Introduction to Econometrics

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Preface

1 Nature and scope of Econometrics

2 Introductory Econometrics

2.1 Install packages

```r
library(wooldridge)
library(readr)
library(stargazer)
library(kableExtra)
library(quantmod)
library(xts)
```

2.2 Useful dataset

```r
earns <- read.csv("data/earns.csv")
gpa1 <- read.csv("data/gpa1.csv")
hprice1 <- read.csv("data/hprice1.csv")
hprice2 <- read.csv("data/hprice2.csv")
hprice3 <- read.csv("data/hprice3.csv")
jtrain <- read.csv("data/jtrain.csv")
nyse <- read.csv("data/nyse.csv")
phillips <- read.csv("data/phillips.csv")
rdchem <- read.csv("data/rdchem.csv")
traffic1 <- read.csv("data/traffic1.csv")
wage1 <- read.csv("data/wage1.csv")
```

2.3 Simple regression model

2.3.1 `Example 1:` A log wage equation

- Load the `wage1` data and check out the documentation.

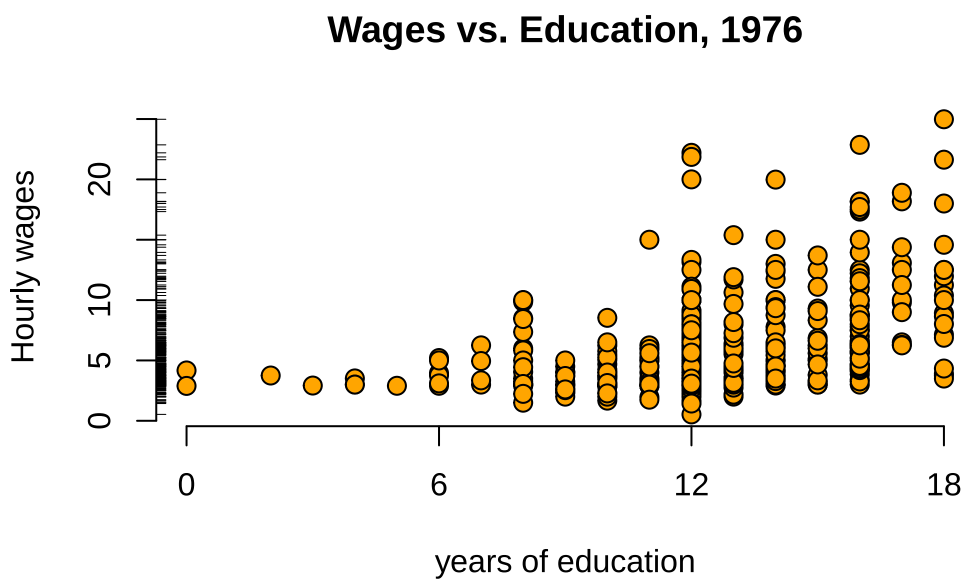
educ: years of education

wage: average hourly earnings

lwage: log of the average hourly earnings

- First, make a scatter-plot of the two variables and look for possible patterns in the relationship between them.

```
plot(y = wage1$wage, x = wage1$educ, col = "black", pch = 21, bg = "orange",
     cex=1.25, xaxt="n", frame = FALSE, main = "Wages vs. Education, 1976",
     xlab = "years of education", ylab = "Hourly wages")
axis(side = 1, at = c(0,6,12,18))
rug(wage1$wage, side=2, col="black")
```



1. It appears that ***on average***, more years of education, leads to higher wages.
2. The example in the text is interested in the *return to another year of education*, or what the ***percentage*** change in wages one might expect for each additional year of education. To do so, one must use the *log*(`wage`). This has already been computed in the data set and is defined as `lwage`.

