Enroll. No.___________

MARWADI UNIVERSITY

Faculty of Technology

CE-FOT1 (MU), CE, BIOINFO FOT(MU)

B.Tech. SEM:3 **MID-SEM. EXAM: I** **SEPTEMBER-2024**

Subject: - (Probability & Statistics) (01CE0309) **Date:- 17/09/2024**

Total Marks:-30 **Time: - 75 Minutes**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Question: 1. [6]

(a) Answer in Short.

1 a 2 a 3 b 4 a 5 b 6 b

Question: 2. [12]

(a) Bag I contain 4 white and 6 black balls while Bag II contains 4 white and 3 black balls. One ball is drawn at random from one of the bags, and it is found to be black. Find the probability that it was drawn from i) Bag I ii) Bag II. [06]

Let B1 be the event of choosing bag I, B2 the event of choosing bag II, and A be the event of drawing a black ball.

Then, P(B1) =P(B2) =1/2

Also,

P(A|B1) = P(drawing a black ball from Bag I) = 6/10 = 3/5

P(A|B2) = P(drawing a black ball from Bag II) = 3/7

By using Bayes' theorem, the probability of drawing a black ball from bag I out of two bags,

$P(B1|A)=(P(A|B1)P(B1))/(P(A|B1)\cdot P(B1)+P(A|B2)\cdot P(B2))$

$= ((3/5)*(1/2))/((3/5)*(1/2)+(3/7)*(1/2))$

$= 7/12$

(b) Find the regression coefficient b_{xy} and b_{yx} , hence find the correlation coefficient between $x \text{ and } y$. [06]

X	4	2	3	4	2
Y	2	3	2	4	4

Solution: $n = 5$

x	y	x^2	y^2	xy
4	2	16	4	8
2	3	4	9	6
3	2	9	4	6
4	4	16	16	16
2	4	4	16	8
$\sum x = 15$	$\sum y = 15$	$\sum x^2 = 49$	$\sum y^2 = 49$	$\sum xy = 44$

$$b_{yx} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}} = \frac{44 - \frac{(15)(15)}{5}}{49 - \frac{(15)^2}{5}} = -0.25$$

$$b_{xy} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum y^2 - \frac{(\sum y)^2}{n}} = \frac{44 - \frac{(15)(15)}{5}}{49 - \frac{(15)^2}{5}} = -0.25$$

$$r = \sqrt{b_{yx} b_{xy}} = \sqrt{(-0.25)(-0.25)} = 0.25$$

Since b_{yx} and b_{xy} are negative, r is negative

$$r = -0.25$$

OR

(b) Calculate the coefficient of correlation [06]

X	9	8	7	6	5	4	3	2	1
Y	15	16	14	13	11	12	10	8	9

$$r = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sqrt{\sum x^2 - \frac{(\sum x)^2}{n}}\sqrt{\sum y^2 - \frac{(\sum y)^2}{n}}}$$

$$r = \frac{597 - \frac{(45)(108)}{9}}{\sqrt{285 - \frac{45^2}{9}}\sqrt{1356 - \frac{108^2}{9}}}$$

$$r = 0.95$$

<u>Question: 3</u>. [12]

a) The competitors in a beauty contest are ranked by three judges in the following order. Use rank correlation coefficient to discuss which pair of judges has the nearest approach to beauty. [08]

1st Judge	**1**	**5**	**4**	**8**	**9**	**6**	**10**	**7**	**3**	**2**
2nd Judge	4	8	7	6	5	9	10	3	2	1
3rd Judge	6	7	8	1	5	10	9	2	3	4

R_1	R_2	R_3	$R_1 - R_2 = d_1$	$R_1 - R_3 = d_2$	$R_2 - R_3 = d_3$	${d_1}^2$	${d_2}^2$	${d_3}^2$
1	4	6	-3	-5	-2	9	25	4
5	8	7	-3	-2	1	9	4	1
4	7	8	-3	-4	-1	9	16	1
8	6	1	2	7	5	4	49	25
9	5	5	4	4	0	16	16	0
6	9	10	-3	-4	-1	9	16	1
10	10	9	0	1	1	0	1	1
7	3	2	4	5	1	16	25	1
3	2	3	1	0	-1	1	0	1
2	1	4	1	-2	-3	1	4	9
						$\Sigma d_1^2 = 74$	$\Sigma d_2^2 = 156$	$\Sigma d_3^2 = 44$

(1) Correlation coefficient between first and second judge $= 1 - \frac{6\sum {d_1}^2}{n(n^2 - 1)}$

$$= 1 - \frac{6(74)}{10(100-1)}$$

$$= 0.55$$

(2) Correlation coefficient between first and third judge $= 1 - \frac{6\sum d_2^2}{n(n^2-1)}$

$$= 1 - \frac{6(156)}{10(100-1)}$$

$$= 0.05$$

(3) Correlation coefficient between second and third judge $= 1 - \frac{6\sum d_3^2}{n(n^2-1)}$

$$= 1 - \frac{6(44)}{10(100-1)}$$

$$= 0.73$$

b) P(A) = 6/30 = 1/5

P(B)= 4/30 = 2/15
P(C)= 10/30 = 1/3
Now, (ii) probability of getting multiple of 3 or 7 = $P(C \cup B)$
Thus, $P(C \cup B) = P(C) + P(B) - P(C \cap B)$
$= 10/30 + 4/30 - 1/30 = 13/30$

OR

a) The number of bacterial cells (y) per unit volume in a culture at different hours (x) is given below. [08]

x	0	1	2	3	4	5	6	7	8	9
y	43	46	82	98	123	167	199	213	245	272

Fit lines of regression of y on x and x on y. Also, estimate the number of bacterial cells after 15 hours.

Solution:

$$b_{yx} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}} = \frac{8924 - \frac{(48)(1488)}{10}}{285 - \frac{(45)^2}{10}} = 27.0061$$

$$b_{xy} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum y^2 - \frac{(\sum y)^2}{n}} = \frac{8924 - \frac{(45)(1488)}{10}}{282290 - \frac{(1488)^2}{10}} = 0.0366$$

$$\bar{x} = \frac{\sum x}{n} = \frac{45}{10} = 4.5$$

$$\bar{y} = \frac{\sum y}{n} = \frac{1488}{10} = 148.8$$

The equation of the line of regression of y on x is

$$y - \bar{y} = b_{yx}(x - \bar{x})$$
$$y - 148.8 = 27.0061(x - 4.5)$$
$$y = 27.0061x + 27.2726$$

The equation of the line of regression of x on y is

$$x - \bar{x} = b_{xy}(y - \bar{y})$$
$$x - 4.5 = 0.0366(y - 148.8)$$
$$x = 0.0366y - 0.9461$$

At $x = 15$ hours,

$$y = 27.0061(15) + 27.2726 = 432.3641$$

b) In a certain assembly plant, three machines B1, B2, and B3 make 30%, 45%, and 25% respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected. What is the probability that it is defective? [04]

We have P(B1) = 0.3, P(B2) = 0.45 and P(B3) = 0.25.
Also, we know that 2%, 3%, and 2% of the products made by each machine, respectively, are defective.
Thus, P(A|B1)= 0.02 , P(A|B2)=0.03 and P(A|B3)= 0.02
Applying the rule of total probability, we can write
P(A) = P(B1) P(A|B1)+ P(B2) P(A|B2) + P(B3) P(A|B3)
= (0.3)(0.02) + (0.45)(0.03) + (0.25)(0.02) = 49/2000 = 0.0245

---Best of Luck---