (B) 120°
(C) 125° (D) 135°

(9) Side of a square is 4 cm. Length of its diagonal is ....

(A) 4 cm (B) 8 cm
(C) $4\sqrt{2}$ cm (D) $8\sqrt{2}$ cm

(10) One side of a parallelogram is 4.8 cm and other side is $\frac{3}{2}$ times the first side, what is the perimeter of the parallelogram?

(A) 12 cm (B) 24 cm
(C) 36 cm (D) 48 cm

(11) □ KLMN is a parallelogram in which KN = 8 cm. Diagonals meet at point P. KP = 6 cm, NP = 4 cm. What is the perimeter of ΔMPL?

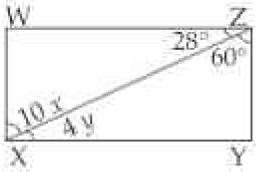
(A) 18 cm (B) 9 cm
(C) 27 cm (D) 15 cm

(12) In ΔPQR , X is midpoint of side PQ. seg XZ ∥ side QR, seg XY ∥ side PR and PQ = 15 cm, YZ = ?

(A) 15 cm (B) 20 cm
(C) 7.5 cm (D) 30 cm

(13) □ WXYZ is a parallelogram value of y = ...

(A) 12 (B) 15
(C) 6 (D) 7



(14) Measures of opposite angles of a parallelogram are $(60 - x)°$ and $(3x - 4)°$. Value of x = ...

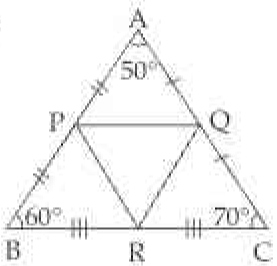
(A) 16 (B) 32
(C) 28 (D) 17

(15) The diagonals of a rectangle ABCD meet at point O. If $\angle$BOC = 44°, then $\angle$OAD = ...

(A) 22° (B) 136°
(C) 68° (D) 90°

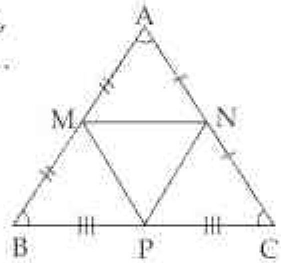
(16) In the adjoining figure, $\angle$PQR = ...

(A) 50°
(B) 70°
(C) 90°
(D) 60°



(17) In the adjoining figure, MN = 3 cm. BP = ...

(A) 6 cm
(B) 1.5 cm
(C) 3 cm
(D) 9 cm



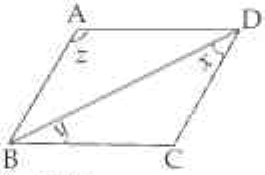
(18) □ ABCD is a rectangle with $m\angle$BAC= 32°. What is $\angle$DBC ?

(A) 32° (B) 58°
(C) 90° (D) 45°

(19) □ ABCD is a rhombus with $\angle$ABC = 56°. $\angle$ACD = ...

(A) 28° (B) 56°
(C) 62° (D) 124°

(20) In the adjoining figure, □ ABCD is a parallelogram. What is the sum of x, y and z?



(A) 140° (B) 150°
(C) 168° (D) 180°

ANSWERS

(1) (B)	(2) (A)	(3) (D)	(4) (C)
(5) (D)	(6) (B)	(7) (D)	(8) (D)
(9) (C)	(10) (B)	(11) (A)	(12) (C)
(13) (D)	(14) (A)	(15) (C)	(16) (D)
(17) (C)	(18) (B)	(19) (C)	(20) (D)

ASSIGNMENT - 5

Time : 1 Hr. **Marks : 20**

Q.1. Attempt the following : **(2)**

(1) □ABCD is a rectangle. AB = 7 cm, BC = 24 cm. Find AC.

(2) □ABCD is a parallelogram. $\angle A = x°$, $\angle B = (3x + 20)°$, find the value of x.

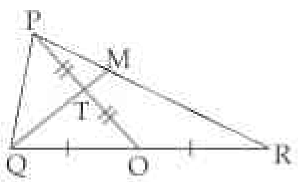
Q.2. Attempt the following : **(4)**

(1) Adjacent sides of a parallelogram are in the ratio 1 : 2. Find the length of each side of this parallelogram if its perimeter is 36 cm.

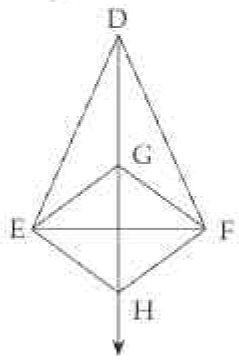
(2) □PQRS is a rhombus. If PQ = 7.5 cm. Find QR. If $\angle QPS = 75°$ then find $\angle PQR$, $\angle SRQ$.

Q.3. Attempt the following : **(6)**

(1) In ΔPQR, seg PO is median. T is midpoint of median PO. seg QT is extended to meet side PR at point M. Prove : $\frac{PM}{PR} = \frac{1}{3}$

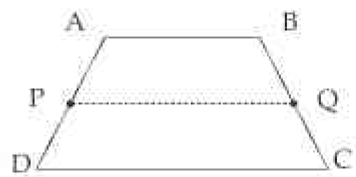


(2) In the adjoining figure, G is the centroid of ΔDEF. H is a point on ray DG such that D - G - H and DG = GH.
Prove that □GEHF is a parallelogram.

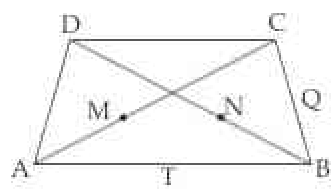


Q.4. Attempt the following : **(8)**

(1) □ABCD is a trapezium in which side AB ∥ side DC.
P and Q are the midpoints of sides AD and BC respectively.
Prove seg PQ ∥ side AB and $PQ = \frac{1}{2}(AB + DC)$



(2) In the adjoining diagram ABCD is a trapezium. AB ∥ DC. Point M and N are midpoints of the diagram AC and diagonal BD respectively.
Prove that MN ∥ AB

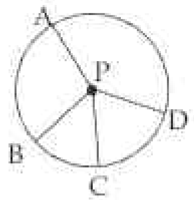


❖ ❖ ❖

6 Circle

Points to Remember:

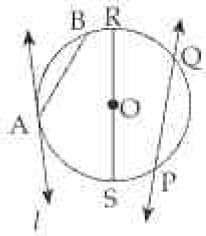
❑ Circle : [Definition]



- Circle is the set of all points in a given plane which are at a constant distance from a fixed point.

  The fixed point is called the **centre.**
- The fixed (constant) distance is called **radius** of the circle. In the above figure, point P is the centre and seg PA, Seg PB, seg PC and seg PD are the radii.

❑ Terms related to circle :



(1) Chord : The segment joining any two points of the circle is called a chord. In the adjoining figure, seg AB is the chord.

(2) Diameter : The chord passing through the centre of the circle is called the diameter.

- Diameter is the longest chord of the circle.
- Diameter is twice the radius of the circle.
- In the above figure seg RS is the diameter.

(3) Secant : A line in the plane of the circle intersecting the circle at two distinct points is called a secant.

- Secant always contains a chord.
- In the above figure, line PQ is a secant.

(4) Tangent : A line in the plane of the circle touching the circle at one and only one point is called a tangent.

- The point at which tangent touches the circle is called the point of contact.
- In the above figure, line *l* is a tangent and A is the point of contact.

❑ Points in the plane of a circle.

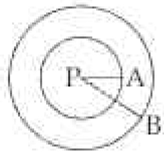
- A circle divides a plane into three disjoint regions. These three regions are the circle itself, its exterior and its interior.
- The union of a circle and its interior is called the circular region.
- If point P is the centre of the circle and '*r*' is the radius of the circle and if

  $d(P, A) = r$, then point A is on the circle.

  $d(P, A) < r$, then point A is in the interior of the circle. and $d(P, A) > r$, then point A is in the exterior of the circle.

❑ Circles in a plane.

(1) Concentric circles :

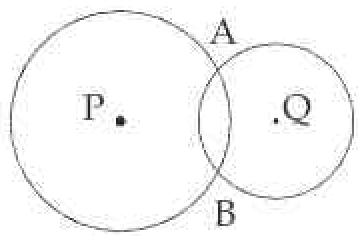


Circles having the same centre but different radii are called concentric circles.

In the adjoining figure, the two circles are concentric circles as they have same centre P and different radii PA and PB.

(2) Intersecting circles :

Coplanar circles having two points in common are called intersecting circles.



In the adjoining figure, circles with centres P & Q are intersecting circles as they have two points A and B in common between them.

(3) Touching Circles : Coplanar circles having one point in common are called touching circles.

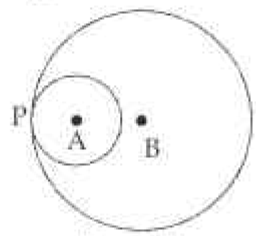


Figure (i)
Internally touching circles

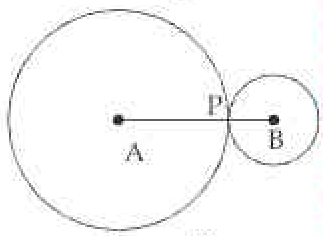
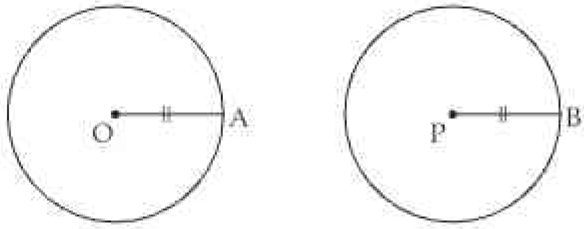


Figure (ii)
Externally touching circles

In figure (i), circles with centres A and B are touching each other **internally** at point P.

In figure (ii), circles with centres A and B are touching each other **externally** at point P.

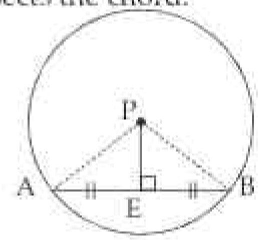
(4) Congruent circles : Circles having equal radii are called congruent circles.



In above figure, circles with centre O and P are congruent as their radii OA and PB are congruent.

Theorem - 1 :

Statement : The perpendicular drawn from the centre of a circle to a chord bisects the chord.



Given : In a circle with centre P, seg PE $\perp$ chord AB such that A - E - B.

To Prove : seg AE $\cong$ seg BE

Construction : Draw seg PA and seg PB.

Proof : In ΔPEA and ΔPEB

$\angle$PEA and $\angle$PEB = 90° ...(Given)

hypotenuse PA $\cong$ hypotenuse PB ...(Radii of same cicle)

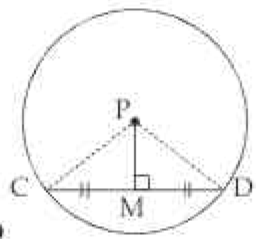
side PE $\cong$ side PE ...(Common side)

$\therefore$ ΔPEA $\cong$ ΔPEB ...(By hypotenuse side test)

$\therefore$ **side AE $\cong$ side BE** ...(c.s.c.t.)

Theorem - 2 :

Statement : The segment joining midpoint of a chord and the centre of a circle is perpendicular to the chord.



Given : In a circle with centre P, M is the midpoint of chord CD.

To Prove : seg PM $\perp$ chord CD

Construction : Draw seg PC and seg PD.

Proof : In ΔPCM and ΔPDM,

side PM $\cong$ side PM ...(Common side)

side CM $\cong$ side DM ...(Given)

side PC $\cong$ side PD ...(Radii of same circle)

$\therefore$ ΔPCM $\cong$ ΔPDM ...(By SSS test)

$\therefore$ $\angle$PMC $\cong$ $\angle$PMD ...(c.a.c.t.)

$\therefore$ $\angle$PMC = $\angle$PMD ...(i)

$\angle$PMC + $\angle$PMD = 180° ...(Angles forming a linear pair)

$\therefore$ $\angle$PMC + $\angle$PMC = 180° ...[From (i)]

$\therefore$ 2 $\angle$PMC = 180

$\therefore$ $\angle PMC = \dfrac{180}{2}$

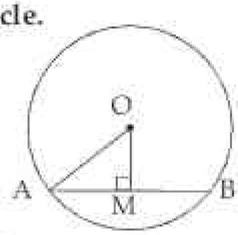
$\therefore$ $\angle$PMC = 90°

$\therefore$ **seg PM $\perp$ chord CD**

MASTER KEY QUESTION SET - 6

PRACTICE SET - 6.1 *(Textbook Page No. 79)*

(1) Distance of chord AB from the centre of a circle is 8 cm. Length of the chord AB is 12 cm. Find the diameter of the circle.



Given :

(i) A circle with centre 'O'

(ii) seg OM is $\perp$ to the chord AB, A - M - B

(iii) OM = 8 cm; AB = 12 cm.

To find : Diameter of the circle.

Solution :

Seg OM $\perp$ Chord AB ...(Given)

$AM = \dfrac{1}{2} AB$

...(Perpendicular drawn from the centre of the circle to the chord bisects the chord.)

$\therefore$ $AM = \dfrac{1}{2} \times 12 = 6$ cm

In ΔOMA, $\angle$OMA = 90° ...(Given)

$\therefore$ $OA^2 = OM^2 + AM^2$ ...(Pythagoras theorem)

$= 8^2 + 6^2$

$= 64 + 36$

$OA^2 = 100$

$\therefore$ OA = 10 cm ...(Taking square roots)

i.e. Radius of the circle is 10 cm

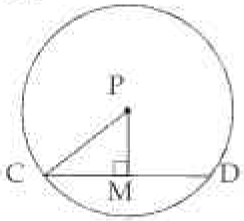
Diameter = 2 × radius

= 2 × 10

$\therefore$ $\boxed{\text{Diameter of the circle is 20 cm}}$

(2) Diameter of a circle is 26 cm and length of the chord of a circle is 24 cm. Find the distance of the chord from the centre.

Given :



(i) A circle with centre 'P' and diameter 26 cm.

(ii) Length of chord CD = 24 cm

(iii) seg PM ⊥ chord CD, C - M - D

To find : PM

Solution :

Diameter of the circle = 26 cm ...(Given)

$\text{Radius} = \frac{\text{Diameter}}{2} = \frac{26}{2}$

∴ Radius of the circle = 13 cm

∴ PC = 13 cm

Seg PM ⊥ chord CD ...(Given)

∴ $CM = \frac{1}{2} CD$ ...(Perpendicular drawn from the centre of the circle to the chord bisects the chord.)

$CM = \frac{1}{2} \times 24 = 12$ cm

In ΔPMC, ∠PMC = 90° ...(Given)

∴ $PC^2 = PM^2 + CM^2$ ...(Pythagoras theorem)

∴ $13^2 = PM^2 + 12^2$

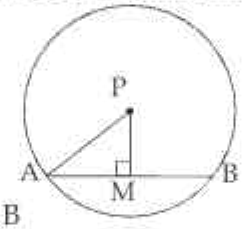
∴ $169 - 144 = PM^2$

∴ $PM^2 = 25$

∴ $\boxed{PM = 5 \text{ cm}}$ ...(Taking square roots)

(3) Radius of the circle is 34 cm and the distance of a chord from the centre is 30 cm. Find the length of the chord.

Given :



(i) A circle with centre 'P' and radius 34 cm.

(ii) seg PM ⊥ chord AB, A - M - B

(iii) l(PM) = 30 cm

To find : l(AB)

Solution : PA = 34 cm ...(Radius of the circle)

In ΔPMA, ∠PMA = 90° ...(Given)

∴ $PA^2 = PM^2 + AM^2$ ...(Pythagoras theorem)

$34^2 = 30^2 + AM^2$

$1156 = 900 + AM^2$

∴ $AM^2 = 1150 - 900$

∴ $AM^2 = 256$

∴ AM = 16 cm ...(Taking square roots)

Seg PM ⊥ chord AB ...(Given)

$AM = \frac{1}{2} \times AB$ ...(Perpendicular drawn from the centre of the circle to the chord bisects the chord.)

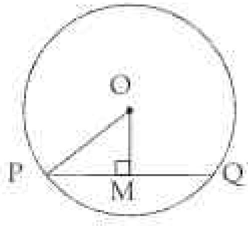
∴ $16 = \frac{1}{2} \times AB$

∴ $AB = 16 \times 2$

∴ $\boxed{AB = 32 \text{ cm}}$

(4) Radius of a circle with centre O is 41 units. Length of a chord PQ is 80 units, find the distance of the chord from the centre of the circle.

Given :



(i) A circle with centre 'O' and radius 41 cm.

(ii) seg OM ⊥ chord PQ, P - M - Q

(iii) l(PQ) = 80 cm

To find : l(OM)

Solution : OP = 41 cm ...(Radius of the circle)

∴ Seg OM ⊥ chord PQ ...(Given)

∴ $PM = \frac{1}{2} PQ$ ...(Perpendicular drawn from the centre of the circle to the chord bisects the chord.)

∴ $PM = \frac{1}{2} \times 80 = 40$ cm

In ΔOMP, ∠OMP = 90° ...(Given)

∴ $OP^2 = OM^2 + PM^2$ ...(Pythagoras theorem)

∴ $41^2 = OM^2 + 40^2$

∴ $1681 = OM^2 + 1600$

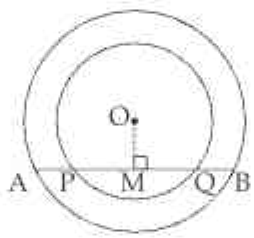
∴ $1681 - 1600 = OM^2$

∴ $OM^2 = 81$

∴ $\boxed{OM = 9 \text{ cm}}$ ...(Taking square roots)

(5) Centre of two circles is O. Chord AB of bigger circle intersects the smaller circle in points P and Q.

Show that AP = BQ.



To prove : AP = BQ

Construction : Draw seg OM ⊥ chord AB, A - M - B.

Proof : In the smaller circle,

Seg OM ⊥ Chord PQ ...(Construction)

∴ PM = MQ ...(i)

(Perpendicular drawn from the centre of the circle to the chord bisects the chord.)

In the bigger circle,

Seg OM ⊥ Chord AB ...(Construction)

∴ AM = MB

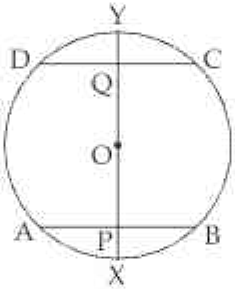
...(Perpendicular drawn from the centre to the chord bisects the chord.)

∴ AP + PM = MQ + QB ...(A - P - M, M - Q - B)

∴ AP + PM = PM + QB ...[From (i)]

∴ **AP = QB**

(6) **If a diameter of a circle bisects two chords of the circle then those two chords are parallel to each other.**



Given :

(i) A circle with centre 'O'

(ii) Diameter XY intersects chord AB and chord CD at points P and Q respectively

(iii) AP = BP and CQ = DQ

To Prove : chord AB || chord CD.

Proof : AP = BP ...(Given)

∴ P is the midpoint of seg AB

∴ seg OP ⊥ chord AB ...(Segment joining centre of the circle and the midpoint of the chord, is perpendiculars to the chords.)

i.e. ∠OPB = 90° ...(i)

CQ = DQ ...(Given)

∴ Q is the midpoint of seg CD

∴ seg OQ ⊥ chord CD ...(Segment joining centre of the circle and the midpoint of the chord, is perpendiculars to the chords.)

i.e. ∠OQC = 90° ...(ii)

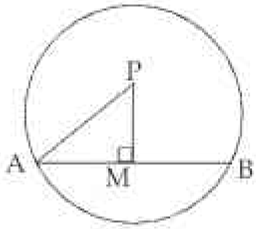
∠OPB + ∠OQC = 90 + 90 = 180 ...[Adding (i) and (ii)]

i.e. ∠QPB + ∠PQC = 180° ...(Q - O - P)

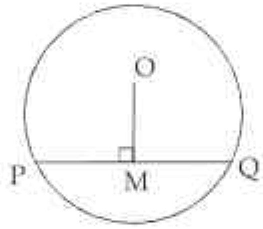
∴ **chord AB || chord CD** ...(Interior Angles Test)

PROBLEMS FOR PRACTICE

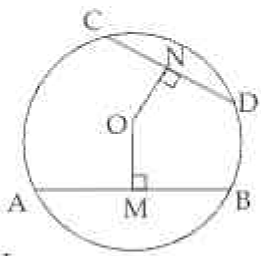
(1) **The radius of a circle with centre P is 25 cm and the length of the chord is 48 cm. Find the distance of the chord from the centre.**



(2) **A chord of length 30 cm is drawn at a distance of 8 cm from the centre of the circle. Find the radius of the circle.**



(3) **In the adjoining figure, O is the centre of the circle. AB = 16 cm CD = 14 cm. Seg OM ⊥ seg AB and ON ⊥ seg CD. If OM = 6 then find seg ON.**



(4) **If chord PQ of a circle have length equal to the radius, then find the distance of the chord from the centre of the circle in terms of radius.**

ANSWERS

(1) 7 cm **(2)** 17 cm **(3)** $\sqrt{51}$ cm

(4) Distance of chord PQ from the centre is $\frac{\sqrt{3}}{2}r$

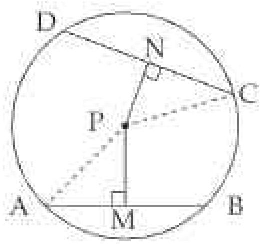
Points to Remember:

❑ **Properties of Congruent Chords**

Theorem - 3 :

Statement : Congruent chords of a circle are equidistant from the centre of the circle.

Given :



(i) A circle with centre 'P'

(ii) Chord AB ≅ chord CD.

(iii) seg PM ⊥ chord AB, A - M - B

(iv) seg PN ⊥ chord CD, C - N - D

To Prove : seg PM ≅ seg PN

Construction : Draw seg PA and seg PC.

Proof : $AM = \frac{1}{2} AB$ ...(i)
$CN = \frac{1}{2} CD$ ...(ii)
(Perpendicular drawn from the centre of the circle to the chord bisects the chord.)

But, AB = CD ...(iii) ...(Given)

∴ AM = CN ...(iv) [From (i), (ii) and (iii)]

∴ In ΔPMA and ΔPNC

∠PMA ≅ ∠PNC ...(Each 90°)

Hypotenuse PA ≅ Hypotenuse PC ...(Radii of same circle)

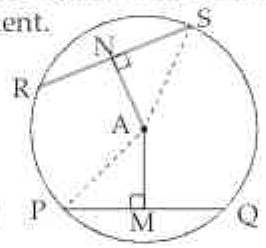
side AM ≅ side CN ...[From (iv)]

∴ ΔPMA ≅ ΔPNC ...(Hypotenuse - side test)

∴ **seg PM ≅ seg PN** ...(c.s.c.t.)

Theorem - 4 :

Statement : Chords which are equidistant from the centre of a circle are congruent.



Given :
(i) A circle with centre 'A'
(ii) seg AM ⊥ chord PQ, P - M - Q
(iii) seg AN ⊥ chord RS, R - N - S
(iv) AM = AN

To Prove : chord PQ ≅ chords RS

Construction : Draw seg AP and seg AS.

Proof : In ΔAMP and ΔANS

∠AMP = ∠ANS ...(Each 90°)

Hypotenuse AP ≅ Hypotenuse AS ...(Radii of same circle)

seg AM ≅ seg AN ...(Given)

ΔAMP ≅ ΔANS...(Hypotenuse - side test)

seg PM ≅ seg SN ...(i) (c.s.c.t.)

$PM = \frac{1}{2} PQ$ ...(ii)
$SN = \frac{1}{2} RS$ ...(iii)
(Perpendicular drawn from the centre of the circle to the chord bisects the chord.)

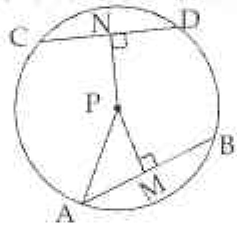
∴ $\frac{1}{2} PQ = \frac{1}{2} RS$ ...[From (i), (ii), (iii)]

∴ PQ = RS

i.e. **chord PQ ≅ chord RS**

PRACTICE SET - 6.2 *(Textbook Page No. 82)*

(1) Radius of circle is 10 cm. There are two chords of length 16 cm each. What will be the distance of these chords from the centre of the circle?