- Build a linear model to estimate the relationship between the *log of wage* (`lwage`) and *education* (`educ`).

$$\widehat{log(wage)} = \beta_0 + \beta_1 educ$$

```r
log_wage_model <- lm(lwage ~ educ, data = wage1)
```

- Print the `summary` of the results.

```r
summary(log_wage_model)
```

```
Call:
lm(formula = lwage ~ educ, data = wage1)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.21158 -0.36393 -0.07263  0.29712  1.52339 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.583773   0.097336   5.998 3.74e-09 ***
educ        0.082744   0.007567  10.935  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4801 on 524 degrees of freedom
Multiple R-squared:  0.1858,	Adjusted R-squared:  0.1843 
F-statistic: 119.6 on 1 and 524 DF,  p-value: < 2.2e-16
```

- Use the `stargazer` package to make beautiful table

```r
stargazer(type = "html", log_wage_model, single.row = TRUE, header = FALSE, digits = 3)
```

Dependent variable:

lwage

educ

0.083*** (0.008)

Constant

0.584*** (0.097)

Observations

526

R2

0.186

Adjusted R2

0.184

Residual Std. Error

0.480 (df = 524)

F Statistic

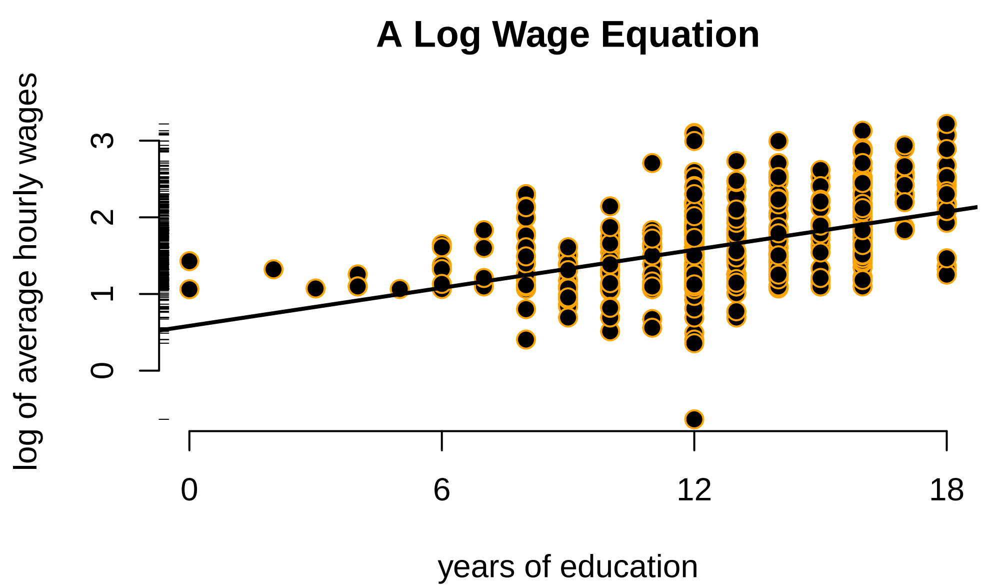
119.582*** (df = 1; 524)

Note:

p<0.1; ***p<0.05;*** p<0.01

- Plot the log(`wage`) vs `educ`. The blue line represents the least squares fit.

```
plot(y = wage1$lwage, x = wage1$educ, main = "A Log Wage Equation",
     col = "orange", pch = 21, bg = "black", cex=1.25,
     xlab = "years of education", ylab = "log of average hourly wages",
     xaxt="n", frame = FALSE)
axis(side = 1, at = c(0,6,12,18))
abline(log_wage_model, col = "black", lwd=2)
rug(wage1$lwage, side=2, col="black")
```



