MODELS OF SEGREGATION

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People get separated along different lines and in different ways. There is segregation by sex, age, income, language, color, taste, comparative advantage, and the accidents of historical location. Some segregation is organized; some is economically determined; some results from specialized communication systems; and some results from the interplay of individual choices that discriminate. This paper is about the segregation that can result from discriminatory individual choices.

My ultimate concern of course is segregation by color in the United States; but at the level of abstraction of this paper, any twofold distinction could constitute an interpretation—whites and blacks, boys and girls, officers and enlisted men. The only requirement in my model is that the distinction be twofold, exhaustive, and recognizable.

Skin color, of course, is neither dichotomous nor even unidimensional; but by convention the distinction is twofold, even in the U.S. census.

Economists are familiar with systems that lead to aggregate results that the individual neither intends nor needs to be aware of, the results sometimes having no recognizable counterpart at the level of the individual. The creation of money by a commercial banking system is one; the way that savings decisions cause depressions or inflations is another. In some cases small incentives can lead to striking results; Gresham's Law is a good example. My conjecture is that the interplay of individual choices, where unorganized segregation is concerned, is a complex system with collective results that bear no close relation to individual intent.

For some purposes an "unseen hand" of comparative advantage may sort people in a way that, though foreseen and intended by no one, corresponds to some socially efficient satisfaction of individual preferences. But we know a good many macrophenomena, like depression and inflation, that do not reflect any universal desire for lower incomes or higher prices. The worth of a new turnpike depends on constraining traffic below the density that would equalize its attractiveness with alternative routes. Typewriter keyboards, the pitches of screws, and left-hand automobile drive can be self-perpetuating in spite of inefficiency until an organized effort brings about concerted change.

A special reason for doubting any social efficiency in aggregate segregation is that the range of choice is so meager. The demographic map of almost any American metropolitan area suggests that it is easy to find residential areas that are all white or nearly so and areas that are all black or nearly so but hard to find localities in which neither whites nor nonwhites are more than, say, three-quarters of the total. And, comparing decennial maps, it is nearly impossible to find an area that, if integrated within that range, will remain integrated long enough for a man to get his house paid for or his children through school. The distribution is so U-shaped that it is virtually a choice of two extremes.

Some aspects of segregation lend themselves to quantitative analysis. Counting blacks and whites in a residential block or on a baseball team will not tell how they get along, but it tells something, especially if numbers and ratios matter to the people who are moving in or out of the block or being recruited for the team. And with quantitative analysis there are usually a few logical constraints, somewhat analogous to the balance-sheet identities in economics. Being logical constraints, they contain no news unless one just never thought of them before.

The simplest constraint is that, within a given set of boundaries, not both groups (colors, sexes) can enjoy numerical superiority. Within the population as a whole, the numerical ratio is determined at any given time; locally, in a city or a neighborhood, a church or a school, either blacks or whites can be a majority. But if each insists on being a local majority, there is only one mixture that will do it: complete segregation.

Relaxing the condition, if whites want to be at least three-fourths and blacks at least one-third, it won't work. If whites want to be at least two-thirds and blacks no fewer than one-fifth, there is a small range of mixtures that meet the conditions; and not everybody can be in the mixtures if the aggregate ratio is outside the range.

Other constraints have to do with small numbers. A classroom can be mixed but the teacher is one color; mixed marriages can occur only in the ratio of one-to-one; a three-man team cannot represent both colors equally, and even in a two-man team each member has company exclusively of one color.

In spatial arrangements, like a neighborhood or a hospital ward, everybody is next to somebody. A neighborhood may be 10 percent black or white; but if you have a neighbor on either side, the minimum nonzero percentage of neighbors of opposite color is fifty. If people draw their boundaries differently, we can have everybody in a minority: at dinner, with men and women seated alternately, everyone is outnumbered two to one locally by the opposite sex but can join a three-fifths majority if he extends his horizon to the next person on either side. If blacks occupy a sixth of the beds in a hospital and there are four beds to a room, at least 40 percent of the whites will be in all-white rooms.

There are several mechanisms by which blacks and whites, or boys and girls, can become segregated through individual choice. Whites may prefer to be among whites and blacks among blacks; whites may merely avoid or escape blacks and blacks avoid or escape whites; whites may prefer the company of whites, while the blacks don't care; and if whites can afford to live or to eat or to belong where the blacks cannot afford to follow, separation can occur.