Given :
(i) A circle with centre P and radius 10 cm.
(ii) chord AB ≅ chord CD
(iii) AB = CD = 16 cm.
(iv) seg PM ⊥ chord AB, A - M - B
(v) seg PN ⊥ chord CD, C - N - D.

To find : PM and PN

Solution : PA = 10 cm ...(Radius of the circle)

seg PM ⊥ chord AB ...(Given)

∴ $AM = \frac{1}{2} AB$ ...(Perpendicular drawn from the centre of the circle to the chord bisects the chord)

∴ $AM = \frac{1}{2} \times 16 = 8$ cm

In ΔPMA, ∠PMA = 90° ...(Given)

∴ $PA^2 = PM^2 + AM^2$ ...(Pythagoras theorem)

∴ $10^2 = PM^2 + 8^2$

∴ $100 - 64 = PM^2$

∴ $PM^2 = 36$

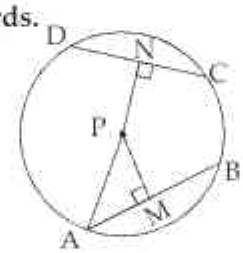
∴ $\boxed{PM = 6 \text{ cm}}$ ...(Taking square roots)

chord AB ≅ chord CD ...(Given)

∴ PM = PN ...(In a circle, congruent chords are equidistant from the centre)

∴ $\boxed{PN = 6 \text{ cm}}$

(2) In a circle with radius 13 cm, two equal chords are at a distance of 5 cm from the centre. Find the lengths of the chords.



Given :
(i) A circle with centre P and radius 13 cm.
(ii) chord AB ≅ chord CD
(iii) seg PM ⊥ chord AB, A - M - B.
(iv) seg PN ⊥ chord CD, C - N - D.
(v) PM = PN = 5 cm

To find : *l*(AB) and *l*(CD)

Solution : PA = 13 cm ...(Radius of the circle)

In ΔPMA, $\angle PMA = 90°$ ...(Given)

$\therefore$ $PA^2 = PM^2 + AM^2$ ...(Pythagoras theorem)

$13^2 = 5^2 + AM^2$

$\therefore$ $169 - 25 = AM^2$

$\therefore$ $AM^2 = 144$

$\therefore$ $AM = 12$ cm ...(Taking square roots)

$\therefore$ $PM \perp$ chord AB ...(Given)

$\therefore$ $AM = \frac{1}{2}$ AB ...(Perpendicular drawn from the centre of the circle to the chord bisects the chord)

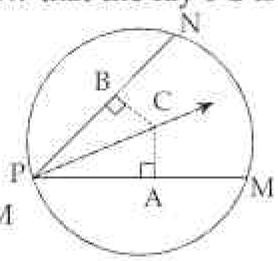
$\therefore$ $12 = \frac{1}{2} \times AB$

$\therefore$ $\boxed{l(AB) = 24 \text{ cm}}$

$\therefore$ $AB = CD$ ...(Given)

$\therefore$ $\boxed{l(CD) = 24 \text{ cm}}$

(3) Seg PM and seg PN are congruent chords of a circle with centre C. Show that the ray PC is the bisector of $\angle NPM$.



Given :

(i) A circle with centre C

(ii) chord PM $\cong$ chord PN

To prove : ray PC bisects $\angle NPM$

Construction :

Draw seg $CA \perp$ chord PM, P - A - M

and seg $CB \perp$ chord PN, P - B - N

Proof :

chord PM $\cong$ chord PN ...(Given)

$\therefore$ CA = CB ...(i)

(In a circle, congruent chords are equidistant from the centre of the circle)

In ΔPAC and ΔPBC

$\angle PAC \cong \angle PBC$ ...(Each 90°)

Hypotenuse PC $\cong$ Hypotenuse PC ...(Common side)

seg CA $\cong$ seg CB ...[From (i)]

$\therefore$ $\Delta PAC \cong \Delta PBC$ ...(Hypotenuse - side test)

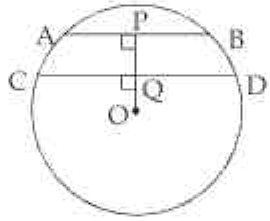
$\therefore$ $\angle CPA \cong \angle CPB$ ...(c.a.c.t.)

$\therefore$ ray PC bisects $\angle BPA$

i.e. **ray PC bisects $\angle NPM$** ...(P - A - M, P - B - N)

PROBLEMS FOR PRACTICE

(1) In the adjoining figure chord AB || chord CD of a circle wih centre O and radius 5 cm. Such that AB = 6 cm and CD = 8 cm.

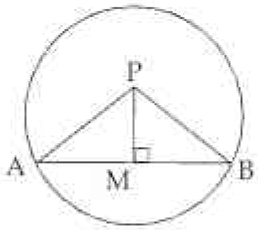


If $OP \perp AB$, $OP \perp CD$ determine PQ

(2) A chord AB of a circle is at a distance of 6 cm from the centre of a circle whose radius is 6 cm less than the chord AB. Find the length of the chord and radius of the circle.

(3) The radius of a circle is 16 cm. The midpoint of a chord of a circle lies on the diameter perpendicular to the chord and its distance from one end of the diameter is 3 cm. Find the length of he chord.

(4) In the adjoining figure, P is the centre. seg AB is smaller than the sum of side PA and side PB by 4 cm. If the perimeter of ΔPAB is 144 cm then find the length of seg PM.



ANSWERS

(1) PQ = 1 cm

(2) length of the chord is 16 cm and radius is 10 cm

(3) The length of the chord is $2\sqrt{87}$ cm

(4) PM = 12 cm

Points to Remember:

Incircle of a Triangle

- A circle which touches all three sides of a triangle is called incircle of a triangle.
- Centre of incircle is called 'INCENTRE' of the triangle.
- As incentre is equidistant from all three sides, it lies on the angle bisectors of all three angles of a triangle.
- While constructing incircle it is sufficient to draw any two angle bisectors to get the incentre.
  ($\because$ Angle bisectors of all three angles of a Δ are concurrent)

❑ Circumicircle of a Triangle

- A circle which passes through all three vertices of a triangle is called circumcircle of the triangle.
- Centre of circumcircle is called 'CIRCUMENTRE' of the triangle.
- As circumcentre is equidistant from all three vertices, it lies on the perpendicular bisectors of all three sides of a triangle.
- While constructing circumcircle it is sufficient to draw the perpendicular bisectors of any two sides.

  ($\because$ Perpendicular bisectors of three sides of a triangle are concurrent)

❑ Position of Incentre and circumcentre

	Type of triangle	Equilateral	Isosceles	Scalene
(1)	Position of incentre	Interior of a triangle	Interior of a triangle	Interior of a triangle
(2)	Position of circumcentre	Interior of a triangle	Interior or exterior or on the side of a triangle	Interior or exterior or on the side of a triangle.

❑ Important to know :

(1) For an equilateral triangle, angle bisector is also perpendicular bisector.

(2) For an equilateral triangle, incentre and circumcentre is one and the same.

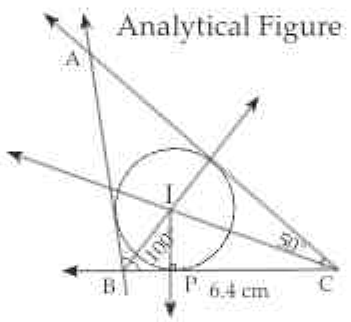
(3) For an equilateral triangle, ratio of circumradius and inradius is 2 : 1.

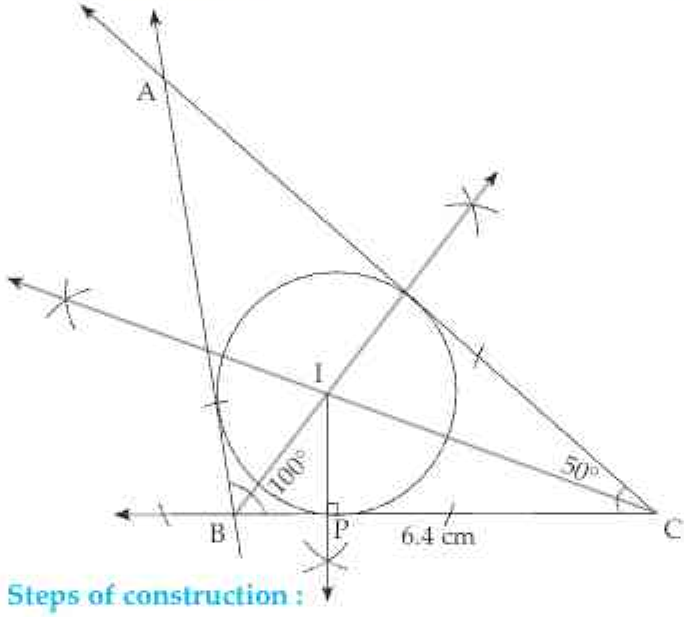
PRACTICE SET - 6.3 *(Textbook Page No. 86)*

(1) **Construct ΔABC such that $\angle B = 100°$, $BC = 6.4$ cm and $\angle C = 50°$ and construct its incircle.**

Solution :

Analytical Figure





Steps of construction :

(1) Draw seg BC of length 6.4 cm

(2) Draw an angle of 100° at vertex B and an angle of 50° at vertex C, name the point of intersection as point A.

(3) Draw bisectors of $\angle ABC$ and $\angle ACB$ which meet at point I.

(4) Draw ray IP $\perp$ to side BC, B - P - C.

(5) Taking 'I' as centre and seg IP as a radius draw a circle.

(2) **Construct ΔPQR such that $\angle P = 70°$, $\angle R = 50°$, QR = 7.3 cm. and construct its circumcircle.**

Solution : In ΔPQR, $\angle P + \angle Q + \angle R = 180°$

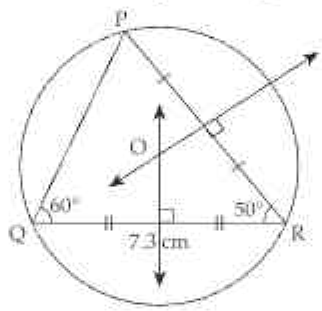
...(sum of measures of all angles of a triangle is 180°)

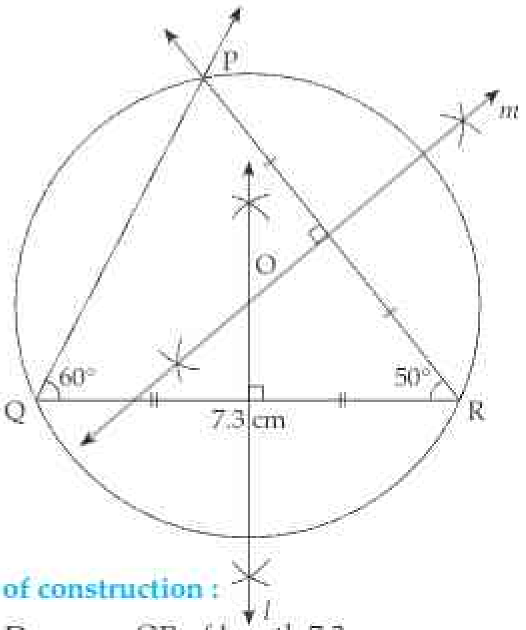
$\therefore \quad 70 + \angle Q + 50 = 180$

$\therefore \quad \angle Q = 180 - 120$

$\therefore \quad \angle Q = 60°$

Analytical Figure





Steps of construction :

(1) Draw seg QR of length 7.3 cm

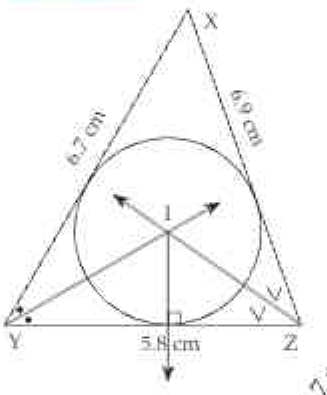
(2) Draw an angle of 60° at vertex Q and an angle of 50° at vertex R, name the point of intersection as point P.

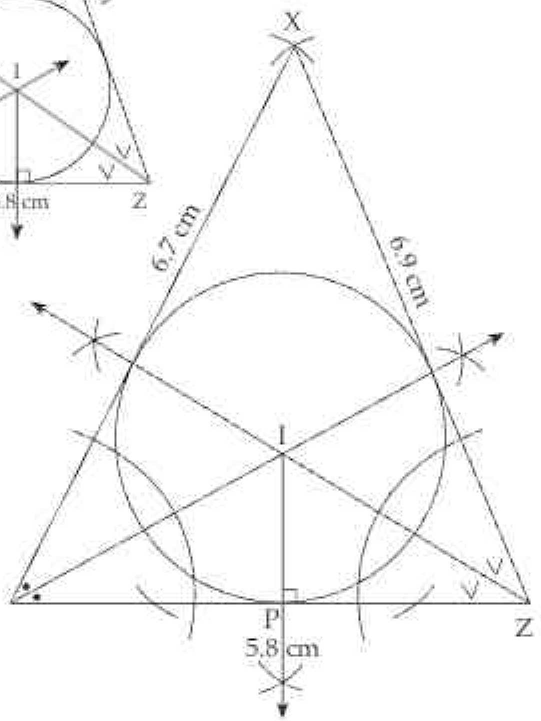
(3) Draw perpendicular bisectors of seg QR and seg PR, let they intersect at point O.

(4) Taking 'O' as a centre and seg OP as a radius, draw a circle.

(3) Construct ΔXYZ such that XY = 6.7 cm, YZ = 5.8 cm, XZ = 6.9 cm. Construct its incircle.

Solution : Analytical Figure





Steps of construction :

(1) Construct ΔXYZ, using lengths of all three sides.

(2) Draw bisectors of $\angle XYZ$ and $\angle XZY$.

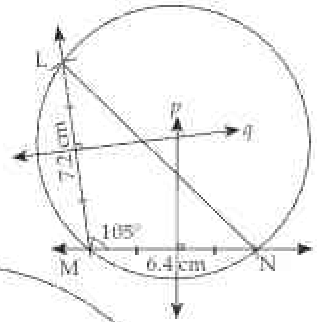
(3) Name the point of intersection of angle bisectors as point I.

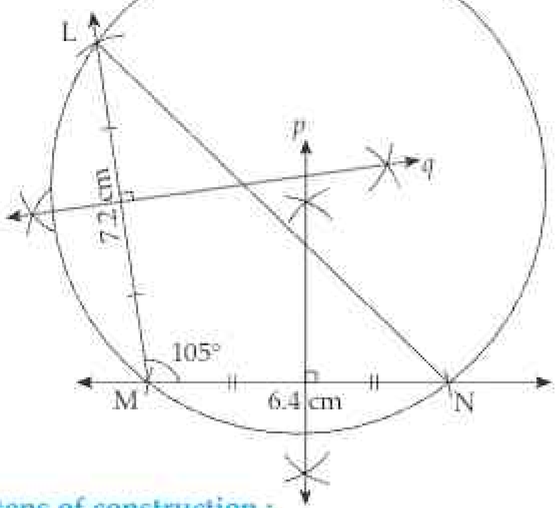
(4) Draw IP $\perp$ side YZ, Y - P - C

(5) Taking 'I' as the centre and seg IP as the radius draw the circle.

(4) In ΔLMN, LM = 7.2 cm, $\angle M = 105°$, MN = 6.4 cm, then draw ΔLMN and construct its circumcircle.

Solution : Analytical Figure





Steps of construction :

(1) Draw ΔLMN as per the given measurements.

(2) Draw line p as perpendicular bisector of side MN.

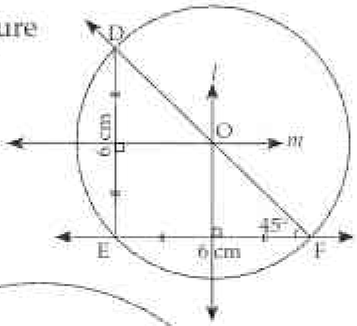
(3) Draw line q as perpendicular bisector of side LM.

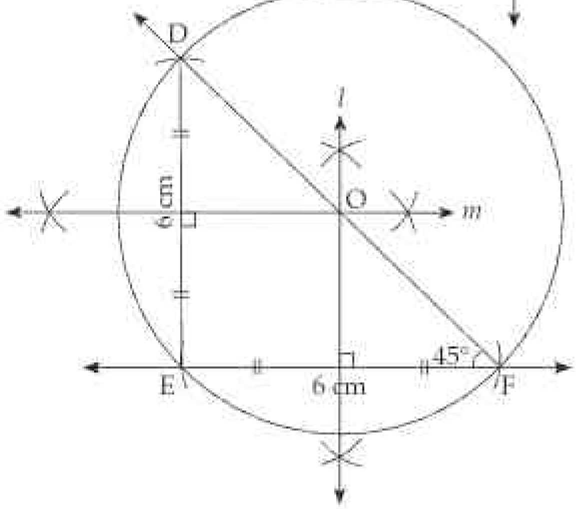
(4) Name the point of intersection of line p and line q as point 'O'

(5) Taking point 'O' as centre and seg OL as a radius, draw a circle.

(5) Construct ΔDEF such that DE = EF = 6 cm, $\angle$F = 45° and construct its circumcircle.

Solution : Analytical Figure





Steps of construction :

(1) Draw ΔDEF as per the given measurements.

(2) Draw perpendicular bisector of seg EF, name the line as line *l*.

(3) Draw perpendicular bisector of seg DF, name the line as line *m*.

(4) Name the point of intersection of line *l* and line *m* as point 'O'.

(5) Taking point 'O' as centre and seg OD as a radius draw the circle.

PROBLEMS FOR PRACTICE

(1) Draw the circumcircle of ΔPMT in which PM = 5.4 cm, $\angle$P = 60°, $\angle$M = 70°

(2) Construct ΔDCE, such that DC = 7.9 cm, $\angle$C = 135°, $\angle$D = 20° and draw its circumcircle.

(3) Construct any right angle triangle and draw its incircle.

(4) Construct Δ SRP such that RP = 6 cm, $\angle$R = 75° and $\angle$P = 55° and draw its incircle.

(5) Construct the circumcircle and incircle at an equilateral ΔXYZ with side 7.3 cm

PROBLEM SET - 6 *(Textbook Page No. 86)*

(1) Choose correct alternative answer and fill in the blanks.

(i) Radius of a circle is 10 cm and distance of a chord from the centre is 6 cm. Hence the length of the chord is ....

(A) 16 cm (B) 8 cm (C) 12 cm (D) 32 cm

Ans. (A)

(ii) The point of concurrence of all angle bisectors of a triangle is called the ....

(A) centroid (B) circumcentre
(C) incentre (D) orthocentre

Ans. (C)

(iii) The circle which passes through all the vertices of a triangle is called ....

(A) circumcircle (B) incircle
(C) congruent circle (D) concentric circles

Ans. (A)

(iv) Length of a chord of a circle is 24 cm. If distance of the chord from the centre is 5 cm, then the radius of that circle is ....

(A) 12 cm (B) 13 cm (C) 14 cm (D) 15 cm

Ans. (B)

(v) The length of the longest chord of the circle with radius 2.9 cm is ....

(A) 3.5 cm (B) 7 cm (C) 10 cm (D) 5.8 cm

Ans. (D)

(vi) Radius of a circle with centre O is 4 cm. l(OP) = 4.2 cm, say where point P will lie.

(A) on the centre (B) inside of the circle
(C) outside of the circle (D) on the circle

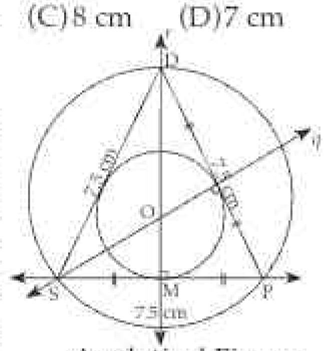
Ans. (C)

(vii) The lengths of parallel chords which are on opposite sides of the centre of a circle are 6 cm and 8 cm. If radius of the circle is 5 cm, then the distance between these chords is ....

(A) 2 cm (B) 1 cm (C) 8 cm (D) 7 cm

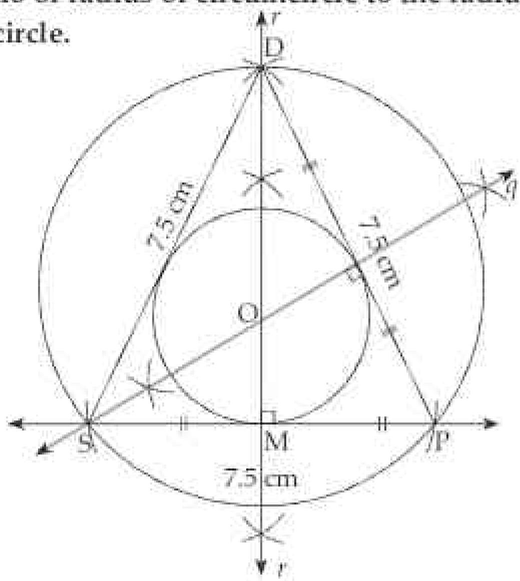
Ans. (D)

(2) Construct incircle and circumcircle of an equilateral ΔDSP with side 7.5 cm. Measure the radii of both the circles and find the



Analytical Figure

ratio of radius of circumcircle to the radius of incircle.



Circumradius = 3.8 cm
Inradius = 1.9 cm
Circumradius : Inradius = 3.8 cm : 1.9 cm = 2 : 1.

Steps of construction :

(1) Draw ΔDSP as per given measurements.

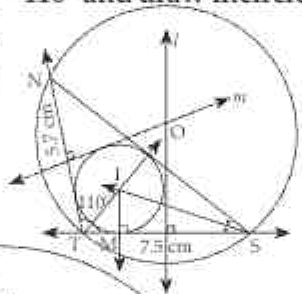
(2) Draw line r and line q as bisectors of sides PS and DP respectively. Name the point of intersection of line r and line q as point O.

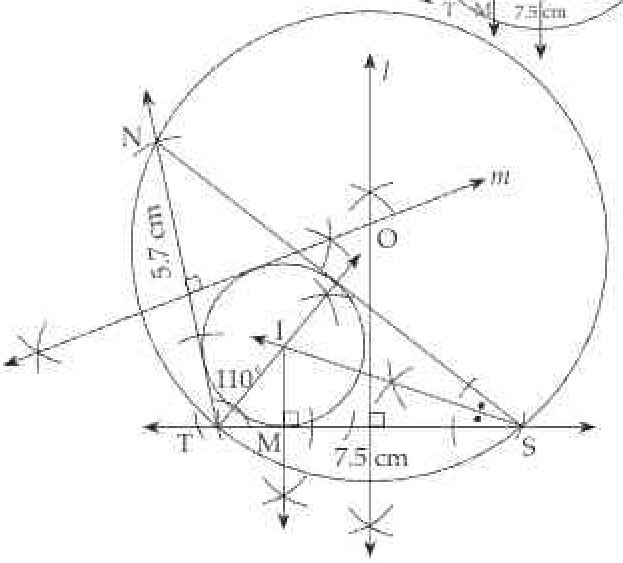
(3) Draw a circle taking 'O' as a centre and OD as a radius.

(4) Draw a circle taking 'O' as a centre and OM as a radius.

(3) Construct ΔNTS in which NT = 5.7 cm, TS = 7.5 cm and $\angle$NTS = 110° and draw incircle and circumcircle of it.

Analytical Figure





Steps of construction :

(1) Draw ΔNTS as per the given measurement.

(2) Draw bisectors of NTS and NST intersecting at point I.

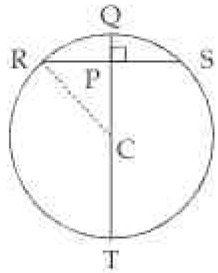
(3) Draw IM $\perp$ to seg TS, T - M - S.

(4) Taking point I as a centre and IM as a radius draw a circle.

(5) Draw line l and line m as perpendicular bisectors of side TS and side TN, intersecting at point 'O'.

(6) Taking point 'O' as a centre and seg ON as a radius draw the circle.