2.4 Multiple regression analysis

2.4.1 `Example 2:` Hourly wage equation

Check the documentation for variable information

lwage: log of the average hourly earnings

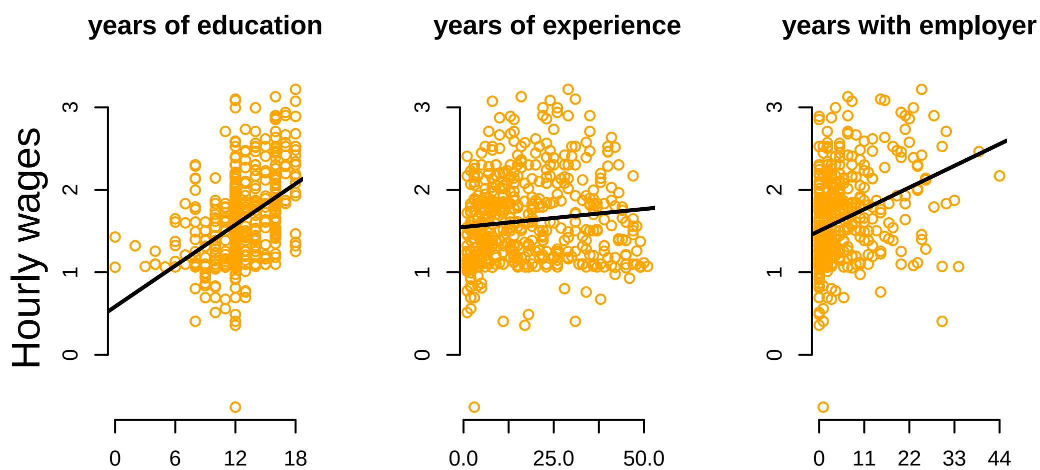
educ: years of education

exper: years of potential experience

tenutre: years with current employer

- Plot the variables against `lwage` and compare their distributions and slope (β) of the simple regression lines

```
par(mfrow=c(1,3))
plot(y = wage1$lwage, x = wage1$educ, col="orange", xaxt="n", frame = FALSE, main = "years
mtext(side=2, line=2.5, "Hourly wages", cex=1.25)
axis(side = 1, at = c(0,6,12,18))
abline(lm(lwage ~ educ, data=wage1), col = "black", lwd=2)
plot(y = wage1$lwage, x = wage1$exper, col="orange", xaxt="n", frame = FALSE, main = "year
axis(side = 1, at = c(0,12.5,25,37.5,50))
abline(lm(lwage ~ exper, data=wage1), col = "black", lwd=2)
plot(y = wage1$lwage, x = wage1$tenure, col="orange", xaxt="n", frame = FALSE, main = "yea
axis(side = 1, at = c(0,11,22,33,44))
abline(lm(lwage ~ tenure, data=wage1), col = "black", lwd=2)
```



- Estimate the model regressing *educ*, *exper*, and *tenure* against *log(wage)*.

$$\widehat{log(wage)} = \beta_0 + \beta_1 educ + \beta_3 exper + \beta_4 tenure$$

```
hourly_wage_model <- lm(lwage ~ educ + exper + tenure, data = wage1)
```

- Print the estimated model coefficients:

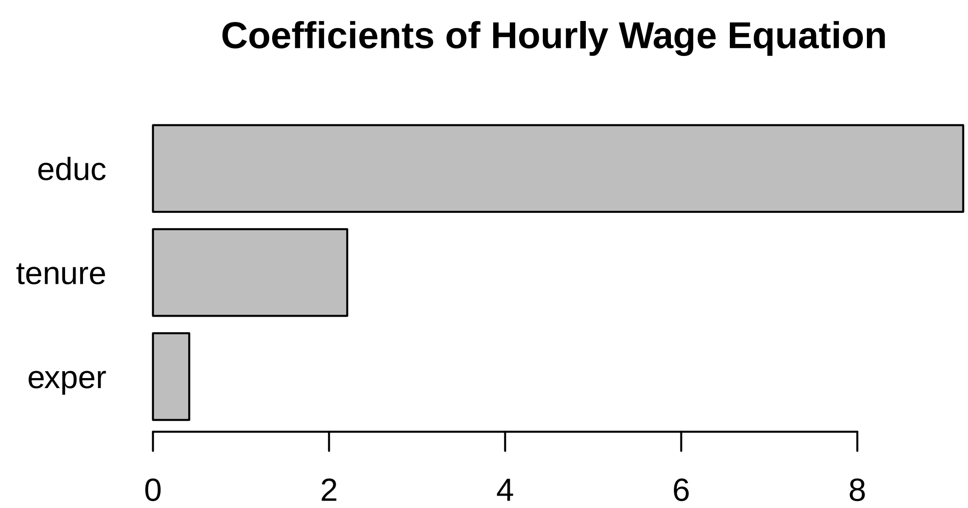
```
coefficients(hourly_wage_model)
```

```
kable(coefficients(hourly_wage_model), digits=4, col.names = "Coefficients", align = 'l')
```

