MEC 320: Numerical Methods in Engineering Design and Analysis (Spring 2022)

Instructor: Professor Foluso Ladeinde
Homework Set **II**
Due February 13, 2024 (Brightspace), 11:59 PM
Please Work Independently. Show Your Work Very Clearly
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(I) **[40 Points]** Using centered finite difference approximations as done in class, solve the equation:

$$\frac{d^2\theta}{dx^2} + Q = 0$$

subject to the boundary conditions shown in the stencil below. Do this for two values of Q: (a) $Q = 0.5$, and (b) $Q = \sqrt{\left|(1+2x)^{e-sinx(\cos(2x)+x-0.5\sqrt{0.001+x}}\right|} + e^{-105*|1+.0089*x^3|} * sin2x + \left(\cos x + e^{x-|5000*sinx|}\right) * x + 1.003x^2$. For Case (a) (that is, $Q = 0.5$), use the stencil in Fig. 1. For Case (b), calculate with both the stencils in Fig. 1 and Fig 2. For all the three cases, show a table as well as a plot of θ versus x. Discuss your results. Hand in any MATLAB codes if you use MATLAB.

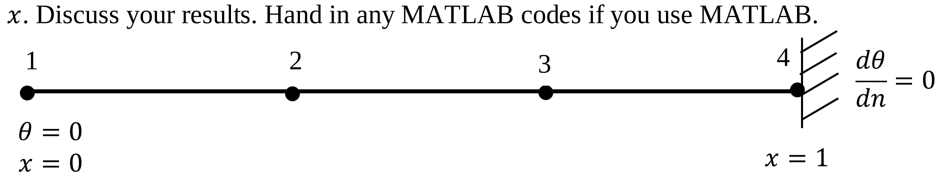

Fig 1

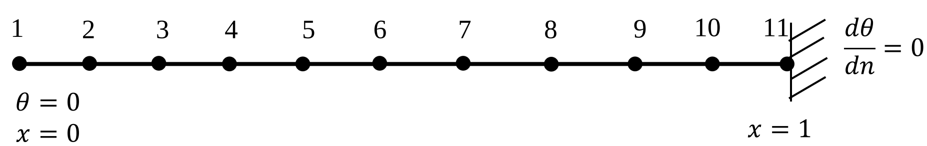

Fig 2

(II) **[60 Points]** Using centered finite difference approximation as done in class, solve the equation:

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + Q = 0$$

subject to the boundary conditions shown in the stencils below. Do this for two values of Q: (a) $Q = .5$, and (b) $Q = 2.5 * x^{3.5} + 1.26 * y^2 - x^{0.0015}y^{0.0019}$. For Case (a) (that is, $Q = 0.5$) use Fig 3. For Case (b), use both Fig. 3 and Fig 4. For all the three cases, show a table as well as the contour plots of θ versus (x, y), and the (x, y) heat flux values at all the nodes on the boundaries $x = 1$ and $y = 1$. Discuss your results. Hand in any MATLAB codes if you use MATLAB. (Note that the domain is $(x, y)\epsilon[0,1] \times [0,1]$.)

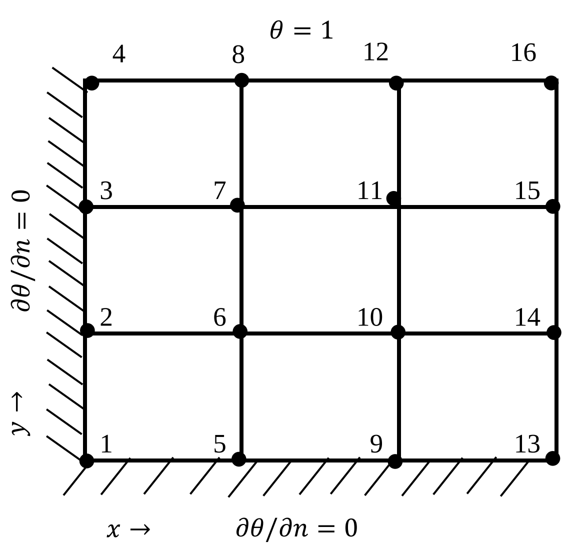

Fig 3

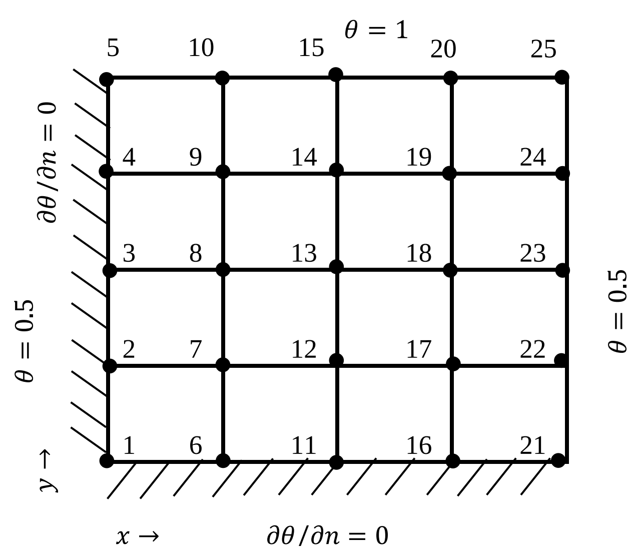

Fig 4

Hint: To calculate heat flux at a node (i, j):
Interior nodes:

$$(q_x)_{i,j} = -k \frac{\theta_{i+1,j} - \theta_{i-1,j}}{2\Delta x} \qquad (1)$$

and

$$(q_y)_{i,j} = -k \frac{\theta_{i,j+1} - \theta_{i,j-1}}{2\Delta y} \qquad (2)$$

where, k is the thermal conductivity.

One-sided differences:
Forward-x:

$$(q_x)_{i,j} = -k \frac{\theta_{i+1,j} - \theta_{i,j}}{\Delta x} \qquad (3)$$

Backward-x:

$$(q_x)_{i,j} = -k \frac{\theta_{i,j} - \theta_{i-1,j}}{\Delta x} \qquad (4)$$

Forward-y:

$$(q_y)_{i,j} = -k \frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta y} \qquad (5)$$

Backward-y:

$$(q_y)_{i,j} = -k \frac{\theta_{i,j} - \theta_{i,j-1}}{\Delta y} \qquad (6)$$