(4) In the adjoining figure, C is the centre of the circle. Seg QT is diameter, CT = 13 cm, CP = 5, find the length of chord RS.



Construction : Draw seg CR.

Solution : CT = CR. ...(Radius of the same circle)

$\therefore$ CR = 13 ...($\because$ CT = 13, given)

In ΔCPR, $\angle$CPR = 90° ...(Given)

$\therefore$ $CR^2 = CP^2 + RP^2$ ...(Pythagoras theorem)

$13^2 = 5^2 + RP^2$

$\therefore$ $169 - 25 = RP^2$

$\therefore$ $RP^2 = 144$

$\therefore$ RP = 12 units ...(Taking square roots)

seg CP $\perp$ chord RS, ...(Given)

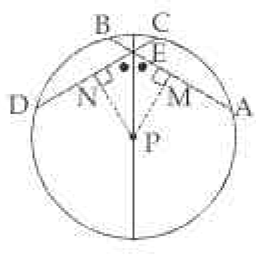
$RP = \frac{1}{2}$ RS ...(Perpendicular drawn from the centre of the circle to the chord bisects the chord)

$\therefore$ $12 = \frac{1}{2} \times$ RS

$\therefore$ $\boxed{\text{RS} = 24 \text{units}}$

(5) In the adjoining figure, P is the centre of the circle. chord AB and chord CD intersect on the diameter at a point E.

If $\angle$AEP $\cong$ $\angle$DEP then prove that AB = CD.



Construction : Draw seg PM $\perp$ chord AB, A - M - B.

Draw seg PN $\perp$ chord CD, C - N - D.

Proof : In ΔPME and ΔPNE,

side PE $\cong$ side PE ...(Common side)

$\angle$PME $\cong$ $\angle$PNE ...(Each 90°)

$\angle$PEM $\cong$ $\angle$PEN

...(Given, A - M - E, D - N - E)

$\therefore$ $\Delta PME \cong \Delta PNE$ ...(SAA test)

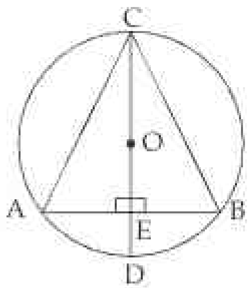
$\therefore$ seg PM $\cong$ seg PN ...(c.s.c.t.)

i.e. chord AB and chord CD are equidistant from the cenre P.

$\therefore$ **chord AB $\cong$ chord CD.**

...(In a circle, if chords are equidistant from the centre then they are congruent.)

(6) In the adjoining figure, CD is a diameter of the circle with centre O. Diameter CD is perpendicular to chord AB at point E. Show that ΔABC is an isosceles triangle.



To prove : ΔABC is an isosceles triangle.

Proof : Diameter CD $\perp$ chord AB ...(Given)

$\therefore$ seg OE $\perp$ chord AB ...(C - O - E - D)

$\therefore$ AE = BE ...(i) (Perpendicular drawn from the centre of the circle to the chord bisects the chord.)

In ΔAEC and ΔBEC,

side CE $\cong$ side CE ...(Common side)

$\angle AEC \cong \angle BEC$ ...(Each 90°)

side AE $\cong$ side BE ...[From (i)]

$\therefore$ $\Delta AEC \cong \Delta BEC$ ...(SAS test)

$\therefore$ side AC $\cong$ side BC ...(c.s.c.t.)

$\therefore$ **ΔABC is an isosceles triangle** ...(Definition)

MCQ's

(1) The segment joining any two points on the circle is called ....

(A) tangent (B) radius

(C) chord (D) none of the above

(2) Diameter is .... of radius.

(A) half (B) twice

(C) thrice (D) one-fourth

(3) Two circles having same centre but different radii are called .... circles.

(A) concentric (B) congruent

(C) intersecting (D) touching

(4) The perpendicular drawn from the centre of a circle to a chord .... the chord.

(A) parallel (B) trisects

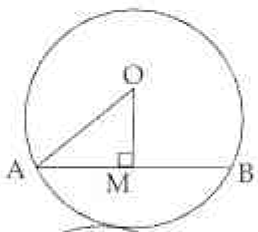
(C) bisects (D) equal

(5) If the radius of the circle is 7.6 cm then its diameter is ....

(A) 13.2 cm (B) 7.6 cm

(C) 3.8 cm (D) 15.2 cm

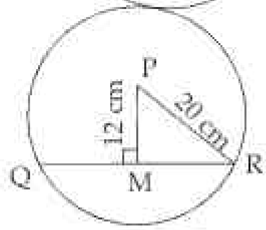
(6) In the adjoining figure, OM $\perp$ AB. OM = 3, AM = 4, then OA = ...?



(A) 25 (B) 5

(C) 2.5 (D) 10

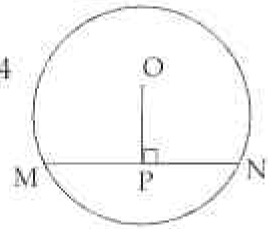
(7) Using the given information, find QR.



(A) 6 (B) 32

(C) 8 (D) 42

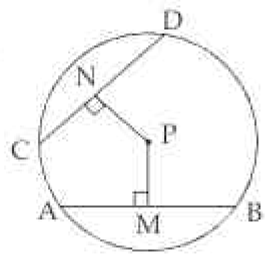
(8) In the adjoining figure, OP $\perp$ MN and MN = 24 then MP = ...?



(A) 24 (B) 6

(C) 12 (D) 4

(9) In the adjoining figure, chord AB $\cong$ chord CD. PM $\perp$ AB, PN $\perp$ CD, then what is the relation between PM and PN ?



(A) PM > PN

(B) PM < PN

(C) PM $\perp$ PN

(D) PM = PN

(10) How many circles can be drawn through two give points?

(A) one (B) two

(C) none (D) infinite

(11) In a circle, chords which are equidistant from the centre are ....

(A) parallel (B) congruent

(C) intersecting (D) all of above

(12) The point of intersection of the perpendicular bisectors of a trianlge is called ....

(A) centroid (B) incentre

(C) circumcentre (D) orthocentre

(13) Incircle of a triangle should touch .... sides of a triangle.

(A) 1 (B) 3

(C) 2 (D) none

(14) For a right angle triangle, the circumcentre lies on .... of the triangle.

(A) on the hypotenuse (B) in the interior

(C) in the exterior (D) on the vertex

(15) For a scalene triangle, the incentre lies .... of the triangle.

(A) in the exterior (B) on the side

(C) in the interior (D) none of the above

(16) Radius of the circle with centre O is 7 cm, l (OP) = 8 cm. Then the position of point P is .... of the circle.

(A) in the exterior (B) in the interior

(C) on the circle (D) on the centre

(17) In which triangle is the circumcentre and incentre the same point?

(A) isosceles (B) equilateral

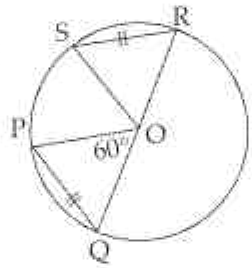
(C) scalene (D) right angled

(18) If the radius of the circle is 3.9 cm, then the length of the longest chord is ....

(A) 3.9 cm (B) 9.3 cm

(C) 1.3 cm (D) 7.8 cm

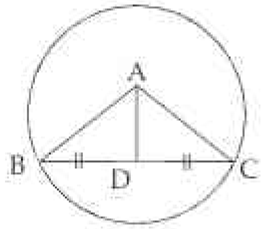
(19) Using the information given in the figure, what is $m\angle$ROS?



(A) 30° (B) 45°

(C) 60° (D) 90°

(20) Using the information given in the figure, find the $m\angle$ADB.



(A) 80° (B) 90°

(C) 60° (D) 70°

ANSWERS

(1) (C)	(2) (B)	(3) (A)	(4) (C)
(5) (D)	(6) (B)	(7) (B)	(8) (C)
(9) (D)	(10) (D)	(11) (B)	(12) (C)
(13) (B)	(14) (A)	(15) (C)	(16) (A)
(17) (B)	(18) (D)	(19) (C)	(20) (B)

ASSIGNMENT - 6

Time : 1 Hr. **Marks : 20**

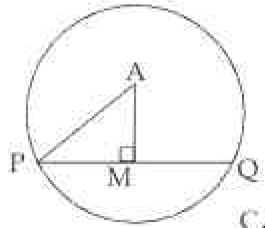
Q.1. Attempt the following : **(2)**

(1) The radius of a circle is $7\sqrt{2}$ cm. Find the length of largest chord of that circle.

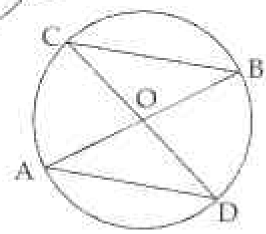
(2) The radius of a circle with centre O is 6 cm. OP = 6.3 cm and OQ = 5.8 cm. State with reason which point lies in the interior of the circle.

Q.2. Attempt the following : **(4)**

(1) In the adjoining figure, AP = 34, AM = 30, AM $\perp$ PQ. Find the length of chord PQ.



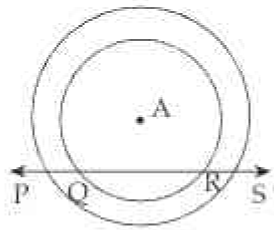
(2) In a circle with centre O, diameter AB and diameter CD are given. Prove that AD = BC



Q.3. Attempt the following : **(6)**

(1) Prove that perpendicular drawn from the centre of a circle to the chord bisects the chord.

(2) If a line intersects two concentric circles with centre A in points P, Q, R and S respectively. Prove that PQ = RS.



Q.4. Attempt the following : **(8)**

(1) Construct the circumcircle of ΔPQR in which QR = 6.5 cm, $\angle Q = 125°$, PQ = 4.4 cm

(2) Construct the incircle of right angled ΔABC, such that AB = 5 cm, BC = 7 cm, $\angle B = 90°$

7 Co-ordinate Geometry

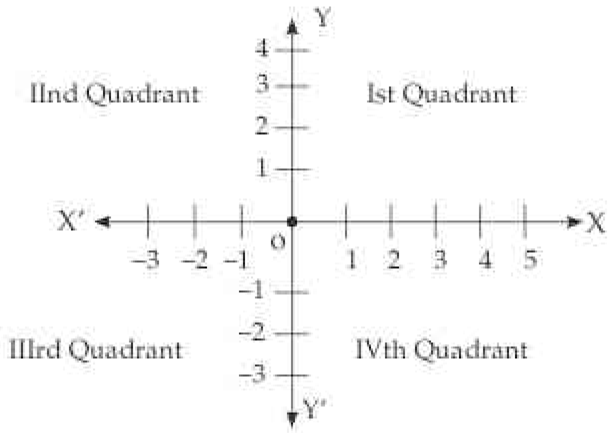
❑ Introduction :

A pair of numbers which represents the position of a point in a plane of two mutually perpendicular lines is called an ordered pair.

The two mutually perpendicular lines are called co-ordinate axes.

❑ Origin, Axes and Quadrants :

The position of a point in a plane is determined with reference to two fixed mutually perpendicular number lines, called as co-ordinate axes.



- The horizontal number line is called X-axis represented as XX'.
- The vertical number line is called Y-axis represented as YY'.
- The point of intersection of these two axes is called the 'origin' and represented as 'O'.
- Ray OX is positive direction and ray OX' is negative direction of X-axis.
- Ray OY is positive direction and ray OY' is negative direction of Y-axis.

The co-ordinate axes divide the plane of the paper into four regions, XOY, X'OY, X'OY' and XOY'. Each of this region is called 'Quadrant'.

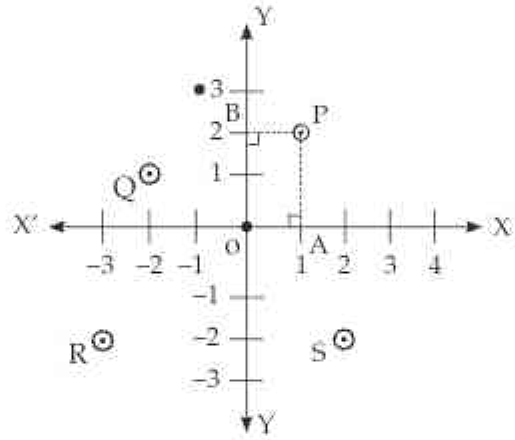
Region XOY is called Ist Quadrdant.

Region X'OY is called IInd Quadrdant.

Region X'OY' is called IIIrd Quadrdant.

Region XOY' is called IVth Quadrdant.

To determine co-ordinates of a point in a quadrant.



To determine co-ordinates of a point in a quadrant we have to draw a perpendicular to each of the axis.

Let us understand with an example. Point P is in the Ist quadrant . From point P we have drawn a ⊥ to X-axis at which intersects X-axis at point A which represents '1'.

∴ X co-ordinate of point P is 1.

From point P we have drawn a ⊥ to Y axis which intersects Y-axis at point B which represents '2'.

∴ Y co-ordinate of point P is 2.

∴ P (1, 2)

Points to Remember:

❑ Coordinates of points in a plane :

When we write co-ordinates of a point, the first number is x-coordinate and the second number is y-coordinate.

x coordinate is also called **'Abscissa'**.

y coordinate is also called **'Ordinate'**.

(1) If a point is in the first quadrant then its both the coordinates are positive. e.g. P(1,2).

(2) If a point is in the second quadrant then its x coordinate is negative and y coordinate is positive. e.g. Q(–2,1).

(3) If a point is in the third quadrant then its both the coordinates are negative. e.g. R(–3, –2).

(4) If a point is in the fourth quadrant then its x coordinate is positive and y coordinate is negative S(2, –2).

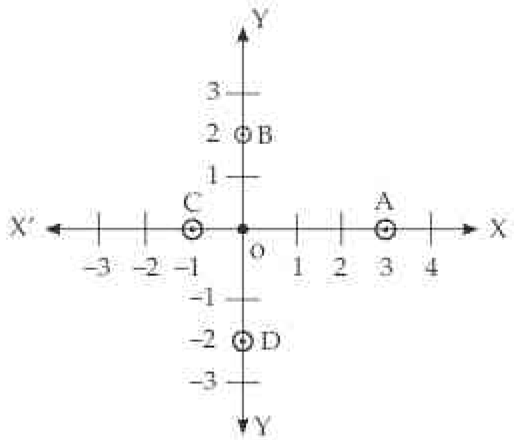
	X-coordinate	Y-coordinate
I Quadrant	+	+
II Quadrant	–	+
III Quadrant	–	–
IV Quadrant	+	–

❑ **Coordinates of points on the Axes.**

(1) y coordinate of any point on x axis is '0'.

(2) x coordinate of any point on y axis is '0'.

(3) Coordinates of the origin are (0, 0).



For point A, x coordinate is 3 and y coordinate is 0.

∴ A(3, 0)

For point B, x coordinate is 0 and y coordinate is 2.

∴ B (0, 2)

For point C, x coordinate is –1 and y coordinate is 0.

∴ C (–1, 0)

For point D, x coordinate is 0 and y coordinate is –2.

∴ D (0, –2)

MASTER KEY QUESTION SET - 7

PRACTICE SET - 7.1 *(Textbook Page No. 93)*

(1) State in which quadrant or on which axis do the following points lie.

A (–3, 2), B (–5, –2), K (3.5, 1.5), D (2, 10), E (37, 35), F (15, –18), G (3, –7), H (0,–5), M (12, 0), N (0, 9), P (0, 2.5), Q (–7, –3)

Solution:

A(–3, 2) IInd Quadrant G (3, –7) IV quadrant

B (–5, –2) IIIrd Quadrant H (0, –5) y-axis.

K (3.5, 1.5) Ist Quadrant M (12, 0) x-axis.

D (2, 10) Ist Quadrant N (0, 9) y-axis.

E (37, 35) Ist Quadrant P (0, 2.5) y-axis.

F(15,–18) IVth Quadrant Q(–7,–3) IIIrd quadrant.

(2) In which quadrant the following points lie?

(i) Whose both coordinates are positive.

Solution:

Ist quadrant.

(ii) Whose both coordinates are negative.

Solution:

IIIrd quadrant.

(iii) Whose x coordinate is positive and y coordinate is negative.

Solution:

IVth quadrant.

(iv) Whose x coordinate is negative and y coordinate is positive.

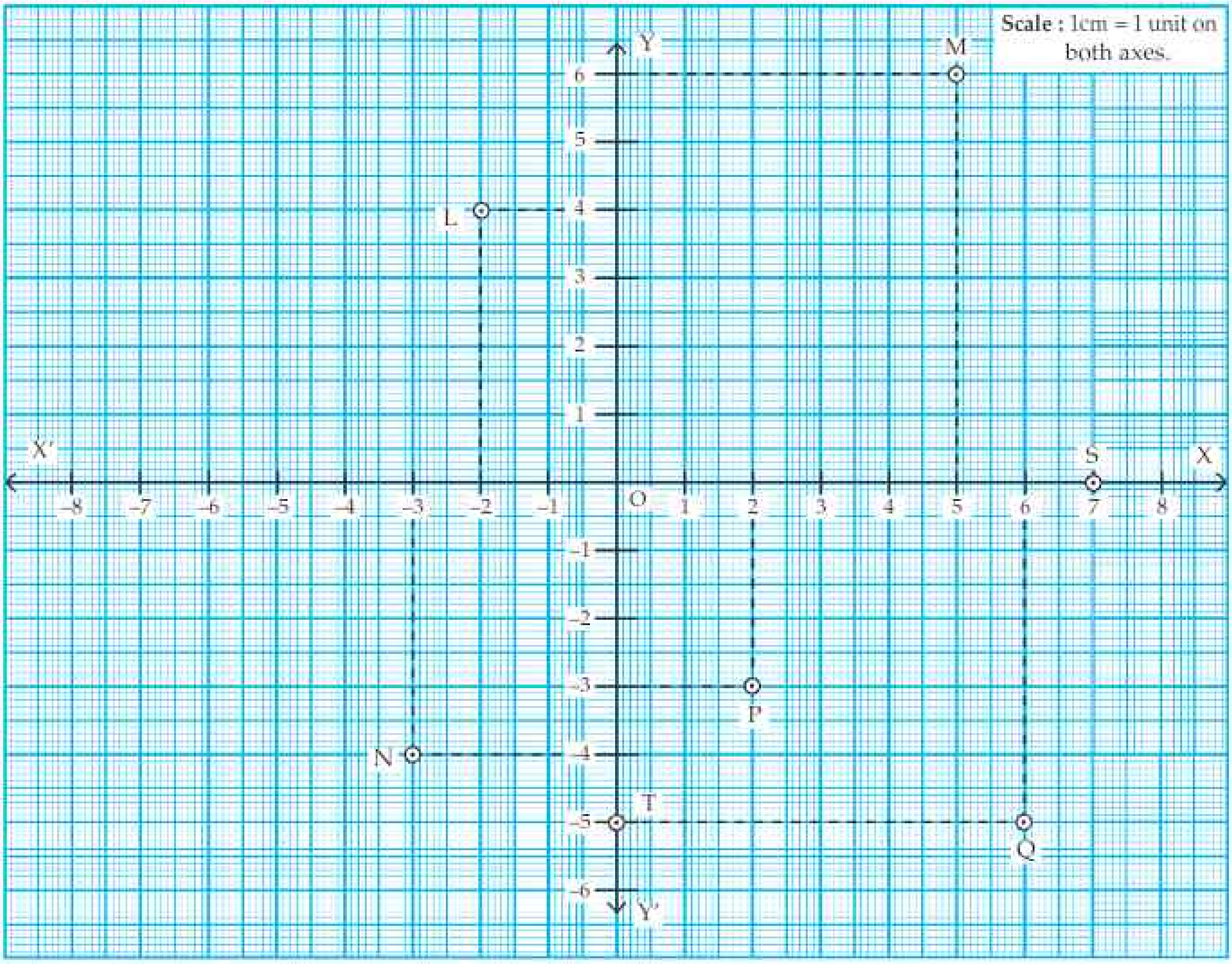
Solution:

IInd quadrant.

(3) Using cartesian coordinate system, plot the following points.

L(–2, 4), M(5, 6), N(–3, –4), P(2, –3), Q(6, –5), S(7, 0), T(0, –5).

Solution:



PROBLEMS FOR PRACTICE

(1) **From the given coordinates, state to which quadrant do the following points belong:**

A(8, 9), B(–4, –1), M(–6.2, 4.3),

S(5, –1), Q(7, 3), T(–2, –2)

(2) **In which quadrant or on which axes do the following points lie if:**

(i) $x < 0$ and $y > 0$

(ii) Both the coordinates are negative

(iii) $x = 0$ and $y < 0$

(iv) $y = 0$ and $x > 0$

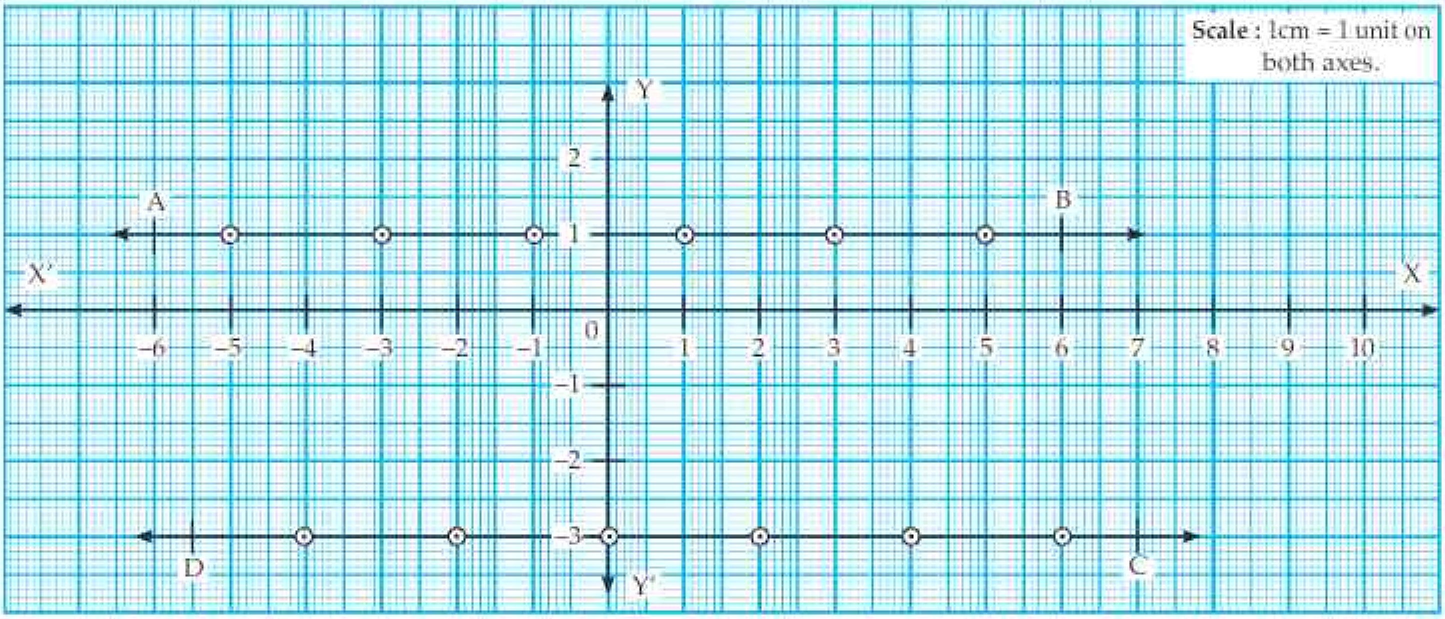
ANSWERS

(1) A - I, B - III, M - II, S - IV, Q - I, T - III

(2) (i) II quadrant (ii) III quadrant (iii) on negative y-axis (iv) on Positive x-axis.

❑ Equation of a line parallel to Axis:

(1) A line parallel to X-axis.



Observe the line AB and coordinates of all the points marked on line AB.

We can see that Y coordinate of all points on line AB is constant i.e. 1.

Now observe line CD and coordinates of all the points marked on line CD.

Here also, we can see that Y coordinate of all points on line CD are constant. i.e. –3.

If we observe carefully then both these lines are parallel to X axis.