	Coefficients
(Intercept)	0.2844
educ	0.0920
exper	0.0041
tenure	0.0221

- Plot the coefficients, representing percentage impact of each variable on $log($`wage`$)$ for a quick comparison.

```
barplot(sort(100*hourly_wage_model$coefficients[-1]), horiz=TRUE, las=1,
        ylab = " ", main = "Coefficients of Hourly Wage Equation")
```



2.5 Multiple regression analysis: inference

2.5.1 `Example 3`: Hourly Wage Equation

Using the same model estimated in **`example 3`**, examine and compare the standard errors associated with each coefficient. Like the textbook, these are contained in parenthesis next to each associated coefficient.

```r
summary(hourly_wage_model)
```

```

Call:
lm(formula = lwage ~ educ + exper + tenure, data = wage1)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.05802 -0.29645 -0.03265  0.28788  1.42809 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.284360   0.104190   2.729  0.00656 ** 
educ        0.092029   0.007330  12.555  < 2e-16 ***
exper       0.004121   0.001723   2.391  0.01714 *  
tenure      0.022067   0.003094   7.133 3.29e-12 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4409 on 522 degrees of freedom
Multiple R-squared:  0.316,	Adjusted R-squared:  0.3121 
F-statistic: 80.39 on 3 and 522 DF,  p-value: < 2.2e-16
```

```r
stargazer(type = "html", hourly_wage_model,  single.row = TRUE, header = FALSE, digits=5)
```

Dependent variable:

lwage

educ

0.09203*** (0.00733)

exper

0.00412** (0.00172)

tenure

0.02207*** (0.00309)

Constant

0.28436*** (0.10419)

Observations

526

R2

0.31601

Adjusted R2

0.31208

Residual Std. Error

0.44086 (df = 522)

F Statistic

80.39092*** (df = 3; 522)

Note:

p<0.1; ***p<0.05;*** p<0.01

For the years of experience variable, or `exper`, use coefficient and Standard Error to compute the t statistic:

$$t_{exper} = \frac{0.004121}{0.001723} = 2.391$$

Fortunately, `R` includes t statistics in the `summary` of model diagnostics.

```
summary(hourly_wage_model)$coefficients
```

```
               Estimate  Std. Error   t value     Pr(>|t|)
(Intercept) 0.284359541 0.104190379  2.729230 6.562466e-03
educ        0.092028988 0.007329923 12.555246 8.824197e-32
exper       0.004121109 0.001723277  2.391437 1.713562e-02
tenure      0.022067218 0.003093649  7.133070 3.294407e-12
```

```
kable(summary(hourly_wage_model)$coefficients, align="l", digits=3)
```

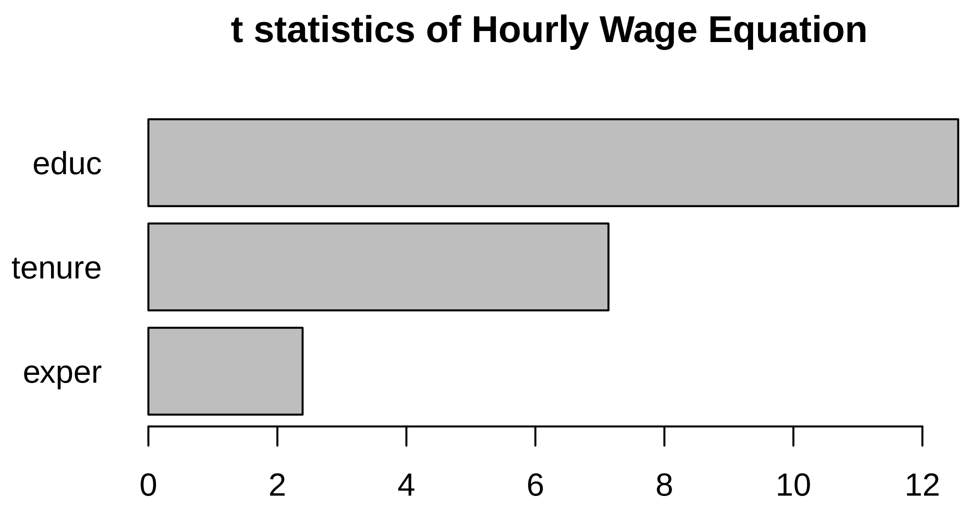
	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.284	0.104	2.729	0.007
educ	0.092	0.007	12.555	0.000
exper	0.004	0.002	2.391	0.017
tenure	0.022	0.003	7.133	0.000