Whites and blacks may not mind each other's presence, even prefer some integration, but, if there is a limit to how small a minority either color is willing to be, initial mixtures more extreme than that will lose their minority members and become all of one color; those who leave may move to where they constitute a majority, increasing the majority there and causing the other color to evacuate.

Evidently if there are any limits to the minority status that either color can tolerate and if initially complete segregation obtains, no individual will move to an area dominated by the other color. Complete segregation is then a stable equilibrium. The concerted movement of blacks into a white area or whites into black could achieve some minimum percentage; but in the absence of concert, somebody has to move first, and nobody will.

Let's examine a few of these mechanisms. Imagine a line along which blacks and whites (or men and women or Catholics and Protestants) have been distributed in equal numbers and random order, as in the line of plusses and zeros shown below. We expect them to be evenly distributed in the large but not in the small. If the colors or sexes or religions represented by plusses and zeros are content to live together in a ratio of about fifty-fifty, each finds himself in a satisfactorily mixed neighborhood if he defines his neighborhood as a long stretch of this line. If instead everybody defines his neighborhood as his own house and the neighbors on either side, a quarter of the whites and a quarter of the blacks are going to be surrounded by neighbors of opposite color. Satisfaction depends on how far one's "neighborhood" extends. For illustration, define everybody's "neighborhood" as extending four neighbors on either side, and suppose that everyone is content if half his "neighbors" are the same color as he. If fewer than half are his color, he moves in either direction to the nearest point (measured in the number he passes on the way) at which half his eight nearest neighbors are the same color as he. The particular row of plusses and zeros shown here corresponds to odd and even numbering in a column of random digits.

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  .       . .     .   .   . .       .     .     .         .                 . . . .
0 + 0 0 0 + + 0 + 0 0 + + 0 0 + + + 0 + + 0 + + 0 0 + + 0 0 + + 0 0 + + 0 + 0 + 0 0
                                                  .   .           . .       .     .   .   .   . .
                                            + + + 0 + + 0 0 0 0 0 + + + 0 0 0 + 0 0 + + 0 + 0 + + 0
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I have put a dot over the individuals that are dissatisfied. It turns out that, of 35 plusses and 35 zeros, 11 plusses and 13 zeros are motivated to move. Two things happen as they move. Some who were content become discontent, as like neighbors move away or unlike ones move near; and some who were discontent become satisfied as like neighbors move near or unlike ones move away. Suppose that the dotted individuals move in turn, starting from the left, if they are still discontent when their turns come: rearrange the plusses and zeros by moving each dotted one to the nearest point where, inserting itself between two others, at least four of its eight neighbors are of its own color. This gives us the rearranged line:

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                        .                   . .     . .       .     .       .
0 0 0 0 0 0 0 0 + + + + 0 + + + + + + + + + 0 0 0 0 + + 0 0 0 + 0 + 0 + + + 0
                                      + + + + + + + + + 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 + + + + + +
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Some who were going to move did not move after all. Eight have become newly discontent. We give them their turn and get this rearranged line:

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0 0 0 0 0 0 0 0 + + + + + + + + + + + + + + + 0 0 0 0 0 0 0 0 0 0
                    + + + + + + + + + + + + + + + 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 + + + + + +
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We end up with six groups of alternating color. Nearly half (thirty) have no neighbors of opposite color within four houses of them. Since we don't allow vacant spots, somebody is at the boundary of every group and has neighbors of opposite color; but, not only is everybody in a local majority as he wished to be, but by the efforts of each to achieve bare majority status they have together achieved an average majority status of more than five to one! This is not mathematical necessity: clusters of five would satisfy everybody at his minimum demands, but the actual clusters average twelve. (Alternating plusses and zeros or alternating pairs would also meet everybody's demands.) If people, though not wanting to be in the minority, prefer mixed neighborhoods, only forty of the seventy achieved it. Furthermore, anyone who wants some neighbors of opposite color, but not more than half, can move nearer the boundary of his cluster but will not move beyond; his movement will not change the clustering.

All of this is too abstract to be a motion picture of whites and blacks or boys and girls choosing houses on a road or even stools along a counter; but it is suggestive of some of the dynamics that could be present in individually motivated segregation.

Turn now to a different model. Suppose there is some area that both blacks and whites would prefer to occupy as long as the ratio of opposite color to one's own color does not exceed some limit. (This could be membership in an organization or occupation as well as a residential location.) We let this limit—call it "tolerance"—differ among the whites and also among the blacks.