$\therefore$ Equation of line AB is $y = 1$

$\therefore$ Equation of line CD is $y = -3$

Points to Remember:

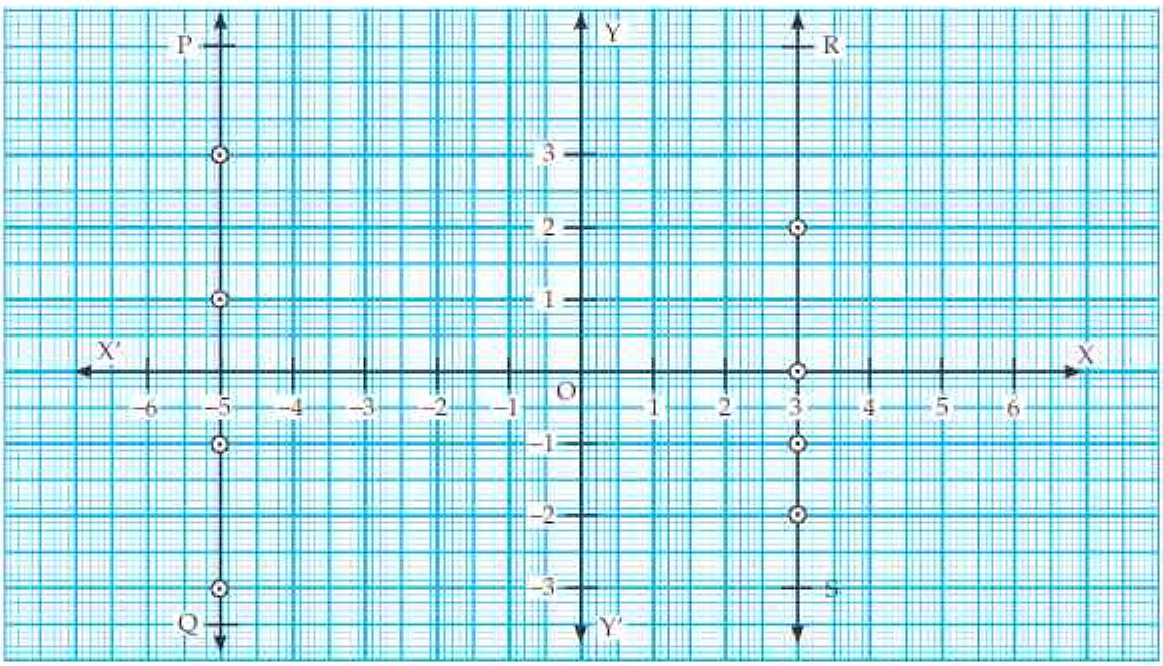
(1) Equation of a line parallel to X axis is $y = b$ where $b \in R$.

(2) If $b > 0$ then line will be parallel to X-axis above the X-axis.

(3) If $b < 0$ then line will be parallel to X-axis below X-axis.

(4) Equation of X axis is $y = 0$.

❑ A line parallel to y-axis.



Observe the line PQ and coordinates of all the points marked on line PQ.

We can see that X coordinate of all points on line PQ is constant i.e. –5.

Now, observe the line RS and coordinates of all the points marked on RS.

Here also, we can see that X coordinate of all points on line RS is constant i.e. 3.

If we observe carefully then both the lines are parallel to Y-axis.

$\therefore$ Equation of line PQ is X = –5.

$\therefore$ Equation of line RS is X = 3.

Points to Remember:

(1) Equation of a line parallel to Y-axis is $x = a$ where $a \in R$.

(2) If $a > 0$ then line will be parallel to y-axis and on right side of y-axis.

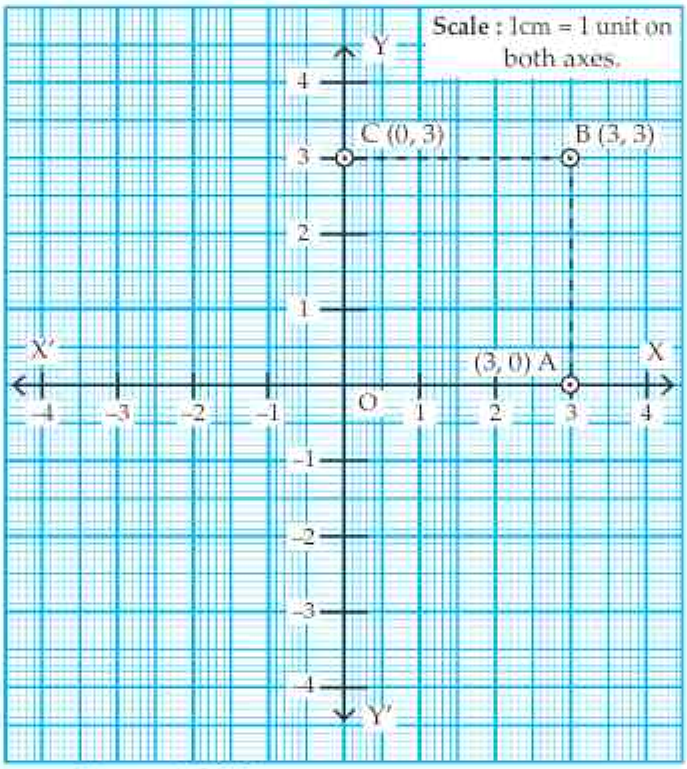
(3) If $a < 0$ then line will be parallel to y-axis and on left side of y-axis.

(4) Equation of y-axis is X = 0.

PRACTICE SET - 7.2 *(Textbook Page No. 97)*

(1) On a graph paper plot points A(3, 0) B(3, 3) and C(0, 3). Join A, B and B, C. What is the figure formed?

Solution:



Square ABCO.

(2) Write the equation of the line parallel to the Y-axis at a distance of 7 units from it to its left.

Solution:

Equation of the required line is $x = -7$.

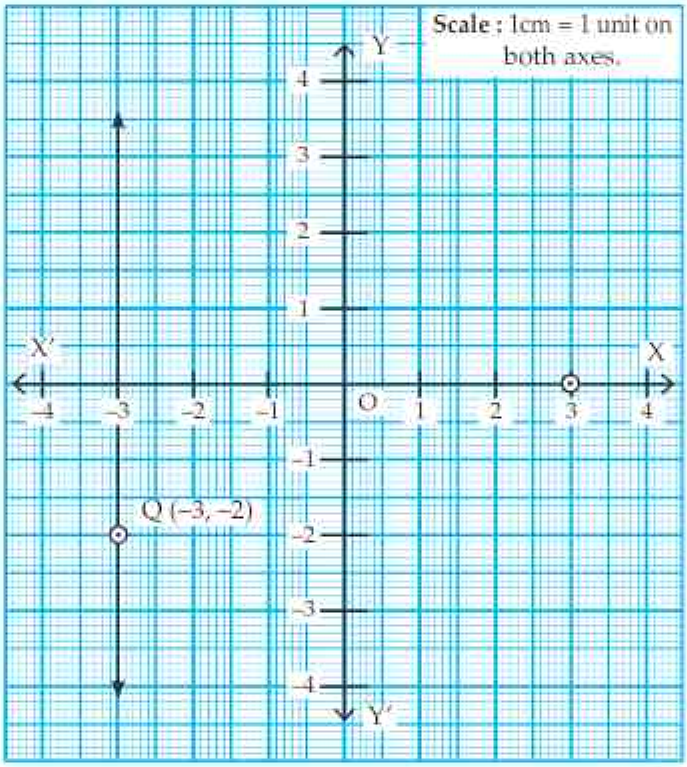
(3) Write the equation of the line parallel to the X-axis at a distance of 5 units from it and below the X-axis.

Solution:

Equation of the required line is $y = -5$.

(4) Point Q(–3, –2) lies on a line parallel to the Y-axis. Write the equation of the line and draw its graph.

Solution:



Equation of the required line is $x = -3$.

(5) Y-axis and line $x = -4$ are parallel lines. What is the distance between them?

Note: **In the text book X-axis is corrected to Y-axis.**

Solution:

Equation of y axis is $x = 0$.

Other line's equation is $x = -4$.

Distance between these two lines $= 0 - (-4) = 0 + 4$

$= 4$ units.

(6) Which of the equations given below have graphs parallel to the X-axis, and which ones have graphs parallel to the Y-axis?

(i) $x = 3$

Solution:

It is parallel to y-axis.

(ii) $y - 2 = 0$

Solution:

$y - 2 = 0$ i.e. $y = 2$. It is parallel to x-axis.

(iii) $x + 6 = 0$

Solution:

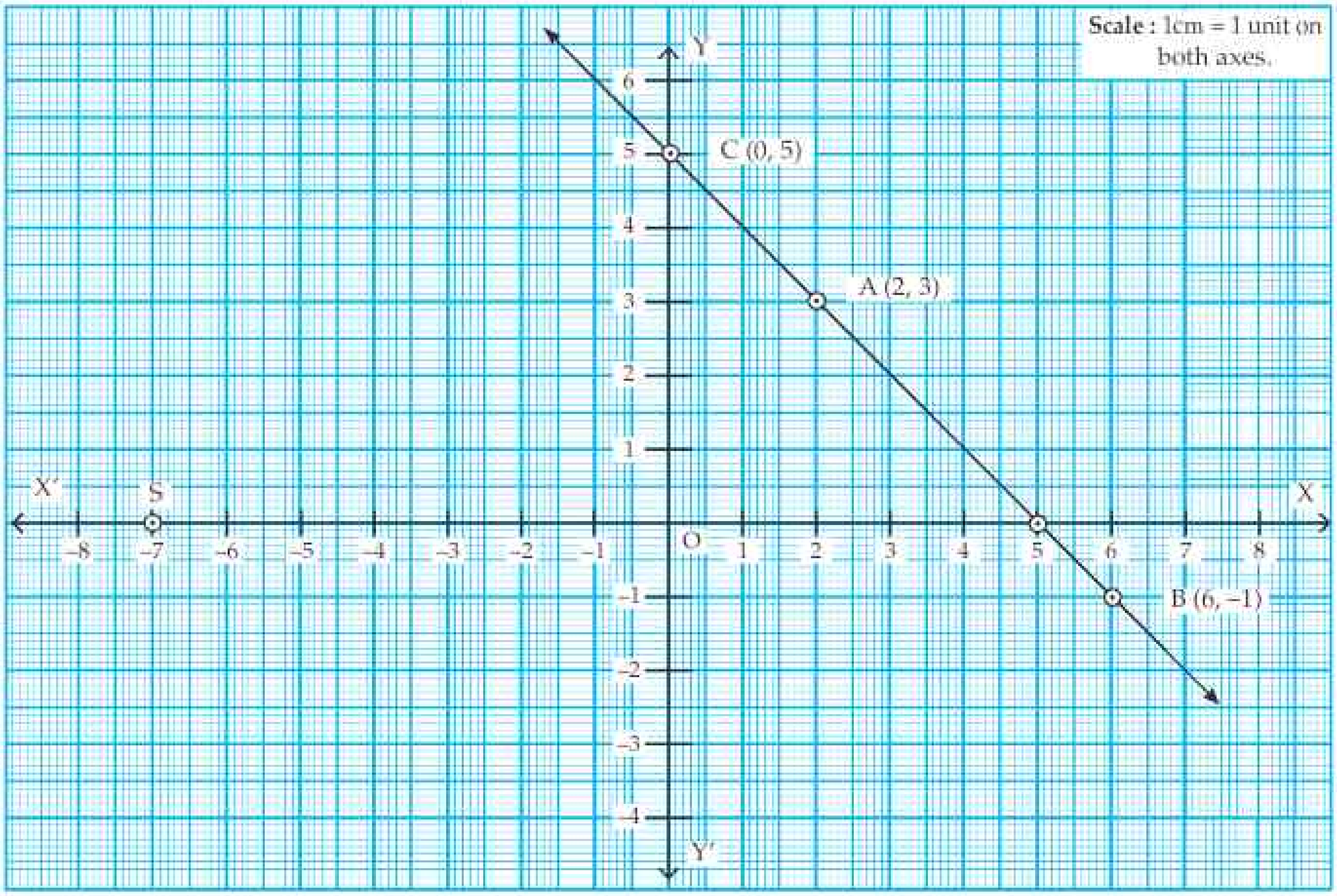
$x + 6 = 0$ i.e. $x = -6$. It is parallel to y-axis.

(iv) $y = -5$

Solution:

It is parallel to x-axis.

(7) **On a graph paper, plot the points A(2, 3) B(6, –1) and C(0, 5). If those points are collinear then draw the line which includes them. Write the co-ordinates of the points at which the line intersects the X-axis and the Y-axis.**



Solution:

Yes, the points are collinear.

Coordinates of points of intersection of the line and x axis is (5, 0)

Coordinates of point of intersection of the line and y axis is (0, 5)

(8) **Draw the graphs of the following equations on the same system of co-ordinates. Write the co-ordinates of their points of intersection.**

$x + 4 = 0$; $y - 1 = 0$; $2x + 3 = 0$; $3y - 15 = 0$

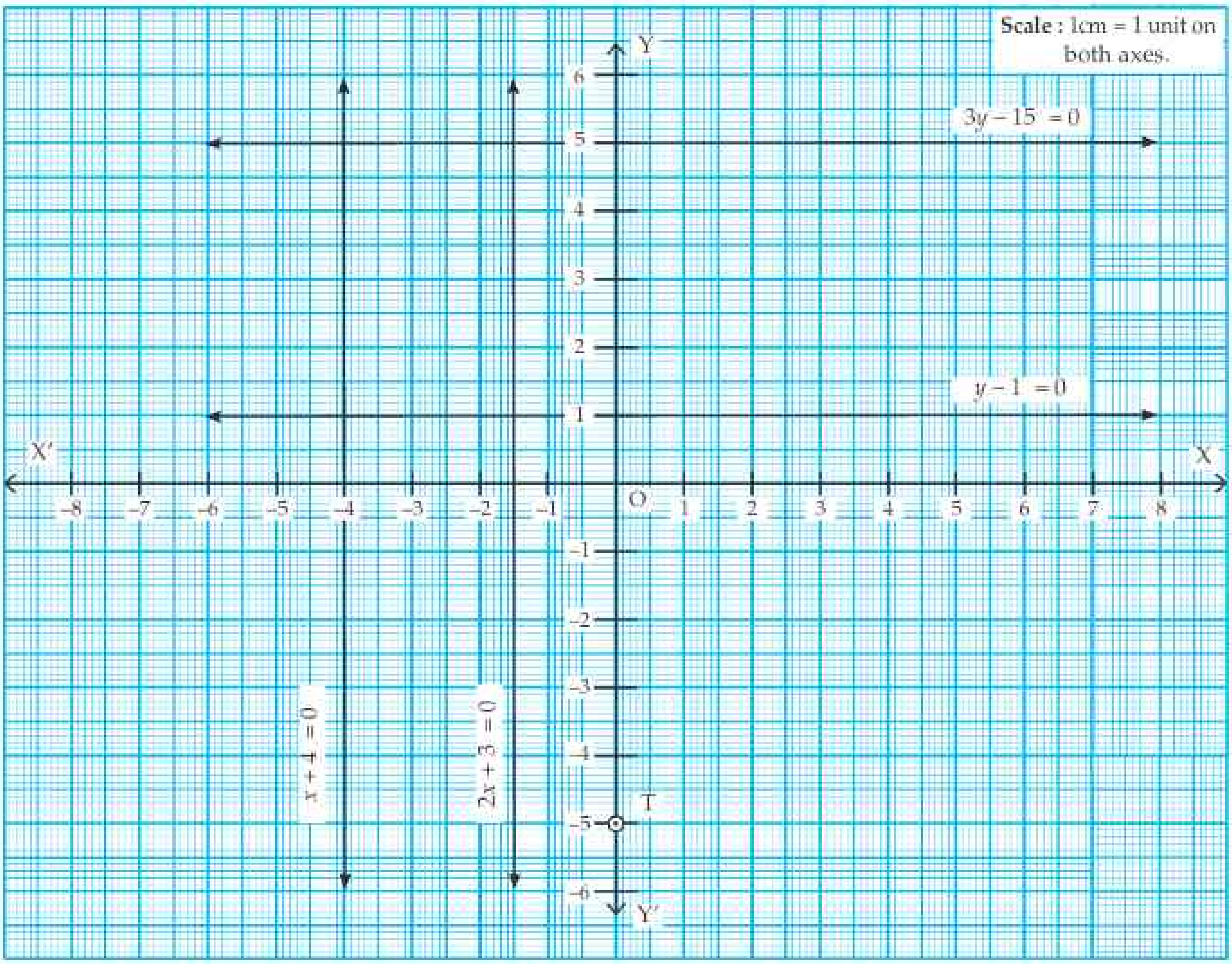
Solution:

(i) $x + 4 = 0 \quad \therefore x = -4$

(ii) $y - 1 = 0 \quad \therefore y = 1$

(iii) $2x + 3 = 0 \quad \therefore 2x = -3 \ \therefore x = \dfrac{-3}{2} \ \therefore x = -1.5$

(iv) $3y - 15 = 0 \quad \therefore 3y = 15 \ \therefore y = \dfrac{15}{3} \ \therefore y = 5$



Point of intersection of lines

x + 4 = 0 and y – 1 = 0 is (–4, 1)

Point of intersection of lines

$x + 4 = 0$ and 3y – 15 = 0 is (–4, 5)

Point of intersection of lines

$2x + 3 = 0$ and $y - 1 = 0$ is (–1.5, 1)

Point of intersection of lines

$2x + 3 = 0$ and $3y - 15 = 0$ is (–1.5, 5)

(9) Draw the graphs of the equations given below.

(i) $x + y = 2$

Solution:

$x + y = 2$ i.e. $y = 2 - x$

x	1	2	3
y	1	0	–1
(x, y)	(1, 1)	(2, 0)	(3, –1)

(a) When $x = 1$

$y = 2 - x$

$\therefore \quad y = 2 - 1 = 1$

(b) When $x = 2$

$y = 2 - x$

$\therefore \quad y = 2 - 2 = 0$

(c) When $x = 3$

$y = 2 - x$

$\therefore \quad y = 2 - 3 = -1$

(ii) $3x - y = 0$

$\therefore y = 3x$

Solution:

x	1	2	–1
y	3	6	–3
(x, y)	(1, 3)	(2, 6)	(–1, –3)

(a) When $x = 1$

$y = 3x$

$\therefore \quad y = 3 \times 1 = 3$

(b) When $x = 2$

$y = 3x$

$\therefore \quad y = 3 \times 2 = 6$

(c) When $x = -1$

$y = 3 \times (-1)$

$\therefore \quad y = -3$

(iii) $\mathbf{2x + y = 1}$

$\therefore \ \mathbf{y = 1 - 2x}$

Solution:

x	1	2	3
y	–1	–3	–5
(x, y)	(1, –1)	(2, –3)	(3, –5)

(a) When $x = 1$

$y = 1 - 2x$

$\therefore \quad y = 1 - (2 \times 1)$

$\therefore \quad y = 1 - 2$

$\therefore \quad y = -1$

(b) When $x = 2$

$y = 1 - 2x$

$\therefore \quad y = 1 - (2 \times 2)$

$\therefore \quad y = 1 - 4$

$\therefore \quad y = -3$

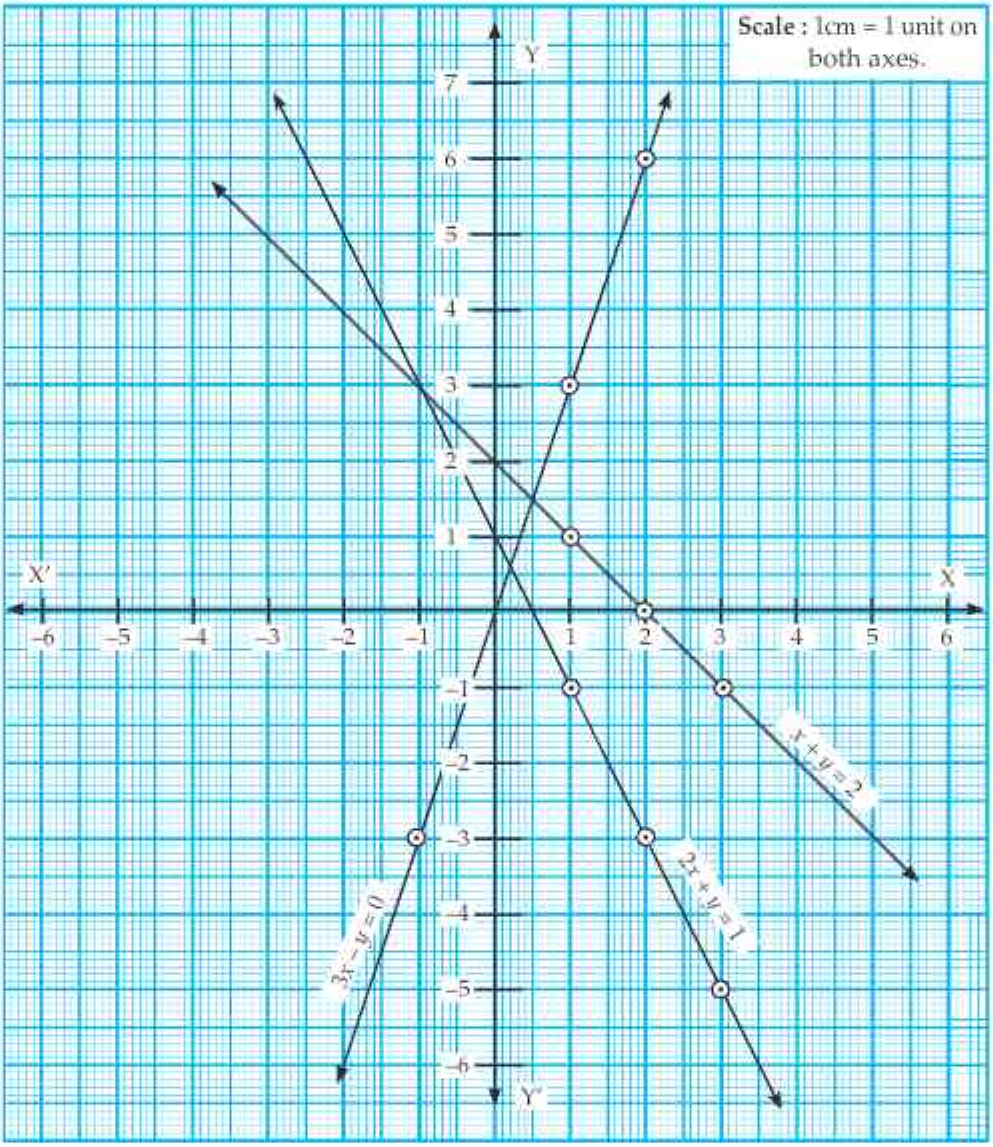
(c) When $x = 3$

$y = 1 - 2x$

$\therefore \quad y = 1 - (2 \times 3)$

$\therefore \quad y = 1 - 6$

$\therefore \quad y = -5$



PROBLEMS FOR PRACTICE

(1) Draw the graphs of the following equations:

(i) $y = -2$ (ii) $3(x + 2) = 2x - 4$

(iii) $2y + 7 = 0$

(2) Write the equations of the following lines:

(i) Parallel to X-axis and 5 units below it.

(ii) Parallel to Y-axis and 1 unit on the right of it.

(iii) Parallel to X-axis and 7 units above it.

(iv) Parallel to Y-axis and 3 units on the left of it.

(3) Draw the graphs of the following equations:

(i) $x + 2y = 0$ (ii) $2x - 3y = 0$ (iii) $3x - 2y = 0$

(4) **Draw the graph of the following equation $3x + 2y = 0$. Find the co-ordinates of the point where the graph intersects the Y-axis.**

(5) **Draw the graphs of the following equations $2x + y = -10$ and $2x - 3y = 6$ on the same graph paper.**

ANSWERS

(2) (i) $y = -5$ (ii) $x = 1$ (iii) $y = 7$ (iv) $x = -3$

PROBLEM SET - 7 *(Textbook Page No. 98)*

(1) **Choose the correct alternative answer for each of the following questions.**

(i) What is the form of co-ordinates of a point on the X-axis.