- lets plot this results

```
par(mfrow=c(2,2))
plot(y = hourly_wage_model$residuals, x = hourly_wage_model$fitted.values , col="orange",
     frame = FALSE, main = "Fitted Values", xlab = "", ylab = "")
mtext(side=2, line=2.5, "Model Residuals", cex=1.25)
abline(0, 0, col = "black", lty=2, lwd=2)
plot(y = hourly_wage_model$residuals, x = wage1$educ, col="green", xaxt="n",
     frame = FALSE, main = "years of education", xlab = "", ylab = "")
axis(side = 1, at = c(0,6,12,18))
abline(0, 0, col = "black", lty=2, lwd=2)
plot(y = hourly_wage_model$residuals, x = wage1$exper, col="gray", xaxt="n",
     frame = FALSE, main = "years of experience", xlab = "", ylab = "")
mtext(side=2, line=2.5, "Model Residuals", cex=1.25)
axis(side = 1, at = c(0,12.5,25,37.5,50))
abline(0, 0, col = "black", lty=2, lwd=2)
plot(y = hourly_wage_model$residuals, x = wage1$tenure, col="blue", xaxt="n",
     frame = FALSE, main = "years with employer", xlab = "", ylab = "")
axis(side = 1, at = c(0,11,22,33,44))
abline(0, 0, col = "black", lty=2, lwd=2)
```

- Plot the t statistics for a visual comparison:

```r
barplot(sort(summary(hourly_wage_model)$coefficients[-1, "t value"]), horiz=TRUE, las=1,
        ylab = " ", main = "t statistics of Hourly Wage Equation")
```



2.5.2 `Example 4`: Effect of Job Training on Firm Scrap Rates

- Load the `jtrain` data set. (From H. Holzer, R. Block, M. Cheatham, and J. Knott (1993), *Are Training Subsidies Effective? The Michigan Experience*, Industrial and Labor Relations Review 46, 625-636. The authors kindly provided the data.)

year : 1987, 1988, or 1989

union : =1 if unionized

lscrap : Log(scrap rate per 100 items)

hrsemp : (total hours training) / (total employees trained)
lsales : Log(annual sales, $)

lemploy : Log(umber of employees at plant)

- First, use the `subset` function and it's argument by the same name to return observations which occurred in **1987** and are not **union**.
- At the same time, use the `select` argument to return only the variables of interest for this problem.

```r
jtrain_subset <- subset(jtrain, subset = (year == 1987 & union == 0), select = c(year, uni
```

- Next, test for missing values. One can "eyeball" these with R Studio's `View` function, but a more precise approach combines the `sum` and `is.na` functions to return the total number of observations equal to `NA`.

```
sum(is.na(jtrain_subset))
```