Evidently the higher these limits, the more blacks and whites will be content to live in the area with each other. Evidently, too, the upper limits for the "most tolerant" whites and blacks must be compatible—their product must exceed one—or no contented mixture of any size is possible. And if nobody can tolerate extreme ratios, like 100 to 1, then, if the area is initially occupied by one color alone, none of the other would enter.

We can experiment with different distributions of tolerance to see what the process is by which the area becomes occupied by blacks or whites or a mixture, and to search for some principles that relate outcomes to the shapes of the curves, the initial positions, and the dynamics of entry and exit. There is no room for many alternatives in this paper, but the process can be illustrated.

What we are dealing with is a frequency distribution, separately for the whites and the blacks, of the upper limits to the ratios of opposite color to own color at which people will live in the area under consideration. The assumption is that anyone whose limiting ratio is exceeded by the prevailing mixture will go elsewhere. For ease of illustration suppose horizontal distributions from 2.0 to zero: for whites, the highest ratio of black to white that anybody can abide is two to one, the median white can tolerate a one-to-one ratio and the least tolerant cannot stand any blacks at all. On a diagram whose horizontal axis measures the white population and whose vertical axis measures the ratio of black to white on an arithmetic scale, the cumulative distribution will be a straight line intersecting the vertical axis at 2 and the horizontal axis at 100. For simplicity suppose the distribution of ratios of white to black that the blacks can tolerate is the same, and suppose that whites and blacks are equal in number.

To examine the dynamics we have to get whites and blacks on the same diagram. We translate the tolerance schedules into graphs expressing the absolute number of blacks whose presence can be tolerated by given numbers of whites, and vice versa. Keeping the whites ordered along the horizontal axis as they were in drawing up the frequency distribution—that is, with the most tolerant whites nearest the origin—we can plot, for a given number of whites, the maximum number of blacks whose presence they can tolerate. (We just multiply the number of whites by the ratio they can tolerate; the cumulative distribution of ratios translates into this absolute-number function exactly as a demand curve translates into a total revenue curve.)

The resulting two parabolas, Figure 1, divide the diagram into four regions. Any point beneath the inverted dish (the curve for whites, labeled W) is a point such that at least that many whites are satisfied with the presence of that many blacks: the whites present will not leave and additional whites will enter. Any point to the left of the blacks' curve (labeled B) represents a point at which at least that

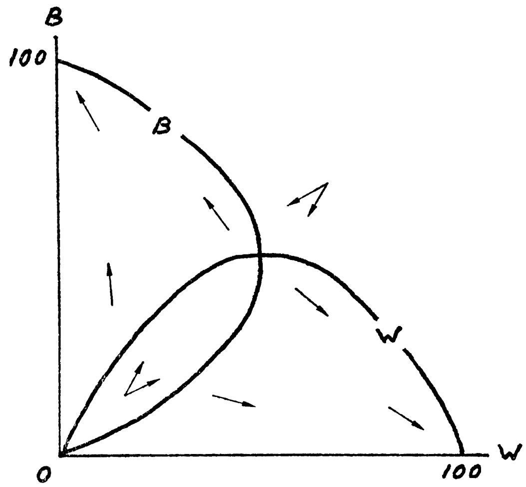

FIGURE 1

many blacks can tolerate the presence of that many whites: blacks will not leave and additional blacks will enter. Where the curves overlap the number of both blacks and whites present will be increasing; outside the curves, the numbers of both will be decreasing. Beneath the whites' curve but to the right of the blacks' curve, blacks will be evacuating and whites coming in; to the left of the blacks' curve but above the whites' curve, whites will be evacuating and blacks coming in.

There are two stable equilibria, one with exclusive occupation by blacks; the other with exclusive occupation by whites. The initial distribution of the two populations and the rates at which they move in or out will determine which one of the two colors eventually occupies and which one evacuates. Up to half of both colors could contentedly coexist at ratios near one to one, but the dynamics of entry prevent any mixture from stabilizing.

If the tolerance schedules are made steeper, the two parabolas can overlay each other as shown in Figure 2 (which corresponds to a slope of 5 and median tolerance of 2.5:1). There is now a stable mixed equilibrium. There are also stable equilibria at the two extremes. Again, which one would be obtained depends on initial conditions and rates of movement.