(A) (b, b) (B) $(0, b)$ (C) $(a, 0)$ (D) (a, a)

Ans. (C)

(ii) Any point on the line $y = x$ is of the form .....

(A) (a, a) (B) $(0, a)$ (C) $(a, 0)$ (D) $(a, -a)$

Ans. (A)

(iii) What is the equation of X-axis?

(A) $x = 0$ (B) $y = 0$ (C) $x + y = 0$ (D) $x = y$

Ans. (B)

(iv) In which quadrant does point $(-4, -3)$ lies?

(A) First (B) Second (C) Third (D) Fourth

Ans. (C)

(v) What is the nature of the line which includes the points $(-5, 5)$, $(6, 5)$, $(-3, 5)$, $(0, 5)$?

(A) Passing through the origin

(B) Parallel to Y-axis (C) Parallel to X-axis

(D) None of the above

Ans. (C)

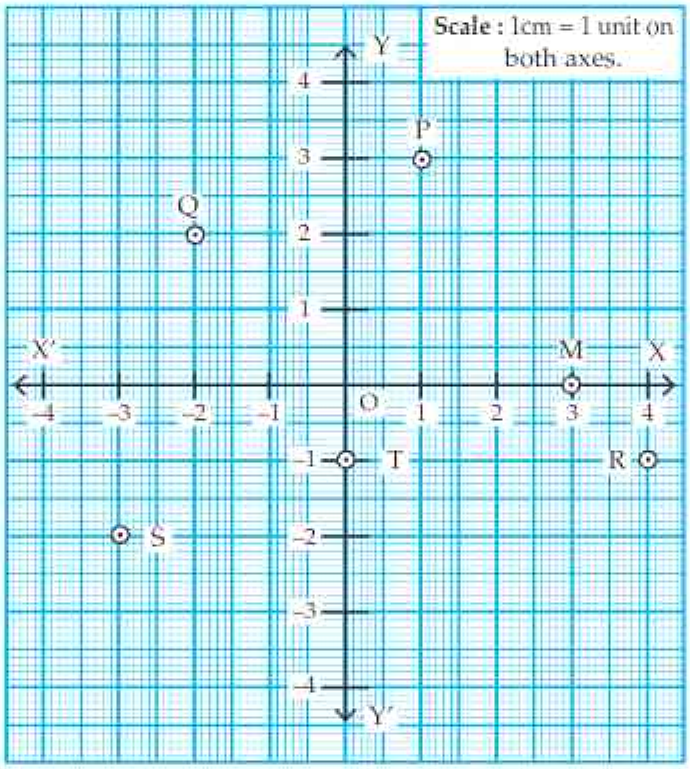
(vi) Which of the points P$(-1, 1)$, Q $(3, -4)$, R $(1, -1)$ S$(-2, -3)$, T $(-4, 4)$ lie in the fourth quadrant?

(A) P and T (B) Q and R

(C) Only S (D) P and R

Ans. (B)

(2) **Observe the following graph and answer the following questions.**



(i) Write the coordinates of point Q and point R.

Ans. Q$(-2, 2)$ and R$(4, -1)$

(ii) Write the coordinates of point T and point M.

Ans. T$(0, -1)$ and M$(3, 0)$

(iii) Which point lies in the IIIrd quadrant?

Ans. Point S

(iv) Which are the points whose x and y coordinat are equal.

Ans. Origin O$(0, 0)$

(3) **Without plotting the following points, state in which quadrant or on which axis does the following point lie.**

(i) $(5, -3)$ (ii) $(-7, -12)$

(iii) $(-23, 4)$ (iv) $(-9, 5)$

(v) $(0, -3)$ (vi) $(-6, 0)$

Ans.

(i) Fourth quadrant (ii) Third quadrant

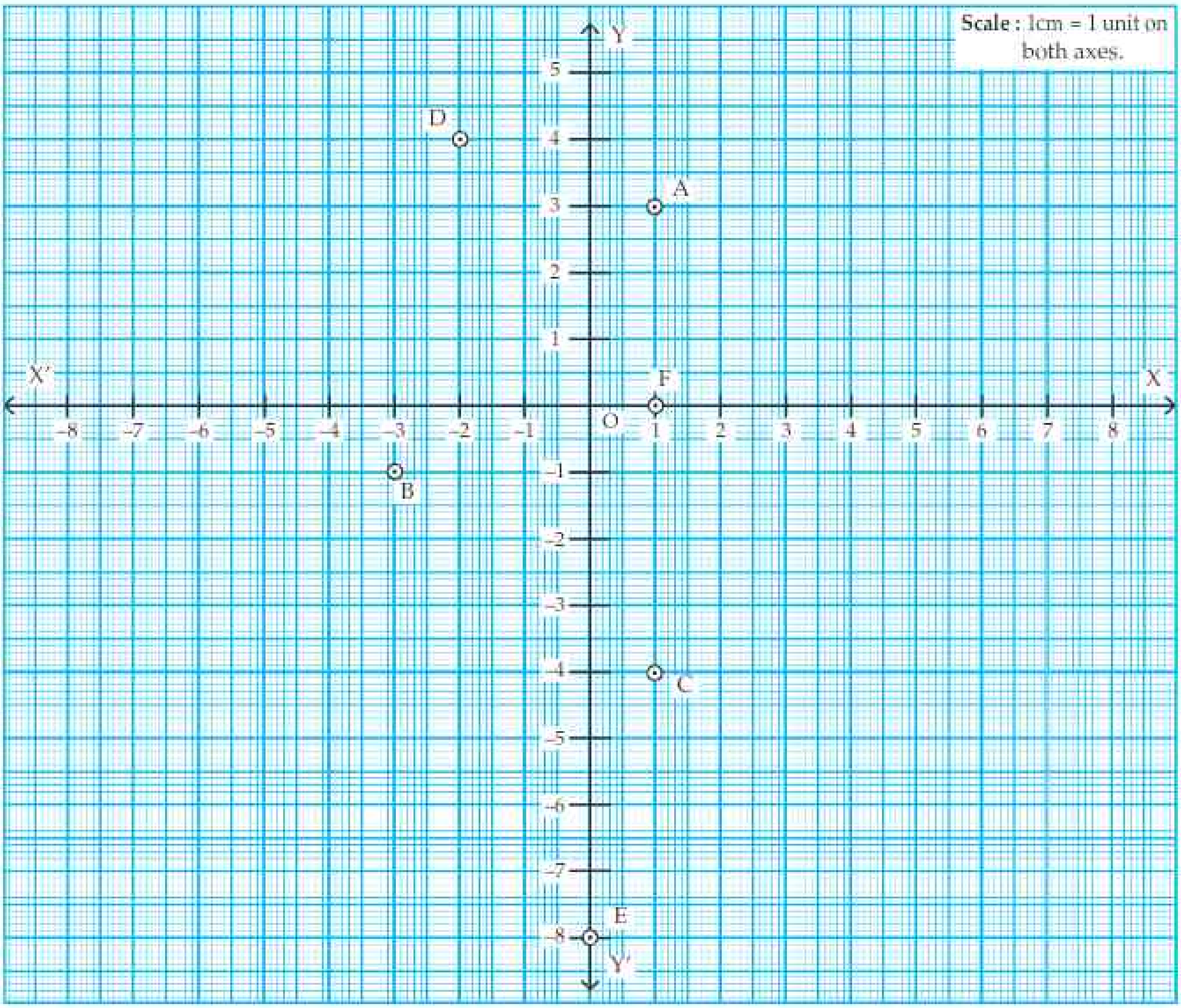
(iii) Second quadrant (iv) Second quadrant

(v) Y-axis (vi) X-axis

(4) **Plot the following points on the one and the same co-ordinate system.**

A $(1, 3)$, B $(-3, -1)$, C $(1, -4)$, D $(-2, 3)$, E $(0, -8)$, F $(1, 0)$

Ans.



(5) In the adjoining figure line LM is parallel to Y-axis.

(i) What is the distance between line LM and Y-axis?

Equation of Y-axis is $x = 0$

Equation of line LM is $x = 3$.

Distance between line LM and Y-axis = 3 – 0 = 3.

(ii) Write the coordinates of points P, Q and R.

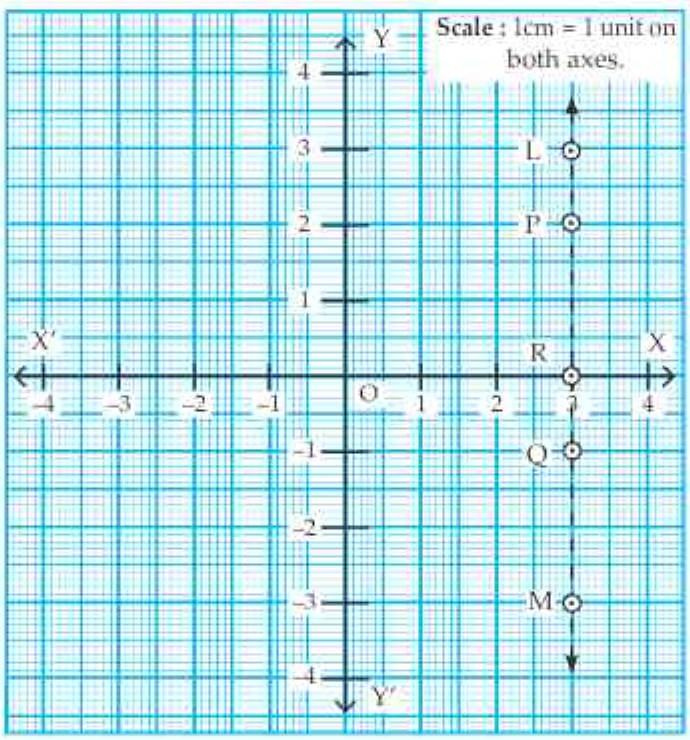
Ans. P(3, 2) Q(3, –1) R(3, 0)

(iii) Find the difference between x - coordiante of point L and point M.

Ans. L(3, 3) M(3, –3) The x-coordiante of point L is 3 and the x-coordinate of point M is 3.

$\therefore$ Difference between x-coordinates = 3 – 3 = 0

Ans.



(6) How many lines are there which are parallel to X- axis and having a distance 5 units?

Ans. There can be two lines.

(i) $y = 5$ (ii) $y = -5$

(7) If a is any real number then find the distance between Y-axis and $x = a$.

Ans. Distance between Y-axis and a line is $|a|$ with equation $x = a$.

MCQ's

(1) If x coordinate is positive and y coordiante is negative the point lies in ....

(A) I quadrant (B) III quadrant
(C) II quadrant (D) IV quadrant

(2) If x coordinate of a point is zero, then the point lies on ....

(A) IV quadrant (B) X-axis
(C) Y-axis (D) Origin

(3) In III quadrant both the coordinates are ....

(A) x -positive, y-negative
(B) x -negative, y-negative
(C) x -positive, y-positive
(D) x -negative, y-positive

(4) If the coordinates of a point are $(0, a)$ where $a = 6$, then the point belongs to ....

(A) I quadrant (B) X-axis
(C) II quadrant (D) Y-axis

(5) The point of intersection of the lines $y = -3$ and $x = 4$ is ....

(A) $(-3, 4)$ (B) $(2, 3)$ (C) $(4, -3)$ (D) $(4, 3)$

(6) For the equation $x - 2y = 0$

When $x = -2$, the coordinate of the point is ....

(A) $(2, 1)$ (B) $(-2, 3)$ (C) $(-2, 2)$ (D) $(-2, 4)$

(7) The equation of a line passing through -3 on the x-axis and parallel to y-axis is ....

(A) $x = -3$ (B) $x = 3$ (C) $y = -3$ (D) $y = 3$

(8) On simplifying the equation $3(x + 1) = 2x - 3$ we get ....

(A) $x = -4$ (B) $x = 4$ (C) $x = 6$ (D) $x = -6$

(9) The point of intersection of the lines $y + 4 = 2y + 7$ and $2x + 4 = 3x + 1$ is ....

(A) $(3, -3)$ (B) $(-3, 3)$ (C) $(2, 1)$ (d) $(1, 2)$

(10) For the equation, $-3x + 4y = 12$, the value of y-coordinate is 3 then the value of x-coordinate is ....

(A) 0 (B) 1 (C) 2 (D) 3

(11) In which quadrant or on which axis will (x, y) lie, if $x < 0$ and $y = 0$.

(A) I quadrant (B) II quadrant
(C) negative X-axis (D) positive X-axis

(12) For the given line $2y = 3x + 2$ if $x = \frac{7}{3}$ then $y = ...$

(A) $\frac{2}{9}$ (B) $\frac{9}{2}$ (C) $\frac{4}{5}$ (D) $\frac{5}{4}$

(13) The equation of a line is $5x - 4y = P$. If $x = 2$ and $y = 0$ then $P =$ ....

(A) 5 (B) 10 (C) -10 (D) -5

(14) The equation of a given line is $8x - 3y + 4 = 0$. If $y = -4$ then $x =$ ....

(A) -1 (B) -2 (C) -3 (D) -4

(15) The equation of a line is $2x - 5y - 7 = 0$. If $y = 0$ then $x =$ ....

(A) 2.5 (B) 2 (C) 3 (D) 3.5

(16) If $x = \frac{4y - 12}{3}$ and the value of $y = 3$ then the value of $x =$ ....

(A) 3 (B) 2 (C) 1 (D) 0

(17) For the given equation $3x + 2y = 4$ the value of y is ....

(A) $\frac{4 + 3x}{2}$ (B) $\frac{3x - 4}{2}$
(C) $\frac{4 - 3x}{2}$ (D) None of above

(18) If $y = \frac{4 - 3x}{2}$ and $x = 1$, then $y =$ ....

(A) 2 (B) 3 (C) 0.5 (D) 4

(19) For the given equation $x = \frac{10 - 7y}{5}$ if $y = 1$ then $x =$ ....

(A) 0.7 (B) 0.6 (C) 1.6 (D) 0.8

(20) If $a + b = -1$ and $-a + b = 3$ Then adding both the equations, the value of b is ....

(A) -1 (B) 1 (C) -2 (D) 2

ANSWERS

(1) (D) (2) (C) (3) (B) (4) (D) (5) (C)
(6) (A) (7) (A) (8) (D) (9) (A) (10) (A)
(11) (C) (12) (B) (13) (B) (14) (B) (15) (D)
(16) (D) (17) (C) (18) (C) (19) (B) (20) (B)

ASSIGNMENT - 7

Time : 1 Hr. **Marks : 20**

Q.1. Attempt the following: **(2)**

(1) From the given co-ordinates, state to which quadrant do the following points belong:

A(–4, –1) B(5, –8)

(2) In which quadrant or on which axis will (x, y) lie if $x > 0$ and $y > 0$

Q.2. Attempt the following: **(4)**

(1) (i) Line PQ is parallel to y-axis and passing through the point (–4, 3). Write the equation of the line PQ.

(ii) Line AB is parallel to x-axis and passing through the point (6, 7). Write the equation of line AB.

(2) Given below are some points with their co-ordinates. Answer the following questions.

Q(–3, 2), R(5, 1), S(0, –4)

(i) What is the y co-ordinate of point S.

(ii) What is the x co-ordinate of point Q.

(iii) What is the y co-ordinate of point R.

Q.3. Attempt the following: **(6)**

(1) Write the co-ordinates of the following points in ordered pairs.

(i) Point B : x co-ordinate is 4 and y co-ordinate is –5.

(ii) Point F : y co-ordinate is 6 and x co-ordinate is 0.

(iii) Point O : origin.

(2) Draw x axis and y axis on a graph paper and plot the following points.

A(1.5, 2.5) B(0, –3) C(4, 0) P(–4, –3) Q(0, 6) R(2, 5)

Q.4. Attempt the following: **(8)**

(1) Draw the graphs of the following equations on the same graph paper.

(i) $2y + 1 = y + 3$ (ii) $3(x + 1) = 2x - 3$

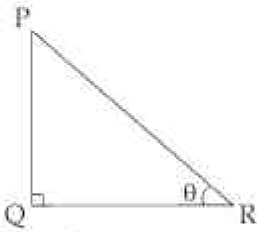
(2) Draw the graphs of the following equations on the same graph paper.

(i) $2x - 3y = 5$ (ii) $2x - 5y - 7 = 0$

❖❖❖

8 Trigonometry

Points to Remember:



In ΔPQR, $\angle PQR = 90^\circ$

$\therefore$ side PR is hypotenuse.

Let $\angle R = \theta$.

$\therefore$ With respect to θ,

side PQ is the opposite side and

side QR is the adjacent side.

Now we shall study three trigonometric ratios, viz sine (sin), cosine (cos) and tangent (tan).

$$\text{sine} = \frac{\text{Opposite side}}{\text{Hypotenuse}}; \ \text{cosine} = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$

$$\text{and tangent} = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

So with respect to above figure,

$$\sin\theta = \frac{\text{Opposite side of } \theta}{\text{Hypotenuse}} = \frac{PQ}{PR}$$

$$\cos\theta = \frac{\text{Adjacent side of } \theta}{\text{Hypotenuse}} = \frac{QR}{PR}$$

$$\tan\theta = \frac{\text{Opposite side of } \theta}{\text{Adjacent side of } \theta} = \frac{PQ}{QR}$$

Note:

Cosecant, secant and cotangent are three more ratios in trigonometry. These ratios are multiplicative inverse of sine, cosine and tangent ratios.

i.e. $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$ and

$$\cot\theta = \frac{1}{\tan\theta}$$

With respect to figure given above

$$\operatorname{cosec}\theta = \frac{PR}{PQ}, \ \sec\theta = \frac{PR}{QR} \text{ and } \cot\theta = \frac{QR}{PQ}$$

MASTER KEY QUESTION SET - 8

PRACTICE SET - 8.1 *(Textbook Page No. 104)*

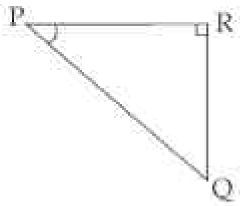
(1) In the figure, $\angle R$ is the right angle of ΔPQR. Write the following ratios.

(i) sin P

(ii) cos Q

(iii) tan P

(iv) tan Q



Solution:

(i) $\sin P = \dfrac{\text{Opposite side of } \angle P}{\text{Hypotenuse}}$

$\therefore$ $\boxed{\sin P = \dfrac{QR}{PQ}}$

(ii) $\cos Q = \dfrac{\text{Adjacent side of } \angle Q}{\text{Hypotenuse}}$

$\therefore$ $\boxed{\cos Q = \dfrac{QR}{PQ}}$

(iii) $\tan P = \dfrac{\text{Opposite side of } \angle P}{\text{Adjacent side}}$

$\therefore$ $\boxed{\tan P = \dfrac{QR}{PR}}$

(iv) $\tan Q = \dfrac{\text{Opposite side of } \angle Q}{\text{Adjacent side}}$

$\therefore$ $\boxed{\tan Q = \dfrac{PR}{QR}}$

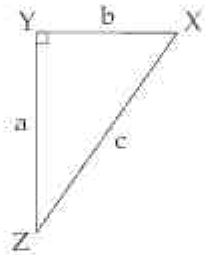
(2) In the right angled ΔXYZ, $\angle XYZ = 90^\circ$ and a, b, c are the lengths of the sides as shown in the figure. Write the following ratios,

(i) sin X

(ii) tan Z

(iii) cos X

(iv) tan X



Solution:

(i) $\sin X = \dfrac{\text{Opposite side of } \angle X}{\text{Hypotenuse}} = \dfrac{YZ}{XZ} = \dfrac{a}{c}$

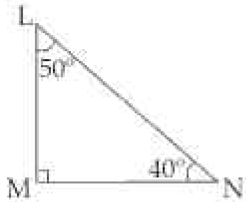
(ii) $\tan Z = \dfrac{\text{Opposite side of } \angle Z}{\text{Adjacent side of } \angle Z} = \dfrac{XY}{YZ} = \dfrac{b}{a}$

(iii) $\cos X = \dfrac{\text{Adjacent side of } \angle X}{\text{Hypotenuse}} = \dfrac{XY}{XZ} = \dfrac{b}{c}$

(iv) $\tan X = \dfrac{\text{Opposite side of } \angle X}{\text{Adjacent side of } \angle X} = \dfrac{YZ}{XY} = \dfrac{a}{b}$

(3) In right angled ΔLMN, $\angle LMN = 90^\circ$ $\angle L = 50^\circ$ and $\angle N = 40^\circ$,
Write the following ratios.

(i) $\sin 50^\circ$
(ii) $\cos 50^\circ$
(iii) $\tan 40^\circ$
(iv) $\cos 40^\circ$



Solution:

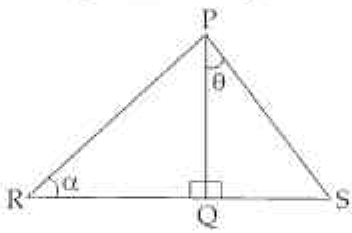
(i) $\sin 50^\circ = \dfrac{\text{Opposite side of } \angle L}{\text{Hypotenuse}} = \dfrac{MN}{LN}$

(ii) $\cos 50^\circ = \dfrac{\text{Adjacent side of } \angle L}{\text{Hypotenuse}} = \dfrac{LM}{LN}$

(iii) $\tan 40^\circ = \dfrac{\text{Opposite side of } \angle N}{\text{Adjacent side of } \angle N} = \dfrac{LM}{MN}$

(iv) $\cos 40^\circ = \dfrac{\text{Adjacent side of } \angle N}{\text{Hypotenuse}} = \dfrac{MN}{LN}$

(4) In the figure, $\angle PQR = 90^\circ$, $\angle PQS = 90^\circ$, $\angle PRQ = \alpha$ and $\angle QPS = \theta$.



Write the following trigonometric ratios.

(i) $\sin \alpha$, $\cos \alpha$, $\tan \alpha$
(ii) $\sin \theta$, $\cos \theta$, $\tan \theta$

Solution:

(i) $\sin \alpha = \dfrac{\text{Opposite side of } \alpha}{\text{Hypotenuse}} = \dfrac{PQ}{PR}$

$\cos \alpha = \dfrac{\text{Adjacent side of } \alpha}{\text{Hypotenuse}} = \dfrac{QR}{PR}$

$\tan \alpha = \dfrac{\text{Opposite side of } \alpha}{\text{Adjacent side of } \alpha} = \dfrac{PQ}{QR}$

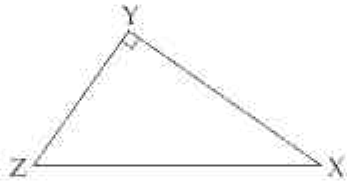
(ii) $\sin \theta = \dfrac{\text{Opposite side of } \theta}{\text{Hypotenuse}} = \dfrac{QS}{PS}$

$\cos \theta = \dfrac{\text{Adjacent side of } \theta}{\text{Hypotenuse}} = \dfrac{PQ}{PS}$

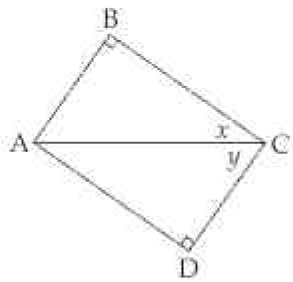
$\tan \theta = \dfrac{\text{Opposite side of } \theta}{\text{Adjacent side of } \theta} = \dfrac{QS}{PQ}$

PROBLEMS FOR PRACTICE

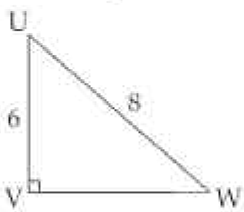
(1) In the following figure, ΔXYZ is a right angled triangle with $\angle Y = 90^\circ$. Write all trigonometric ratio of $\angle X$ and $\angle Z$.



(2) In the following figure, write $\tan x$, $\cos(90 - y)$, $\sin y$, $\tan(90 - x)$, $\sin(90 - y)$, $\cos x$.



(3) In ΔUVW, $m\angle UVW = 90^\circ$, UV = 6, UW = 8. Find trigonometric ratios of $\angle W$.