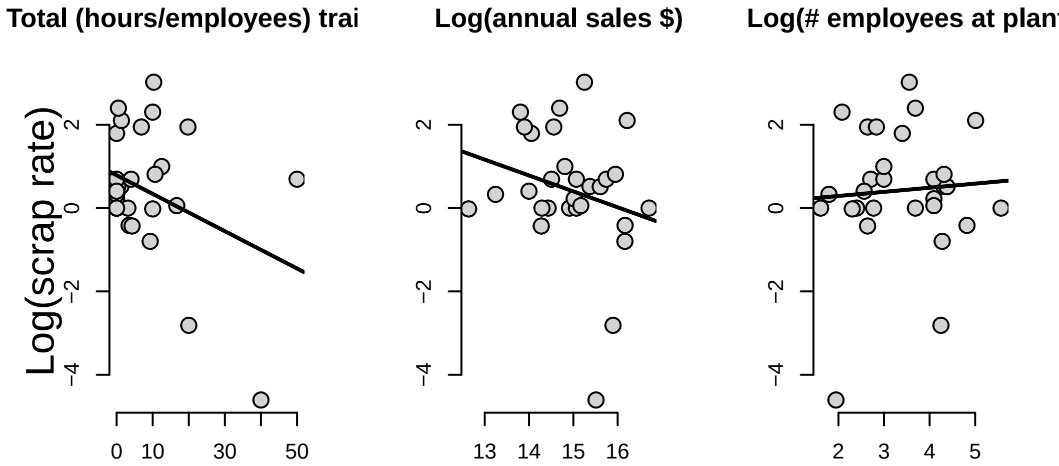
```
[1] 156
```

- While `R`'s `lm` function will automatically remove missing `NA` values, eliminating these manually will produce more clearly proportioned graphs for exploratory analysis. Call the `na.omit` function to remove all missing values and assign the new `data.frame` object the name `jtrain_clean`.

```
jtrain_clean <- na.omit(jtrain_subset)
```

- We use `jtrain_clean` to plot the variables of interest against `lscrap`. Visually observe the respective distributions for each variable, and compare the slope (β) of the simple regression lines.

```
par(mfrow=c(1,3))
point_size <- 1.60
plot(y = jtrain_clean$lscrap, x = jtrain_clean$hrsemp, frame = FALSE,
main = "Total (hours/employees) trained", ylab = "", xlab="", pch = 21, bg = "lightgrey",
mtext(side=2, line=2, "Log(scrap rate)", cex=1.25)
abline(lm(lscrap ~ hrsemp, data=jtrain_clean), col = "black", lwd=2)
plot(y = jtrain_clean$lscrap, x = jtrain_clean$lsales, frame = FALSE, main = "Log(annual s
abline(lm(lscrap ~ lsales, data=jtrain_clean), col = "black", lwd=2)
plot(y = jtrain_clean$lscrap, x = jtrain_clean$lemploy, frame = FALSE, main = "Log(# emplo
abline(lm(lscrap ~ lemploy, data=jtrain_clean), col = "black", lwd=2)
```



- Now create the linear model regressing **hrsemp**(total hours training/total employees trained), **lsales**(log of annual sales), and **lemploy**(the log of the number of employees), against **lscrap**(the log of the scrape rate).

$$lscrap = \alpha + \beta_1 hrsemp + \beta_2 lsales + \beta_3 lemploy$$

```r
linear_model <- lm(lscrap ~ hrsemp + lsales + lemploy, data = jtrain_clean)
```

- Finally, print the complete summary diagnostics of the model.

```r
summary(linear_model)
```

```
Call:
lm(formula = lscrap ~ hrsemp + lsales + lemploy, data = jtrain_clean)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.6301 -0.7523 -0.4016  0.8697  2.8273 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)  
(Intercept) 12.45837    5.68677   2.191   0.0380 *
hrsemp      -0.02927    0.02280  -1.283   0.2111  
lsales      -0.96203    0.45252  -2.126   0.0436 *
lemploy      0.76147    0.40743   1.869   0.0734 .
```

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.376 on 25 degrees of freedom
Multiple R-squared:  0.2624,	Adjusted R-squared:  0.1739 
F-statistic: 2.965 on 3 and 25 DF,  p-value: 0.05134
```

- Use `stargazer` to create representative table