If whites outnumber blacks by two to one, the parabolas of Figure 2 will look as in Figure 3; the equilibrium mixture has disappeared. Whites numerically over-

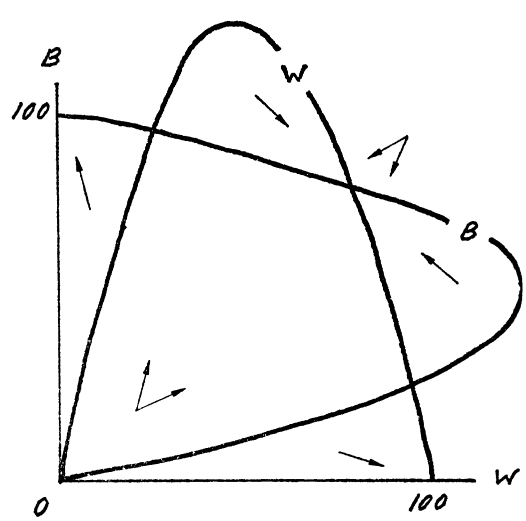

FIGURE 2

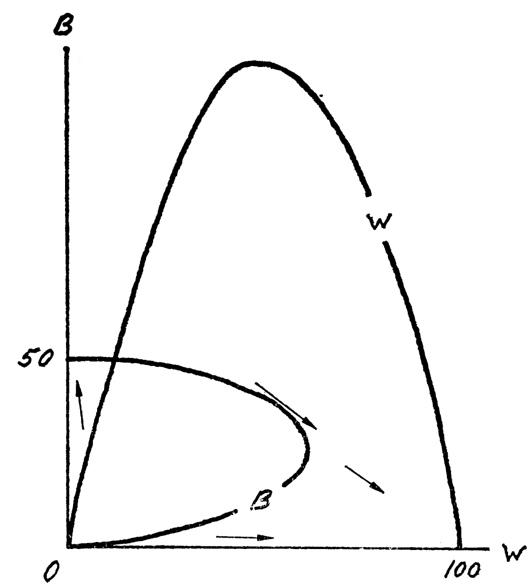

FIGURE 3

whelm the blacks, who evacuate (unless the initial mixture is in the thin upper-left slice). Limiting white entry can restore the stable mixed equilibrium. Interestingly, excluding some whites has the same effect as supposing the least tolerant whites to be more intolerant. Whether we limit whites in the area to half their total number or suppose that half the whites cannot tolerate any blacks at all, the tolerance schedule falls vertically at fifty whites, yielding intersecting curves (at thirty-six blacks) as in Figure 4.

This is but a small sample of possible results, using straight-line schedules and simple dynamics. There are no expectations in the model, no speculation, no concerted action, no restriction on the alternative localities available.

Just to mention two somewhat unexpected results: first, as we just saw, the polarized equilibria often come about because one color overwhelms the other; it is not the case, within the confines of this model, that the prospects for a stable mixed population are necessarily enhanced by an increase in the tolerance of one color for the other. (Make the least tolerant 60 percent of blacks and whites absolutely intolerant in Figure 1 and a stable equilibrium will occur at forty apiece.)

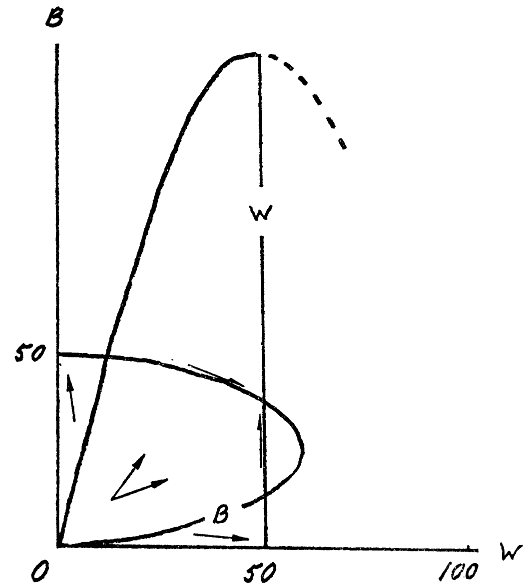

FIGURE 4

Second, the results do not depend on each color's having a preference for the absence of the other. We can equally suppose that most blacks and most whites prefer a color mixture, and reinterpret their tolerances as merely the upper limits to the ratios at which their preference for integrated residence is outweighed by numerical imbalance. The model fits both interpretations and produces the same results either way.