ANSWERS

(1) (i) $\sin X = \dfrac{YZ}{XZ}$; $\cos X = \dfrac{XY}{XZ}$; $\tan X = \dfrac{YZ}{XY}$

(ii) $\sin Z = \frac{XY}{XZ}$; $\cos Z = \frac{YZ}{XZ}$; $\tan Z = \frac{XY}{YZ}$

(2) (i) $\tan x = \frac{AB}{BC}$ (ii) $\cos(90 - y) = \frac{AD}{AC}$

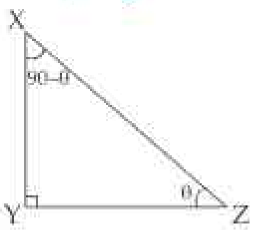
(iii) $\sin y = \frac{AD}{AC}$ (iv) $\tan(90 - x) = \frac{BC}{AB}$

(v) $\sin(90 - y) = \frac{CD}{AC}$ (vi) $\cos x = \frac{BC}{AC}$

(3) (i) $\sin W = \frac{3}{4}$; $\cos W = \frac{\sqrt{7}}{4}$; $\tan W = \frac{3}{\sqrt{7}}$

Points to Remember:

❑ Relation between Trigonometric ratios of complementary angles:



In ΔXYZ, $\angle XYZ = 90^\circ$

$\therefore$ seg XZ is hypotenuse

$\angle Z = \theta$; $\angle X = 90 - \theta$ (Remaining angle)

With respect to θ

Opposite side is XY, Adjacent side is YZ

With respect to $(90 - \theta)$

Opposite side is YZ

Adjacent side is XY

(i) $\sin\theta = \frac{XY}{XZ}$ (ii) $\cos\theta = \frac{YZ}{XZ}$

(iii) $\tan\theta = \frac{XY}{YZ}$ (iv) $\sin(90 - \theta) = \frac{YZ}{XZ}$

(v) $\cos(90 - \theta) = \frac{XY}{XZ}$ (vi) $\tan(90 - \theta) = \frac{YZ}{XY}$

Three relations:

(i) $\sin\theta = \cos(90 - \theta)$ ... [From (i) and (v)]

(ii) $\cos\theta = \sin(90 - \theta)$ ... [From (ii) and (iv)]

(iii) $\tan\theta \times \tan(90 - \theta) = \frac{XY}{YZ} \times \frac{YZ}{XY} = 1$

... [From (iii) and (vi)]

❑ For additional information:

(1) $\sin\theta \times \operatorname{cosec}\theta = 1$ i.e. $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$

(2) $\cos\theta \times \sec\theta = 1$ i.e. $\cos\theta = \frac{1}{\sec\theta}$

(3) $\tan\theta \times \cot\theta = 1$ i.e. $\tan\theta = \frac{1}{\cot\theta}$

(4) $\sec\theta = \operatorname{cosec}(90 - \theta)$

(5) $\operatorname{cosec}\theta = \sec(90 - \theta)$

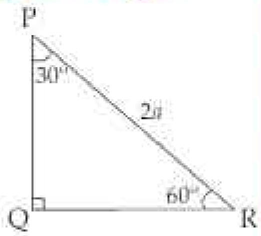
(6) $\tan\theta = \cot(90 - \theta)$

(7) $\cot\theta = \tan(90 - \theta)$

❑ Trigonometric Ratios of special angles:

(I) In ΔPQR, $\angle P = 30^\circ$, $\angle Q = 90^\circ$ and $\angle R = 60^\circ$, $PR = 2a$

ΔPQR is 30°-60°-90° triangle



$QR = \frac{1}{2} \times PR$ (Side opposite to 30°)

$\therefore$ $QR = \frac{1}{2} \times 2a = a$

$PQ = \frac{\sqrt{3}}{2} \times PR$ (Side opposite to 60°)

$\therefore$ $PQ = \frac{\sqrt{3}}{2} \times 2a = \sqrt{3}a.$

(A) Trigonometric ratios of angle measuring 30°

(i) $\sin 30^\circ = \frac{QR}{PR} = \frac{a}{2a} = \frac{1}{2}$

(ii) $\cos 30^\circ = \frac{PQ}{PR} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$

(iii) $\tan 30^\circ = \frac{QR}{PQ} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$

(B) Trigonometric ratios of angle measuring 60°

(i) $\sin 60^\circ = \frac{PQ}{PR} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$

(ii) $\cos 60^\circ = \frac{QR}{PR} = \frac{a}{2a} = \frac{1}{2}$

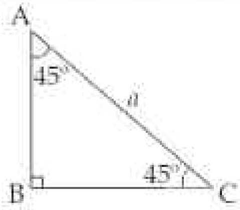
(iii) $\tan 60^\circ = \frac{PQ}{QR} = \frac{\sqrt{3}a}{a} = \sqrt{3}$

(II) In ΔABC,

$\angle A = \angle C = 45^\circ$,

$\angle B = 90^\circ$, $AC = a$

ΔABC is 45°-45°-90° triangle.



$\therefore$ $AB = BC = \dfrac{1}{\sqrt{2}} \times AC$ (Side opposite to 45°)

$\therefore$ $AB = BC = \dfrac{1}{\sqrt{2}} \times a = \dfrac{a}{\sqrt{2}}$

(C) Trigonometric ratios of angle measuring 45°:

(i) $\sin 45^\circ = \dfrac{AB}{AC} = \dfrac{a}{\sqrt{2}} \div a = \dfrac{a}{\sqrt{2}} \times \dfrac{1}{a} = \dfrac{1}{\sqrt{2}}$

(ii) $\cos 45^\circ = \dfrac{BC}{AC} = \dfrac{a}{\sqrt{2}} \div a = \dfrac{a}{\sqrt{2}} \times \dfrac{1}{a} = \dfrac{1}{\sqrt{2}}$

(iii) $\tan 45^\circ = \dfrac{AB}{BC} = \dfrac{a}{\sqrt{2}} \div \dfrac{a}{\sqrt{2}} = 1$

(D) Trigonometric ratios of angle measuring 0° and 90°:

(i) $\sin 0^\circ = 0$

(ii) $\cos 0^\circ = 1$

(iii) $\tan 0^\circ = \dfrac{\sin 0^\circ}{\cos 0^\circ} = \dfrac{0}{1} = 0$

(iv) $\sin 90^\circ = 1$

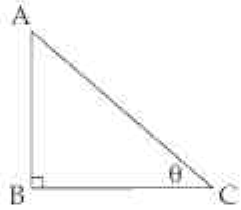
(v) $\cos 90^\circ = 0$

(vi) $\tan 90^\circ = \dfrac{\sin 90^\circ}{\cos 90^\circ} = \dfrac{1}{0} =$ Not defined

We summarise all the values calculated in (A), (B), (C) and (D) in the following table.

	0°	**30°**	**45°**	**60°**	**90°**
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined

❑ **Trigonometric Identity:**



In ΔABC, $\angle B = 90^\circ$, $\angle C = \theta$

$\therefore$ $\sin\theta = \dfrac{AB}{AC}$ ...(i)

and $\cos\theta = \dfrac{BC}{AC}$ ...(ii)

In ΔABC, $\angle B = 90^\circ$

$\therefore$ $AB^2 + BC^2 = AC^2$ ... [Pythagoras theorem]

Dividing throughout by AC^2

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

$\therefore$ $(\sin\theta)^2 + (\cos\theta)^2 = 1$ ... [From (i) and (ii)]

Note:

$(\sin\theta)^2$ means square of $\sin\theta$ which is conventionally written as $\sin^2\theta$.

Hence above identity can be written as

$\mathbf{\sin^2\theta + \cos^2\theta = 1}$

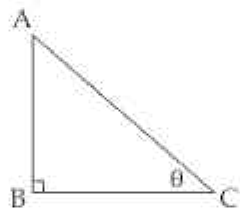
PRACTICE SET - 8.2 *(Textbook Page No. 112)*

(1) In the following table one of the trigonometric ratio is given. Using this find remaining trigonometric ratios.

sin θ		$\frac{11}{61}$		$\frac{1}{2}$				$\frac{3}{5}$	
cos θ	$\frac{35}{37}$				$\frac{1}{\sqrt{3}}$				
tan θ			1			$\frac{21}{20}$	$\frac{8}{15}$		$\frac{1}{2\sqrt{2}}$

Solution:

(i) Method I



In the above adjoining, in ΔABC

$\angle B = 90^\circ$ and $\angle C = \theta$

$\therefore \quad \cos\theta = \dfrac{BC}{AC}$ ... (i) (By definition)

But $\cos\theta = \dfrac{35}{37}$ ... (ii) (Given)

$\therefore \quad \dfrac{BC}{AC} = \dfrac{35}{37}$ ... [From (i) and (ii)]

Let the common multiple be k ($k \neq 0$)

$\therefore \quad BC = 35k;\ AC = 37k$

In ΔABC, $\angle B = 90^\circ$ ... (Given)

$\therefore \quad AC^2 = AB^2 + BC^2$ ... (Pythagoras theorem)

$\therefore \quad (37k)^2 = AB^2 + (35k)^2$

$\therefore \quad (37k)^2 - (35k)^2 = AB^2$

$\therefore \quad AB^2 = (37k + 35k)(37k - 35k)$

$\therefore \quad AB^2 = 72k \times 2k$

$\therefore \quad AB^2 = 144k^2$

$\therefore \quad AB = 12k$ ... (Taking square roots)

$\sin\theta = \dfrac{AB}{AC}$

$\therefore \quad \sin\theta = \dfrac{12k}{37k}$

$$\boxed{\therefore \sin\theta = \dfrac{12}{37}}$$

$\tan\theta = \dfrac{AB}{BC}$

$\therefore \quad \tan\theta = \dfrac{12k}{35k}$

$$\boxed{\therefore \tan\theta = \dfrac{12}{35}}$$

(i) Method II

$\sin^2\theta + \cos^2\theta = 1$ ... (Trigonometric identity)

$\therefore \quad \sin^2\theta + \left(\dfrac{35}{37}\right)^2 = 1$

$\therefore \quad \sin^2\theta = 1 - \dfrac{35^2}{37^2}$

$\therefore \quad \sin^2\theta = \dfrac{37^2 - 35^2}{37^2}$

$\therefore \quad \sin^2\theta = \dfrac{(37+35)(37-35)}{37^2}$

$= \dfrac{72 \times 2}{37^2}$

$\sin^2\theta = \dfrac{144}{37^2}$

$$\boxed{\therefore \sin\theta = \dfrac{12}{37}}$$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{12}{37} \div \dfrac{35}{37}$

$\therefore \quad \tan\theta = \dfrac{12}{37} \times \dfrac{37}{35}$

$$\boxed{\therefore \tan\theta = \dfrac{12}{35}}$$

(ii) $\mathbf{\sin\theta = \dfrac{11}{61}}$ ... (Given)

$\sin^2\theta + \cos^2\theta = 1$ ... (Trigonometric identity)

$\therefore \quad \left(\dfrac{11}{61}\right)^2 + \cos^2\theta = 1$

$\therefore \quad \cos^2\theta = 1 - \dfrac{11^2}{61^2}$

$\therefore \quad \cos^2\theta = \dfrac{61^2 - 11^2}{61^2}$

$\therefore \quad \cos^2\theta = \dfrac{(61+11)(61-11)}{61^2}$

$= \dfrac{72 \times 50}{61^2}$

$\cos^2\theta = \dfrac{3600}{61^2}$

$\therefore \quad \boxed{\mathbf{\cos\theta = \dfrac{60}{61}}}$ ... (Taking square roots)

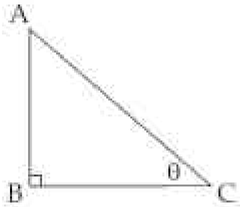
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{11}{61} \div \dfrac{60}{61}$

$= \dfrac{11}{61} \times \dfrac{61}{60}$

$$\boxed{\mathbf{\tan\theta = \dfrac{11}{60}}}$$

(iii) In the adjoining figure,



in ΔABC, $\angle B = 90^\circ$

$\angle C = \theta$

$\tan\theta = 1$ ... (i) (Given)

$\therefore \tan\theta = \dfrac{AB}{BC}$ ... (ii) (By definition)

$\therefore \dfrac{AB}{BC} = \dfrac{1}{1}$ [From (i) and (ii)]

Let the common multiple be k ($k \neq 0$)

$\therefore$ AB = k, BC = k

In ΔABC, $\angle B = 90^\circ$

$\therefore AC^2 = AB^2 + BC^2$ ... (Pythagoras theorem)

$\therefore AC^2 = k^2 + k^2$

$\therefore AC^2 = 2k^2$

$\therefore AC = \sqrt{2}\,k$ ... (Taking square roots)

$\sin\theta = \dfrac{AB}{AC} = \dfrac{k}{\sqrt{2}\,k} = \dfrac{1}{\sqrt{2}}$

$\cos\theta = \dfrac{BC}{AC} = \dfrac{k}{\sqrt{2}\,k} = \dfrac{1}{\sqrt{2}}$

(iv) $\mathbf{\sin\theta = \dfrac{1}{2}}$

$\sin^2\theta + \cos^2\theta = 1$ ... (Trigonometric identity)

$\therefore \left(\dfrac{1}{2}\right)^2 + \cos^2\theta = 1$

$\dfrac{1}{4} + \cos^2\theta = 1$

$\therefore \cos^2\theta = 1 - \dfrac{1}{4}$

$\therefore \cos^2\theta = \dfrac{4-1}{4} = \dfrac{3}{4}$

$\therefore \boxed{\cos\theta = \dfrac{\sqrt{3}}{2}}$ ... (Taking square roots)

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1}{2} \div \dfrac{\sqrt{3}}{2}$

$\therefore \tan\theta = \dfrac{1}{2} \times \dfrac{2}{\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\therefore \boxed{\tan\theta = \dfrac{1}{\sqrt{3}}}$

(v) $\mathbf{\cos\theta = \dfrac{1}{\sqrt{3}}}$

$\sin^2\theta + \cos^2\theta = 1$ ... (Trigonometric identity)

$\sin^2\theta + \left(\dfrac{1}{\sqrt{3}}\right)^2 = 1$

$\therefore \sin^2\theta + \dfrac{1}{3} = 1$

$\therefore \sin^2\theta = 1 - \dfrac{1}{3}$

$\therefore \sin^2\theta = \dfrac{3-1}{3}$

$\therefore \sin^2\theta = \dfrac{2}{3}$

$\therefore \boxed{\sin\theta = \dfrac{\sqrt{2}}{\sqrt{3}}}$ ... (Taking square roots)

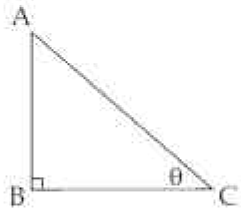
$\therefore \tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\tan\theta = \dfrac{\sqrt{2}}{\sqrt{3}} \div \dfrac{1}{\sqrt{3}}$

$= \dfrac{\sqrt{2}}{\sqrt{3}} \times \dfrac{\sqrt{3}}{1}$

$\therefore \boxed{\tan\theta = \dfrac{\sqrt{2}}{1}}$

(vi) In the adjoining figure,



in ΔABC,

$\angle B = 90^\circ$,

$\angle C = \theta$

$\tan\theta = \dfrac{21}{20}$ ... (i) (Given)

$\tan\theta = \dfrac{AB}{BC}$ ... (ii) (By definition)

$\therefore \dfrac{AB}{BC} = \dfrac{21}{20}$ ... [From (i) and (ii)]

Let the common multiple be k. ($k \neq 0$)

$\therefore$ AB = $21k$ and BC = $20k$.

In ΔABC, $\angle ABC = 90^\circ$

$\therefore AC^2 = AB^2 + BC^2$ ... (Pythagoras theorem)

$= (21k)^2 + (20k)^2$

$= 441k^2 + 400k^2$

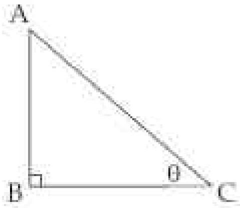
$AC^2 = 841k^2$

$\therefore AC = \sqrt{841k^2} = 29k$ ... (Taking square roots)

$$\sin\theta = \frac{AB}{AC} = \frac{21k}{29k} = \frac{21}{29}$$

$$\cos\theta = \frac{BC}{AC} = \frac{20k}{29k} = \frac{20}{29}$$

(vii) In the adjoining figure, in ΔABC, $\angle B = 90^\circ$, $\angle C = \theta$



$\tan\theta = \dfrac{8}{15}$ ... (i) (Given)

$\tan\theta = \dfrac{AB}{BC}$ ... (ii) (By definition)

$\therefore\ \dfrac{AB}{BC} = \dfrac{8}{15}$ ... [From (i) and (ii)]

Let the common multiple be k.

$\therefore\ AB = 8k$ and $BC = 15k$.

In ΔABC, $\angle B = 90^\circ$

$\therefore\ AC^2 = AB^2 + BC^2$ ... (Pythagoras theorem)

$\therefore\ AC^2 = (8k)^2 + (15k)^2$

$= 64k^2 + 225k^2$

$AC^2 = 289k^2$

$\therefore\ AC = \sqrt{289k^2} = 17k$ ... (Taking square roots)

$$\sin\theta = \frac{AB}{AC} = \frac{8k}{17k} = \frac{8}{17}$$

$$\cos\theta = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$$

(viii) $\sin\theta = \dfrac{3}{5}$

$\sin^2\theta + \cos^2\theta = 1$ ... (Trigonometric identity)

$\left(\dfrac{3}{5}\right)^2 + \cos^2\theta = 1$

$\dfrac{9}{25} + \cos^2\theta = 1$

$\therefore\ \cos^2\theta = 1 - \dfrac{9}{25}$

$\therefore\ \cos^2\theta = \dfrac{25-9}{25} = \dfrac{16}{25}$

$\therefore\ \boxed{\cos\theta = \dfrac{4}{5}}$ ... (Taking square roots)

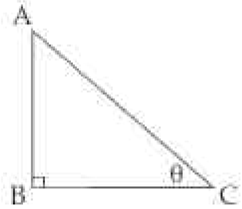
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\therefore\ \tan\theta = \dfrac{3}{5} \div \dfrac{4}{5}$

$\tan\theta = \dfrac{3}{5} \times \dfrac{5}{4}$

$\therefore\ \boxed{\tan\theta = \dfrac{3}{4}}$

(ix) In the adjoining figure, in ΔABC, $\angle B = 90^\circ$, $\angle C = \theta$



$\tan\theta = \dfrac{1}{2\sqrt{2}}$ ... (i) (Given)

$\tan\theta = \dfrac{AB}{BC}$ ... (ii) (By definition)

$\therefore\ \dfrac{AB}{BC} = \dfrac{1}{2\sqrt{2}}$ ... [From (i) and (ii)]

Let the common multiple be k. ($k \neq 1$)

$AB = k$; $BC = 2\sqrt{2}\,k$.

In ΔABC, $\angle B = 90^\circ$

$\therefore\ AC^2 = AB^2 + BC^2$ ... (By Pythagoras theorem)

$= k^2 + (2\sqrt{2}\,k)^2$

$= k^2 + 8k^2$

$AC^2 = 9k^2$

$\therefore\ AC = 3k$ ... (Taking square roots)

$$\sin\theta = \frac{AB}{AC} = \frac{k}{3k} = \frac{1}{3}$$

$$\cos\theta = \frac{BC}{AC} = \frac{2\sqrt{2}\,k}{3k} = \frac{2\sqrt{2}}{3}$$

(2) Find the values of:

(i) $5\sin 30^\circ + 3\tan 45^\circ$

Solution:

$5\sin 30^\circ + 3\tan 45^\circ$

$= 5 \times \left(\dfrac{1}{2}\right) + 3 \times (1)$

$= \dfrac{5}{2} + 3$

$= \dfrac{5+6}{2}$

$= \dfrac{11}{2}$

$\therefore\ \boxed{5\sin 30^\circ + 3\tan 45^\circ = \dfrac{11}{2}}$

(ii) $\frac{4}{5}\tan^2 60^\circ + 3\sin^2 60^\circ$

Solution:

$$\frac{4}{5}\tan^2 60^\circ + 3\sin^2 60^\circ$$
$$= \frac{4}{5}\times(\sqrt{3})^2 + 3\times\left(\frac{\sqrt{3}}{2}\right)^2$$
$$= \frac{4}{5}\times 3 + 3\times\frac{3}{4}$$
$$= \frac{12}{5}+\frac{9}{4} = \frac{48+45}{20} = \frac{93}{20}$$
$$\therefore \boxed{\frac{4}{5}\tan^2 60^\circ + 3\sin^2 60^\circ = \frac{93}{20}}$$

(iii) $2\sin 30^\circ + \cos 0^\circ + 3\sin 90^\circ$

Solution:

$$2\sin 30^\circ + \cos 0^\circ + 3\sin 90^\circ$$
$$= 2\times\left(\frac{1}{2}\right) + (1) + 3\times(1)$$
$$= 1+1+3$$
$$= 5$$
$$\therefore \boxed{2\sin 30^\circ + \cos 0^\circ + 3\sin 90^\circ = 5}$$

(iv) $\frac{\tan 60}{\sin 60 + \cos 60}$

Solution:

$$\frac{\tan 60}{\sin 60 + \cos 60}$$
$$= (\sqrt{3}) \div \left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right)$$
$$= (\sqrt{3}) \div \left(\frac{\sqrt{3}+1}{2}\right)$$
$$= \sqrt{3}\times\frac{2}{\sqrt{3}+1}$$
$$= \frac{2\sqrt{3}}{\sqrt{3}+1}$$
$$\therefore \boxed{\frac{\tan 60}{\sin 60 + \cos 60} = \frac{2\sqrt{3}}{\sqrt{3}+1}}$$

(v) $\cos^2 45^\circ + \sin^2 30^\circ$

Solution:

$$\cos^2 45^\circ + \sin^2 30^\circ$$
$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2$$
$$= \frac{1}{2}+\frac{1}{4}$$
$$= \frac{2+1}{4}$$
$$= \frac{3}{4}$$
$$\therefore \boxed{\cos^2 45^\circ + \sin^2 30^\circ = \frac{3}{4}}$$

(vi) $\cos 60^\circ \times \cos 30^\circ + \sin 60^\circ \times \sin 30^\circ$

Solution:

$$\cos 60^\circ \times \cos 30^\circ + \sin 60^\circ \times \sin 30^\circ$$
$$= \frac{1}{2}\times\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\times\frac{1}{2}$$
$$= \frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4}$$
$$= \frac{2\sqrt{3}}{4}$$
$$= \frac{\sqrt{3}}{2}$$
$$\therefore \boxed{\cos 60^\circ \times \cos 30^\circ + \sin 60^\circ \times \sin 30^\circ = \frac{\sqrt{3}}{2}}$$

(3) If $\sin\theta = \frac{4}{5}$ then find $\cos\theta$

Solution:

$$\sin^2\theta + \cos^2\theta = 1 \quad \text{... (Trigonometric identity)}$$
$$\therefore \left(\frac{4}{5}\right)^2 + \cos^2\theta = 1$$
$$\therefore \cos^2\theta = 1 - \frac{16}{25}$$
$$\therefore \cos^2\theta = \frac{25-16}{25}$$
$$\therefore \cos^2\theta = \frac{9}{25}$$
$$\therefore \boxed{\cos\theta = \frac{3}{5}} \quad \text{... (Taking square roots)}$$

(4) If $\cos\theta = \frac{15}{17}$ then find $\sin\theta$

Solution:

$$\sin^2\theta + \cos^2\theta = 1 \quad \text{... (Trigonometric identity)}$$
$$\sin^2\theta + \left(\frac{15}{17}\right)^2 = 1$$

$\therefore \quad \sin^2\theta + \frac{225}{289} = 1$

$\therefore \quad \sin^2\theta = 1 - \frac{225}{289}$

$\therefore \quad \sin^2\theta = \frac{289 - 225}{289}$

$\therefore \quad \sin^2\theta = \frac{64}{289}$

$\therefore \quad \boxed{\sin\theta = \frac{8}{17}}$ ... (Taking square roots)

PROBLEMS FOR PRACTICE

(1) **If $\cos\theta = \frac{\sqrt{3}}{2}$, find other trigonometric ratios of θ.**

(2) **Find the value of $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$**

(3) **If $\sin\theta = \frac{45}{53}$, find $\cos\theta$.**

(4) **Find the value of $\frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \cdot \tan 30^\circ}$.**

(5) **Find the value of:**
$2(\cos^2 45^\circ + \tan^2 60^\circ) - 6(\sin^2 45^\circ - \tan^2 30^\circ)$

ANSWERS

(1) $\sin\theta = \frac{1}{2}$; $\tan\theta = \frac{1}{\sqrt{3}}$

(2) 2 (3) $\frac{28}{53}$

(4) $\frac{1}{\sqrt{3}}$ (5) 6

PROBLEM SET - 8 *(Textbook Page No. 113)*

(1) **Choose the correct alternative answers for the following multiple choice questions.**

(i) Which of the following statement is true?
(A) $\sin\theta = \cos(90 - \theta)$ (B) $\cos\theta = \tan(90 - \theta)$
(C) $\sin\theta = \tan(90 - \theta)$ (D) $\tan\theta = \tan(90 - \theta)$

Ans. (A)

(ii) Which of the following is the value of $\sin 90^\circ$?