```
stargazer(type = "html", linear_model, single.row = TRUE, header = FALSE, digits=5)
```

Dependent variable:

lscrap

hrsemp

-0.02927 (0.02280)

lsales

-0.96203** (0.45252)

lemploy

0.76147* (0.40743)

Constant

12.45837** (5.68677)

Observations

29

R2

0.26243

Adjusted R2

0.17392

Residual Std. Error

1.37604 (df = 25)

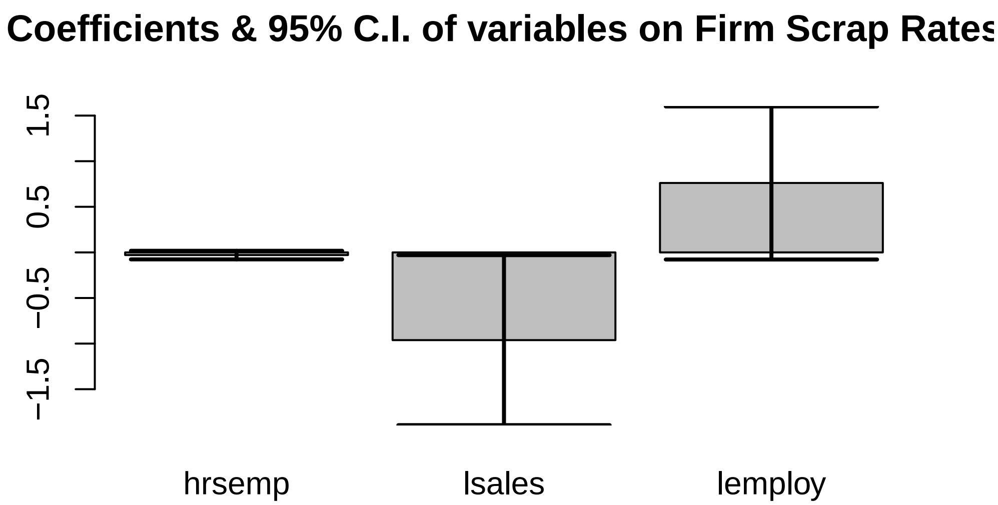
F Statistic

2.96504* (df = 3; 25)

Note:

p<0.1; ***p<0.05;*** p<0.01

```
coefficient <- coef(linear_model)[-1]
 confidence <- confint(linear_model, level = 0.95)[-1,]
graph <- drop(barplot(coefficient, ylim = range(c(confidence)),
              main = "Coefficients & 95% C.I. of variables on Firm Scrap Rates"))
arrows(graph, coefficient, graph, confidence[,1], angle=90, length=0.55, col="black", lwd=
arrows(graph, coefficient, graph, confidence[,2], angle=90, length=0.55, col="black", lwd=
```



2.6 Chapter 5: Multiple Regression Analysis: OLS Asymptotics

2.6.1 `Example`: Housing Prices and Distance From an Incinerator

- We will use the `hprice3` data set.

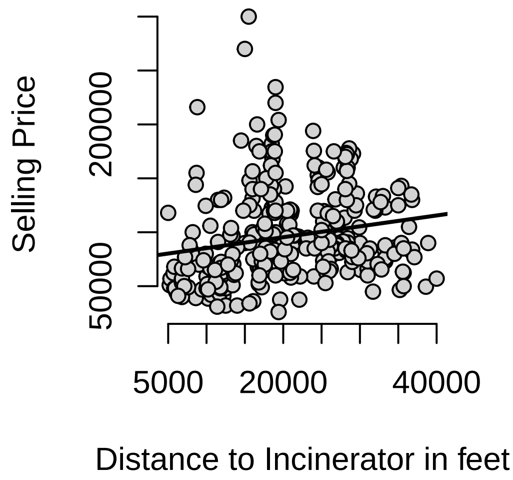
lprice : Log(selling price)

ldist : Log(distance from house to incinerator, feet)

larea : Log(square footage of house)

- Graph the prices of housing against distance from an incinerator:

```
par(mfrow=c(1,2))
plot(y = hprice3$price, x = hprice3$dist, main = " ", xlab = "Distance to Incinerator in f
abline(lm(price ~ dist, data=hprice3), col = "black", lwd=2)
```



- Next, model the log(`price`) against the log(`dist`) to estimate the percentage relationship between the two.

$$price = \alpha + \beta_1 dist$$

```
price_dist_model <- lm(lprice ~ ldist, data = hprice3)
```

- Create another model that controls for "quality" variables, such as square footage `area` per house.

$$price = \alpha + \beta_1 dist + \beta_2 area$$

```
price_area_model <- lm(lprice ~ ldist + larea, data = hprice3)
```

- Compare the coefficients of both models. Notice that adding `area` improves the quality of the model, but also reduces the coefficient size of `dist`.