(A) $\frac{\sqrt{3}}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) 1

Ans. (D)

(iii) $2\tan 45^\circ + \cos 45^\circ - \sin 45^\circ = ?$

(A) 0 (B) 1 (C) 2 (D) 3

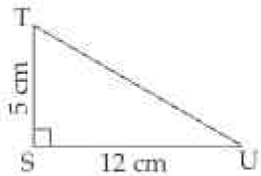
Ans. (C)

(iv) What is the value of $\frac{\cos 28^\circ}{\sin 62^\circ} = ?$

(A) 2 (B) -1 (C) 0 (D) 1

Ans. (D)

(2) **In right angled Δ TSU, TS = 5, $\angle$S = 90°, SU = 12 then find sin T, cos T, tan T. Similarly find sin U, cos U, tan U.**



Solution:

In Δ UST, $\angle S = 90^\circ$

$\therefore \quad US^2 + ST^2 = UT^2$ ... (Pythagoras theorem)

$\therefore \quad 12^2 + 5^2 = UT^2$

$\therefore \quad 144 + 25 = 169 = UT^2$

$\therefore \quad UT = 13$ ... (Taking square roots)

$\sin T = \frac{US}{UT} = \frac{12}{13}$

$\cos T = \frac{ST}{UT} = \frac{5}{13}$

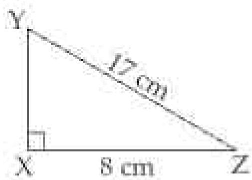
$\tan T = \frac{US}{ST} = \frac{12}{5}$

$\sin U = \frac{ST}{UT} = \frac{5}{13}$

$\cos U = \frac{US}{UT} = \frac{12}{13}$

$\tan U = \frac{ST}{US} = \frac{5}{12}$

(3) **In right angled Δ YXZ, $\angle$X = 90°, XZ = 8 cm, YZ = 17 cm, find sin Y, cos Y, tan Y, sin Z, cos Z, tan Z.**



Solution:

In Δ YXZ, $\angle X = 90^\circ$

XZ = 8 cm; YZ = 17 cm

$\therefore \quad YZ^2 = XY^2 + XZ^2$ ... (Pythagoras theorem)

$\therefore \quad 17^2 = XY^2 + 8^2$

$\therefore \quad 289 - 64 = XY^2$

$\therefore \quad XY^2 = 225$

$\therefore \quad XY = 15$ cm ... (Taking square roots)

$\sin Y = \frac{XZ}{YZ} = \frac{8}{17}$

$\cos Y = \frac{XY}{YZ} = \frac{15}{17}$

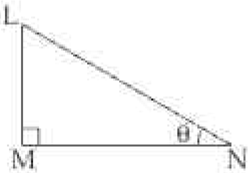
$\tan Y = \frac{XZ}{XY} = \frac{8}{15}$

$\sin Z = \frac{XY}{YZ} = \frac{15}{17}$

$\cos Z = \frac{XZ}{YZ} = \frac{8}{17}$

$\tan Z = \frac{XY}{XZ} = \frac{15}{8}$

(4) In the adjoining figure, in Δ LMN, $\angle N = \theta$, $\angle M = 90°$ $\cos\theta = \frac{24}{25}$. Find remaining trigonometric ratios of θ.



Also, find $(\sin\theta)^2 + (\cos\theta)^2$.

Solution:

In Δ LMN, $\angle M = 90°$

$\therefore \quad \cos\angle N = \frac{MN}{LN}$ (By definition)

i.e. $\cos\theta = \frac{MN}{LN}$ $[\because \angle N = \theta]$ ... (i)

But $\cos\theta = \frac{24}{25}$ (Given) ... (ii)

$\therefore \quad \frac{MN}{LN} = \frac{24}{25}$ ... [From (i) and (ii)]

Let the common multiple be k, $(k \neq 0)$

$\therefore \quad MN = 24k, LN = 25k$

In Δ LMN, $\angle M = 90°$ ... (Given)

$\therefore \quad LN^2 = LM^2 + MN^2$ [Pythagoras theorem]

$\therefore \quad (25k)^2 = LM^2 + (24k)^2$

$\therefore \quad 576k^2 = LM^2 + 625k^2$

$\therefore \quad 625k^2 - 576k^2 = LM^2$

$\therefore \quad LM^2 = 49k^2$

$\therefore \quad LM = 7k$ ... (Taking square roots)

$\sin\theta = \frac{LM}{LN} = \frac{7k}{25k} = \frac{7}{25}$

$\tan\theta = \frac{LM}{MN} = \frac{7k}{24k} = \frac{7}{24}$

$$\sin^2\theta + \cos^2\theta = \left(\frac{7}{25}\right)^2 + \left(\frac{24}{25}\right)^2$$

$$= \frac{49}{625} + \frac{576}{625}$$

$$= \frac{49+576}{625} = \frac{625}{625} = 1$$

$\therefore \quad \boxed{\sin^2\theta + \cos^2\theta = 1}$

(5) Fill in the blanks:

(i) $\sin 20° = \cos \square°$

(ii) $\tan 30° \times \tan \square° = 1$

(iii) $\cos 40° = \sin \square°$

Ans. (i) 70° (ii) 60° (iii) 50°

MCQ's

(1) Trigonometry is applicable only in a ....

(A) Equilateral triangle

(B) Right angled triangle

(C) Obtuse angled triangle

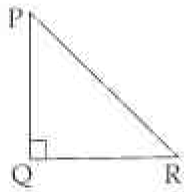
(D) Acute angled triangle

(2) For any acute angle of a right angled triangle there are .... trigonometric ratios.

(A) one (B) three (C) four (D) six

(3) In the given figure,

cos P = ...



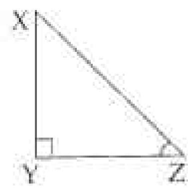
(A) $\frac{PQ}{QR}$ (B) $\frac{PQ}{PR}$ (C) $\frac{PR}{PQ}$ (D) $\frac{QR}{PR}$

(4) $\tan\theta$ = ...

(A) $\frac{\text{Adjacent side}}{\text{Opposite side}}$ (B) $\frac{\text{Adjacent side}}{\text{Hypotenuse}}$

(C) $\frac{\text{Hypotenuse}}{\text{Opposite side}}$ (D) $\frac{\text{Opposite side}}{\text{Adjacent side}}$

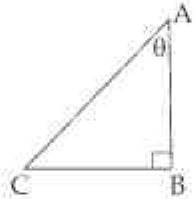
(5) In Δ XYZ, $m\angle Y = 90°$ then for $\angle Z$, what is the trigonometric ratio for $\frac{XY}{XZ}$?



(A) cos Z (B) tan Z

(C) sin Z (D) None of above

(6)



In Δ ABC, $\angle B = 90°$, $\angle A = \theta$. $\angle C$ = ...

(A) $(90 + \theta)$ (B) θ

(C) $(180 - \theta)$ (D) $(90 - \theta)$

(7) If $\sin\theta = \frac{\sqrt{3}}{2}$, then $\text{cosec}\,\theta =$ ....

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{2}{\sqrt{3}}$ (C) $\frac{2\sqrt{3}}{3}$ (D) $\frac{2\sqrt{3}}{4}$

(8) If $\tan\theta = \frac{44}{21}$ and $\cos\theta = \frac{61}{66}$, then $\sin\theta =$ ....

(A) $\frac{3}{2}$ (B) $\frac{11}{4}$ (C) $\frac{2}{3}$ (D) $\frac{3}{11}$

(9) Find the value of $2\sin 60^\circ . \cos 30^\circ$

(A) $\frac{5}{2}$ (B) $\frac{3}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{4}$

(10) Find the value of $2\sin^2 45^\circ + \cos^2 45^\circ$.

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{3}{\sqrt{2}}$

(11) Find the value of $2\tan^2 45^\circ + \cos^2 30^\circ + \sin^2 60^\circ$

(A) $\frac{9}{2}$ (B) $\frac{9}{4}$ (C) $\frac{11}{4}$ (D) $\frac{7}{2}$

(12) cos .... $= \frac{1}{2}$.

(A) 0° (B) 45° (C) 90° (D) 60°

(13) If $\tan\theta = \sqrt{3}$, then $\theta =$ ....

(A) 60° (B) 30° (C) 45° (D) 90°

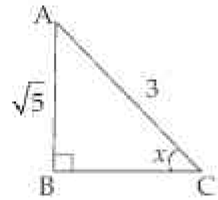
(14) $\tan 52^\circ =$ ....

(A) $\tan 38^\circ$ (B) $\text{cosec}\, 38^\circ$ (C) $\cot 38^\circ$ (D) $\sec 38^\circ$

(15) $\sin 21^\circ =$ ....

(A) $\cos 21^\circ$ (B) $\sin 69^\circ$ (C) $\cos 69^\circ$ (D) $\tan 69^\circ$

(16) In the adjoining figure, find $\sin x$



(A) $\frac{\sqrt{5}}{2}$ (B) $\frac{\sqrt{5}}{3}$ (C) $\frac{2}{3}$ (D) $\frac{\sqrt{13}}{2}$

(17) If $\sin\theta = \cos\theta$, then $\tan\theta =$ ....

(A) 2 (B) 3 (C) 1 (D) 0

(18) What is the value of $\sin^2\theta + \cos^2\theta$?

(A) $\tan^2\theta$ (B) 0 (C) 1 (D) 2

(19) If $\sqrt{3}\tan x - 1 = 0$. What is the value of x?

(A) 30° (B) 60° (C) 90° (D) 45°

(20) What is the value of cosA, if $A = 30^\circ$?

(A) 0 (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{2}$

(D) Not defined

ANSWERS

(1)	(B)	(2)	(D)	(3)	(B)	(4)	(D)
(5)	(C)	(6)	(D)	(7)	(B)	(8)	(C)
(9)	(B)	(10)	(B)	(11)	(D)	(12)	(D)
(13)	(A)	(14)	(C)	(15)	(C)	(16)	(B)
(17)	(C)	(18)	(C)	(19)	(A)	(20)	(B)

ASSIGNMENT - 8

Time : 1 Hr. **Marks : 20**

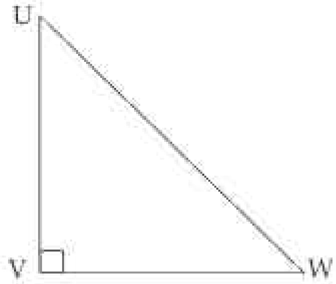
Q.1. Attempt the following: **(2)**

(1) If $\sin\theta = \frac{\sqrt{2}}{3}$, $\cos\theta = \frac{1}{2}$ then find $\tan\theta$.

(2) Evaluate: $\sin 30^\circ + \cos 60^\circ$

Q.2. Attempt the following: **(4)**

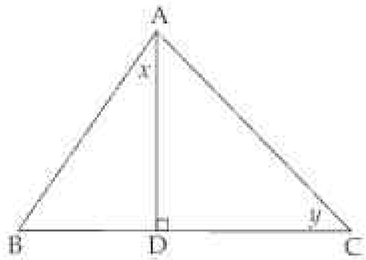
(1) In the adjoining figure, ΔUVW is a right angled triangle and $\angle UVW = 90^\circ$, UV = 6 cm, VW = 8 cm. Find the sinU and tanW.



(2) Find the value of the following: $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

Q.3. Attempt the following: **(6)**

(1) In the adjoining figure, Find the following ratios:
$\tan x$, $\cos(90 - y)$, $\sin y$, $\cos(90 - x)$, $\tan(90 - x)$ and $\sin x$



(2) If $\angle A = 30^\circ$, then show that $\sin A = \sqrt{\frac{1-\cos 2A}{2}}$

Q.4. Attempt the following: **(8)**

(1) If $\angle A = 30^\circ$, then show that $\cos 3A = 4\cos^3 A - 3\cos A$

(2) If $\cos\theta = \frac{\sqrt{3}}{2}$, Find $\sin\theta$ and $\tan\theta$ and also prove that $\sin^2\theta + \cos^2\theta = 1$

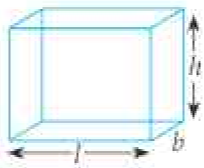
❖ ❖ ❖

9 Surface Area and Volume

Points to Remember:

❑ **Introduction:**

(I) Cuboid:



In the adjoining cuboid, length, breadth and height are l, b, and h respectively.

(1) Perimeter of base = $2(l + b)$.

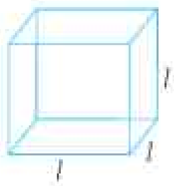
(2) Area of base = $l \times b$.

(3) Vertical surface area = $2(l + b) \times h$.

(4) Total surface area = $2(lb + bh + lh)$.

(5) Volume = $l \times b \times h$.

(II) Cube:



In the adjoining cube, each side has length l.

(1) Perimeter of base = $4l$

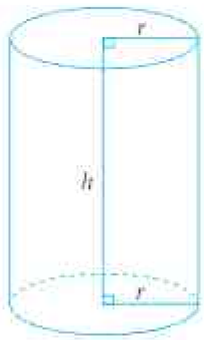
(2) Area of base = l^2

(3) Vertical surface area = $4l^2$

(4) Total surface area = $6l^2$

(5) Volume = l^3

(III) Cylinder: (Right Circular Cylinder)



In the adjoining right circular cylinder,
radius = r and height = h.

(1) Circumference of base = $2\pi r$.

(2) Area of base = πr^2

(3) Curved surface area = $2\pi rh$.

(4) Total surface area = $2\pi r(r + h)$.

(5) Volume = $\pi r^2 h$

MASTER KEY QUESTION SET - 9

PRACTICE SET - 9.1 *(Textbook Page No. 115)*

(1) Length, breadth and height of a cuboid shape box of medicine is 20cm, 12 cm and 10 cm respectively. Find the surface area of vertical faces and total surface area of this box.

Solution:

Vertical surface area of the box = $2(l + b) \times h$

$= 2(20 + 12) \times 10$

$= 2 \times 32 \times 10$

$= 640 \text{ cm}^2$

Total surface area of the box = $2(lb + bh + lh)$

$= 2(20 \times 12 + 12 \times 10 + 20 \times 10)$

$= 2(240 + 120 + 200)$

$= 2 \times 560$

$= 1120 \text{ cm}^2$

(1) Vertical surface area of box is 640 cm^2.
(2) Total surface area of box is 1120 cm^2.

(2) Total surface area of a box of cuboid shape is 500 sq. unit. Its breadth and height is 6 unit and 5 unit respectively. What is the length of that box?

Solution:

Total surface area = $2(lb + bh + lh)$

$\therefore \quad 500 = 2(l \times 6 + 6 \times 5 + l \times 5)$

$\therefore \quad \dfrac{500}{2} = 6l + 30 + 5l$

$\therefore \quad 250 = 11l + 30$

$\therefore \quad 250 - 30 = 11l$

$\therefore \quad 220 = 11l$

$\therefore \quad \dfrac{220}{11} = l$

$\therefore \quad l = 20$ unit

Length of the box is 20 units.

(3) Side of a cube is 4.5 cm. Find the surface area of all vertical faces and total surface area of the cube.

Solution:

Vertical surface area of the cube = $4l^2$

$= 4 \times 4.5 \times 4.5$

$= 81 \text{cm}^2$

Total surface area of the cube = $6l^2$

$= 6 \times 4.5 \times 4.5$

$= 121.5 \text{ cm}^2$

(1) Vertical surface area of cube is 81 cm^2.
(2) Total surface area of cube is 121.5 cm^2.

(4) Total surface area of a cube is 5400 sq. cm. Find the surface area of all vertical faces of the cube.

Solution:

Total surface area of the cube = $6l^2$

$\therefore \quad 5400 = 6 \times l^2$

$\therefore \quad \dfrac{5400}{6} = l^2$

$\therefore \quad l^2 = 900 \text{ cm}^2$

Vertical surface area of cube = $4l^2$.

$= 4 \times 900$

$= 3600 \text{ cm}^2$

Vertical surface area of cube is 3600 cm^2.

(5) Volume of a cuboid is 34.50 cubic metre. Breadth and height of the cuboid is 1.5m and 1.15m respectively. Find its length.

Solution:

Volume of a cuboid = $l \times b \times h$

$\therefore \quad 34.5 = l \times 1.5 \times 1.15$

$\therefore \quad \dfrac{345}{10} = l \times \dfrac{15}{10} \times \dfrac{115}{100}$

$\therefore \quad \dfrac{345 \times 10 \times 100}{10 \times 15 \times 115} = l$

$\therefore \quad l = 20\ m$

Length of the cuboid is 20 m

(6) What will be the volume of a cube having length of edge 7.5 cm?

Solution:

Volume of a cube $= l^3$

$= 7.5 \times 7.5 \times 7.5$

$= 421.875 \text{ cm}^3$

Volume of the cube is 421.875 cm^3

(7) Radius of base of a cylinder is 20 cm and its height is 13 cm, find its curved surface area and total surface area. (π = 3.14)

Solution:

Curved surface area of a cylinder = $2\pi rh$

$= 2 \times 3.14 \times 20 \times 13$

$= 1632.8 \text{ cm}^2$

Total surface area of a cylinder = $2\pi r(r + h)$

$= 2 \times 3.14 \times 20 \times (20 + 13)$

$= 2 \times 3.14 \times 20 \times 33$

$= 4144.8 \text{ cm}^2$

(1) Curved surface area is 1632.8 cm^2
(2) Total surface area is 4144.8 cm^2

(8) Curved surface area of a cylinder is 1980 cm² and radius of its base is 15 cm. Find the height of the cylinder $\left(\pi = \dfrac{22}{7}\right)$.

Solution:

Curved surface area of a cylinder = $2\pi rh$

$\therefore \quad 1980 = 2 \times \dfrac{22}{7} \times 15 \times h$

$\therefore \quad \dfrac{1980 \times 7}{2 \times 22 \times 15} = h$

$\therefore \quad h = \dfrac{90 \times 7}{30}$

$\therefore \quad h = 21 \text{ cm}$

Height of the cylinder is 21 cm

PROBLEMS FOR PRACTICE

(1) The dimensions of a cuboid in cm are $16 \times 14 \times 20$. Find its total surface area.

(2) The cuboid water tank has length 2 m, breadth 1.6 m and height 1.8 m. Find the capacity of the tank in litres.

(3) A fish tank is in the form of a cuboid, external measures of that cuboid are 80 cm × 40 cm × 30 cm. The base, side faces and back face are to be covered with a coloured paper. Find the area of the paper needed.

(4) The length, breadth, and height of a cuboid are in the 5 : 4 : 2. If the total surface area is 1216 cm^2, find the dimension of the solid.

(5) The volume of a cube is 1000 cm^3. Find the total surface area.

(6) The radius and height of a cylinder are in same ratio 3 : 7 and its volume is 1584 cm^3. Find its radius.

(7) A cylindrical hole of diameter 30 cm is bored through a cuboid wooden block with side 1 metre. Find the volume of the object so formed. (π = 3.14)

(8) **A building has 8 cylindrical pillars whose cross sectional diameter is 1 m and whose height is 4.2 m. Find the expenditure to paint these pillars at the rate of ₹ 24 per sq.m.**

ANSWER

(1)	1648 cm^2	(5)	5760 litres
(3)	8000 cm^2	(4)	20 cm, 16 cm, 8 cm
(5)	600 cm^2	(6)	6 cm
(7)	929350 cm^3	(8)	₹ 2534.40

Points to Remember:

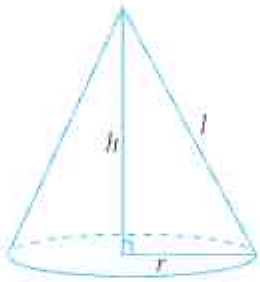
(IV) Right Circular Cone:

In the adjoining figure, for a cone,

Radius = r,

Perpendicular height = h,

Slant height = l.



(1) $l^2 = r^2 + h^2$

(2) Circumference of base = $2\pi r$

(3) Area of base = πr^2

(4) Curved surface area = πrl

(5) Total surface area = $\pi r (r + l)$

(6) Volume = $\frac{1}{3} \pi r^2 h$

PRACTICE SET - 9.2 *(Textbook Page No. 119)*

(1) **Perpendicular height of cone is 12 cm and its slant height is 13 cm. Find the radius of the base of the cone.**

Solution:

$l^2 = r^2 + h^2$

$\therefore$ $13^2 = r^2 + 12^2$

$\therefore$ $169 = r^2 + 144$

$\therefore$ $169 - 144 = r^2$

$\therefore$ $r^2 = 25$

$\therefore$ $r = 5$ cm

Radius of the cone is 5 cm

(2) **Find the volume of a cone, if its total surface area is 7128 sq.cm. and radius of base is 28 cm.** $\left(\pi = \frac{22}{7}\right)$

Solution:

Total surface area of a cone = $\pi r (r + l)$

$\therefore$ $7128 = \frac{22}{7} \times 28 \times (28 + l)$

$\therefore$ $\frac{7128 \times 7}{22 \times 28} = 28 + l$

$\therefore$ $\frac{324}{4} = 28 + l$

$\therefore$ $81 = 28 + l$

$\therefore$ $l = 81 - 28$

$\therefore$ $l = 53$ cm

$l^2 = r^2 + h^2$

$\therefore$ $53^2 = 28^2 + h^2$

$\therefore$ $53^2 - 28^2 = h^2$

$\therefore$ $h^2 = (53 + 28)(53 - 28)$

$\therefore$ $h^2 = 81 \times 25$

$\therefore$ $h = 9 \times 5$ (Taking square roots)

$\therefore$ $h = 45$ cm

Volume = $\frac{1}{3} \pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 28 \times 28 \times 45$

V $= 36960$ cm^3

Volume of the cone is 36960 cm^3

(3) **Curved surface area of a cone is 251.2 cm^2 and radius of its base is 8 cm. Find its slant height and perpendicular height. ($\pi = 3.14$)**

Solution:

Curved surface area of a cone = $\pi \times r \times l$

$\therefore$ $251.2 = 3.14 \times 8 \times l$

$\therefore$ $\frac{2512}{10} = \frac{314}{100} \times 8 \times l$

$\therefore$ $\frac{2512 \times 100}{10 \times 314 \times 8} = l$

$\therefore$ $\frac{314 \times 10}{314} = l$

$\therefore$ $l = 10$ cm

$l^2 = r^2 + h^2$

$\therefore$ $10^2 = 8^2 + h^2$

$\therefore$ $100 - 64 = h^2$

$\therefore \quad h^2 = 36$

$\therefore \quad h = 6$ cm

(1) Slant height of the cone is 10 cm
(2) Perpendicular height of the cone is 6 cm

(4) What will be the cost of making a closed cone of tin sheet having radius of base 6 m and slant height 8 m if the rate of making is ₹ 10 per sq.m?

Solution:

Total surface area of a cone = $\pi r (r + l)$

$$= \frac{22}{7} \times 6 \times (6 + 8)$$

$$= \frac{22}{7} \times 6 \times 14$$

$= 264$ sq.m

Cost = Area × Rate

= 264 × 10 = ₹2640

Cost of making closed cone is ₹ 2640

(5) Volume of a cone is 6280 cubic cm and base radius of the cone is 30 cm. Find its perpendicular height. ($\pi = 3.14$)

Solution:

Volume of a cone = $\frac{1}{3} \pi r^2 h$

$$\therefore \quad 6280 = \frac{1}{3} \times 3.14 \times 30 \times 30 \times h$$

$$\therefore \quad 6280 = \frac{1}{3} \times \frac{314}{100} \times 30 \times 30 \times h$$

$$\therefore \quad \frac{6280 \times 3 \times 100}{314 \times 30 \times 30} = h$$

$$\therefore \quad h = \frac{20}{3}$$

$\therefore \quad h = 6.67$ cm

Perpendicular height of a cone is 6.67 cm

(6) Surface area of a cone is 188.4 sq.cm and its slant height is 10 cm. Find its perpendicular height. ($\pi = 3.14$)

Solution:

Surface area of a cone = $\pi \times r \times l$

$\therefore \quad 188.4 = 3.14 \times r \times 10$

$$\therefore \quad \frac{1884}{10} = \frac{314}{100} \times r \times 10$$

$$\therefore \quad \frac{1884 \times 100}{314 \times 10 \times 10} = r$$

$\therefore \quad r = 6$ cm

$l^2 = r^2 + h^2$

$\therefore \quad 10^2 = 6^2 + h^2$

$\therefore \quad h^2 = 100 - 36$

$\therefore \quad h^2 = 64$

$\therefore \quad h = 8$ cm

Height of the cone is 8 cm

(7) Volume of a cone is 1232 cm³ and its height is 24 cm. Find the surface area of the cone. $\left(\pi = \frac{22}{7}\right)$

Solution:

Volume of a cone = $\frac{1}{3} \pi r^2 h$

$$\therefore \quad 1232 = \frac{1}{3} \times \frac{22}{7} \times r^2 \times 24$$

$$\therefore \quad \frac{1232 \times 7 \times 3}{22 \times 24} = r^2$$

$$\therefore \quad r^2 = \frac{154 \times 7}{22} = 7 \times 7$$

$\therefore \quad r^2 = 49$

$\therefore \quad r = 7$ cm

$l^2 = r^2 + h^2$

$= 7^2 + 24^2$

$= 49 + 576$

$\therefore \quad l^2 = 625$

$\therefore \quad l = 25$ cm

Curved surface area of a cone = $\pi r l$

$$= \frac{22}{7} \times 7 \times 25$$

$= 550$ sq. cm

Curved surface area of cone is 550 sq. cm

(8) The curved surface area of a cone is 2200 sq. cm and its slant height is 50 cm. Find the total surface area of cone. $\left(\pi = \frac{22}{7}\right)$

Solution:

Curved surface area of a cone = $\pi r l$

$$\therefore \quad 2200 = \frac{22}{7} \times r \times 50$$

$$\therefore \quad \frac{2200 \times 7}{50 \times 22} = r$$

$\therefore \quad r = 14$ cm

Total surface area of a cone = $\pi r (r + l)$

$$= \frac{22}{7} \times 14 \times (14 + 50)$$

$= 44 \times 64$

$= 2816$ sq. cm

$l^2 = r^2 + h^2$

$\therefore \quad 50^2 = 14^2 + h^2$

$\therefore \quad 50^2 - 14^2 = h^2$

$\therefore \quad h^2 = (50 + 14)(50 - 14)$

$\therefore \quad h^2 = 64 \times 36$

$\therefore \quad h = 8 \times 6$ (Taking square roots)

$\therefore \quad h = 48$ cm

Volume of a cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times 48$

V $= 9856$ cubic cm.

(1) Total surface area of cone is 2816 sq.cm. (2) Volume of cone is 9856 cubic cm

(9) There are 25 persons in a tent which is conical in shape. Every person needs an area of 4 sq.m. of the ground inside the tent. If height of the tent is 18 m, find the volume of the tent.

Solution:

Area of base of tent = Area occupied by a person × Number of persons

$= 4 \times 25$

Area of base of tent = 100 sq. m.

Volume of tent $= \frac{1}{3} \times$ Area of base of tent × Height of tent

$= \frac{1}{3} \times 100 \times 18$

$= 600$ cubic m.

Volume of the tent is 600 cubic metre

(10) In a field, dry fodder for the cattle is heaped in a conical shape. The height of the cone is 2.1m and diameter of base is 7.2 m. Find the volume of the fodder if it is to be covered by polythene in rainy season then how much minimum polythene sheet is needed? $\left(\pi = \frac{22}{7} \text{ and } \sqrt{17.37} = 4.17\right)$

Solution:

Diameter of conical shape = 7.2 m

Radius of conical shape $(r) = \frac{7.2}{2} = 3.6$ m

Perpendicular height $(h) = 2.1$ m.

Volume of fodder in Conical shape =

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 3.6 \times 3.6 \times 2.1$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{36}{10} \times \frac{36}{10} \times \frac{21}{10}$

$= \frac{28512}{1000}$

$= 28.512$ cubic m.

$l^2 = r^2 + h^2$

$= 3.6^2 + 2.1^2$

$= 12.96 + 4.41$

$l^2 = 17.37$

$l = 4.17$ m (Taking square roots)

Area of polythene required $= \pi r l$

$= \frac{22}{7} \times 3.6 \times 4.17$

$= 47.18\ m^2$

Area of polythene needed is $47.18 m^2$

PROBLEMS FOR PRACTICE

(1) A cone of height 24 m has a plane base of surface area 154 cm^2. Find its volume.

(2) Curved surface area of a cone with base radius 40 cm is 1640 π sq. cm. Find the height of the cone.

(3) The total surface area of a cone is 71.28 cm^2. Find the height of this cone if the diameter of the base is 5.3 cm.

(4) The volume of a cone of height 5 cm is 753.6 cm^3. This cone and a cylinder have equal radii and height. Find the total surface area of cylinder. (π = 3.14)

(5) The diameter of the base of metallic cone is 2 cm and height is 10 cm. 900 such cones are melted to form 1 right circular cylinder whose radius is 10 cm. Find height of the right circular cylinder so formed.

ANSWER

(1) 1232 cm^3 (2) 9 cm

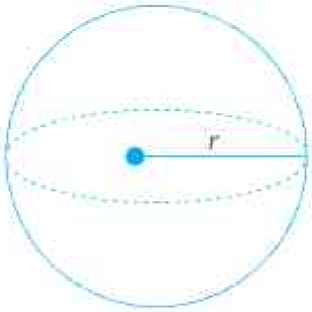
(3) 4.5 cm (4) 1281.12 cm^2

(5) 30 cm

Points to Remember:

(V) Sphere:

In adjoining figure, for the given sphere radius is r.



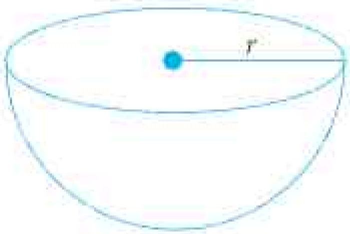
(1) Surface area of sphere = $4 \pi r^2$

(2) Volume of sphere = $\frac{4}{3} \pi r^3$

Note: Sphere has only one surface, so surface area of sphere is often called as curved surface area or total surface area.

(VI) Hemisphere:

In the adjoining figure for the given hemisphere radius is r.



(1) Circumference of flat surface = $2 \pi r$

(2) Area of flat surface = πr^2

(3) Curved surface area = $2 \pi r^2$

(4) Total surface area = $3 \pi r^2$

(5) Volume of hemisphere = $\frac{2}{3} \pi r^3$

PRACTICE SET - 9.3 *(Textbook Page No. 123)*

(1) Find the surface areas and volumes of sphere of the following radii.

(i) 4 cm (ii) 9 cm (ii) 3.5 cm ($\pi = 3.14$)

Solution:

(i) Surface area of a sphere $= 4 \pi r^2$

$= 4 \times 3.14 \times 4 \times 4$

$= 200.96 \text{ cm}^2$

Volume of a sphere $= \frac{4}{3} \pi r^3$

$= \frac{4}{3} \times 3.14 \times 4 \times 4 \times 4$

$V = 267.95 \text{ cm}^3$

(1) Curved surface area of sphere = 200.96 cm^2
(2) Volume of sphere = 267.95 cm^3

(ii) Surface area of a sphere $= 4 \pi r^2$

$= 4 \times 3.14 \times 9 \times 9$

$= 1017.36 \text{ cm}^2$

Volume of a sphere $= \frac{4}{3} \pi r^3$

$= \frac{4}{3} \times 3.14 \times 9 \times 9 \times 9$

$V = 3052.08 \text{ cm}^3$

(1) Curved surface area of sphere = 1017.36 cm^2
(2) Volume of sphere = 3052.08 cm^3

(iii) Surface area of a sphere $= 4 \pi r^2$

$= 4 \times 3.14 \times 3.5 \times 3.5$

$= 153.86 \text{ cm}^2$

Volume of a sphere $= \frac{4}{3} \pi r^3$

$= \frac{4}{3} \times 3.14 \times 3.5 \times 3.5 \times 3.5$

$V = 179.50 \text{ cm}^3$

(1) Curved surface area of sphere = 153.86 cm^2
(2) Volume of sphere is = 179.50 cm^3

(2) If the radius of a solid hemisphere is 5 cm, then find its curved surface area and total surface area. ($\pi = 3.14$)

Solution:

Curved surface area of Hemisphere $= 2 \pi r^2$

$= 2 \times 3.14 \times 5 \times 5$

$= 157 \text{ cm}^2$

Total surface area of Hemisphere $= 3 \pi r^3$

$= 3 \times 3.14 \times 5 \times 5$

$= 235.5 \text{ cm}^2$

(1) Curved surface area of hemisphere is 157 cm^2
(2) Total surface area of hemisphere is 235.5 cm^3

(3) If the surface area of a sphere is 2826 cm^2 then, find its volume. ($\pi = 3.14$)

Solution:

Surface area of sphere $= 4 \pi r^2$

$\therefore \quad 2826 = 4 \times 3.14 \times r^2$

$\therefore \quad 2826 = 4 \times \frac{314}{100} \times r^2$

$\therefore \quad \frac{2826 \times 100}{4 \times 314} = r^2$

$\therefore \quad r^2 = 9 \times 25$

$r = 3 \times 5$

$r = 15$

Volume of a sphere $= \frac{4}{3} \pi r^3$

$= \frac{4}{3} \times 3.14 \times 15 \times 15 \times 15$

$= \frac{4}{3} \times \frac{314}{100} \times 15 \times 15 \times 15$

$= 14130 \text{ cm}^3$

Volume of the sphere is 14130 cm^3

(4) Find the surface area of a sphere, if its volume is 38808 cubic cm $\left(\pi = \frac{22}{7}\right)$

Solution:

Volume of a sphere $= \frac{4}{3} \pi r^3$

$\therefore \quad 38808 = \frac{4}{3} \times \frac{22}{7} \times r^3$

$\therefore \quad \frac{38808 \times 3 \times 7}{4 \times 22} = r^3$

$\therefore \quad \frac{4{,}851 \times 21}{11} = r^3$

$\therefore \quad 441 \times 21 = r^3$

$\therefore \quad r^3 = 21 \times 21 \times 21$

$\therefore \quad r = 21 \text{ cm}$

Surface area of a sphere $= 4 \pi r^2$

$= 4 \times \frac{22}{7} \times 21 \times 21$

$= 5544 \text{ cm}^2$

Surface area of sphere is 5544 cm^2

(5) Volume of a hemisphere is 18000 π cubic cm. Find its diameter.

Solution:

Volume of a hemisphere $= \frac{2}{3} \pi r^3$

$\therefore \quad 18000\pi = \frac{2}{3} \pi r^3$

$\therefore \quad \frac{18000 \times \pi \times 3}{2 \times \pi} = r^3$

$\therefore \quad 3 \times 3 \times 10 \times 10 \times 10 \times 3 = r^3$

$\therefore \quad r = 30 \text{ cm}$

$d = 2r$

$\therefore \quad d = 2 \times 30$

$\therefore \quad d = 60 \text{ cm}$

Diameter of hemisphere is 60 cm

PROBLEMS FOR PRACTICE

(1) Find the volume and surface area of a sphere of radius 4.2 cm.

(2) The volumes of two spheres are in the ratio 27 : 64. Find their radii if the sum of their radii is 28 cm.

(3) The surface area of a sphere is 616 cm^2. What is its volume?

(4) The curved surface area of a hemisphere is $905\frac{1}{7}$ cm^2, what is its radius?

(5) If the radius of a sphere is doubled, what will be the ratio of its surface area to that of the first sphere.

ANSWER

(1) 310.46 cm^3; 221.76 cm^2 (2) 12 m, 16 m

(3) 1437.33 cm^3 (4) 12 cm

(5) 4 : 1

PROBLEM SET - 9 *(Textbook Page No. 123)*

(1) If diameter of a road roller is 0.9 m and its length is 1.4 m, how much area of a field will be pressed in its 500 revolutions. $\left(\pi = \frac{22}{7}\right)$

Solution:

Diameter of roller $= 0.9$ m

$\therefore$ Radius of roller (r) $= \frac{0.9}{2} = 0.45 \text{ m} = \frac{45}{100}$ m

Length of roller (h) $= 1.4 \text{ m} = \frac{14}{10}$ m

Area pressed by roller in 1 revolution

= curved surface area of roller.

$\therefore$ Area pressed by roller in 500 revolution

= 500 × curved surface area of roller

$= 500 \times 2 \pi rh$

$= 500 \times 2 \times \frac{22}{7} \times \frac{45}{100} \times \frac{14}{10}$

$= 1000 \times \frac{22}{7} \times \frac{45 \times 14}{1000}$

= 1980 sq. m.

Area pressed by the roller in 500 revolutions is 1980 sq. m.

(2) To make an open fish tank, a glass sheet of 2 mm gauge is used. The outer length, breadth and height of the tank are 60.4 cm, 40.4 cm and 40.2 cm respectively. How much maximum volume water will be contained in it?

Solution:

External length of fish tank = 60.4 cm

External breadth of fish tank = 40.4 cm

External height of fish tank = 40.2 cm

Thickness of glass = 2 mm = 0.2 cm

Inner length of fish tank (l) = 60.4 – 0.4

= 60 cm

Inner breadth of fish tank (b) = 40.4 – 04

= 40 cm

Inner height if fish tank (h) = 40.2 – 0.2

= 40cm

Inner volume of fish tank = $l \times b \times h$

= $60 \times 40 \times 40$

= 96,000 cm^3

Maximum quantity of water

fish tank can hold = $\frac{96000}{1000}\ l$ $\boxed{\because l = 1000\ cm^3}$

= 96 l

Maximum quantity of water fish tank can hold is 96 l.

(3) If the ratio of radius of base and height of a cone is 5 : 12 and its volume is 314 cubic metre. Find its perpendicular height and slant height (π = 3.14)

Solution:

Let the radius, perpendicular height and slant height be r, h and l respectively $r : h = 5 : 12$

Let the common multiple be x.

$\therefore r = 5x$; $h = 12x$.

Volume of cone $= \frac{1}{3}\pi r^2 h$.

$\therefore \quad 314 = \frac{1}{3} \times 3.14 \times 5x \times 5x \times 12x$

$314 = \frac{1}{3} \times \frac{314}{100} \times (5x)(5x)(12x)$

$\therefore \quad \frac{314 \times 3 \times 100}{314 \times 5 \times 5 \times 12} = x^3$

$\therefore \quad x^3 = \frac{100}{100}$

$\therefore \quad x^3 = 1$

$\therefore \quad x = 1$

$\therefore \quad r = 5x = 5 \times 1 = 5$ m

$h = 12x = 12 \times 1 = 12\ m$

$l^2 = r^2 + h^2$

$\therefore \quad l^2 = 5^2 + 12^2$

$\therefore \quad l^2 = 25 + 144$

$\therefore \quad l^2 = 169$

$\therefore \quad l = 13$ m

Perpendicular height is 12 m and slant height is 13 m

(4) Find the radius of a sphere if its volume is 904.32 cubic cm. (π = 3.14)

Solution:

Volume of sphere $= \frac{4}{3}\pi r^3$

$\therefore \quad 904.32 = \frac{4}{3} \times 3.14 \times r^3$

$\therefore \quad \frac{90432}{100} = \frac{4}{3} \times \frac{314}{100} \times r^3$

$\therefore \quad \frac{90432}{100} \times \frac{3}{4} \times \frac{100}{314} = r^3$

$\therefore \quad 72 \times 3 = r^3$

$\therefore \quad r^3 = 216$

$\therefore \quad r = 6$ cm (Taking cube roots)

Radius of a sphere is 6 cm

(5) Total surface area of a cube is 864 sq. cm. Find its volume.

Solution:

Total surface area of cube = $6l^2$

$\therefore \quad 864 = 6l^2$

$\frac{864}{6} = l^2$

$\therefore \quad l^2 = 144$

$\therefore \quad l = 12$ cm (Taking square roots)

Volume of cube = l^3

= 12^3

$\therefore$ Volume of cube = 1728 cubic cm

Volume of a cube is 1728 cubic cm

(6) Find the volume of a sphere, if its surface area is 154 sq.cm.

Solution:

Total surface area of sphere = $4\pi r^2$

$\therefore \quad 154 = 4 \times \frac{22}{7} \times r^2$

$\therefore \quad \frac{154 \times 7}{4 \times 22} = r^2$

$\therefore \quad r^2 = \frac{49}{4}$

$\therefore \quad r = \frac{7}{2} = 3.5$ cm (Taking square roots)

Volume of sphere $= \frac{4}{3} \pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} \times \frac{35}{10}$

$= \frac{3773}{21} = 179.67$ cubic cm

Volume of sphere is 179.67 cubic cm

(7) Total surface area of a cone is 616 sq. cm. If the slant height of the cone is three times the radius of its base, find its slant height.

Solution:

For a cone, $l = 3r$ ..... (i) (Given)

Total surface area of a cone $= \pi r (r + l)$

$\therefore \quad 616 = \pi r \times (r + 3r)$ (From (i))

$616 = \frac{22}{7} \times r \times 4r$

$\therefore \quad \frac{616 \times 7}{22 \times 4} = r^2$

$\therefore \quad r^2 = 49$

$\therefore \quad r = 7$ cm (Taking square roots)

$l = 3r$

$\therefore \quad l = 3 \times 7$

$\therefore \quad l = 21$ cm

Slant height of the cone is 21 cm

(8) The inner diameter of a well is 4.20 metre and its depth is 10 metre. Find the inner surface area of the well. Find the cost of plastering if from inside at the rate ₹ 52 per sq. m.

Solution:

Internal diameters of cylindrical well = 4.20 m

$\therefore$ Internal radius of cylindrical well (r) = 2.10 m

Depth of the cylindrical well (h) = 10 m,

Internal curved surface area (Sc) $= 2 \pi rh$ (Formula)

$= 2 \times \frac{22}{7} \times 2.10 \times 10$

$\therefore \quad = 132$ sq. m.

Cost of repairing the well = Area × Rate

$= 132 \times 52$

= ₹ 6,864

(9) The length of a road roller is 2.1 m and its diameter is 1.4 m. For levelling a ground 500 rotations of the road roller were required. How much area of ground was levelled by the road roller? Find the cost of levelling at the rate of ₹ 7 per sq. m.

Solution:

Length of the road roller (h) = 2.1 m

Diameter of the road roller (d) = 1.4 m

$\therefore$ Radius of the road roller (r) = 0.7 m

Area pressed in one revolution

= Curved surface area of road roller

$\therefore$ Area pressed in 500 revolutions

= 500 × curved surface area of road roller

$= 500 \times 2 \pi rh$

$= 500 \times 2 \times \frac{22}{7} \times 0.7 \times 2.1$

$= 5 \times 2 \times \frac{22}{7} \times \frac{7}{10} \times \frac{21}{10}$

Area pressed in 500 revolutions = 4620 sq.m

Cost of pressing the ground = Area × Rate

$= 4620 \times 7$

= ₹ 32340

The cost of levelling the ground is ₹32340

MCQ's

(1) The area of the four walls of a room is 80cm² and its height is 4 m. Then the perimeter of the floor of the room is ....

(A) 16 m (B) 5 m (C) 20 m (D) 10 m

(2) The capacity of a cuboidal tank is 50,000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.

(A) 3 m (B) 2 m (C) 4 m (D) 2.5 m

(3) Volume of a right circular cone of height 14 cm is 168π cm³. The radius of the cone is ....

(A) 6 cm (B) 12 cm (C) 8 cm (D) 10 cm

(4) The ratio of the volumes of the two spheres is 1 : 27. The ratio of their radii is ....

(A) 1 : 3 (B) 1 : 9 (C) 3 : 1 (D) 1 : 27

(5) The volume and curved surface of a sphere are numerically equal. The radius of the sphere is ....

(A) $\frac{1}{3}$ cm (B) 3 cm (C) 4 cm (D) 7 cm

(6) In a cylinder, radius is doubled and height is

halved, curved surface area will be ....

(a) halved (b) doubled

(c) same (d) four times

(7) For which solid figure is the TSA same as its CSA?

(A) Cone (B) Sphere

(C) Cylinder (D) Hemisphere

(8) A semi-circle is folded to form a cone. The radius will form ....

(A) height of the cone

(B) slant height of the cone

(C) circumference of the base

(D) half of the circumference of the base

(9) If the height of a cone is doubled then its volume is increased by ....

(A) 100% (B) 200% (C) 300% (D) 400%

(10) If A, B and C denote the areas of three adjacent faces of a cuboid, then its volume is ....

(A) ABC (B) 2ABC

(C) $\sqrt{ABC}$ (D) A + B + C

(11) A well of diameter 8 m has been dug out to a depth of 21 m. The volume of earth (in m^3) dug out is ....

(A) 4224 (B) 1056 (C) 528 (D) 2112

(12) If the sum of the all the edges of a cube is 36 cm, then the volume of the cube is ....

(A) 9 cm^3 (B) 27 cm^3 (C) 216 cm^3 (D) 1728 cm^3

(13) Three solid cubes of side 4 cm each are joined to form a cuboid. Then its surface area is ....

(A) 285 cm^2 (B) 200 cm^2 (C) 192 cm^2 (D) 224 cm^2

(14) If height of a right circular cylinder is 15 cm and volume is 2310 cm^3, then radius of the base will be ....

(A) 7 cm (B) 10 cm (C) 14 cm (D) 12 cm

(15) The base radii of the two cones are the same but their volumes are 4π m^3 and 9π m^3 respectively. The ratio of their heights is ....

(A) $\sqrt{2} : \sqrt{3}$ (B) 2 : 3

(C) 4 : 9 (D) 16 : 81

ANSWER

(1)	(C)	(2)	(B)	(3)	(A)	(4)	(A)
(5)	(B)	(6)	(C)	(7)	(B)	(8)	(B)
(9)	(B)	(10)	(C)	(11)	(B)	(12)	(B)
(13)	(D)	(14)	(A)	(15)	(C)		

ASSIGNMENT - 9

Time : 1 Hr. **Marks : 20**

Q.1. Solve the following: 2

(1) Find slant height of a cone, if its base radius is 12 cm and height is 16 cm.

(2) Area of the base of a cylinder is 154 cm^2 and its height is 7 cm. Find its volume.

Q.2. Solve the following: 4

(1) Total surface area of a box of cuboid shape is 500 sq. units. Its breadth and height is 6 units and 5 units respectively What is the length of that box?

(2) Find the radius of a sphere whose volume is 113040 cubic cm. ($\pi = 3.14$)

Q.3. Solve the following: 6

(1) Total surface area of a cone is 616 sq. cm. If the slant height of the cone is three times the radius of its base, find its slant height.

(2) The inner diameter of a well is 4.2 metre and its depth is 10 metre. Find the inner surface area of the well. Find the cost of plastering it from inside at the rate ₹ 52 per sq. m.

Q.4. Solve the following: 8

(1) There is 25 persons in a tent which is conical in shape. Every person needs an area of 4 sq. m of the ground inside the tent. If height of the tent is 18 m, find the volume of the tent.

(2) The length of a road roller is 21 m and its diameter is 1.4 m. For levelling a ground 500 rotations of the road roller were required. How much area of ground was levelled by the road roller? Find the cost of levelling at the rate of ₹ 7 per sq. m.

❖ ❖ ❖