```
summary(price_dist_model)
```

```
Call:
lm(formula = lprice ~ ldist, data = hprice3)
```

```
Residuals:
     Min       1Q   Median       3Q      Max 
-1.22356 -0.28076 -0.05527  0.27992  1.29332 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  8.25750    0.47383  17.427  < 2e-16 ***
ldist        0.31722    0.04811   6.594 1.78e-10 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4117 on 319 degrees of freedom
Multiple R-squared:  0.1199,	Adjusted R-squared:  0.1172 
F-statistic: 43.48 on 1 and 319 DF,  p-value: 1.779e-10
```

```r
summary(price_area_model)
```

```
Call:
lm(formula = lprice ~ ldist + larea, data = hprice3)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.23380 -0.18820 -0.01723  0.21751  0.86039 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  3.49394    0.49065   7.121 7.18e-12 ***
ldist        0.19623    0.03816   5.142 4.77e-07 ***
larea        0.78368    0.05358  14.625  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3188 on 318 degrees of freedom
Multiple R-squared:  0.4738,	Adjusted R-squared:  0.4705 
F-statistic: 143.2 on 2 and 318 DF,  p-value: < 2.2e-16
```

- Use *stargazer* for better table

```r
stargazer(type = "html",price_dist_model, price_area_model,  single.row = TRUE, header = F
```

Dependent variable:

lprice

(1)

(2)

ldist

0.31722*** (0.04811)

0.19623*** (0.03816)

larea

0.78368*** (0.05358)

Constant

8.25750*** (0.47383)

3.49394*** (0.49065)

Observations

321

321

R2

0.11994

0.47385

Adjusted R2

0.11718

0.47054

Residual Std. Error

0.41170 (df = 319)

0.31883 (df = 318)

F Statistic

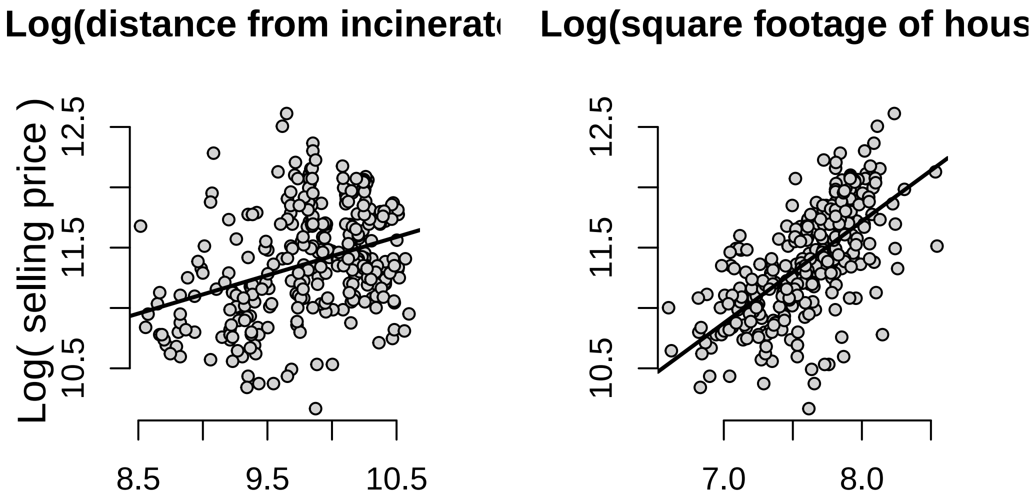
43.47673*** (df = 1; 319)

143.19470*** (df = 2; 318)

Note:

p<0.1; ***p<0.05;*** p<0.01

- Graphing illustrates the larger coefficient for `area`



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